GRAVITATIONAL LENSING AND RECENT CONTRIBUTIONS FROM RADIO STUDIES

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1. INTRODUCTION

1.1 Preamble

The year 1979 saw the discovery of the first example of "gravitational lensing" in the form of multiple imaging of a background quasar by a foreground galaxy (or group of galaxies), and the fulfilment of an implicit prediction of Einstein's general theory of relativity over sixty years before. Before 1970, the subject produced a few papers per year in the astronomical literature. Since then, the literature has grown considerably with the discovery and interpretation of many more lens systems. It is difficult to tell how much effect the discovery had, as the beginning of the rise depends on whether the number of papers is plotted on a linear *[Refsdal, 1993]* or log scale *[Press, 1996]*.

Lens systems are a uniquely powerful astrophysical tool in that they allow the masses and mass distributions of cosmologically distant objects to be determined independent of the light they emit. They are thus crucial in constraining "dark matter" on scales from stellar masses to the largest scales in the universe. Moreover, lensing observations have brought within our grasp some of the great prizes of cosmology. These include the determination of the Hubble constant, which relates distance to redshift and fixes the age and scale of the universe, and of the cosmological constant which, if it exists, causes the acceleration of the expansion of the universe. The cosmological constant corresponds to a universal vacuum energy, and confirmation of its existence would virtually guarantee the ultimate fate of the universe as an everlasting dimming and expansion. In all lens studies, from the first gravitational lens discovery onwards, radio observations have played a vital role due to their routinely high resolution and to the ability of radio waves to pass unobscured through dusty regions of the universe which have extremely high optical depths to shorter-wavelength visible light.

The standard reference work for gravitational lensing studies is the monograph by *Schneider, Ehlers & Falco [1992]* which contains a comprehensive description of lensing theory and a complete summary of the observational status as of 1992. Other major reviews have been written by *Blandford & Narayan [1992]* on the cosmological implications of lensing, by *Refsdal & Surdej [1994]* on the theory of lensing and by *Paczynski [1996]* on lensing by stars. More recent reviews include *Narayan [1998]*, on general astrophysical results from gravitational lensing, and *Schechter [2001]* who gives a sceptical review of Hubble constant determinations from lensing studies. In addition, IAU Symposium 173 (1996, eds. *Kochanek & Hewitt*) contains a comprehensive range of papers on all aspects of lensing.

The aim of this review is to give an overview of current developments in the subject, and in particular of lensing by galaxies and what radio observations can contribute. The review starts with a concise introduction to lensing (Section 1) and a description of lens searches to date (Section

2). In Section 3 we describe the mass models used to characterise lens systems, concentrating on the galaxy-mass lensing systems most accessible to radio interferometers and in particular to developments in the last three years. In Section 4 we describe the efforts made to derive cosmological parameters from lenses, in particular from radio-selected gravitational lenses which we argue give the cleanest samples. Section 5 is a review of some new fields which are currently being opened up by radio observations of microlensing and of the Faraday depths of lensing galaxies. Finally, in Section 6 we describe the future prospects for lensing studies using the new generation of instruments – in particular the Atacama Large Millimetre Array (ALMA) and the proposed Square Kilometre Array (SKA). Throughout we concentrate on the major developments in lensing since 1998, when the last major reviews were written.

1.2 Basic geometry of lensing

If a ray of light passes close to a point mass M, it is deflected by the gravitational field of the mass through a small angle

$$\alpha = 4GM/bc^2$$

where b is the impact parameter. This is a standard result of general relativity, and also one of the first to be tested. Eddington in 1919, during an expedition to a total solar eclipse, measured the predicted small gravitational deflection of light (of about $2''$) from stars passing close to the line of sight to the limb of the sun during the eclipse.

Measuring very small deflections is not easy. A light ray passing through the solar system at an impact parameter similar to the radius of the Earth's orbit will be deflected by only $0\overset{''}{.}01$ by the Sun's gravitational field. Whole galaxies often produce more significant deflections; the same light ray would be deflected by about $1''$ by the Galactic gravitational field during its whole passage through the plane of our Galaxy.

For any given system in which a background light source is subjected to gravitational deflection by a foreground massive object, we can work out the appearance of the background source to the observer by use of Fermat's principle: the path followed by a ray of light must be an extremum (i.e. a maximum, minimum or saddle point). In the case where no lens is present, the problem is of course very simple; the path actually followed is a Fermat minimum, corresponding to a straight line. If a lensing mass is introduced, however, a compromise must be made between a straight path, which involves an extra time delay due to the traversal of a deep gravitational potential well (the "Shapiro time delay"), and a path deviating considerably from a straight line, which incurs a time penalty corresponding to the extra path length. In between these extremes there will be a path corresponding to a Fermat minimum, and an image of the background source will therefore be seen, but offset from the line of sight that would be obtained if no lens were present. In the completely symmetric case, where lens and source are precisely aligned, a ring image will be seen – known as an "Einstein ring" in honour of Einstein's general-relativistic prediction of its radius for a given mass of deflector. In the more general case, there will be multiple Fermat extrema in the field of the lensed system, and multiple images will be produced.

The radius of the Einstein ring can be calculated using simple theory. Consider the diagram shown in Figure 1. If we write θ for the apparent separation of the lens and lensed image, D_l and D_s for the distances[1] to the lens and source, respectively, and D_{ls} for the separation of lens and source, simple geometry shows that

[1]To be precise, these are what is known in cosmology as "angular size distances", defined such that the angle θ subtended by unit length within an object is given by $\theta = 1/D$. They are different by a factor of $(1+z)^2$, where z is the redshift, from the "luminosity distance" which relates absolute and apparent brightness. Consult any cosmology text, for example *Peacock [2000]* for further details.

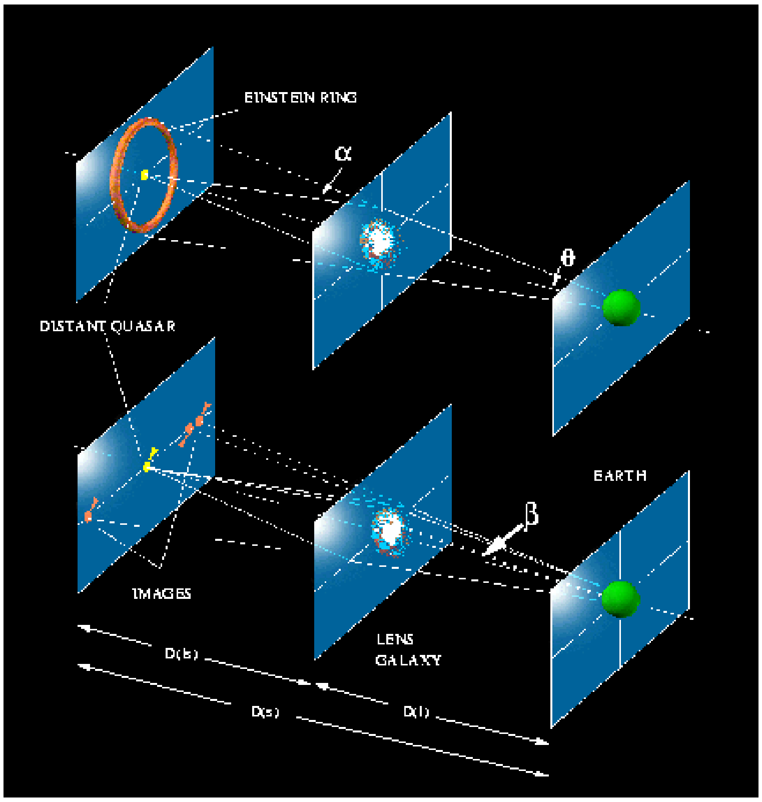

Figure 1: The formation of gravitationally lensed images. At the top is shown the formation of an Einstein ring of a distant quasar by a foreground galaxy directly along the line of sight. The light rays are deflected by an angle α, and the apparent separation of the galaxy and lensed images from the observer's point of view is θ. The bottom panel shows the formation of images from a quasar by a galaxy which is offset by a small angle β from the line of sight to the quasar. The picture also gives the definitions of the distances D_l, D_s and D_{ls} discussed in the text. [A colour version of this figure appears in the CD-ROM version of this chapter].

