Determining cosmological parameters from individual gravitational lens systems

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Abstract

We introduce a new method to calculate the time delay for double or multiple image gravitational lens systems from the potential. This method leads to a formalism providing simple linear relations between observables and cosmological parameters to be determined. The well known mass sheet degeneracy and its breaking by independent measurements of the mass scale is easily recognized and the consequences for the determination of cosmological parameters are shown. As an example, the proposed $D_\mathrm{s}/D_\mathrm{ds}$ method is applied to 0957+561.

1 Introduction

The possibility of determining the Hubble constant H_0 from the time delay of the gravitational lens effect has been known since Refsdal (1964b). The time delay of the double quasar 0957+561 has finally been determined with high accuracy providing a value for H_0 with small internal errors. New observational data for the velocity dispersion of the lensing galaxy and the surrounding cluster allow to determine the scaling factor of the mass density independently of the lensing effect. Since cosmological parameters enter differently in both determinations of the mass scale, the relation of the two can be used to constrain the possible values of Ω_0 and especially λ_0.

For the following considerations, we assume the image positions as given. Therefore the relations between light deflection and time delay on one hand and the mass distribution on the other hand will be linear which allows a very simple formulation of the theory.

2 The wavefront method in the lens plane

There are two well known methods to determine the time delay of a gravitational lens system from the potential or equivalently the deflection angle as a function of the position in the lens plane. The method of Cooke & Kantowski (1975) calculates the time delay by integrating along the light paths, while the second one, which was used by Refsdal (1964a, 1966) for compact deflectors and generalized by Kayser & Refsdal (1983) for arbitrary lenses, evaluates the time delay from geometrical considerations on the wavefronts near the observer. A third method, invented by Borgeest (1995), uses the geometry of wavefronts near the lens plane. This method leads to an expression for the time delay which is especially well suited for our further considerations.

We use the standard approximation of the thin lens with a weak potential, small angles and geometrical optic.

In the following, the lens "plane" is defined as a surface of constant light travel time to the observer. This surface has by assumption a weak curvature and can be treated as a plane in the calculations. The basic geometry of an incoming wavefront W_in and an outgoing wavefront W_out is shown in figure 1.

In the approximation of the thin lens, the effect of the deflecting potential can be described as a Shapiro delay τ_p occuring in the lens plane. Because the curvature of the wavefronts and

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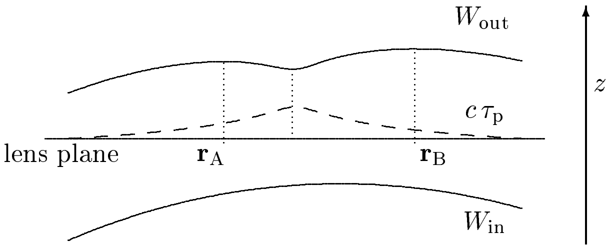

Figure 1: Wavefronts near the lens plane

all deflection angles are small, the wavefront near the lens plane is shifted by this delay in the negative z-direction,

$$W_{\text{out}}(\mathbf{r}) = \text{const} + W_{\text{in}}(\mathbf{r}) - c\,\tau_{\text{p}}(\mathbf{r}) \ . \tag{1}$$

The light travel time to the observer will be τ as measured in the system of the lens plane:

$$\tau(\mathbf{r}) = \text{const} - \frac{W_{\text{out}}(\mathbf{r})}{c} \tag{2}$$

$$= \text{const} - \frac{W_{\text{in}}(\mathbf{r})}{c} + \tau_{\text{p}}(\mathbf{r}) \tag{3}$$

The time delay in the system of the observer will be the redshifted difference of the light travel times in the system of the lens plane:

$$\Delta t = (1+z_{\text{d}})\big(\tau(\mathbf{r}_{\text{B}}) - \tau(\mathbf{r}_{\text{A}})\big) \tag{4}$$

$$= (1+z_{\text{d}})\big(\Delta\tau_{\text{pot}} + \Delta\tau_{\text{geom}}\big) \tag{5}$$

$$\Delta\tau_{\text{pot}} = \tau_{\text{p}}(\mathbf{r}_{\text{B}}) - \tau_{\text{p}}(\mathbf{r}_{\text{A}}) \tag{6}$$

$$\Delta\tau_{\text{geom}} = \frac{W_{\text{in}}(\mathbf{r}_{\text{A}}) - W_{\text{in}}(\mathbf{r}_{\text{B}})}{c} \tag{7}$$

The total time delay is the sum of the potential part (6) and the geometrical part (7). Since the true position of the source and thus W_{in} is not known, it seems appropriate to find an expression for the geometrical part which contains only the Shapiro delay τ_{p} and the positions of the images $\mathbf{r}_{\text{A}}$ and $\mathbf{r}_{\text{B}}$.

