Asteroid mass determination & Contribution to GR tests

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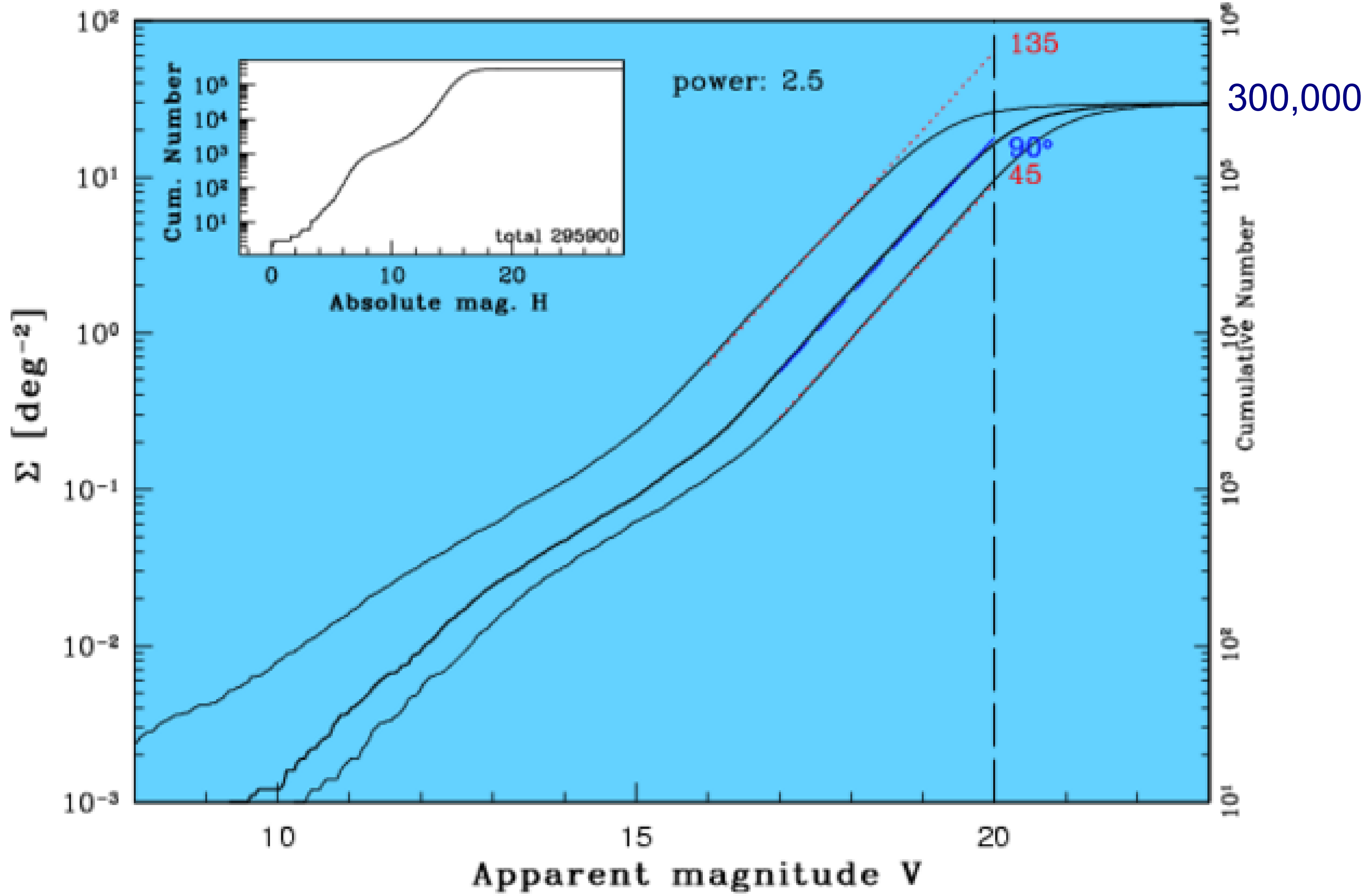
Outline

- Gaia observation of asteroids (overview)
- Orbit improvement (precision)
- Determination of asteroids mass
- GR tests (local)

