Cosmic shear in strong lensing events and the equivalent single-plane lens

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Abstract

The gravitational lens equation resulting from a single (non-linear) mass concentration (the main lens) plus inhomogeneities of the large-scale structure is equivalent to the single-plane gravitational lens equation without the cosmological perturbations. The deflection potential (and, by applying the Poisson equation, also the mass distribution) of the equivalent single-plane lens is derived. If the main lens is described by elliptical isopotential curves plus a shear term, the equivalent single-plane lens will be of the same form. Due to this equivalence, the determination of the Hubble constant from time delay measurements is affected by the same mass-sheet invariance transformation as for the single-plane lens. If the lens strength is fixed (e.g., by measuring the velocity dispersion of stars in the main lens), the determination of H_0 is affected by inhomogeneous matter between us and the lens. The orientation of the mass distribution relative to the image positions is the same for the cosmological lens situation and the single-plane case. In particular this implies that cosmic shear cannot account for a misalignment of the observed galaxy orientation relative to the best-fitting lens model.

1 Introduction

The tidal gravitational field caused by the large scale distribution of matter in the Universe affects the propagation of light rays. This effect is inevitably present, of course also close to lines-of-sight to multiple QSOs, and will affect the imaging properties of the corresponding lenses. This fact has been noticed in several papers, see e.g., Bar-Kana (1996, hereafter B-K96). Taking a fixed gravitational lens and adding this cosmic shear can lead to a qualitative change of the lensing properties, and a source that was imaged twice without cosmic shear may have four images when shear is included.

If we want to understand the lensing geometry of a given system, generally a parametrized lens model is fitted to the observations. Let L_1 be such a model with the best fitting parameters, with no cosmic shear included. Now assume that in this system we have some cosmic shear γ, and let L_2 be the lens model which is obtained by adding γ to L_1. We shall notice, depending on the magnitude of γ and the image configuration in this system, that L_2 does no longer provide a satisfactory fit to the observations, as L_1 did. Therefore, we would change the parameters of our lens model such that they describe the observations best in the presence of the cosmic shear, and we obtain a new lens model L_3. The difference between the predictions of L_1 and L_2 can be substantial, as shown in B-K96 (e.g., changing from 2 to 4 image configurations), but that should not bother us too much – the observer will know whether she has two or four images, and so cosmic shear can not fool us with that. What is important, however, is the difference between L_1 and L_3, as both yield good fits to the data. In particular, how different are the predictions of $H_0\,\Delta t$ from these two models? And is it possible to identify the presence of cosmic shear in a given lens system?

As it turns out, the uncertainty in lens modelling caused by cosmic shear is considerably smaller than might be feared. In this contribution, I will show the following: Given a gravitational lens model plus cosmic shear (L_3), one can find a lens model without cosmic shear that yields the same observables, i.e., image configuration, relative magnification matrices, and time delays. This equivalent single-plane lens model is not necessarily L_1, as it may not be member of the same family of parametrized lens models. However, if the family of models is chosen as elliptical potentials plus a shear matrix (not necessarily with vanishing trace), then L_1 will be member of the same family. Note that such models have been applied to observed lens systems, and it appears that an (external) shear in addition to the ellipticity of the lens galaxy is needed in most cases where enough observational constraints are available (e.g., four-image systems) – see Keeton, Kochanek & Seljak (1996), Witt & Mao (1997). In that case, the observed image positions relative to the observable major axis of the matter distribution in the main lens is the same in the cosmic lens situation L_3 and in the equivalent single-plane situation L_1, in contrast to the possibility indicated by Blandford & Kundić (1996) who suspected possible misalignment of the images with respect to the observed orientation of the galaxy.

This contribution summarizes a recent preprint (Schneider 1997) to which the reader is referred for more details.