$$\Delta\tau_{\text{geom}} = \frac{1}{c}\int_{\mathbf{r}_{\text{B}}}^{\mathbf{r}_{\text{A}}} d\mathbf{r} \bullet \boldsymbol{\nabla}_{\!r} W_{\text{in}}(\mathbf{r}) \tag{8}$$

The incoming wavefront is assumed to be sufficiently smooth to be described as a surface of second order in $\mathbf{r}$, leading to a linear dependence of $\boldsymbol{\nabla} W_{\text{in}}$ on $\mathbf{r}$. The integral can be written as the mean value of the function multiplied by the length of the integration interval. For a linear function, the mean value is simply the mean of the values at the endpoints.

$$\Delta\tau_{\text{geom}} = \frac{\mathbf{r}_{\text{A}} - \mathbf{r}_{\text{B}}}{c} \bullet \frac{\boldsymbol{\nabla}_{\!r} W_{\text{in}}(\mathbf{r}_{\text{A}}) + \boldsymbol{\nabla}_{\!r} W_{\text{in}}(\mathbf{r}_{\text{B}})}{2} \tag{9}$$

$$= \frac{\mathbf{r}_{\text{A}} - \mathbf{r}_{\text{B}}}{2} \bullet \left(\frac{\boldsymbol{\nabla}_{\!r} W_{\text{out}}(\mathbf{r}_{\text{A}}) + \boldsymbol{\nabla}_{\!r} W_{\text{out}}(\mathbf{r}_{\text{B}})}{c} + \boldsymbol{\nabla}_{\!r}\tau_{\text{p}}(\mathbf{r}_{\text{A}}) + \boldsymbol{\nabla}_{\!r}\tau_{\text{p}}(\mathbf{r}_{\text{B}})\right) \tag{10}$$

From the theorem of Fermat (Kovner 1990) we know that the light travel time has to be stationary at the positions of real images,

$$\boldsymbol{\nabla} W_{\text{out}}(\mathbf{r}_{\text{A}}) = 0 \ , \qquad\qquad \boldsymbol{\nabla} W_{\text{out}}(\mathbf{r}_{\text{B}}) = 0 \ . \tag{11}$$

Therefore the W_{out} terms in equation (10) vanish, leading to the following total time delay:

$$\Delta t = (1+z_{\text{d}})\left(\tau_{\text{p}}(\mathbf{r}_{\text{B}}) - \tau_{\text{p}}(\mathbf{r}_{\text{A}}) + (\mathbf{r}_{\text{A}} - \mathbf{r}_{\text{B}}) \bullet \frac{\boldsymbol{\nabla}_{\!r}\tau_{\text{p}}(\mathbf{r}_{\text{A}}) + \boldsymbol{\nabla}_{\!r}\tau_{\text{p}}(\mathbf{r}_{\text{B}})}{2}\right) \tag{12}$$

3 Relation to observables

To determine cosmological parameters from (12), we have to find relations between the Shapiro delay $\tau_{\rm p}$, $\mathbf{r}_{\rm A} - \mathbf{r}_{\rm B}$ and obervable quantities. From the definition of the angular size distance $D_{\rm d}$ (figure 2) we have:

$$\mathbf{r} = D_{\rm d}\,\boldsymbol{\theta} \tag{13}$$

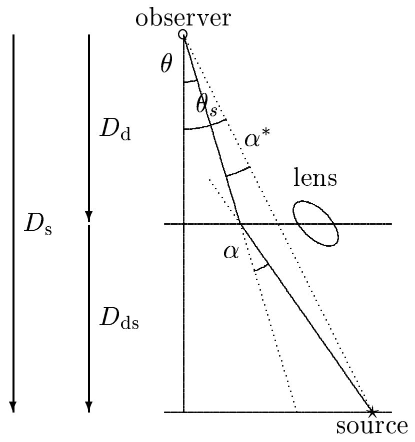

Figure 2: Angles and distances in a lens system

To measure the surface mass density σ independent of the lens effect, we can use the velocity dispersion of the lensing galaxy. This will supply the density multiplied by the distance to the deflector. Analysis of the X-ray gas in a galaxy cluster allows the determination of the same combination of $D_{\rm d}$ and σ. Therefore we define a scaled and dimensionless surface mass density $\tilde{\sigma}$ which is measurable by different means without using the lens effect.

$$\tilde{\sigma} = \frac{G\,D_{\rm d}}{c^2}\sigma \tag{14}$$

The Poisson equation for $\tau_{\rm p}$ reads:

$$\boldsymbol{\nabla}_r^2 \tau_{\rm p}(\mathbf{r}) = -\frac{8\pi G}{c^3}\sigma(\mathbf{r}) \tag{15}$$