$$\theta - \beta = \alpha D_{ls}/D_s$$

(the so-called *lens equation*), where β is the angle on the sky, in the absence of lensing, separating the lines of sight to the source and lens. For an Einstein ring to be seen, $\beta = 0$ and, since $b = \theta D_l$, we can combine the two equations with Einstein's original equation for the deflection angle in terms of the impact parameter b to obtain

$$\theta = \sqrt{\frac{4GMD_{ls}}{c^2 D_l D_s}}.$$

For typical galaxy masses, $M \sim 10^{11}$–$10^{12} M_\odot$, at cosmological distances, one obtains Einstein radii of about $1''$. For typical stellar masses, Einstein radii are of the order of a few microarcseconds at cosmological distances, or about a milliarcsecond at distances of $\sim 1\,\mathrm{kpc}$ within our galaxy; with current technology, such small splittings are difficult to observe in the optical, although such "microlensing" events are recognisable by increases in magnification. However, most of this review will concentrate on arcsecond-scale lensing by galaxies, as this is the area in which most progress has been made by radio techniques, though radio observations are just beginning to be made of stellar-scale "microlensing" events and will be discussed in some detail in section 5. Lensing on larger scales, by clusters of galaxies, is also possible. This gives information on mass distributions in clusters of galaxies and large-scale structure We mention this only briefly, in section 6.2 on the SKA, but an extensive review is given by *Mellier [1999]*.

1.3 Observables in real systems

In order to get an idea of what might actually be observed, we make some plausible assumptions. First, real lensing galaxies normally lie at redshifts of $\sim$0.5. At this distance, most of a typical galaxy's stellar light subtends an angle of about $1''$ or less. Therefore measuring the light distribution requires the Hubble Space Telescope or at least adaptive optics on ground-based telescopes. But what really matters in gravitational lensing studies is the *mass* distribution in the lensing galaxy and here our lack of knowledge becomes embarrassing. It has been known for some time that galaxies contain a large amount of "dark matter" over and above that which can be accounted for by normal luminous stars (see *Rubin [2000]* for a recent review). There is evidence for dark matter existing on many scales, from within galaxies themselves, in which dynamical studies indicate much more mass than is observed as luminous mass, to clusters, where the dynamics of the galaxies within the cluster are dominated by dark matter. How this mass is distributed is not clear, although recent simulations of galaxy formation have begun to illuminate this area (e.g. *Navarro, Frenk & White [1996]*).

Knowledge of the mass distribution is required for gravitational lensing studies because the lens equation (which relates θ and α) simply states the geometrical conditions for images to be seen. In order to make further progress with physical modelling we need a relation between θ and α peculiar to the mass distribution we are considering. Einstein's original relation does this for a point mass, but this is not a particularly realistic representation of a galaxy! In section 3 we discuss the impact on lensing studies of galaxy mass models; here we examine the general properties of such models.

In Figure 2 the lens equation (the dotted straight line) and the variation of α with θ for different galaxy mass models are plotted. Lensed images occur only when both equations are satisfied simultaneously, and it can be seen that three images may be produced, provided that β is small enough. This latter condition of course corresponds to the lines of sight to lens and source being close enough to each other. Another condition is also necessary; the slope of the α — θ relation must be sufficiently high in the central region to allow three intercepts rather than one. Since the deflection angle α is related to the projected gravitational potential ψ by the equation

$$\alpha = \nabla\psi$$

this introduces the additional requirement that the potential well should be sufficiently deep, steep or both to allow multiple-image lensing to occur. It can be shown that this condition corresponds to having a surface mass density exceeding a critical value (Σ_{crit}, which at cosmological distances is about 1 gram cm^{-2}). Objects with less than this critical density may distort and magnify background objects, but they will not generally produce multiple images. In practice, massive galaxies often possess central mass densities such that $\Sigma > \Sigma_{crit}$ by factors of a few, and stars by factors of many thousands. Clusters of galaxies can fall below this surface mass density limit, although many do produce multiple images.

There are situations in which more than three images of the background object will be seen. First, the above analysis has been done in one dimension (see e.g. Figure 2). Proper analysis in two dimensions shows that for small enough impact parameters and non-circularly-symmetric potentials, five images, not three, will be produced (the reader is referred to standard texts such as *Schneider, Ehlers & Falco [1992]* for the full details; see also Fig. 3). Secondly, it is a standard

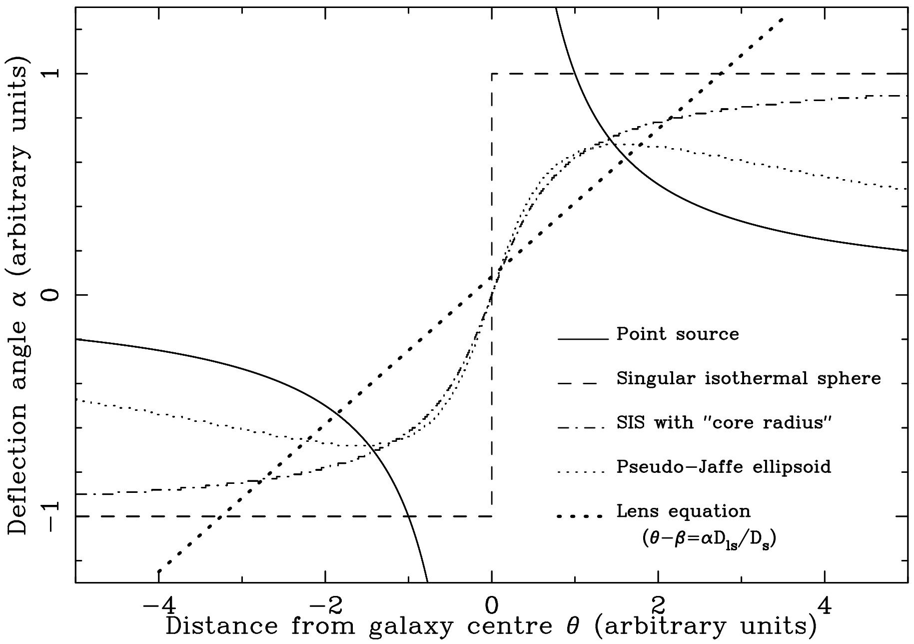

Figure 2: "Bend angle diagram" for a gravitational lens. This shows the deflection suffered by a ray of light as a function of impact parameter from the centre of the lensing galaxy. For a point mass, this is given by $1/b$. Three other, more plausible, mass distributions in common use for lens modelling are also plotted, including a variant of the *Jaffe [1983]* model. In addition, a straight line corresponding to the Lens Equation is also plotted. For any mass distribution, images will be formed when the bend-angle line crosses the lens equation line. *Keeton [2001]* summarises many other mass distributions.

result of geometrical optics that lenses, including gravitational lenses, produce distortion (changing areas and shapes) of background objects during the imaging process yet always conserve surface brightness. The production of larger or smaller images with constant surface brightness therefore corresponds to production of images which have larger or smaller flux densities than the original image. Most of the images in a gravitational lens are magnified by the lensing, by factors of up to 10 in typical galaxies. However, the central image, which is always produced near the centre of the lens, is demagnified, usually by factors of a few hundred, to the point where it falls below detection thresholds. This results in typical galaxy-mass lens systems having either two or four detectable images[2], depending on the mass of the lensing galaxy and the size of the impact parameter.