2 The cosmological lens equation

We shall assume to have a single strong deflector ('the lens') along the line-of-sight to a QSO, whereas the foreground and background mass distribution acts on larger scales, so that the deflection caused by it can linearized across the multiple-image region of the main deflector; this is the same situation as considered by Kovner (1987). In that case, the lens equation can be written as

$$\boldsymbol{\eta} = \mathcal{D}_{\rm s}\boldsymbol{\theta} - \mathcal{D}_{\rm ds}\boldsymbol{\alpha}(\boldsymbol{\xi}) \quad , \quad \boldsymbol{\xi} = \mathcal{D}_{\rm d}\boldsymbol{\theta}\ ; \tag{1}$$

here, $\boldsymbol{\theta}$ is the angle between a light ray under consideration and a fiducial light ray between the 'center' of the lens and the observer. $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are the proper transverse separation vectors between these two rays at the redshift of the lens and the source, respectively, and $\boldsymbol{\alpha}(\boldsymbol{\xi})$ is the deflection angle of the main lens. The $\mathcal{D}$'s are 2×2-matrices which describe the influence of matter inhomogeneities along the line-of-sight to the QSO; they are obtained in general from a propagation equation which contains the LSS as a source term – see Seitz, Schneider & Ehlers (1994; also Blandford et al. 1991, Kaiser 1992, and references therein). In the absence of LSS perturbations, the $\mathcal{D}$'s would become proportional to the unit matrix, multiplied by the corresponding angular-diameter distances. In general, the $\mathcal{D}$'s are non-symmetric matrices, though owing to the smallness of the cosmic shear the asymmetry is expected to be small along most lines-of-sight. The non-zero trace-less components of the $\mathcal{D}$'s imply that the angular diameter distances are in general non-isotropic.

We rewrite the lens equation (1) by applying a linear transformation to the source coordinates, $\boldsymbol{\beta} = \mathcal{D}_{\rm ds}^{-1}\boldsymbol{\eta}$; then,

$$\boldsymbol{\beta} = D_{\rm ds}^{-1}\mathcal{D}_{\rm s}\mathcal{D}_{\rm d}^{-1}\boldsymbol{\xi} - \boldsymbol{\alpha}(\boldsymbol{\xi}) =: C_\xi\boldsymbol{\xi} - \boldsymbol{\alpha}(\boldsymbol{\xi}) \quad . \tag{2}$$

As shown in Schneider (1997), the matrix C_ξ multiplying $\boldsymbol{\xi}$ in (2) is symmetric, so that the matrix $\partial\boldsymbol{\beta}/\partial\boldsymbol{\xi}$ is symmetric. The symmetry of C_ξ is in complete analogy to the symmetry of the 'telescope matrix' introduced by Kovner (1987) in the frame of a multiple deflection gravitational lens system with at most one non-linear deflector; see also Schneider, Ehlers & Falco (1992), Sect. 9.3. The explicit algebraic proof for the symmetry of the 'telescope matrix' has been given in Seitz & Schneider (1994). The symmetry of C_ξ to first order in the cosmological density fluctuations was noted by B-K96. The symmetry of C_ξ to all orders is due to the fact that the $\mathcal{D}$'s satisfy the same Jacobi differential equation; this fact is heavily used in the proof. We also note in passing that the magnification theorem (Schneider 1984; Seitz & Schneider 1992) is still satisfied: at least one of the images formed by the gravitational lens of any source is brighter than the source would appear in the same direction (i.e., with the same optical tidal matrix) in the absence of the lens, due to the non-negativity of the durface mass density $\Sigma(\boldsymbol{\xi})$ of the main deflector. However, it should be noted that the source can appear fainter than the same source at the same redshift would appear in the corresponding homogeneous Universe, since the cosmological density fluctuation $\delta\rho$ can be negative; indeed, $\langle\delta\rho\rangle = 0$ by definition.

We write the lens equation (2) in terms of the angle $\boldsymbol{\theta}$, by multiplying (2) from the left by $\mathcal{D}_{\rm d}^{\rm T}$, and by defining $\boldsymbol{\eta}' = \mathcal{D}_{\rm d}^{\rm T}\boldsymbol{\beta}$:

$$\boldsymbol{\eta}' = \mathcal{D}_{\rm d}^{\rm T}\mathcal{D}_{\rm ds}^{-1}\mathcal{D}_{\rm s}\boldsymbol{\theta} - \mathcal{D}_{\rm d}^{\rm T}\boldsymbol{\alpha}\left(\mathcal{D}_{\rm d}\boldsymbol{\theta}\right) =: C_\theta\boldsymbol{\theta} - \mathcal{D}_{\rm d}^{\rm T}\boldsymbol{\alpha}\left(\mathcal{D}_{\rm d}\boldsymbol{\theta}\right) \quad . \tag{3}$$

Since $C_\theta = \mathcal{D}_{\rm d}^{\rm T}C_\xi\mathcal{D}_{\rm d}$, the symmetry of C_ξ implies that C_θ is also symmetric.