It is convenient to define in the same way a "measurable potential" ψ:

$$\psi = \frac{c}{D_{\rm d}}\tau_{\rm p} \tag{16}$$

$$\boldsymbol{\nabla}_\theta^2 \psi(\boldsymbol{\theta}) = -8\pi\tilde{\sigma}(\boldsymbol{\theta}) \tag{17}$$

Equation (17) can be inverted to obtain ψ from $\tilde{\sigma}$:

$$\psi(\boldsymbol{\theta}) = -4\int d^2\theta'\,\tilde{\sigma}(\boldsymbol{\theta}')\ln|\boldsymbol{\theta} - \boldsymbol{\theta}'| \tag{18}$$

To separate the effect of the Hubble constant H_0 from the cosmological parameters Ω_0 and λ_0, we use the reduced distances d_{ij}.

$$d_{ij} = \frac{H_0}{c}D_{ij} \tag{19}$$

$$= d(z_i, z_j, \Omega_0, \lambda_0) \tag{20}$$

Combining equations (12), (13), (16) and (19) results in a simple relation between $H_0\,\Delta t$, d_d and the potential ψ,

$$H_0\,\Delta t = (1+z_\mathrm{d})\,d_\mathrm{d}\,G_\psi[\psi]\ , \tag{21}$$

$$G_\psi[\psi] = \psi(\boldsymbol{\theta}_\mathrm{B}) - \psi(\boldsymbol{\theta}_\mathrm{A}) + \boldsymbol{\theta}_\mathrm{AB} \cdot \frac{\boldsymbol{\nabla}_\theta\psi(\boldsymbol{\theta}_\mathrm{A}) + \boldsymbol{\nabla}_\theta\psi(\boldsymbol{\theta}_\mathrm{B})}{2}\ , \tag{22}$$

where the angular image separation is defined as $\boldsymbol{\theta}_\mathrm{AB} = \boldsymbol{\theta}_\mathrm{A} - \boldsymbol{\theta}_\mathrm{B}$. To find the direct relation between $\tilde{\sigma}$ and $H_0\,\Delta t$ we combine equation (22) with (18):

$$G_\sigma[\tilde{\sigma}] = G_\psi[\psi] \tag{23}$$

$$= 4\int d^2\theta\,\tilde{\sigma}(\boldsymbol{\theta})\,g(\boldsymbol{\theta}) \tag{24}$$

$$g(\boldsymbol{\theta}) = \ln\frac{|\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{A}|}{|\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{B}|} + \frac{\boldsymbol{\theta}_\mathrm{AB}}{2} \cdot \left(\frac{\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{A}}{|\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{A}|^2} + \frac{\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{B}}{|\boldsymbol{\theta}-\boldsymbol{\theta}_\mathrm{B}|^2}\right) \tag{25}$$

The plot of g in figure 3 shows the contribution of mass elements at different positions to G_σ and $H_0\,\Delta t$.

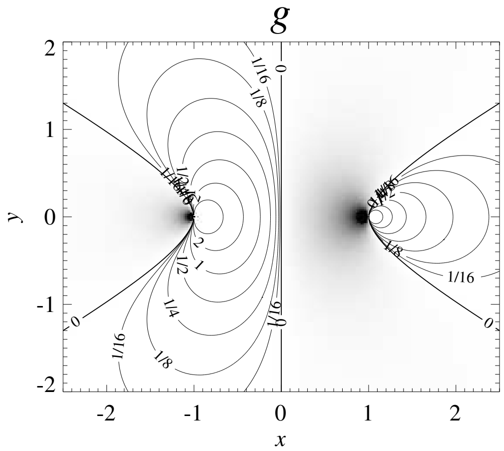

Figure 3: The function g using $\boldsymbol{\theta}_\mathrm{A} = (+1, 0)$ and $\boldsymbol{\theta}_\mathrm{B} = (-1, 0)$. Negative values are shown as grayscale, positive ones as contour lines.

4 The image separation

To learn which parameters of the mass distribution $\tilde{\sigma}$ can be constrained by modelling we have to find relations between $\tilde{\sigma}$ and all observable quantities of the lens system. The most fundamental observable is the image separation $\boldsymbol{\theta}_\mathrm{AB}$ which can be derived using figure 2.

$$\boldsymbol{\theta}_\mathrm{s} = \boldsymbol{\theta} + \frac{D_\mathrm{ds}}{D_\mathrm{s}}\boldsymbol{\alpha} \tag{26}$$

$$\boldsymbol{\theta}_\mathrm{AB} = \frac{D_\mathrm{ds}}{D_\mathrm{s}}(\boldsymbol{\alpha}_\mathrm{B} - \boldsymbol{\alpha}_\mathrm{A}) \tag{27}$$