In Figure 3 we illustrate the configuration of images formed as a source approaches the line of sight to a lensing galaxy. Far from the line of sight, only one image is formed although it is in general slightly magnified and distorted. As the source crosses the outer caustic, a second faint image appears close to the lensing galaxy, corresponding to the appearance of an extremum in the Fermat surface of the system. As the line of sight moves closer, the source crosses the inner caustic. At this point another extremum appears, which splits into two producing two highly magnified, distinct images which gradually separate as the source moves closer in. This four-image configuration, with two close, bright images and two more distant images, is characteristic of lensing systems. To search for gravitational lenses one therefore looks for the characteristic two- or four-image configurations within any sample of interest.

2. LENS SEARCHES

[2]In fact, more than four images can in principle be generated for complicated deflectors such as multiple galaxies. *Keeton, Mao & Witt [2000]* discuss such systems, and one has actually been found *[Rusin et al., 2000*; *Rusin et al., 2001]*

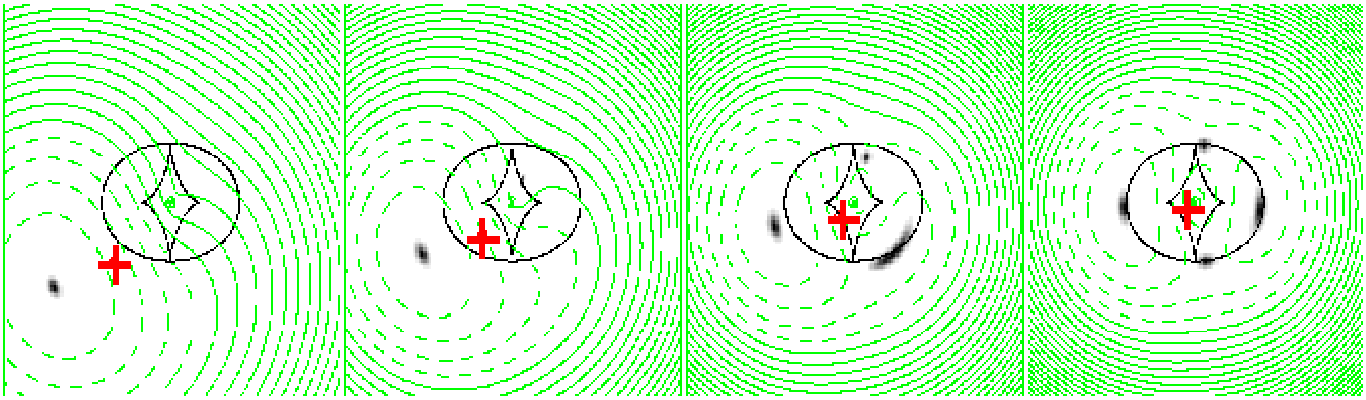

Figure 3: This figure shows the formation of images by a lensing galaxy as the source, of which the undeflected position is indicated by a cross, moves progressively closer to the line of sight. At large distances (left panel) the image of the source, shown in greyscale, is distorted but not multiply imaged. The green contours show the Fermat surfaces, which are loci of points requiring equal travel times for light rays that pass through them, and it can be seen that a single image forms at the extremum. The inner (diamond-shaped) and outer (elliptical) lens caustics are also shown in all four panels. As the source crosses the outer caustic of the lens (panel 2) the Fermat surfaces pucker and a second image is formed. As the source crosses the inner caustic (panel 3) a new double image is formed, which separates to a more symmetrical four-image structure (right panel) as the source and the galaxy become more aligned. An odd image forms near the centre of the galaxy, but does not appear in this figure due to strong demagnification. [A colour version of this figure appears in the CD-ROM version of this chapter].

2.1 Serendipity: the first lens system

The first gravitational lens system was discovered in 1979 by a combination of systematic observing technique and good fortune. In 1972/73 the University of Manchester's Mk1A (now Lovell) 250-foot radio telescope at Jodrell Bank was used to conduct a radio source survey at a frequency of 966 MHz of a strip of northern sky. The Jodrell Bank group were particularly interested in quasars, which are now known to involve violent ejection of synchrotron-emitting radio plasma from a small region ($\sim$1 parsec) surrounding a large central black hole in the centre of a distant galaxy. That these regions could be very small had been demonstrated by interferometric observations in the 1960s, using single-baseline interferometers involving the MkI telescope at Jodrell Bank together with other smaller telescopes at successively greater distances. VLBI in the 1970s showed that these regions could be milliarcseconds in size.

It was necessary to improve on the accuracy of the survey radio positions in order to make the optical identifications required to separate the quasars from other objects. An interferometer consisting of the Jodrell Bank Mk1A and Mk2 ($\sim$25 m) telescopes was used for this purpose giving positions accurate to $\sim 1''$ for compact objects. There were some fields, however, for which the interferometer did not give unambiguous results; either the target was very extended and resolved by the interferometer or there was more than one radio source in the telescope beam. Such fields were reobserved with the NRAO 300-foot radio telescope operating at a frequency of 5 GHz. One of the fields with multiple sources in the MkIA beam was close to the galaxy NGC3079. *Walsh [1989]* describes what happened next; the NRAO 300-foot radio observations *[Porcas et al., 1980]* found a weak source, 0957+561, below the survey limit in addition to (and initially, instead of) the strong source NGC3079 with which it had been confused. 0957+561 would never have made it into the survey had it not been for this coincidence. When optical identifications were undertaken, and optical spectroscopy obtained to determine the redshift, the radio source was found to correspond to two optical components, about 6 arcseconds apart. It was in principle possible that this represented a chance coincidence of two quasars. However, the optical spectra of the two quasars were virtually identical, a fact which argued strongly against the coincidence hypothesis – resulting in the first claim *[Walsh, Carswell & Weymann 1979]* of the observation of a multiple image gravitational lens system. Confirmation in the form of the double nature of the radio source *[Pooley et al. 1979]* and the presence of a lensing galaxy *[Young et al. 1980]* soon followed, and the observational gravitational lensing era was born.

2.2 Systematic searches

Systematic searches for lenses soon followed. Many of these have relied on the same brute-force principle — namely, the imaging of many background sources in order to pick out the small fraction (now known to be about 1:500) which have lensing galaxies sufficiently close to the line of sight to produce the multiple imaging characteristic of a lens system. A full catalogue of known galaxy-mass lenses and a gallery of Hubble Space Telescope images is given on the CASTLeS (CFA-Arizona Space Telescope Lens Survey) website *[Kochanek et al., 2001]*.

The early history of lens searches is described in detail in chapter 2 of *Schneider, Ehlers & Falco [1992]*. The first major survey, the MIT-Greenbank (MG) Survey, was carried out at radio wavelengths by the group based at MIT [*Bennett et al., 1986; Lawrence et al., 1986; Hewitt et al., 1988]* and consisted of high-resolution observations with the VLA[3] of sources picked up with lower resolution using the 300-foot Green Bank radio telescope. Five lenses were eventually discovered by this survey, some including Einstein ring images which arise when parts of an extended radio source lie exactly behind the lensing galaxy. *Lehár et al. [2001]* describe a more efficient new survey which examines extended radio structures coincident with optical identifications of extended galaxies which may be lensing them[4].