3 Relation to observables

The 'source position' $\boldsymbol{\eta}'$ is obtained from the 'true' source position $\boldsymbol{\eta}$ through a linear transformation. However, this transformation does not affect the observables. Let $\boldsymbol{\theta}^i$ be the observed image positions, $1 \le i \le N$, then the lens equation predicts that they have to satisfy

$$C_\theta\left(\boldsymbol{\theta}^i - \boldsymbol{\theta}^j\right) = \mathcal{D}_{\rm d}^{\rm T}\left[\boldsymbol{\alpha}\left(\mathcal{D}_{\rm d}\boldsymbol{\theta}^i\right) - \boldsymbol{\alpha}\left(\mathcal{D}_{\rm d}\boldsymbol{\theta}^j\right)\right] \quad , \tag{4}$$

for $i \ne j$, a relation not containing $\boldsymbol{\eta}'$. Secondly, whereas the magnification of the images depends on the linear transformation in the source plane, the observable magnification ratios are unaffected by it. What can be observed are the relative magnification matrices $\mathcal{A}_{ij}$ between images,

$$\mathcal{A}_{ij} = \left(\frac{\partial\boldsymbol{\eta}'(\boldsymbol{\theta}^i)}{\partial\boldsymbol{\theta}}\right)\left(\frac{\partial\boldsymbol{\eta}'(\boldsymbol{\theta}^j)}{\partial\boldsymbol{\theta}}\right)^{-1} \quad , \tag{5}$$

which are unchanged by a linear transformation in the source plane. Note that

$$\frac{\partial \boldsymbol{\eta}'}{\partial \boldsymbol{\theta}} = C_\theta - \mathcal{D}_\mathrm{d}^\mathrm{T} U\left(\mathcal{D}_\mathrm{d}\boldsymbol{\theta}\right)\mathcal{D}_\mathrm{d} \quad , \tag{6}$$

where

$$U(\boldsymbol{\xi}) = \frac{\partial \boldsymbol{\alpha}(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} \quad . \tag{7}$$

Since U is symmetric, so is $\partial\boldsymbol{\eta}'/\partial\boldsymbol{\theta}$.

We next consider the time delay function T. For that we first note that the potential part of the time delay is (Cooke & Kantowski 1975)

$$cT_\mathrm{pot} = (1+z_\mathrm{d})\Psi(\boldsymbol{\xi}) \quad , \tag{8}$$

where

$$\Psi(\boldsymbol{\xi}) = \frac{4G}{c^2}\int_{\mathbb{R}^2} \mathrm{d}^2\xi' \; \Sigma(\boldsymbol{\xi}') \, \ln\left(\frac{|\boldsymbol{\xi}-\boldsymbol{\xi}'|}{\xi_*}\right) \quad , \tag{9}$$

is the deflection potential, and ξ_* is an arbitrary length scale; changing ξ_* changes Ψ by an additive constant which does therefore not affect the measureable time delays. From Fermat's principle (Schneider 1985, Kovner 1990) one knows that the lens equation is equivalent to $\nabla T = \mathbf{0}$, which determines T up to an affine transformation. The multiplicative constant is determined by that of T_pot, so that

$$cT = (1+z_\mathrm{d})\left[\boldsymbol{\theta}\cdot(C_\theta\boldsymbol{\theta})/2 - \boldsymbol{\eta}'\cdot\boldsymbol{\theta} - \Psi(\mathcal{D}_\mathrm{d}\boldsymbol{\theta})\right] \quad , \tag{10}$$

and noting that $\partial\Psi/\partial\boldsymbol{\theta} = \mathcal{D}_\mathrm{d}^\mathrm{T}\boldsymbol{\alpha}(\mathcal{D}_\mathrm{d}\boldsymbol{\theta})$, one sees that $\partial T/\partial\boldsymbol{\theta} = \mathbf{0}$ is equivalent to (3). The time delay between any pair of images is then $\Delta t = T\left(\boldsymbol{\theta}^i\right) - T\left(\boldsymbol{\theta}^j\right)$. Whereas T as written in the form (10) contains the source position, one notes that $\boldsymbol{\eta}'$ is substituted in terms of the observed image positions using the lens equation (3). Therefore, the linear transformation in the source plane does not affect the calculation of the time delay.

