Measuring the Hubble constant with lens time delays in an inhomogeneous universe

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Abstract

The effects of a locally inhomogeneous universe on the uncertainty of the Hubble constant as determined from measured time delays in gravitational lens systems is discussed. The effect has been described adequately in the literature, but it is usually not taken into account when discussing measurements of H_0 using gravitational lens time delays. Depending on the cosmological model and the redshifts of the particular lens system considered, the effect of local inhomogeneity can significantly increase the uncertainty in the determination of H_0, and in 'probable' cosmological models can be the dominant uncertainty.

a. Introduction

The idea of measuring the Hubble constant H_0 using the time delay between images of a source which is multiply imaged due to the gravitational lens effect was introduced by Refsdal (1964), who also discussed the higher-order dependence on the other main cosmological parameters, in modern notation the cosmological constant λ_0 and the density parameter Ω_0 (Refsdal 1966). In particular, Refsdal (1966) introduced the 'cosmological correction function' T which describes these higher-order effects. Kayser & Refsdal (1983) showed that this same formalism also applies in the case of an arbitrary lens mass distribution and in the extreme case of a locally inhomogeneous universe, the so-called empty-cone approximation. Since the cosmological correction function depends only on the redshifts of the lens and source and on the distances involved, it is straightforward to generalise even further, using the formalism and methods set out in Kayser *et al.* (1997), to cases intermediate between the traditional approach (which assumes an idealised universe consisting of a perfect fluid) and the empty-cone approximation.

Recently, not only has the general idea of measuring H_0 by lens time delays become more acceptable, but (partly the cause of this) other uncertainties, such as measuring (and interpreting!) the time delay itself (see Pelt *et al.* (1996) and references therein) and modelling the lens mass distribution have become better understood, so that now the dominant uncertainties are cosmological—the values of λ_0 and Ω_0 and the parameter η discussed below, which describes local inhomogeneity.

b. Basic theory

i. Time delay

One can write an expression for the time delay (*cf.* Kayser & Refsdal (1983))

$$H_0 = (\Delta t)^{-1} T f \qquad (1)$$

where H_0 is the Hubble constant, Δt the time delay, T the cosmological correction function and f is a function of observational quantities and the mass distribution of the lens and will not be further discussed here. The cosmological correction function

$$T = \frac{H_0}{c}\frac{D_\mathrm{d} D_\mathrm{s}}{D_\mathrm{ds}}(1 + z_D)\frac{z_\mathrm{s} - z_\mathrm{d}}{z_\mathrm{d} z_\mathrm{s}} \qquad (2)$$

is defined so that $T \rightarrow 0$ for $z_\mathrm{s} \rightarrow 0$.

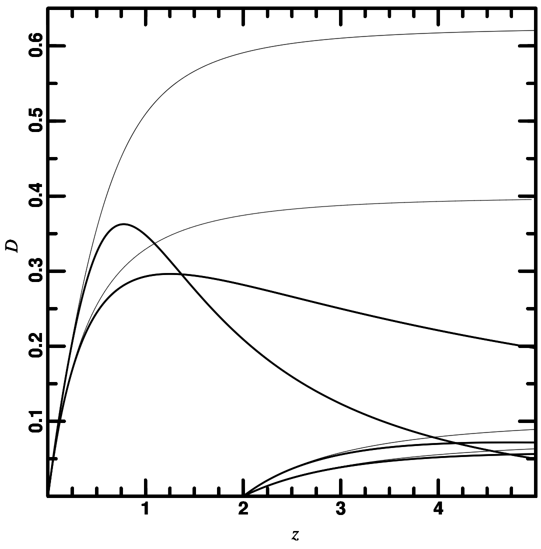

Figure 1 Dependence of the angular size distance D on λ_0 and η
The angular size distance from the observer and from an object at $z = 2$ to another at higher redshift as a function of the redshift z for different cosmological models. Thin curves are for $\eta = 0$, thick for $\eta = 1$. The upper curves near $z = 0$ ($z = 2$ at lower right) are for $\lambda_0 = 2$, the lower for $\lambda_0 = 0$. $\Omega_0 = 1$ for all curves. The distances are given in units of c/H_0.