Gaia Observation of Asteroids

- About 300,000 asteroids
  - (8 ≤) V ≤ 20
- Scanning law
  - Observations around quadratures and to low elongations, including NEOs (or IEOs)
    - 45° ≤ L ≤ 135°
  - No pointing, and varying sequences of observations
    - Approx 50 observations per target over 5 years
- One-dimensional, sub-mas to mas precision

Gaia Observation of Asteroids



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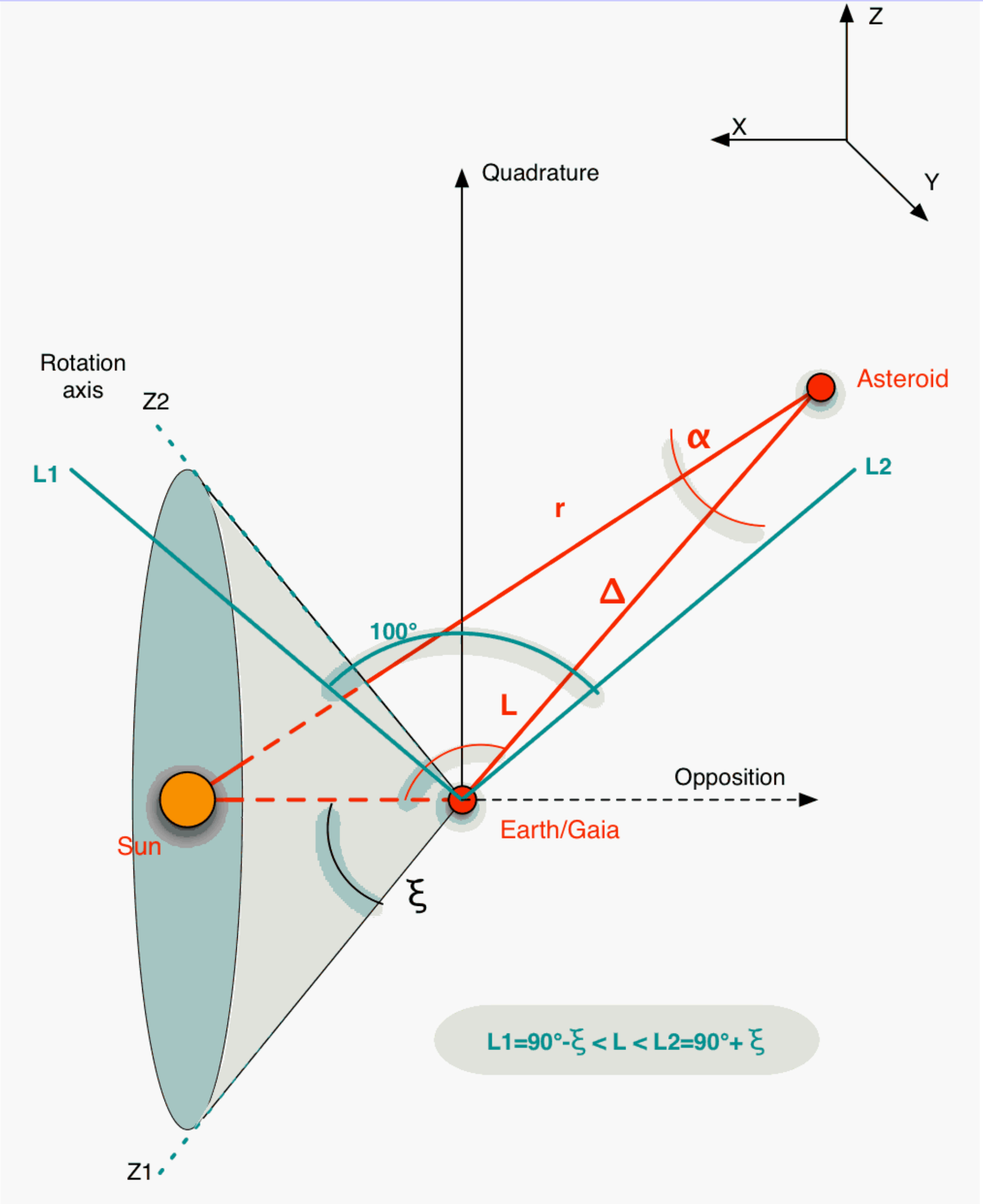
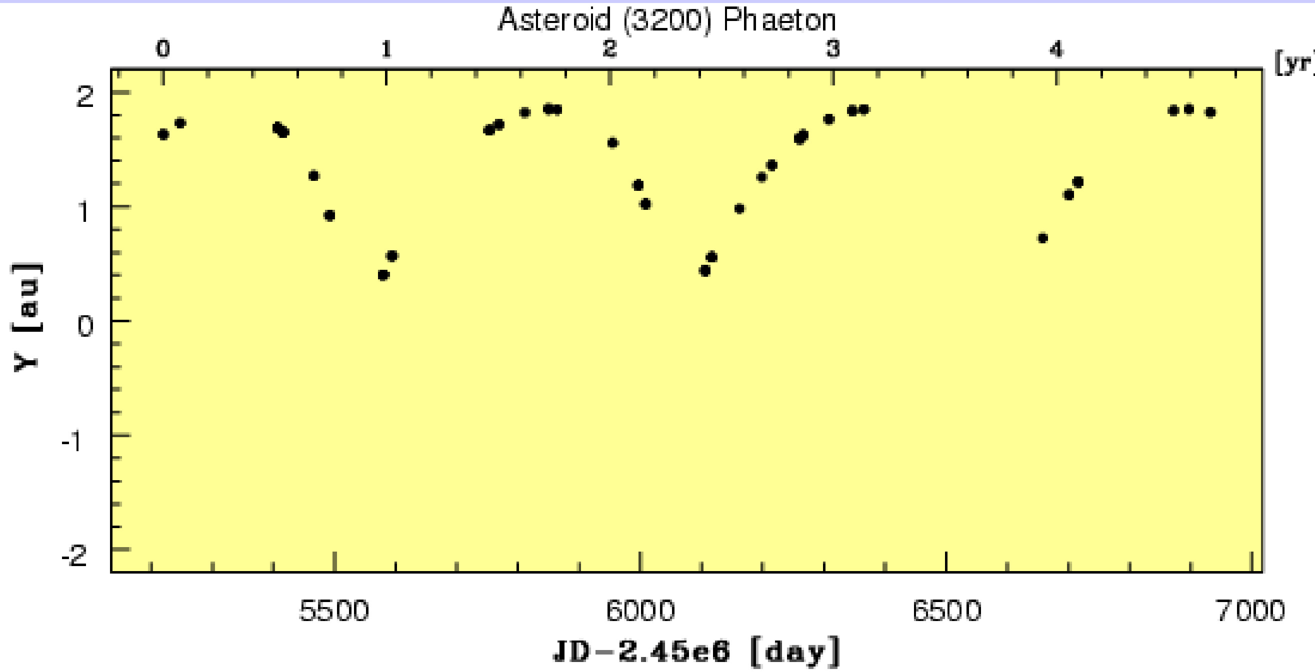
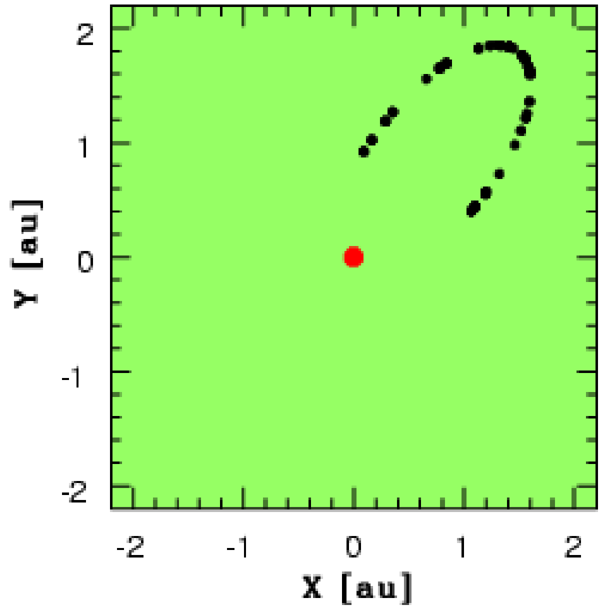
Gaia Scanning Law