The deflection angle $\boldsymbol{\alpha}$ can be determined geometrically from figure 1:

$$\boldsymbol{\alpha} = c\,\boldsymbol{\nabla}_r\tau_\mathrm{p} \tag{28}$$

$$= \boldsymbol{\nabla}_\theta\psi \tag{29}$$

The relation between the image separation and the mass distribution is linear and can be written using another functional $\mathbf{H}$. The following equations become much more compact, if we treat the two-dimensional vectors as complex numbers allowing us to divide by "vectors".

$$\overline{\mathbf{H}}_\alpha[\boldsymbol{\alpha}] = \frac{\boldsymbol{\alpha}_\mathrm{B} - \boldsymbol{\alpha}_\mathrm{A}}{\boldsymbol{\theta}_\mathrm{AB}} \tag{30}$$

$$\overline{\mathbf{H}}_\psi[\psi] = \frac{\boldsymbol{\nabla}_\theta \psi(\boldsymbol{\theta}_\mathrm{B}) - \boldsymbol{\nabla}_\theta \psi(\boldsymbol{\theta}_\mathrm{A})}{\boldsymbol{\theta}_\mathrm{AB}} \tag{31}$$

$$\mathbf{H}_\sigma[\tilde{\sigma}] = \frac{8}{|\boldsymbol{\theta}_\mathrm{AB}|^2} \int d^2\theta \, \tilde{\sigma}(\boldsymbol{\theta}) \, \mathbf{h}(\boldsymbol{\theta}) \tag{32}$$

$$\mathbf{h}(\boldsymbol{\theta}) = \frac{\boldsymbol{\theta}_\mathrm{AB}}{2} \left(\frac{1}{\boldsymbol{\theta} - \boldsymbol{\theta}_\mathrm{B}} - \frac{1}{\boldsymbol{\theta} - \boldsymbol{\theta}_\mathrm{A}} \right) \tag{33}$$

With these definitions we write equation (27) as follows:

$$1 = \frac{D_\mathrm{ds}}{D_\mathrm{s}} \mathbf{H}_\sigma[\tilde{\sigma}] \tag{34}$$

As mentioned in the introduction, the positions of the images are treated as given and are not allowed to change. If equation (34) is fulfilled, the mass distribution in fact leads to the given image separation. Unity on the left hand side corresponds to the scaled observed image separation. The contribution of a mass element at position $\boldsymbol{\theta}$ to the image separation is given by $\mathbf{h}(\boldsymbol{\theta})$, see figure 4. Please note that (34) is a complex equation equivalent to two real equations.

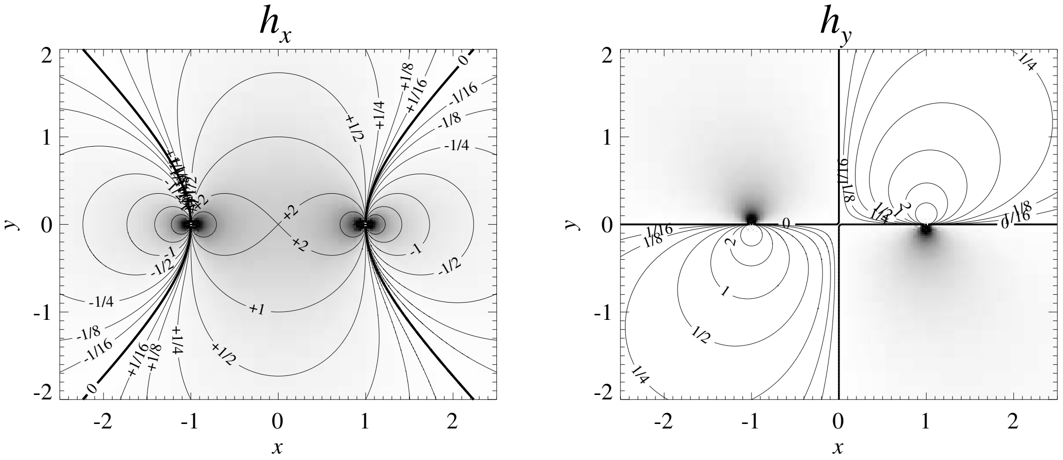

Figure 4: Real and imaginary part of $\mathbf{h}$ using $\boldsymbol{\theta}_\mathrm{A} = (+1, 0)$ and $\boldsymbol{\theta}_\mathrm{B} = (-1, 0)$. The absolute value of h_x is shown as grayscale, h_y is shown as grayscale for negative values and as contour lines for positive ones.