ii. Cosmological distances and the effects of a locally inhomogeneous universe

See, *e. g.*, Kayser *et al.* (1997) for an overview of cosmological distances and for a method of taking inhomogeneities into account when calculating cosmological distances. Figures 1–3 show the dependence of the angular size distance (the relevant distance for gravitational lensing) on the cosmological model and on the inhomogeneity parameter η, which is the fraction of smoothly distributed matter within the light cone which determines the distance; $1 - \eta$ is the fraction of the matter distributed clumpily. Here it is assumed that all clumps are outside the light cone ('clumps' inside having been taken into account explicitly as a gravitational lens effect) and far enough away so that the effects of shear can be ignored.

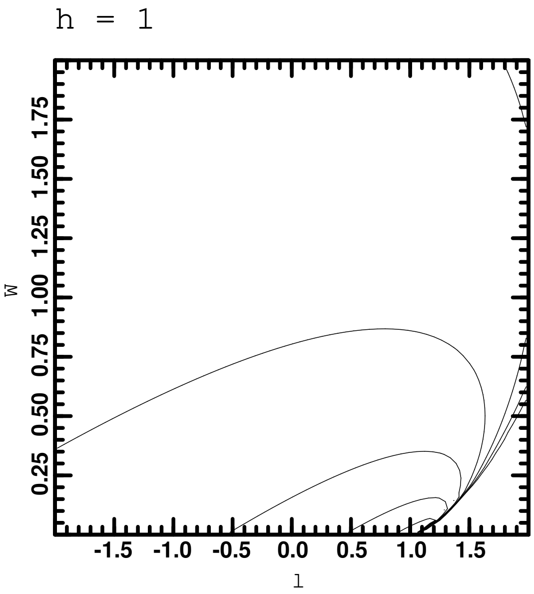

Figure 2 Dependence of D on λ_0 and Ω_0
For $\eta = 1$ $D(\lambda_0, \Omega_0)$ is plotted. The source redshift is $z = 2$. Starting from $(\lambda_0, \Omega_0) = (1, 0)$ and spiraling clockwise, contours are at $0.6, 0.5, 0.4, 0.3, 0.2, 0.1, b$ where b separates the cosmological models with and without a big bang (in the latter the distance is not defined for $z = 2$).

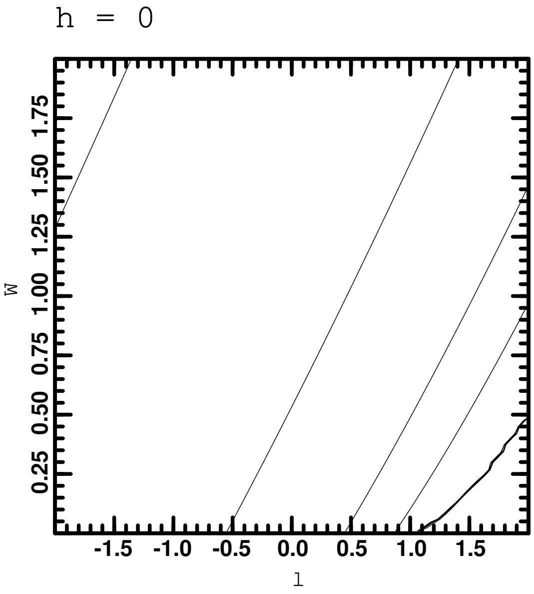

Figure 3 Dependence of D on λ_0 and Ω_0
The same as Fig. 2 but for $\eta = 0$. From upper left to lower right, contours are at $0.3, 0.4, 0.5, 0.6, b$.