- Sun aspect ξ=50°
- Observations in a given range of elongations from L1 to L2



Gaia Scanning Law





Observations distribution
for the NEA Phaeton
(a=1.27   e=0.9)

orbit in EC-J2000

79 obs.

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Orbit Improvement

- Linearized least-squares, variance analysis
  - **A . dq** = O-C = **dλ**
- Jacobian matrix of PD **A** from
  - ⇨ analytical (2 body approximation, elliptic elements)
  - variational equations (numerical integration, (**x**, d**x**/dt))
- Unknown correction vector $\mathbf{dq}=(\mathbf{dq}_i,\mathbf{dq}_g)$
  - $\mathbf{dq}_i$ per asteroid
  - $\mathbf{dq}_g$ global parameters

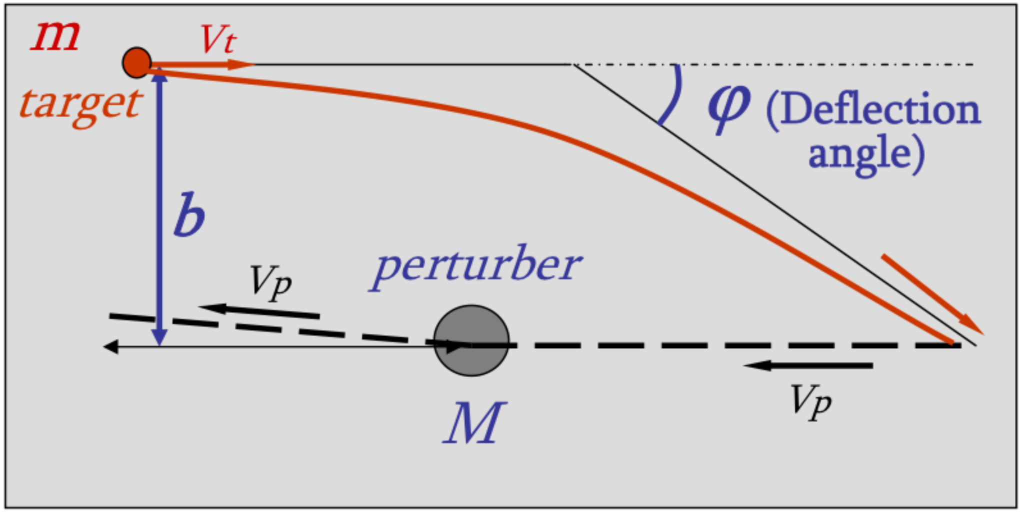
Orbit Improvement (cont.)

- **dq_i** per asteroid
  - osculating elements ($da/a, de, dl_o+dr$, $dp, dq, e.dr$)
  - photocenter offset $C(\alpha) = R.(a.\alpha+b)$
  - etc.
- **dq_g** global parameters
  - global frame rotation (ecliptic and γ)
  - solar J_2
  - GR
  - secular variations
  - asteroid mass m_j, etc.

Determination of Mass

- Masses from close approaches (binaries too)
- One massive perturber vs. several small targets

$$\tan\frac{\varphi}{2} = -\frac{G(M+m)}{bV^2}$$



TWO-BODY HYPERBOLIC APPROXIMATION

Determination of Mass

- About 100 potential perturbers
- Partial derivatives from variational equations.
- Exemple
  - **dq** for Mass only, over 5 years
  - one mass (Ceres) from 19 small targets

Formal precision on the mass of Ceres:

$$\sigma(m_c) \approx 4.8 \times 10^{-14} M_{\odot} \qquad \frac{\sigma(m_c)}{m_c} \approx 0.01\%$$

  - perturbations taken into account even if σ(M) large

Tests of GR

- Sensitivity of orbits e.dω/dt
- Icarus, Phaeton ≈ Mercury (Sitarski) not radar though!
- ~1550 asteroids in present simulation

150 Trojans,
1200 MBAs,
200 NEAs

- β : a (1-e^2)