5 The relative magnification matrix

For small but extended sources we linearize the lens equation (26) to find a linear mapping between two images of the source. Again it is convenient to use complex formalism. Convergence κ and shear $\boldsymbol{\gamma}$ are easily recognized as Wirtinger derivatives of the deflection angle. For an introduction

[0]The bar denotes complex conjugation and is used here for convention. The function $\mathbf{h}$ becomes simpler if we define it this way.

to complex analysis in lensing theory, see Schramm & Kayser (1995).

$$d\boldsymbol{\theta}_{\mathrm{s}} = (1-\kappa)\, d\boldsymbol{\theta} - \boldsymbol{\gamma}\, d\overline{\boldsymbol{\theta}} \tag{35}$$

$$\kappa = -\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{\theta}} \tag{36}$$

$$\boldsymbol{\gamma} = -\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \frac{\partial \boldsymbol{\alpha}}{\partial \overline{\boldsymbol{\theta}}} \tag{37}$$

Written in vector notation:

$$d\boldsymbol{\theta}_{\mathrm{s}} = \mathcal{M}_{\boldsymbol{\theta}}\, d\boldsymbol{\theta} \tag{38}$$

$$\mathcal{M}_{\boldsymbol{\theta}} = \begin{pmatrix} 1-\kappa(\boldsymbol{\theta})-\gamma_x(\boldsymbol{\theta}) & -\gamma_y(\boldsymbol{\theta}) \\ -\gamma_y(\boldsymbol{\theta}) & 1-\kappa(\boldsymbol{\theta})+\gamma_x(\boldsymbol{\theta}) \end{pmatrix} \tag{39}$$

As the true deflection angle is linear in $\tilde{\sigma}$, the same is true for the convergence κ and the shear $\boldsymbol{\gamma}$.

$$\kappa(\boldsymbol{\theta}) = -\frac{1}{2}\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \boldsymbol{\nabla}_{\theta}^2 \psi \tag{40}$$

$$= 4\pi \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \tilde{\sigma}(\boldsymbol{\theta}) \tag{41}$$

$$\overline{\boldsymbol{\gamma}}(\boldsymbol{\theta}) = -2\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \frac{\partial^2 \psi}{\partial \boldsymbol{\theta}^2} \tag{42}$$

$$= -4\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \int d^2\theta' \frac{\tilde{\sigma}(\boldsymbol{\theta}')}{(\boldsymbol{\theta}-\boldsymbol{\theta}')^2} \tag{43}$$

To emphasize the algebraical relations between $\tilde{\sigma}$ and κ and $\boldsymbol{\gamma}$, we introduce another linear (here matrix-valued) functional $\boldsymbol{m}$, which contains the explicit analytical relations of equations (41) and (43).

$$\mathcal{M}_{\boldsymbol{\theta}} = \mathbf{1} - \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \boldsymbol{m}_{\boldsymbol{\theta}}[\tilde{\sigma}] \tag{44}$$

The intrinsic structure of the source is in general not known, making the magnification matrices $\mathcal{M}_{\mathrm{A}}$ and $\mathcal{M}_{\mathrm{B}}$ unobservable. What can be observed is the relative magnification matrix $\mathcal{M}_{\mathrm{BA}}$ describing the mapping between two images.

$$d\boldsymbol{\theta}_{\mathrm{B}} = \mathcal{M}_{\mathrm{BA}}\, d\boldsymbol{\theta}_{\mathrm{A}} \ , \qquad \mathcal{M}_{\mathrm{BA}} = \mathcal{M}_{\mathrm{B}}^{-1} \mathcal{M}_{\mathrm{A}} \tag{45}$$

In the case of 0957+561 the relative magnification matrix provides very important constraints for the modelling of the lens. Grogin & Narayan (1996a, 1996b) use the VLBI data of Garrett et al. (1994) which allowed the determination of $\mathcal{M}_{\mathrm{BA}}$ and the derivatives of the eigenvalues along the jet.

6 Influence of an additional mass sheet on the observables

In a typical lensing situation, the primary lens will be a massive galaxy possibly surrounded by a cluster of galaxies, which may contribute appreciably to the lensing effect. As the typical scale length of the cluster will be much larger than the image separation, the influence of the cluster can be described by a constant convergence κ and shear $\boldsymbol{\gamma}$ to a good approximation. The treatment of the convergence, which is equivalent to a mass sheet of constant density, is most important, because it leads to a degeneracy in the possible lens models.

A mass sheet of constant surface density $\tilde{\sigma}_0$ contributes in the following way to the functionals related to the observable quantities:

$$G_0 = 0 \ , \qquad \mathbf{H}_0 = 4\pi\tilde{\sigma}_0 \ , \tag{46}$$

$$\boldsymbol{m}_0 = -\kappa_0\, \mathbf{1} \ , \qquad \gamma_0 = 0 \ , \qquad \kappa_0 = 4\pi \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \tilde{\sigma}_0 \tag{47}$$

7 Applications

Let us assume a lens consisting of a surface mass density $\tilde{\sigma} = \sigma_v^2\,\hat{\sigma}$, where the scaling factor σ_v^2 (the squared velocity dispersion) may be unknown and a constant mass sheet κ_0.