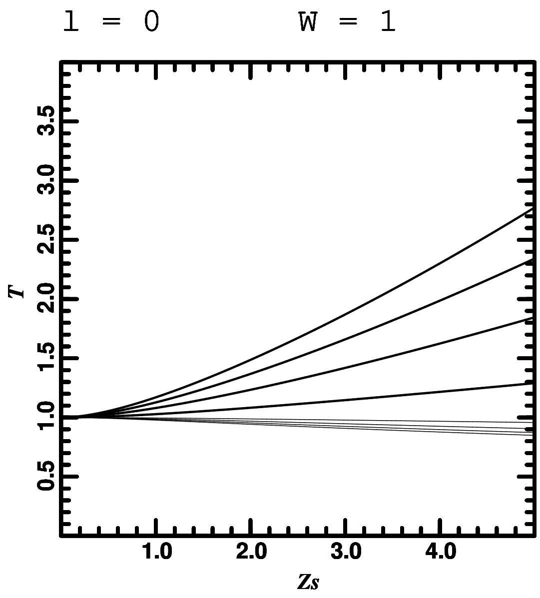

Figure 4 Dependence of T on D_{s}, D_{ds} and η

For a fixed cosmological model ($\lambda_0 = 0$ and $\Omega_0 = 1$, as indicated) $T(z_s)$ is plotted. Thin curves correspond to $\eta = 1$, thick to $\eta = 0$. From top to bottom, $z_d/z_s = 0.7, 0.5, 0.3, 0.1, 0.1, 0.3, 0.5, 0.7$.

c. The cosmological correction function

Figures 4–15 show the dependence of T on the cosmological model. The parameter space examined *roughly* corresponds to cosmological models which cannot be ruled out observationally. Thus, the spread of T gives an idea of the uncertainty in H_0 when determined from a measured time delay, in addition to any uncertainties in (the interpretation of) the measurement itself and the lens model. Alternatively, if H_0 and the lens models are well-constrained by other means, each lens system with a measured time delay provides an independent constraint on λ_0 and Ω_0. The dependence of T on the cosmological parameters comes solely from the influence of the latter on the angular size distances. Since $\eta = 0$ is an extreme case, one could then rule out world models above a contour line such as in Fig. 11; this is interesting since the direction of these contours is such that a degeneracy present in many other cosmological tests—lensing statistics, m-z relation, age of the universe—can be broken.

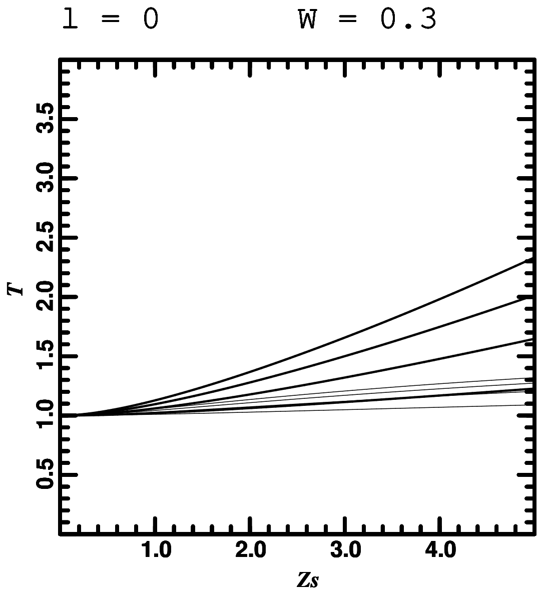

Figure 5 Dependence of T on D_{s}, D_{ds} and η

The same as Fig. 4 but for a different values of λ_0 and Ω_0.