J_2 : a^2(1-e^2)^2

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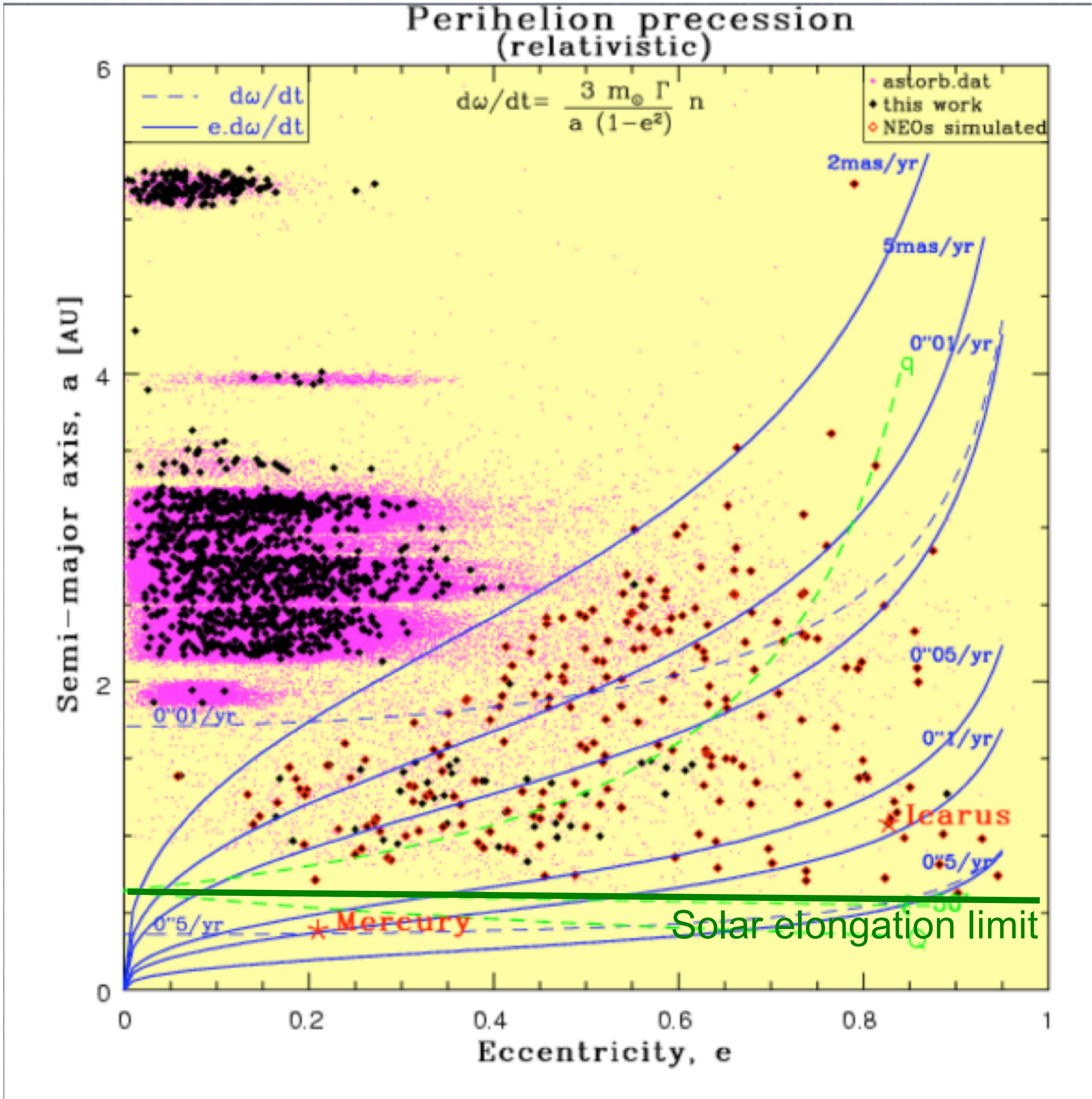


Tests of GR

- PPN formalism – Local test
- Assuming γ is known (Cassini, GAIA, ...)
- Simultaneous determination of PPN β and solar quadrupole J_2
  - Correlation (β+1/4e4.J_2 , J_2)=0.14
- Rotation and rotation rate