We can choose $\tilde{\sigma}$, ψ or $\boldsymbol{\alpha}$ as the function describing the mass distribution. Since we are not concerned with the explicit analytical form of the functionals, we call the chosen function $\mu = \sigma_v^2\,\hat{\mu}$ and use the functionals related to μ.

$$H_0\,\Delta t = \sigma_v^2\,(1+z_\mathrm{d})\,d_\mathrm{d}\,G[\hat{\mu}] \tag{48}$$

$$1 = \frac{\sigma_v^2}{1-\kappa_0}\frac{D_\mathrm{ds}}{D_\mathrm{s}}\mathbf{H}[\hat{\mu}] \tag{49}$$

The equation involving the relative magnification matrix can be written in the same linear form:

$$\mathcal{M}_\mathrm{BA} - \mathbf{1} = \frac{\sigma_v^2}{1-\kappa_0}\frac{D_\mathrm{ds}}{D_\mathrm{s}}\left(\boldsymbol{m}_\mathrm{B}[\hat{\mu}]\mathcal{M}_\mathrm{BA} - \boldsymbol{m}_\mathrm{A}[\hat{\mu}]\right) \tag{50}$$

Using equation (48), it is possible to determine the Hubble constant H_0 if we know d_d and the mass distribution of the lens μ. Turning this calculation around, it is possible to get restrictions on Ω_0 and λ_0 from d_d if we know $H_0\,\Delta t$ and $\tilde{\sigma}$.

The last two equations can be used to determine the scaling factor σ_v^2 and to confine the parameters of $\hat{\mu}$, i.e. for modelling the mass distribution. Unfortunately the equations do not allow the determination of both σ_v^2 and $1-\kappa_0$ but only of $\sigma_v^2/(1-\kappa_0)$. This fact is due to a fundamental degeneracy of lens models first described by Falco, Gorenstein & Shapiro (1985) and Gorenstein, Shapiro & Falco (1988). This degeneracy shows quite clearly in our equations.

7.1 Determining H_0 without knowledge of σ_v

If no measurements of the velocity dispersion σ_v are available, we can use equation (49) or (50) to determine σ_v^2 implicitly. Dividing (48) by (49) eliminates the unknown factor in the following way:

$$H_0\,\Delta t = (1-\kappa_0)\,(1+z_\mathrm{d})\,d\,\frac{G[\hat{\mu}]}{\mathbf{H}[\hat{\mu}]}\ , \tag{51}$$

where the effective distance d is defined as

$$d = \frac{d_\mathrm{d}\,d_\mathrm{s}}{d_\mathrm{ds}}\ . \tag{52}$$

Equation (50) can now be used to constrain the possible mass models of the lens. The same is true for the imaginary part of (49).

If the convergence κ_0 is not known but can be assumed to be less than unity, this equation allows determination of an upper limit for H_0.

The dependence of d on the cosmological parameters Ω_0 and λ_0 is rather weak which would make it difficult to determine the parameters from this equation if H_0 and Δt were known. On the other hand, the influence of the unknown parameters on H_0 is also weak. Figure 5 shows d as a function of Ω_0 and λ_0 for the homogeneous universe ($\eta = 1$) and the empty cone model ($\eta = 0$) respectively.[1] The plot of d is restricted to that area of the Ω_0/λ_0-plane which, following Czoske (1995), can not be excluded by other observations.

[1] All distances calculated with the `ANGSIZ` program. (Kayser, Helbig & Schramm 1997)

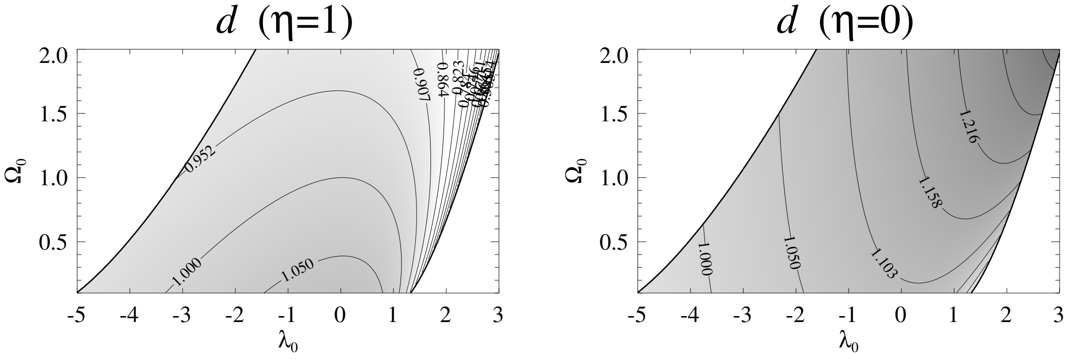

Figure 5: The effective distance d for 0957+561 normalized to the Einstein-de Sitter model $d_{\mathrm{EdS}} = 1$. The contour lines differ by 5 % each. The left plot is valid for a homogeneous universe, the right one for the empty cone model.