4 The equivalent single-plane lens

We shall now show that the lens equation (3) is equivalent to a single-plane lens situation without 'cosmic shear'. The single-plane lens equation reads

$$\tilde{\boldsymbol{\eta}} = D_\mathrm{s}\boldsymbol{\theta} - D_\mathrm{ds}\tilde{\boldsymbol{\alpha}}(D_\mathrm{d}\boldsymbol{\theta}) \quad , \tag{11}$$

where D_d, D_ds, and D_s are the angular diameter distances in a homogeneous Universe. We characterize quantities in the single-plane lens with a tilde, to distinguish them from those of the cosmological lens equation. As before, $\tilde{\boldsymbol{\alpha}}(\tilde{\boldsymbol{\xi}})$ can be obtained from a deflection potential

$$\tilde{\boldsymbol{\alpha}}(\tilde{\boldsymbol{\xi}}) = \frac{\mathrm{d}\tilde{\Psi}(\tilde{\boldsymbol{\xi}})}{\mathrm{d}\tilde{\boldsymbol{\xi}}} \quad . \tag{12}$$

Theorem: For every mass distribution $\Sigma(\boldsymbol{\xi})$ and matrices $\mathcal{D}_\mathrm{d}$, $\mathcal{D}_\mathrm{s}$ and $\mathcal{D}_\mathrm{ds}$ giving rise to the cosmological lens equation (3), there exists a mass distribution $\tilde{\Sigma}(\tilde{\boldsymbol{\xi}})$ for which the single-plane lens equation (11) yields the same observables.

This will be shown by construction. To simplify notation we define the matrices

$$\mathcal{R}_{\rm d} = \frac{\mathcal{D}_{\rm d}}{D_{\rm d}} \quad ; \quad \mathcal{R}_{\rm s} = \frac{\mathcal{D}_{\rm s}}{D_{\rm s}} \quad ; \quad \mathcal{R}_{\rm ds} = \frac{\mathcal{D}_{\rm ds}}{D_{\rm ds}} \quad , \tag{13}$$

which describe the deviation of the light propagation in an inhomogeneous Universe from that in the homogeneous one; i.e., in a homogeneous Universe these three matrices would become the unit matrix. Define the deflection potential

$$\tilde{\Psi}(\tilde{\boldsymbol{\xi}}) := \Psi\left(\mathcal{R}_{\rm d}\tilde{\boldsymbol{\xi}}\right) + \frac{1}{2}\tilde{\boldsymbol{\xi}} \cdot \left(B\tilde{\boldsymbol{\xi}}\right) \quad , \tag{14}$$

where B is the matrix

$$B = \frac{D_{\rm s}}{D_{\rm d}} \left(\mathcal{R}_{\rm d}^{\rm T}\mathcal{R}_{\rm ds}^{-1}\mathcal{R}_{\rm s} - \mathcal{I}\right) \quad ; \tag{15}$$

then the deflection angle becomes, according to (11),

$$\tilde{\boldsymbol{\alpha}}(\tilde{\boldsymbol{\xi}}) = \mathcal{R}_{\rm d}^{\rm T}\boldsymbol{\alpha}(\mathcal{R}_{\rm d}\tilde{\boldsymbol{\xi}}) + B\tilde{\boldsymbol{\xi}} \quad . \tag{16}$$

This is now inserted into the lens equation (11) to yield

$$\tilde{\boldsymbol{\eta}} = D_{\rm s}\boldsymbol{\theta} - D_{\rm ds}\mathcal{R}_{\rm d}^{\rm T}\boldsymbol{\alpha}(\mathcal{D}_{\rm d}\boldsymbol{\theta}) + D_{\rm d}B\boldsymbol{\theta} \quad . \tag{17}$$

If we now define $\tilde{\boldsymbol{\eta}}' \equiv (D_{\rm d}/D_{\rm ds})\tilde{\boldsymbol{\eta}}$, then the lens equation (16) becomes after multiplication with $(D_{\rm d}/D_{\rm ds})$

$$\tilde{\boldsymbol{\eta}}' = C_\theta\boldsymbol{\theta} - \mathcal{D}_{\rm d}^{\rm T}\boldsymbol{\alpha}(\mathcal{D}_{\rm d}\boldsymbol{\theta}) \quad , \tag{18}$$

which is obviously the same equation as (3), except for a linear transformation of the source coordinates. In particular, this single-plane lens yields the same relations (4), (5) and (10) between the image positions, the relative magnification matrices and the time delay function as the cosmological lens mapping. The surface mass density corresponding to the equivalent single-plane lens is obtained from (14) via Poisson's equation. The first term in (14) then yields a contribution to $\tilde{\Sigma}(\tilde{\boldsymbol{\xi}})$ which is obtained from $\Sigma(\boldsymbol{\xi})$ in a non-trivial way, although the corresponding potential contributions are related by a simple linear transformation. The second term in (14) yields a uniform surface mass density. Note that both contributions are not guaranteed to be non-negative.