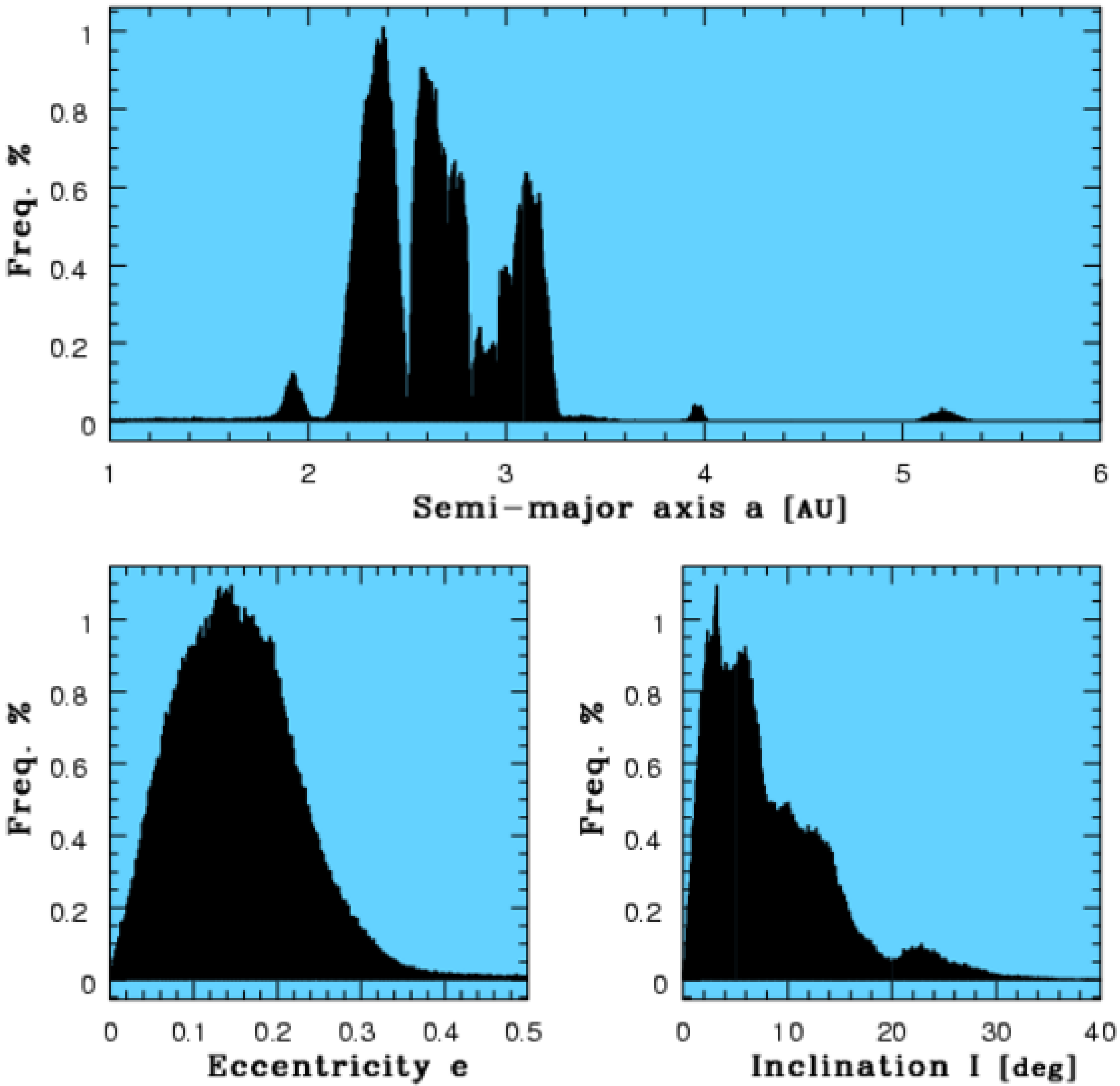
β	J2	W [mas]	dW/dt [mas/yr]
2d-3	1.5d-7	0.02 0.02 0.05	0.005 0.005 0.02