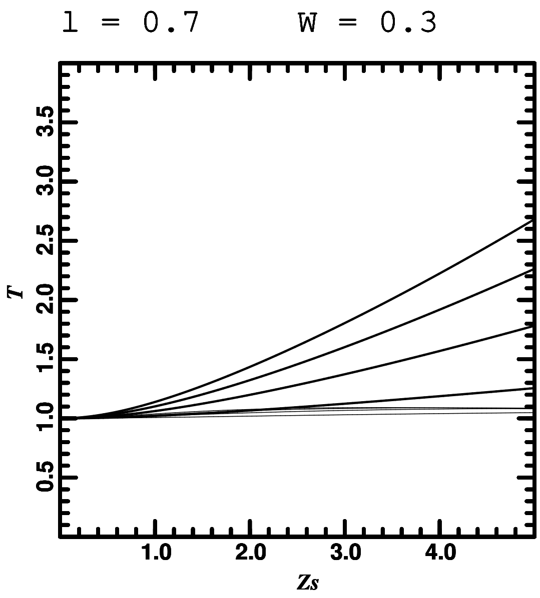

Figure 6 Dependence of T on D_{s}, D_{ds} and η

The same as Fig. 4 but for a different values of λ_0 and Ω_0.

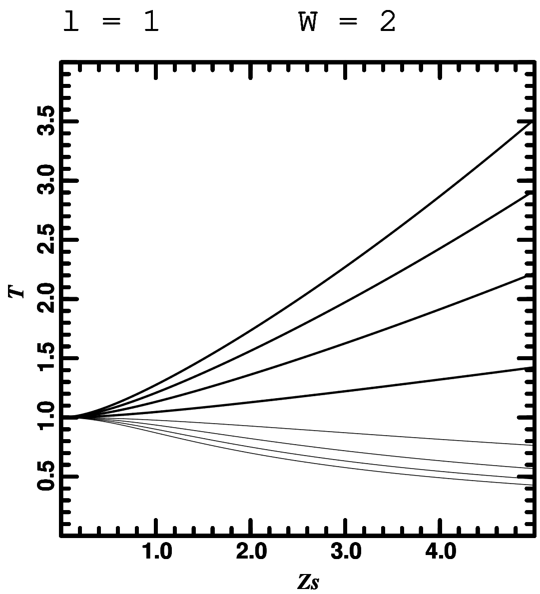

Figure 7 Dependence of T on $D_{\rm s}$, $D_{\rm ds}$ and η
The same as Fig. 4 but for a different values of λ_0 and Ω_0.

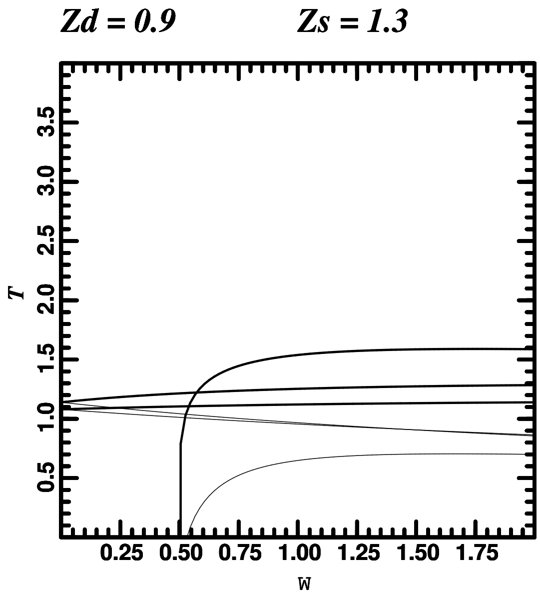

Figure 8 Dependence of T on λ_0, Ω_0 and η
For fixed source and lens redshifts ($z_{\rm d} = 0.9$ and $z_{\rm s} = 1.3$) $T(\Omega_0)$ is plotted. As in Fig. 4, thin curves correspond to $\eta = 1$, thick to $\eta = 0$. The curves for which $T < 0$ for $\Omega_0 < 0.5$ are for $\lambda_0 = 2$; in this case lower values of Ω_0 correspond to the so-called bounce models (see, *e.g.*, Kayser *et al.* (1997)). For the other curves, from top to bottom $\lambda_0 = 1.0, 0.0, 0.0, 1.0$.