If we use the angular source position $\tilde{\boldsymbol{\beta}} = \tilde{\boldsymbol{\eta}}/D_{\rm s}$, the lens equation takes the more familiar form

$$\tilde{\boldsymbol{\beta}} = \mathcal{R}_{\rm d}^{\rm T}\mathcal{R}_{\rm ds}^{-1}\mathcal{R}_{\rm s}\boldsymbol{\theta} - \frac{D_{\rm ds}}{D_{\rm s}}\mathcal{R}_{\rm d}^{\rm T}\boldsymbol{\alpha}(\mathcal{D}_{\rm d}\boldsymbol{\theta}) =: \tilde{\Gamma}\boldsymbol{\theta} - \frac{D_{\rm ds}}{D_{\rm s}}\mathcal{R}_{\rm d}^{\rm T}\boldsymbol{\alpha}(\mathcal{D}_{\rm d}\boldsymbol{\theta}) \quad , \tag{19}$$

so that in fact the equivalent single-plane lens consists of the shear matrix $\tilde{\Gamma}$ plus a term corresponding to the 'main lens'. Since $\tilde{\Gamma}$ is proportional to C_θ it is also symmetric.

In general, the trace of $\tilde{\Gamma}$ is different from 2, which means that the shear matrix contains an equivalent surface mass density term (which is not necessarily positive). As was pointed out by Gorenstein, Falco & Shapiro (1988), adding a homogeneous surface mass density to a lens and at the same time rescaling the lens strength leaves all observables invariant except the time delay. This mass-sheet degeneracy is therefore also present in the single-plane lens and is not a novel feature of the cosmological lens equation.

5 Special case: Elliptical isopotentials

We now consider the case that the lens is described by elliptical isopotential curves, so that

$$\Psi(\boldsymbol{\xi}) = F(u) \quad , \tag{20}$$

with

$$u = \xi^2 \left(1 - \epsilon \cos\left[2\left(\varphi - \vartheta\right)\right]\right) \quad ; \tag{21}$$

here, $\boldsymbol{\xi}$ has been expressed in polar coordinates $\boldsymbol{\xi} = \xi(\cos\varphi, \sin\varphi)$, ϵ describes the ellipticity of the potential, and ϑ is the direction of the major axis of the potential. This kind of lens model has been introduced by Blandford & Kochanek (1987) and investigated in detail by several authors (e.g., Kassiola & Kovner 1993). The corresponding mass distribution has no elliptical isodensity curves, and depending on the shape of the radial profile $F(u)$ and the ellipticity ϵ, dumb bell shaped isodensity curves or even negative surface mass densities are obtained from Ψ. Leaving aside these potential difficulties, models of the form (20) have often been used for modeling gravitational lens systems. Whereas the surface mass density is not elliptical, one can still define a major and minor axis, which coincide with the major and minor axes of Ψ.

Expressed in terms of the observable $\boldsymbol{\theta}$, $\Psi(\mathcal{D}_\mathrm{d}\boldsymbol{\theta})$, the observed major axis of the lens does not coincide with ϑ, but is rotated by the matrix $\mathcal{D}_\mathrm{d}$. But the same is true for the equivalent single-plane lens, see (14). Note that the first term in (14) is again an elliptical potential in $\tilde{\boldsymbol{\xi}}$, however with different ellipticity and orientation, and a modified (i.e., stretched) radial profile. However, the matrix which rotates $\tilde{\Psi}(D_\mathrm{d}\boldsymbol{\theta})$ with respect to Ψ is the same as that rotating the intrinsic lens direction ϑ into the observed one. Hence, the observer in the inhomogeneous Universe sees the same orientation of the lens as the corresponding observer in the homogeneous Universe sees of the equivalent single-plane lens. Since the image positions $\boldsymbol{\theta}^i$ are the same in both cases, this implies that the positions of the images relative to the observed orientation of the lens are the same in both cases. In fact, not only is the observed orientation the same in both situations, but the potentials do agree in both cases when expressed in terms of $\boldsymbol{\theta}$! Note that this is not the case for the surface mass densities.