1,500 objects

Mainly from NEAs

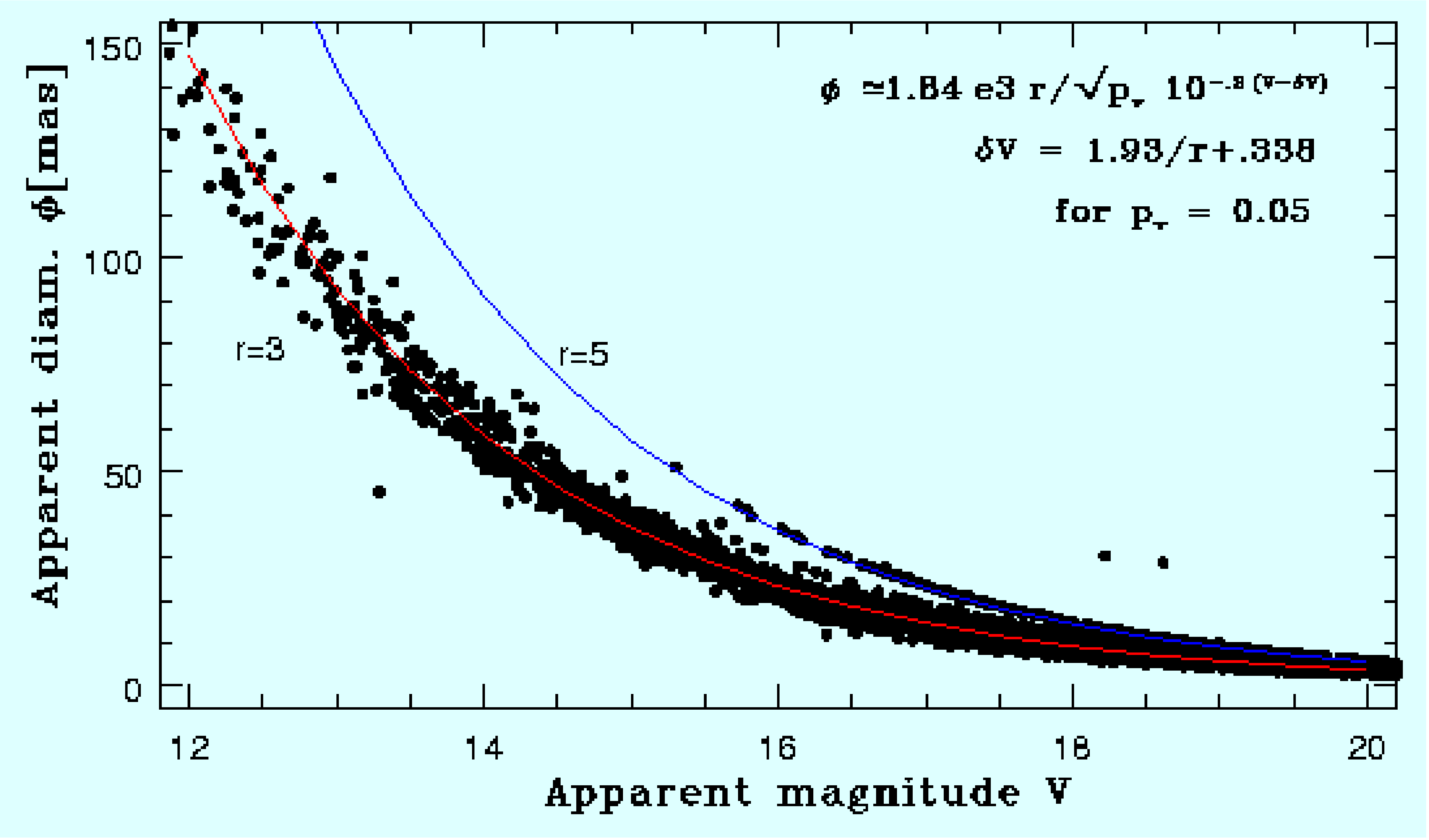
Perspectives

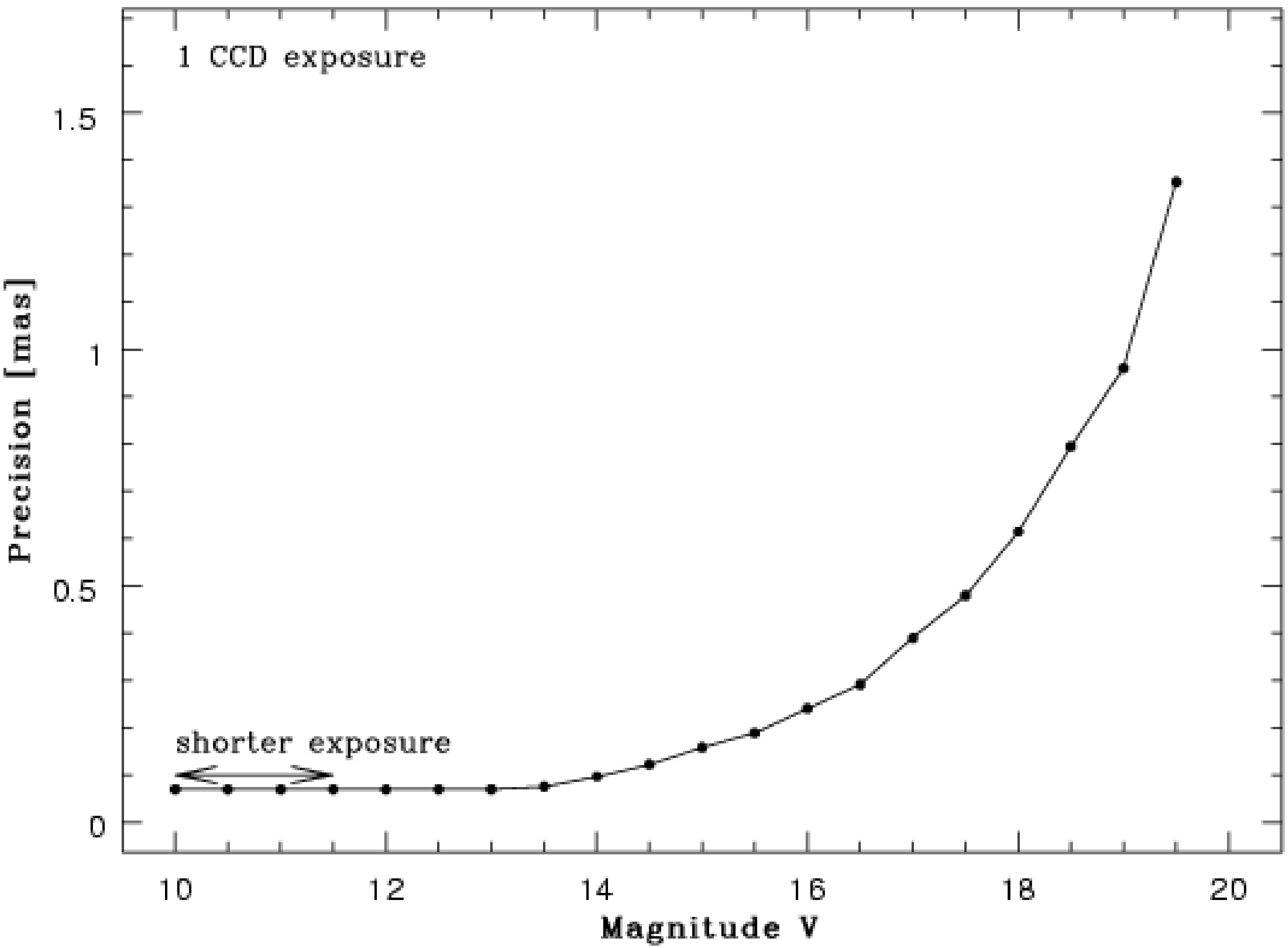
- Some parameters depend on $1/\sqrt{N}$
  - dJ2/dt possibly $10^{-7} yr^{-1}$
  - $\dot{G}/G$ in fact d(G.M)/dt possibly $<10^{-11} yr^{-1}$
  - Global rotation
- Consider extensive simulation with 300,000 objects (code to //)
- All PD from var. eqs. (no approx. from 2 body)
- Consider Nordtvedt η from Trojans (?, Orellana & Vucetictch), β_2 for dG (?..)



Freq. %
Semi-major axis a [AU]
Eccentricity e
Inclination I [deg]

Size-mag (albedo) relation




1 CCD exposure
shorter exposure
Precision [mas]
Magnitude V
0
0.5
1
1.5
10
12
14
16
18
20

Sparse matrix

least-squares procedure :

$var(\mathbf{dq}_i) \approx (\mathbf{B}_i'\mathbf{B}_i)^{-} \sigma_o^2 + \dots$

$var(\mathbf{dg}) = \mathbf{U}^{-1} \sigma_o^2$ where

$\mathbf{U} = \Sigma_i \left[(\mathbf{A}_i'\mathbf{A}_i)^{-1} - \mathbf{A}_i'\mathbf{B}_i (\mathbf{B}_i'\mathbf{B}_i)^{-1} \mathbf{B}_i'\mathbf{A}_i \right]$

$$\begin{vmatrix} \mathbf{B}_1 & 0 & 0 & \mathbf{A}_1 \\ 0 & \mathbf{B}_i & 0 & \mathbf{A}_i \\ 0 & 0 & \mathbf{B}_N & \mathbf{A}_N \end{vmatrix} \begin{vmatrix} \mathbf{dq}_1 \\ \mathbf{dq}_i \\ \mathbf{dq}_N \\ \mathbf{dg} \end{vmatrix} = \begin{vmatrix} d\lambda \end{vmatrix}$$