Early optical searches [*Crampton, McLure & Fletcher, 1992; Surdej et al., 1993; Jaunsen et al., 1995; Kochanek, Falco & Schild, 1995]* discovered a few lenses between them, although the surveys were limited to the resolution available to ground-based optical telescopes. This resolution is typically about $1''$, comparable with the Einstein radius of galaxy-mass gravitational lens systems which encloses only the central region of a typical galaxy. This resolution is barely adequate for detection of the majority of systems. With the coming of the HST, routine high-resolution ($0\farcs1$) optical imaging capable of detecting lensed images with sub-arcsecond separations was also possible. A "snapshot" survey was instigated *[Bahcall et al., 1992; Maoz et al., 1992]* which with its successors has produced 12 lenses to date. Many authors have pointed out *[Mortlock & Webster, 2000; Richards et al., 1999]* that future optical spectroscopic programmes such as the Sloan Digital Sky Survey *[York et al., 2000]* will discover many more gravitationally lensed quasars.

The most extensive lens survey has been the JVAS/CLASS (Jodrell Bank-VLA Astrometric Survey *[King et al., 1999]*/ Cosmic Lens All-Sky Survey *[Jackson et al., 1995; Myers et al., 2001]* based at Jodrell Bank Observatory, University of Manchester, UK and involving NFRA, Dwingeloo and the University of Groningen (Netherlands) and Caltech, NRAO and the University of Pennsylvania (USA). This radio survey has discovered 18 lenses to July 2001 (Fig. 4), and has contributed the largest homogeneous lens sample of all. It differs from earlier radio surveys in that it concentrates on compact, flat-spectrum radio sources rather than the (typically) extended, steep-spectrum objects studied in the MG survey. Although Einstein rings are therefore less com-

[3] Very Large Array, an array of 27 radio telescopes in New Mexico, and part of the U.S. National Radio Astronomy Observatory

[4] When the source size becomes comparable to the Einstein radius the lensing probability depends both on the lens and source properties.

mon, the JVAS/CLASS methodology makes lenses much easier to recognise reliably – which has important advantages for cosmological investigations (section 4).

JVAS/CLASS selects flat-spectrum sources from existing low-resolution radio catalogues of the northern sky, which are then observed at $0\farcs2$ resolution with the VLA. Most flat-spectrum radio sources are the synchrotron-self-absorbed cores of active galaxies, and are intrinsically very small ($\leq$1 mas). This means that any source in which the VLA reveals structure on arcsecond scales becomes a candidate gravitational lens unless its structure is inconsistent with the lensing hypothesis (usually because the secondary component is resolved). 97% of objects are rejected at this stage. Still higher resolution observations using the longer-baseline MERLIN array[5] must, however, be undertaken for the remaining 3% in order to distinguish structure characteristic of gravitational lensing from "normal" source structure, for example collimated radio jets ejected from the compact core. This procedure rejects 80% of the remaining lens candidates *[King et al., 1999]*, leaving only a few targets for the final high-resolution radio observation to be done with very long baseline interferometers, in particular the VLBA[6], to confirm them as lens systems. Reliable lens confirmation is thus achieved by radio means alone.

JVAS/CLASS is a northern sky survey. Recently a number of groups have begun to apply the same method to the southern sky, although a major handicap there is the lack of long-baseline radio interferometers in the southern hemisphere; the lack of a southern MERLIN and the VLA's southern declination limit of $\sim -40°$ makes the process particularly difficult. Nevertheless, the first successes are being achieved by these groups *[Winn et al., 2000; Winn et al., 2001]*.

2.3 The current position: radio and optical searches

The CASTLeS website currently (2001 July 01) lists 63 secure, or reasonably secure, cases of gravitational lensing by galaxy-mass objects. There are approximately equal numbers of lenses discovered by radio and optical searches. However, radio searches have a number of distinct advantages, the first three of which are important in allowing sample completeness and clean statistics:

- *Lens recognition.* Many radio sources exist which have intrinsically simple structures when observed with moderate dynamic range and $0\farcs2$ resolution. This means that one can hope to identify all multiple-image lens systems in any given sample of sources, unlike the case of optical searches where galaxies with intrinsically complex structures can be lensed to give extended images which are difficult to recognise.

- *Resolution.* The initial search with the VLA can be made with a consistent resolution of $0\farcs2$ or better, which is difficult to achieve optically without large expenditure of scarce HST time. This is important given the small scale ($0\farcs5$ – $2''$) of most single-galaxy lens systems.

- *Freedom from obscuration by dust.* By their nature, lens systems involve passage of light through the medium of the lensing galaxy. In some cases there is evidence for considerable reddening, which makes lensed images difficult to recognise in optical pictures, and requires

[5]The MERLIN array is an interferometer system of six radio telescopes in England *[Thomasson, 1986]* based at Jodrell Bank Observatory, University of Manchester

[6]The Very Long Baseline Array *[Kellermann & Thompson, 1985]*is an interferometer system consisting of 10 radio telescopes throughout the continental USA, Hawaii and Puerto Rico, and is part of the US National Radio Astronomy Observatory

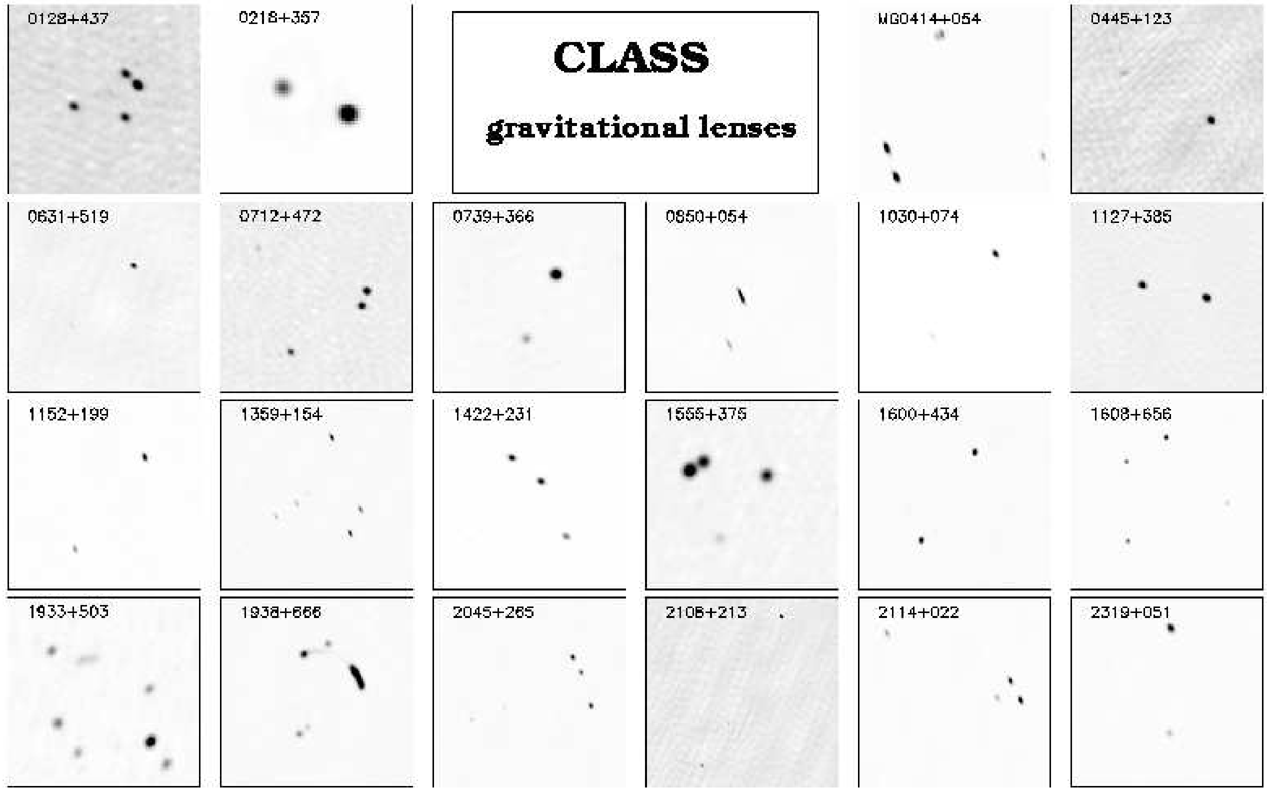

Figure 4: The JVAS/CLASS survey lenses, observed with MERLIN. Note the predominance of simple 2-image and 4-image systems. Some lenses, however, have Einstein rings (e.g. JVAS B0218+357) and arcs (e.g. JVAS B1938+666); Others have more than four images, either due to multiple background sources (CLASS B1933+503) or multiple deflectors (CLASS B1359+154). It is expected that the JVAS/CLASS survey will produce several more lenses when it is completed in 2001.

infrared imaging at $2\,\mu$m before the dust becomes more transparent. Radio studies can therefore be more complete than those in the optical, in the sense of identifying all lenses within any sample.