7.2 Determining H_0 using σ_v

If the velocity dispersion of the lensing galaxy has been measured, equation (48) can be used directly to determine H_0 without any influence of the cluster convergence κ_0. This method thus not only provides an upper limit of H_0 but a definite value. The influence of the cosmological parameters on d_{d} (and on H_0) can be seen in figure 6. The direct measurement of σ_v breaks

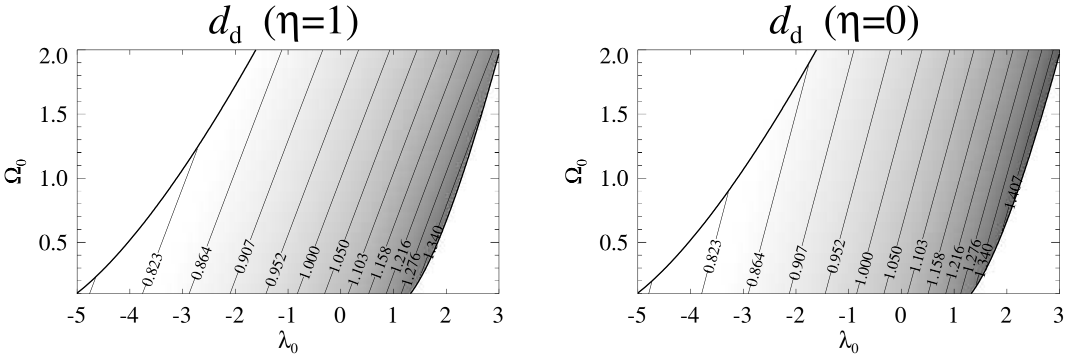

Figure 6: The distance d_{d} for 0957+561 normalized to the Einstein-de Sitter model $d_{\mathrm{dEdS}} = 1$. The contour lines differ by 5 % each.

the degeneracy of the possible lens models, which was realized by Narayan (1991). Within our formalism, the breaking of the degeneracy is even more fundamental than the degeneracy itself.

The other equations can now be used to model the relative mass distribution or to determine cosmological parameters using the method described in the next section.

7.3 Determining cosmological parameters

As noted before, the dependence of the effective distance d on the cosmological parameters is too weak to constrain the values of Ω_0 and λ_0 from equation (51) if $H_0\,\Delta t$ and κ_0 are not known with great accuracy. Therefore this method to determine the parameters, which was already mentioned by Refsdal (1966), seems not to be very promising.

A better candidate is equation (48) using d_{d} to constrain Ω_0 and λ_0 because the sensitivity of d_{d} for small changes of the cosmological parameters is stronger than that of d. Another advantage is the independence of κ_0.

A method to constrain the cosmological parameters without any knowledge of H_0 or Δt uses equations (49) and (50) to determine $D_{\mathrm{s}}/D_{\mathrm{ds}}$. The idea behind this method is to scale the

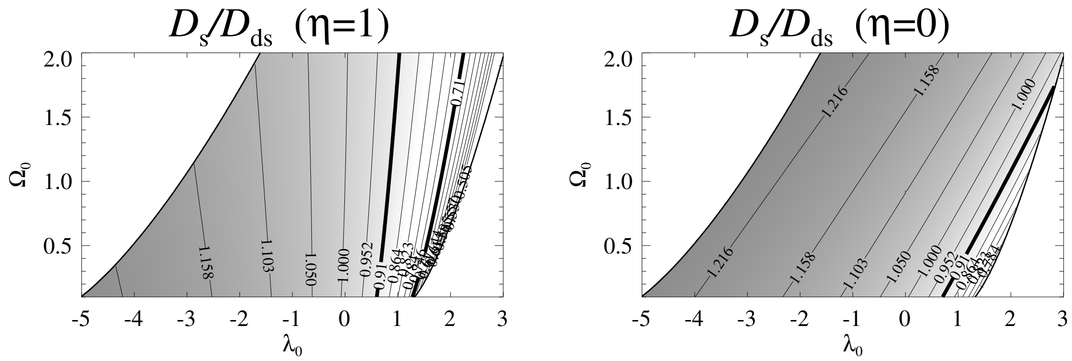

Figure 7: The distance relation $D_\mathrm{s}/D_\mathrm{ds}$ for 0957+561 normalized for the Einstein-de Sitter model $(D_\mathrm{s}/D_\mathrm{ds})_\mathrm{EdS} = 1$. The contour lines differ by 5 % each. The lines for 0.71 and 0.91 are enhanced.

mass distribution independently by the lensing effect, which provides $(D_\mathrm{ds}/D_\mathrm{s})\,\tilde{\sigma}$, and by the measurement of the velocity dispersion, which provides $\tilde{\sigma}$ directly. Figure 7 shows how $D_\mathrm{s}/D_\mathrm{ds}$ depends on the cosmological parameters for the double quasar 0957+561. Please note that the influence of λ_0 is stronger than that of Ω_0 which makes this method suitable to determine λ_0.