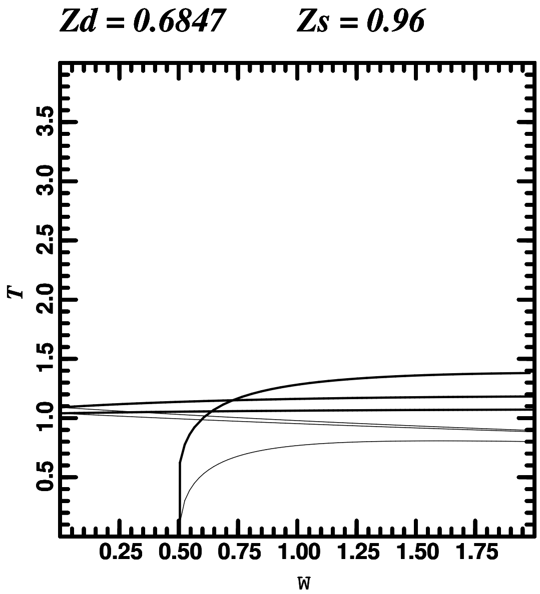

Figure 9 Dependence of T on λ_0, Ω_0 and η
The same as Fig. 8 but with $z_{\rm d}$ and $z_{\rm s}$ equal to the values in the gravitational lens system $0218 + 357$.

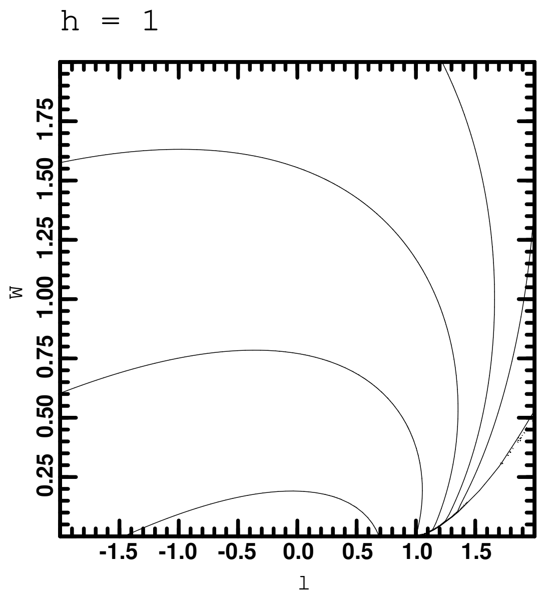

Figure 10 Dependence of T on λ_0 and Ω_0
For fixed source and lens redshifts ($z_{\rm s} = 1.3$ and $z_{\rm d} = 0.9$) $T(\lambda_0, \Omega_0)$ is plotted, here for the case of $\eta = 1$. From $(\lambda_0, \Omega_0) = (0, 0)$ spiraling clockwise, contours are at $1.1, 1.0, 0.9, 0.8, 0.7, b$.