- *Variability.* Many radio sources are variable and high-resolution radio imaging allows us to track the variability of individual components. As we shall see in section 4, this allows us to derive the Hubble constant.

3. LENS MODELLING

3.1 How models are made

The aim of lens modelling is to use observations of gravitational lens systems to determine the mass distribution of the lensing galaxy or galaxies, both for its intrinsic astrophysical interest (see Section 2) and in order to exploit the lens systems for other investigations. Quite frequently a lens mass model is also an important step in confirming a lensed system. No other technique tells us so much about the mass distributions of high-redshift galaxies.

The modelling process consists of a mapping between the source plane, which contains the lensed object, and the observed image plane. In the simplest case, that of a lensed point source, we demand of a "good model" that the observed image positions should all map back to the same point on the source plane, given the model parameters which describe the lensing galaxy. In addition, the observed flux ratios of the images should agree with the relative magnification of the images required by the model.

In many cases there are only very limited constraints on the mass model. Free parameters in the model typically include the position of the source and the position, mass, ellipticity and axial ratio of the mass model for the lens. The constraints are obtained from the image positions and flux densities, including VLBI imaging of milliarcsecond-scale structure, together with the galaxy position (if this is known, usually via an HST image). VLBI imaging has also been made in polarization *[Patnaik et al., 1999]* giving additional constraints and also demonstrating clearly the expected mirror symmetry between different images in the lens system. In many cases, however, the observations leave few degrees of freedom for constraining the detailed mass profile of the galaxy; as we shall see a large range of mass distributions is consistent with the observational constraints. In general, four-image lens systems are more valuable for mass modelling, because of the greater number of constraints available from the lensed images.

An interesting and more radical approach has been taken by *Saha & Williams [1997]*. Instead of using parametric models, they perform pixellated free-form fits to recover the shape of the lensing galaxy, subject to only a few constraints such as rotational symmetry and monotonically decreasing mass density with radius. Results consistent with the data are obtained in which the nature of the lensing galaxy is highly degenerate with the Hubble constant H_0 (see section 4). An intermediate position is taken by *Trotter, Winn & Hewitt [2000]* who decompose the potential by multipole Taylor expansion. Future work in lenses with more observational constraints may diminish the freedom allowed by such models.

3.2 Types of lensing galaxy

Because of their generally larger masses and high central concentration, giant elliptical galaxies are the major contributors to gravitational lensing *[Turner, Ostriker & Gott, 1984]*. *Kochanek et al. [2000a]*, *Lehár et al. [2000]* , and *Keeton, Kochanek & Falco [1998]* in a study of lensing galaxies with the HST conclude that they are indeed generally elliptical galaxies which lie on the so-called "fundamental plane"; this is a locus in the space defined by the velocity dispersion, the surface brightness within effective radius and luminosity, in which elliptical galaxies congregate *[Dressler et al., 1987]*. However, a few lensing galaxies have been shown to be spiral systems (e.g. JVAS B0218+357 *[Wiklind & Combes, 1995]* ; CLASS B1600+434, *[Jaunsen & Hjorth, 1997]*).

3.3 Types of mass profile

As previously suggested, we do not have a very good idea of the mass distributions of galaxies at cosmological redshifts. Lacking such knowledge, we can ask instead about the light distributions. For nearby elliptical galaxies, reasonable fits to the light distributions are given by the de Vaucouleurs "$r^{1/4}$" law for the surface brightness Σ: $\Sigma(r) = \Sigma_0 \exp\left(-7.67((r/r_0)^{1/4} - 1)\right)$ where r_0 and Σ_0 are free parameters and r is the distance from the centre of the galaxy. For spiral galaxies, the "exponential disk" model, $\Sigma(r) = \Sigma_0 e^{-r/r_0}$, appears to fit the data better. However, neither of these parametrizations works particularly well for the central regions of high-redshift galaxies imaged by the HST. *Faber et al. [1997]* suggest instead a "cusp" distribution of light that follows

$I \propto r^{-\gamma}$, with $\gamma \leq 0.3$, for the inner part of the galaxy and a steeper power-law in the outer parts; this distribution has recently been adopted for mass modelling by e.g. *Muñoz et al. [2001]*.

Some of the distributions proposed for the projected surface mass density Σ and corresponding deflection angles α include the following (see also Fig. 2):

- the "singular isothermal sphere (SIS)", for which $\Sigma = \Sigma_0/r$ and α = constant $\times$ `sign` (θ), where Σ is the projected surface mass density;

- the "singular isothermal ellipsoid (SIE)", which is identical to the SIS except for different scale lengths along the x and y axes;

- the "pseudo-Jaffe model" *[Jaffe, 1983*; *Muñoz, Kochanek & Keeton, 2001]* for which $\Sigma = k((r^2 + r_s^2)^{-1/2} + (r^2 + a^2)^{-1/2})$, where k, r_s and a are constants;

- the "Navarro, Frenk & White" profile *[Navarro et al., 1996*; *Bartelmann 1996]*, for which the three-dimensional density is given by $\rho = \rho_s(r_s/r).(1 + r/r_s)^{-2}$, where ρ_s and r_s are constants.

- the generic cusped model, corresponding to $\rho_s r^{-\gamma} a^n (r^2 + a^2)^{(\gamma-n)/2}$, where additional constants γ and n are introduced; several other models are special cases of this distribution *[Muñoz, Kochanek & Keeton, 2001]*.

A full catalogue of these and many other models is given by *Keeton [2001]*.

In practice, few lens systems provide sufficient constraints to distinguish between the various mass models. This is particularly true for lenses in which companion galaxies or galaxy clusters are present, whose mass field provides an extra contribution to lensing (e.g. *Hogg & Blandford [1994]*). There may also be a contribution from the shape of the dark matter halo of the primary lensing galaxy *[Keeton, Kochanek & Seljak, 1997]*. Whatever its origin, nearly all models which fit the data even tolerably well require the parametrisation of some external perturbation (e.g. *Cohn et al. [2001]*).

For a good model, we therefore need extra information. This can be provided by VLBI imaging of the lensed images; the higher resolution can reveal extra components in the lensed images *[Patnaik, Porcas & Browne, 1995*; *Ros et al., 2000]*) and extra modelling constraints [*Trotter et al., 2000]*. However, in some cases even extensive VLBI information does not discriminate between a wide range of models (e.g. 0957+561 *[Barkana et al., 1999]*).