Determination of $D_\mathrm{s}/D_\mathrm{ds}$ was already proposed by Narayan (1991) as a tool to determine Ω_0 for $\lambda_0 = 0$. Since the dependence on Ω_0 is very weak, no useful bounds for Ω_0 could be found.

8 Estimate of λ_0 from the double quasar

To examine the usefulness of the $D_\mathrm{s}/D_\mathrm{ds}$ method to determine cosmological parameters, we use the model of Grogin & Narayan (1996a, 1996b), which provides the scaling of the mass distribution as a velocity dispersion $(\sigma_v^2)_\mathrm{mod}$ for the Einstein-de Sitter universe with $\Omega_0 = 1$ and $\lambda_0 = 0$, and the observed velocity dispersion of $(\sigma_v^2)_\mathrm{obs}$ Falco et al. (1997).

We write (49) for the lens model in the Einstein-de Sitter universe

$$\left(\frac{D_\mathrm{s}}{D_\mathrm{ds}}\right)_\mathrm{EdS} = (\sigma_v^2)_\mathrm{mod}\mathbf{H}[\hat{\mu}] \ , \tag{53}$$

and for the real universe

$$\frac{D_\mathrm{s}}{D_\mathrm{ds}} = \frac{(\sigma_v^2)_\mathrm{obs}}{1-\kappa_0}\mathbf{H}[\hat{\mu}] \ . \tag{54}$$

The mass model can be eliminated by dividing the two equations:

$$\frac{D_\mathrm{s}}{D_\mathrm{ds}} = \left(\frac{D_\mathrm{s}}{D_\mathrm{ds}}\right)_\mathrm{EdS} \frac{1}{1-\kappa_0}\frac{(\sigma_v^2)_\mathrm{obs}}{(\sigma_v^2)_\mathrm{mod}} \tag{55}$$

With $(\sigma_v)_\mathrm{mod} = 330\,\mathrm{km\,s^{-1}}$ from Grogin & Narayan (1996b) for the FGS-model and the mean value of $(\sigma_v)_\mathrm{obs} = 279\,\mathrm{km\,s^{-1}}$ from Falco et al. (1997), we get a lower limit for $D_\mathrm{s}/D_\mathrm{ds}$:

$$\frac{D_\mathrm{s}}{D_\mathrm{ds}} \gtrsim 0.714\left(\frac{D_\mathrm{s}}{D_\mathrm{ds}}\right)_\mathrm{EdS} \tag{56}$$

$$\gtrsim 1.192 \tag{57}$$

No detailed error estimate has yet been done, the statistical error should be in the 10 % range. Comparison with figure 7 tells that, even for $\eta = 1$, only very high values of $\lambda_0 \gtrsim 1.5$ can be excluded.

To find a definite value for $D_\mathrm{s}/D_\mathrm{ds}$, we have to estimate the convergence of the cluster κ_0. To exclude the influence of the cosmological parameters from the determination of κ_0, we use the velocity dispersion $\sigma_v = 715\,\mathrm{km\,s^{-1}}$ of the cluster from Angonin-Willaime, Soucail & Vanderriest (1994) and the position $\theta \approx 22''$ and core radius $\theta_\mathrm{c} \approx 5''$ from Fischer et al. (1997). With these data, we get

$$\tilde{\sigma}_0 \approx 0.026 \ . \tag{58}$$

With equation (47) and (55) we have:

$$\frac{D_\mathrm{s}}{D_\mathrm{ds}} = \left(\frac{D_\mathrm{s}}{D_\mathrm{ds}}\right)_\mathrm{EdS} \frac{(\sigma_v^2)_\mathrm{obs}}{(\sigma_v^2)_\mathrm{mod}} + 4\pi\,\tilde{\sigma}_0 \tag{59}$$

$$\approx 1.52 \tag{60}$$

$$\approx 0.91 \left(\frac{D_\mathrm{s}}{D_\mathrm{ds}}\right)_\mathrm{EdS} \tag{61}$$

The error is expected to be slightly higher than in (56) because of the uncertainties of $\tilde{\sigma}_0$.

9 Conclusions

Gravitational lenses are suitable for determination not only of the Hubble constant but also of the cosmological parameters Ω_0 and λ_0. Using a simple linear formalism, it is easy to find different methods of determining the cosmological parameters. The application of the $D_\mathrm{s}/D_\mathrm{ds}$ method is the most promising candidate to determine λ_0 without knowing H_0 and Δt. The possibility of doing cosmology with individual lens systems even without the time delay proves the importance of modelling lens systems and measuring the mass distributions independently from the lens effect. A first application to 0957+561 provides an estimate of $\lambda_0 \approx 0$–1.

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