Finally, it can be easily seen that if one considers a family of models where the primary lens has elliptical potential curves and in addition a shear matrix is included, then the cosmological contributions C_θ and $\mathcal{R}_\mathrm{d}$ yield an equivalent single-plane lens which is included within this family.

6 Summary and discussion

In this contribution the effects of large-scale matter inhomogeneities on gravitational lensing by galaxies were considered. The only approximation entering the lens equation (1) in an inhomogeneous Universe is that the length scale of the matter inhomogeneities is much larger than the linear size of a light bundle which encloses all light rays from the source to the observer (i.e., for a typical multiply imaged QSO, this size is of order 20 kpc). In that case, light propagating from the source to the lens and from the lens to the observer can be described by the Jacobi differential equation. Note that for the validity of (1) one does not need to assume that the inhomogeneities are weak – or in other words, that the matrices $\mathcal{R}_\mathrm{d}$, $\mathcal{R}_\mathrm{s}$ and $\mathcal{R}_\mathrm{ds}$ are close to the unit matrix. The symmetry of the matrix C_ξ is essential for showing that for arbitrary propagation matrices $\mathcal{R}_{\rm d}$, $\mathcal{R}_{\rm s}$ and $\mathcal{R}_{\rm ds}$ caused by the large-scale structure, there exists an equivalent single-plane gravitational lens such that all observables are the same – image positions, relative magnification matrices and time-delays. However, in general this equivalent single-plane lens contains a homogeneous matter sheet, not necessarily with positive surface mass density. Therefore, the mass-sheet degeneracy as discussed in Gorenstein, Falco & Shapiro (1988) does apply.

To put these results in the context of the Introduction, a theoretician who wants to model a lens system with observed image positions and fluxes would construct a lens model L_1, corresponding to $\tilde{\Psi}$, since she would probably assume the isotropic angular diameter distances, i.e., ignoring cosmic shear, though the appropriate lens model is L_3, corresponding to Ψ. Both of these lenses yield exactly the same image positions and flux ratios, as well as time delay ratios, and the only difference is the product $H_0\,\Delta t$, which is affected by the equivalent homogeneous surface mass density. But such a homogeneous surface mass density can be added to L_1 anyway and is a principal source of uncertainty for determining the Hubble constant from lensing.

For a single lens plane, this mass-sheet degeneracy can be broken if independent observational information about the lens can be obtained. For example, if the lens is well described by an isothermal mass distribution, the measurement of the velocity dispersion determines the lens strength and thus fixes the mass-sheet transformation. If this is done, the time delay then depends on the matrix $\mathcal{R}_{\rm d}$, and thus matter inhomogeneities between us and the lens affect the determination of the Hubble constant. This fact was noted by Narayan (1991) who showed that the measurement of the time delay determines the angular diameter distance to the lens, which is affected by $\mathcal{R}_{\rm d}$, in agreement with B-K96, where the size of this effect has been calculated using the non-linear evolution of the power spectrum. Thus, lensing by the large-scale structure does affect the determination of the Hubble constant, although the magnitude of the effect (i.e., the rms deviation of $\mathcal{R}_{\rm d}$ from the unit matrix) is of the order of a few percent; see Fig. 2 of B-K96. Therefore, in agreement with the conclusion of Surpi, Harari & Frieman (1996), cosmic shear does not seriously compromise the lensing method for the determination of H_0.

Whereas the mass distribution of the equivalent single-plane lens depends non-trivially on the lens mass distribution and the propagation matrices $\mathcal{R}$, it has been shown that if the lens is chosen to have elliptical isopotentials then the equivalent single-plane lens consists of a main component with elliptical isopotentials plus a shear matrix. Furthermore, if a family of lens models is considered, consisting of an elliptical isopotential component and a shear matrix, then the equivalent single-plane lens is also within this family. For this case it was shown that the image positions relative to the orientation of the mass distribution as seen by an observer is the same in the inhomogeneous Universe and in the single-plane lensing situation. Note that lens models consisting of an elliptical lens (either elliptical isodensity contour or elliptical isopotentials) plus external shear seems to be required for those lens systems in which there is a large number of observational constraints (e.g., Keeton, Kochanek & Seljak 1996, Witt & Mao 1997).

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