A few lenses do provide good additional constraints. For example, the lens CLASS B1933+503 (Figure 4) has ten images, formed by the 4-image lensing of two components of the background source and the double imaging of a third. *Cohn et al. [2001]* find the best model for B1933+503 is quite close to the simplest mass model, being slightly shallower than a singular isothermal ellipsoid. The pseudo-Jaffe profile (Fig. 3) and cusp models also fit the data, provided that the free parameters are adjusted to be quite close to an isothermal ellipsoid *[Muñoz, Kochanek & Keeton, 2001]*.

A good constraint is the presence of an Einstein ring, since this effectively gives information on the mass distributions along many lines of sight through the lensing galaxy. *Kochanek, Keeton & McLeod [2001]* discuss Einstein rings and derive a constraint for the system PG1115+080; once again models which fit the data well approximate to the simplest, isothermal-profile, mass distribution. A further constraint comes from the fact that odd (third or fifth) images are not seen in the vast majority of lens systems. *Rusin & Ma [2001]* argue that this implies that the mass profiles of lensing galaxies are not much shallower than the isothermal profile. Alternatively, the data can be used as a constraint on the radius of any mass "core" that is present *[Narasimha,* *Subrahmanian & Chitre 1986*; *Blandford & Kochanek 1987*; *Wallington & Narayan 1993*; *Norbury et al., 2001]*.

3.4 Spiral galaxies

Turner, Ostriker & Gott [1984] predicted that $\sim 94\%$ of lenses associated with normal galaxies should occur in the range $0''.3$–$6''$, with a peak at $\sim 1''.5$ and that ellipticals should dominate the lensing cross-section, with only 20% being contributed by spirals. These predictions are broadly consistent with the presently observed distribution.

However, spiral galaxies should produce image separations predominantly in the relatively unexplored range $0''.1$–$0''.3$. What are the prospects for detection of a useful number of systems in this separation range? The cross-sections are dominated by edge-on disks and the lensing rate depends on: (i) the balance between the masses of the disks and those of their associated dark matter haloes and on (ii) whether or not there is significant evolution of the spiral population between $z = 1$ and the present. *Keeton & Kochanek [1998]* predict that, when averaged over all inclinations, there should be little change in the contribution of spirals over the predictions of *Turner et al. [1984]*. In contrast the models of *Blain, Möller & Maller [1999]*, *Bartelmann and Loeb [1998]* and *Bartelmann [2000]* which invoke maximal disks and also consider the effect of evolution, predict enhancements of the spiral contribution by factors of two or more compared with *Turner et al. [1984]*. *Bartelmann* (private communication) estimates that between 10% and 20% of *all* galaxy-mass lenses could have separations in the range $0''.1$–$0''.3$. Taken with the $1:600$ JVAS/CLASS lensing rate for arcsecond-scale separations, these calculations suggest that the lensing rate in this image separation range should be one per few thousand background objects searched.

These recent calculations have been motivated by the prospect of lens searches at 0.1-arcsecond resolution in the sub-mm (with ALMA) and infra-red (with NGST). Radio-based surveys are also well-suited to an unbiased search for spiral galaxy lensing since they are also unaffected by dust obscuration in edge-on systems. A pilot radio search, largely based on JVAS and sensitive to image separations in the range 150–300 milliarcsec, has yielded a null result for a sample of 1665 sources *[Augusto, Wilkinson & Browne, 1998]*. In prospect, however, is a 10000–source survey based on the CLASS catalogue which has a high probability of detecting small–separation lensing and which would place significant constraints on the uncertain disk/halo mass ratio in spiral galaxies at the redshifts ($z \sim 0.5$) appropriate to lensing.

3.5 Dark galaxies

Recently it has been proposed that very massive (10^{12-13} $M_\odot$) dark objects could give rise to quasar pairs with separations ~ 10 arcseconds by gravitational lensing *[Hawkins, 1997]*. There is, however, no supporting evidence for such a population of massive dark objects. *Kochanek, Falco and Muñoz, [1998]* argue that a comparison of the radio and optical properties of the pairs rules out the massive dark lens hypothesis. And HST imaging of confirmed arcsecond-scale lenses found in the JVAS/CLASS surveys always shows a lensing galaxy with a relatively normal mass–to–light ratio between the images *[Jackson et al., 1998]*.

Phillips, Browne & Wilkinson [2001] use the lack of larger separation ($6'' - 15''$) gravitational lenses to investigate mass distributions on larger ($\sim 10^{13} M_\odot$) scales. In particular, the lack of observed lenses with separations on this angular scale means that groups and clusters of galaxies must have substantially softer central potential wells than would be expected for singular isothermal sphere models. On smaller scales, *Trentham, Möller & Ramirez-Ruiz [2001]* have pointed out that current "cold-dark matter" models of cosmology require numerous condensations of matter on scales smaller than galaxies, possibly 10^7–$10^{10} M_\odot$. Such condensations would be likely to produce little or no starlight; future lensing studies on sub-mas scales could find and count them, but only if they are more centrally concentrated than current theories suggest.

4. COSMOLOGICAL PARAMETERS

4.1 The Hubble constant, H_0

Hubble's discovery of the expansion of the universe, in 1929, resulted from his observation that nearby galaxies were receding from us with a velocity v proportional to their distance d. The velocity was measured by the shift in optical spectral lines from their expected wavelength $\lambda_{\rm lab}$ to the observed wavelength $\lambda_{\rm obs}$, by the usual non-relativistic Doppler formula

$$\frac{v}{c} = \frac{\lambda_{\rm obs} - \lambda_{\rm lab}}{\lambda_{\rm lab}} \equiv z$$

where z is the redshift. Since the empirical relation between recession velocity and redshift is of the form $v = H_0 d$, where H_0 is a constant, we have $d = cz/H_0$ for nearby galaxies. For more distant cases the relation between distance and redshift is more complicated and involves knowledge of the global topology of the universe, determined mainly by the matter density Ω_m and cosmological constant. Nevertheless, the Hubble constant, H_0, is a vital number as it allows distances to be calculated from easily observable quantities in the case of objects at enormous distances. It is also related to the age of the Universe; in most simple cosmological models $t_{\rm univ} \sim H_0^{-1}$.

The Hubble constant is not an easy quantity to determine as it requires the measurement of the distance of far-away objects. The traditional method relies on a number of local "distance indicators" such as moving clusters of stars and a special class of variable stars known as Cepheids whose brightness variations have a well-determined relationship to their absolute luminosity. If the brightness variations in Cepheids are measured and the luminosity inferred, this, together with a measurement of flux density, allows an immediate determination of distance. This in principle allows the Hubble constant to be measured for galaxies in which Cepheid variables can be resolved. Unfortunately, despite the devotion of considerable observational resources including long observations with the Hubble Space Telescope, the current position is still one of controversy, with recent estimates ranging from 53 ± 7 km s^{-1}Mpc^{-1} *[Sandage, 1999]* to the Hubble Space Telescope Key Project value of 71 ± 6 km s^{-1}Mpc^{-1} *[Mould et al., 2000]*. Worse still, "traditional" methods determine H_0 only for relatively short distances by cosmological standards, usually to the nearest clusters of galaxies at distances of a few tens of Mpc.

Gravitational lenses in principle allow a clean determination of H_0 on cosmological scales, as was pointed out by *Refsdal [1964]*. The key requirement is that the lensed object be variable in flux density. If this is the case, the lensed images will of course also vary. However, each lensed image results from light that has taken a slightly different path through the lens; each image will therefore show a variation at a slightly different time, reflecting the different propagation times of light on these different paths. The delay between variations of the image allows us to measure the difference in lengths of light paths – the latter quantity will just be c$\Delta\tau$, where $\Delta\tau$ is the measured time delay (which results from the sum of the geometrical and Shapiro time delays). The delay is proportional to the square of the image separation in the system and ranges from 10 days to just over 400 days in systems examined so far. This result allows us to calculate in parsecs a distance within the system; knowing all the angles within the system, we can then immediately derive the distances to the lens and lensed object. If we know the redshifts of the lens and source, Hubble's constant follows from the definition given above. In principle, if the Hubble constant could be determined accurately from lens systems at different redshifts, this would even allow determination of other cosmological parameters such as Ω_m, because these parameters affect the redshift–distance relation at high redshift.