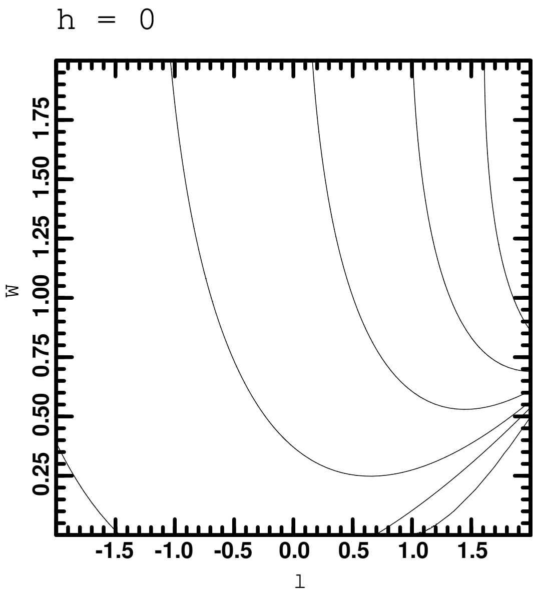

Figure 11 Dependence of T on λ_0 and Ω_0 The same as Fig. 10 but for $\eta = 0$. From lower left to upper right, contours are at $1.1, 1.2, 1.3, 1.4, 1.5$. The contour at lower right is b, the one next to it 1.1.

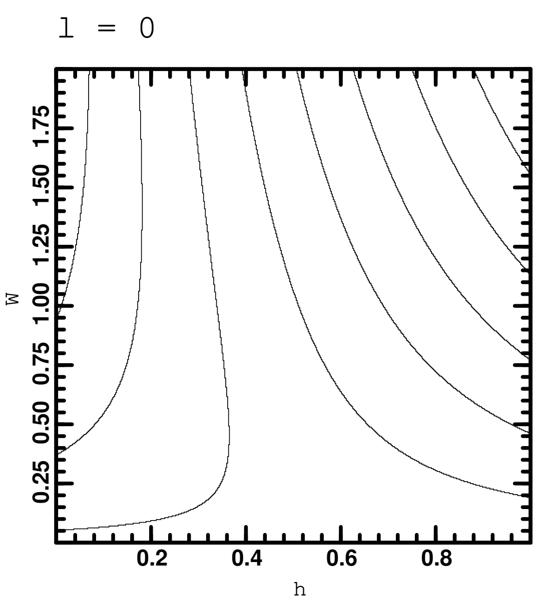

Figure 12 Dependence of T on η and Ω_0 For fixed source and lens redshifts ($z_{\rm s} = 1.3$ and $z_{\rm d} = 0.9$) $T(\eta, \Omega_0)$ is plotted, here for the case of $\lambda_0 = 0$. From left to right, contours are at $1.25, 1.20, 1.15, 1.10, 1.05, 1.00, 0.95, 0.90$.

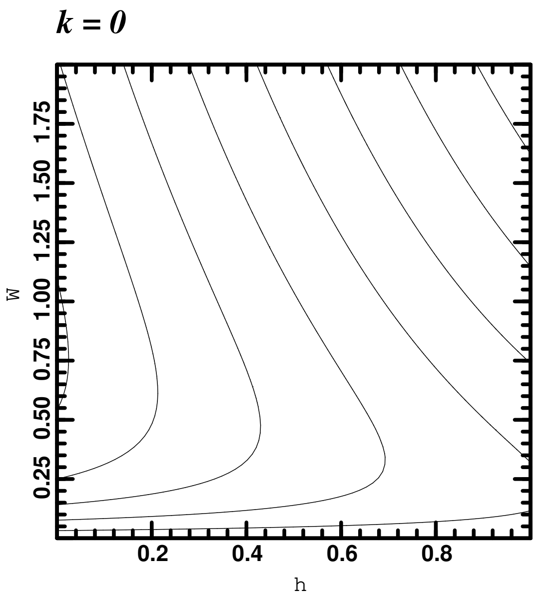

Figure 13 Dependence of T on η and Ω_0 The same as Fig. 12 but for $k = 0$. Contours as in Fig. 12.