There are three problems with this approach. The major difficulty is that for all the angles to be determined, we must have a good knowledge of the mass distribution of the lens, since this affects the deflection angle α. For example, an overestimate of the central mass in the lensing galaxy would cause an overestimate of α, which in turn would result in a distance estimate that was too small and consequently an overestimate of H_0. As previously discussed, the correct mass distribution of lenses still has significant uncertainties, although some constraints are available by considering the positions and flux densities of the lensed images.

The other two problems are observational. First, the redshifts of both lens and source must be determined, which requires observations of optical spectral lines and hence an optical identification for both lens and source. In practice, it has proved difficult to measure redshifts for very weak radio sources and our group is currently following up seven "difficult" source redshifts out of a sample of 19 lenses. Second, the time delays must be determined accurately enough to give useful constraints on H_0.

The ideal lens for H_0 determination, known in the business as a "golden lens", would have a number of characteristics. The main requirements are a highly variable source; a single lensing galaxy with no nearby field galaxies for ease of modelling; an Einstein ring in addition to the lensed images, for best constraints on the model; many lensed images; and a relatively long time delay, for accurate $\Delta\tau$ determination given typical experimental errors $\sigma_{\Delta\tau}$ ~1 day. Unfortunately, long time delays imply large image separations, which in turn tend to be produced by multiple galaxies which are difficult to model; in such cases simpler, smaller lenses are to be preferred as the random error in delay is easier to tackle than the systematic errors introduced by problems with the mass model.

Although no lens so far is perfect, radio-selected gravitational lenses approximate much better to golden lenses. Many contain images of flat-spectrum, intrinsically variable radio sources, some of which are variable both in total intensity and polarization, and radio interferometry achieves the resolution necessary to separate variations in individual images with the necessary accuracy. Radio observations also allow much better sampling of the variability, as they can often be done 24 hours per day from a given site and are less subject to censorship by bad weather or (in the case of longer-timescale variability) by closeness of objects to the Sun for several months per year. It is no coincidence that the majority of the existing H_0 determinations have been made for radio-selected gravitational lenses.

Useful time delay and H_0 determinations have been made for seven lenses so far. Table 1 shows the current best estimates for each of these lenses, using standard SIEs as the mass model. It also attempts to show the major systematic problems and to give a qualitative idea of the likely uncertainties each will introduce.

	Time delay (see refs[1])	Galaxy position	Cluster or nearby gals.	Single lens?	Micro-lensing?	Constraints available?	H_0 (see refs[1])
Golden lens	**<1%**	**yes**	**no**	**yes**	**no**	**YES**	**???**
JVAS B0218+357	3%	no	no	yes	no	YES	$69^{+13}_{-19}(2\sigma)$
0957+561	<1%	yes	YES	yes	yes	yes	$64\pm13(2\sigma)$
HE1104−1805	34%	yes?	(no)	yes	yes	no	
PG1115+080	6%	yes	YES	yes	no	YES	42 ± 6
B1600+434	4%	no	yes	yes	yes	no	$57^{+14}_{-11}(2\sigma)$
B1608+656	4%	no	yes	no	no	yes	$59^{+8}_{-7}(2\sigma)$
PKS1830−211	17%	no	yes	yes	no	YES	65^{+15}_{-9}

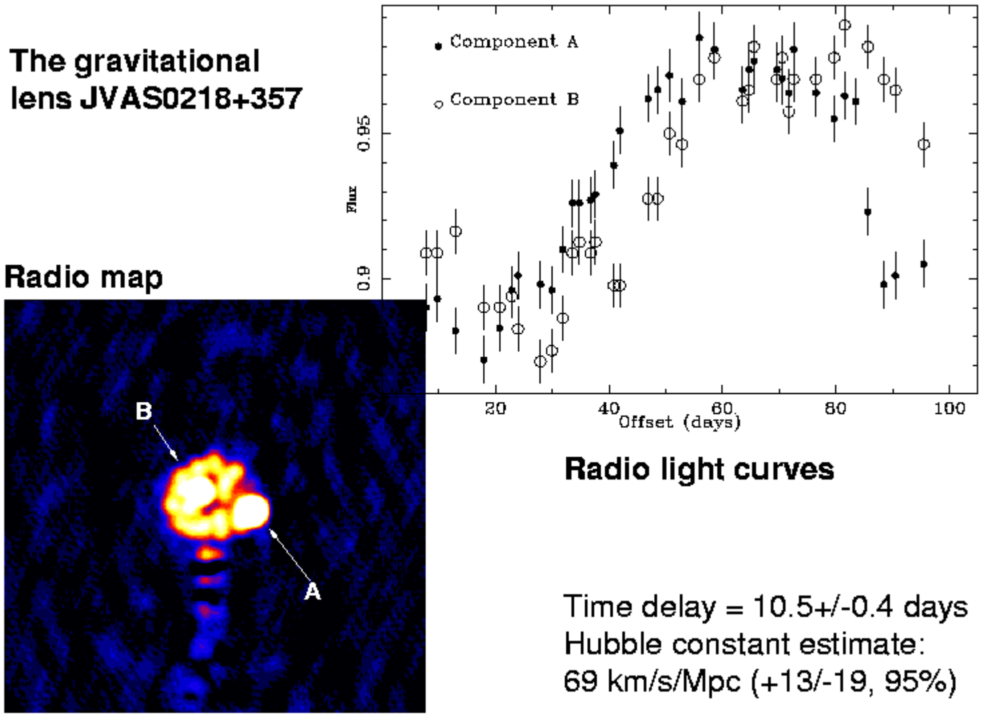

Figure 5: Hubble Constant determination *[Biggs et al., 1999]* from time delays between variations in the lensed images in JVAS B0218+357, shown here in a MERLIN/European VLBI Network radio map. Residual errors in the Hubble Constant are mainly due to uncertainty about the exact position of the lensing galaxy; the uncertainty without this is 5% (1σ). [A colour version of this figure appears in the CD-ROM version of this chapter].

Caption to table 1: Summary of the authors' assessment of suitability of existing gravitational lens systems (column 1) with determined time delays (column 2) for the determination of H_0. References for the time delays in column 2 are as follows: JVAS B0218+357 *Biggs et al., [1999]*; B0957+561 *Kundic et al., [1997]*; HE1104-180 *Wisotzki et al., [1998]*; PG1115+080 *Schechter et al., [1997]*; *Barkana, [1997]*; CLASS B1600+434 *Koopmans et al., [2000]*, *Burud et al., [2000]*; CLASS B1608+656 *Fassnacht et al., [1999]*; PKS1830−211 *Lovell et al., [1998]*, *Lidman et al., [1999]*. In column 3 we give our assessment of whether the galaxy position is sufficiently well known to avoid major errors in H_0 determination, and in column 4 we assess whether nearby galaxies or clusters produce systematic errors in the mass model. Column 5 indicates whether the lens consists of one or more galaxies, column 6 shows whether evidence is available for microlensing affecting the observed fluxes and column 8 indicates whether good observational constraints are available for the mass model, from >2 images or VLBI structure or both. Systems with an Einstein ring are marked YES in this column. Finally, column 9 gives an H_0 estimate. Many lenses have been investigated by a number of authors, and the table generally gives the value quoted by the paper which contains the first measurement of the time delay. Authors have generally derived values assuming (Ω_m, Ω_Λ) = (1,0). For the currently favoured (0.3,0.7) universe, H_0 estimates increase by factors of 5-10%, and by 20% in the higher-redshift system PKS1830−211.