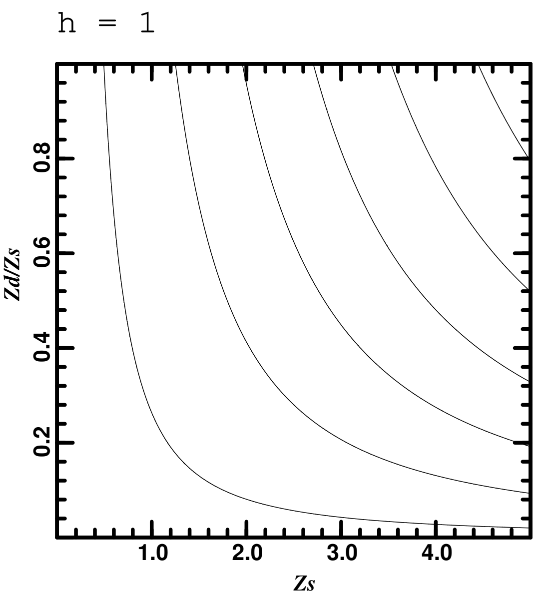

Figure 14 Dependence of T on $z_{\rm s}$ and $z_{\rm d}$ For a fixed cosmological model, ($\lambda_0 = 0$ and $\Omega_0 = 1$) $T(z_{\rm d}, z_{\rm s})$ is plotted, here for $\eta = 1$. From lower left to upper right, contours are at $0.99, 0.96, 0.93, 0.90, 0.87, 0.84$.

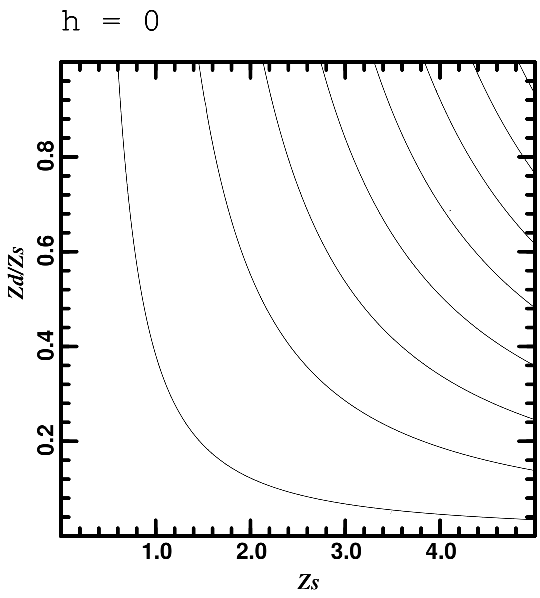

Figure 15 Dependence of T on z_s and z_d The same as Fig. 12 but for $\eta = 0$. From lower left to upper right, contours are at $1.1, 1.4, 1.7, 2.0, 2.3, 2.6, 2.9, 3.2$.

d. Summary and conclusions

The uncertainty due to cosmological considerations, parametrised by the cosmological correction function T, in the value of H_0 as derived from a measured time delay generally behaves as follows when the other parameters are held constant:[1]

- $|T|$ increases with increasing z_d
- $|T|$ increases with decreasing η
- T increases with z_s for $\eta = 1$ and decreases for $\eta = 0$
- $|T|$ increases with increasing Ω_0
- $|T|$ increases with increasing λ_0 except when Ω_0 is small

in order of generally decreasing importance. Thus, if one is interested in minimising this uncertainty, one should measure the time delay preferentially in systems where $z_\mathrm{d}/z_\mathrm{s}$ is relatively low and, less important, where z_s itself is small. Should η prove to be ≈ 1 then the need for small source and (relatively) small lens redshifts is less urgent, and the dependence on λ_0 would be made even smaller than it already generally is. Similarly, a small value for Ω_0 would decrease the uncertainties due to η and λ_0. Of course, if one knows H_0 already, then the criteria for desirable source and lens redshifts and for desirable values of the other cosmological parameters are reversed, since then one could use the observations to constrain λ_0 and Ω_0.

It is interesting to contrast the dependency of T on the cosmological parameters λ_0, Ω_0 and η with that of the statistics of multiply-imaged systems in surveys (see, *e. g.*, Fukugita *et al.* (1992)): the order of decreasing importance in the latter case is λ_0, Ω_0 and η, just the opposite as for the case of T considered here.

[1] See also Kayser & Refsdal (1983)

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