Overall, most lenses have major systematic problems; in JVAS B0218+357 the galaxy position is not well enough known *[Lehár et al., 2000]*, in 0957+561 the cluster mass profile is probably not well enough understood *[Barkana et al., 1999*, but see also *Keeton et al., 2000]*; HE1104−180 is a double with not enough modelling constraints for complete security and as yet no spectroscopic lens redshift, although improved determination of the lens galaxy centre has recently been made *[Courbin, Lidman & Magain 1998]*; in PG1115+080 more constraints are needed on the galaxy and surrounding group [*Impey et al., 1998]* to tie down the mass model *[Keeton & Kochanek 1997]*; *[Courbin et al. 1997]*; CLASS B1600+434 has a spiral lens galaxy whose centre is not well constrained, few modelling constraints in the absence of significant VLBI structure in both images and additionally suffers from the problem of a bright nearby galaxy *[Koopmans, de Bruyn & Jackson 1998]*. In CLASS B1608+656 the principal lensing galaxy is a double whose centre-of-mass position is highly uncertain *[Jackson, Nair & Browne, 1997]*; in PKS1830−211 the galaxy position is uncertain enough to introduce a substantial degeneracy into the mass model *[Lehár et al., 2000]* and the lens galaxy lies in a small cluster. In the authors' opinion, based on the entries in this table, JVAS B0218+357 is the nearest approximation to a "golden lens" with the single caveat that its small size makes the centre of the lensing galaxy difficult to pin down.

The way forward obviously involves the removal of systematic uncertainties in the lensed systems. Unfortunately, the systems with the best-constrained mass models are often not the ones with the measured time delays and the search for the "golden lens" continues. In the authors' opinion one of the best prospects is JVAS B0218+357 (Figure 5), which has the single disadvantage, due to its small size, that the centre of the lensing galaxy, even with current HST imaging is not accurately fixed with respect to the radio components.

4.2 The Cosmological Constant and matter density parameter

There are two very important cosmological parameters which control the overall topology and future development of the universe: Ω_m, the matter density parameter and Λ_0, the "cosmological constant". Ignoring Λ_0, Ω_m expresses the density of matter as a fraction of the density needed to "close" the universe – that is, eventually to halt the universal expansion and bring the universe back to a Big Crunch. In terms of luminous matter $\Omega_m \ll 1$, but there is ample evidence from dynamical studies of galaxies and clusters for a great deal of "dark matter" which contributes to Ω_m but does not emit significant electromagnetic radiation.

The cosmological constant Λ_0 (see *Carroll, [2000]* for a comprehensive review) represents an energy density which is not due to matter, but which can instead be associated with the vacuum – hence its alternative name of "vacuum energy". It is allowed by Einstein's equation of general relativity, although Einstein himself famously repudiated it as an unnecessary complication. Positive values of Λ_0 usually imply a universe which does not recollapse[7] but instead follows a path of accelerating expansion. It can be expressed in the same dimensionless units as Ω_m, in which case the condition for a "flat" universe without curvature can be written as $\Omega_m + \Omega_\Lambda = 1$. Results from the Boomerang and MAXIMA experiments, which measured fluctuations in the cosmic microwave background (CMB) *[Hanany et al., 2000*; *de Bernardis et al., 2000]*, point towards this condition being satisfied. Further constraints have been derived by studies of supernovae *[Perlmutter et al., 1999*; *Riess et al., 1998]*; in the Ω_m vs. Ω_Λ plane the supernovae constraints are approximately orthogonal to those from the CMB. The intersection of the constraints is approximately at $\Omega_m = 0.3$, $\Lambda_0 = 0.7$. In other words, provided that systematic errors affect neither result, we are living in a Universe destined for eternal and accelerating expansion.

In principle, gravitational lensing statistics can also be used to tie down both cosmological parameters *[Fukugita et al., 1992*; *Kochanek, 1996a,b]*. Consider a source at distance l; the optical depth to lensing, and hence the probability of lensing, $P(\text{lens})$, is obtained by integrating along the light path

$$P(\text{lens}) = \int_0^l n(l)\sigma dl$$

where n is the number density of lenses and σ is the cross-sectional area of each lens. This can be written as

$$P(\text{lens}) = \int_0^{z_s} n(z)\sigma c \frac{dt}{dz_l} dz_l$$

where z_l and z_s are the redshifts of the lens and source respectively, c is the speed of light and dt is the time increment along the path. If we measure $P(\text{lens})$ for a complete sample of lenses – that is, one for which all lenses within the sample have been identified – we can then compare this with the right-hand side of the equation. This depends on a number of quantities. First, it is sensitive to the number density $n(z)$ of lensing galaxies and its evolution with redshift z. Second, it depends on the cross-sectional area of individual lenses, σ, which can be calculated if we assume simple models for lens mass distributions. Third, it depends on lengths within the universe, via quantities such as dt/dz_l. Quantities related to length in the universe are in turn sensitive to Ω_m and Ω_Λ, and if all other quantities can be measured by other means, these cosmological parameters can be inferred.

The major problem for radio studies in fact lies with $P(\text{lens})$, as this involves dividing the number of lenses found by the number of sources which are potentially lensed. The former quantity is subject only to Poisson errors, assuming that radio selection allows one to select all the lenses in any sample. The latter, however, requires that one knows the number distribution $n(S, z)$ of radio sources as a function of radio flux density S and redshift z. For faint radio sources, this is difficult to determine as the redshifts of these objects must be measured. A flat-spectrum radio source with $S \sim 25\,\text{mJy}$ is likely to be faint (a visual magnitude $V \sim 25$ is not uncommon), making samples very difficult to study optically, even with the new generation of 8-to-10-m telescopes. Knowing $n(S, z)$ is crucial, as errors as small as 0.1 in the mean redshift, which is typically between 1.0 and 1.5, significantly disturb the best-fit regions for Ω_m and Ω_Λ.

[7] The only exception to this is for small values of Λ_0 where $\Omega_m > 1$; see e.g. *Peacock, [2000]*.

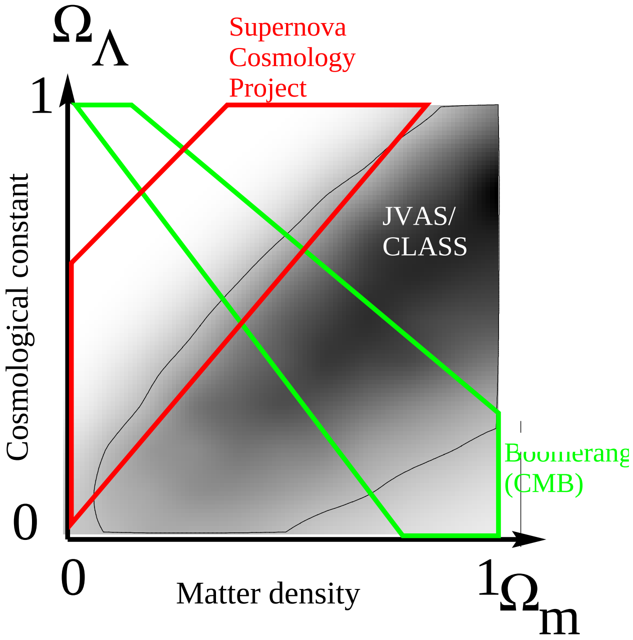

Figure 6: Omega-Lambda plane diagram, showing 2-σ constraints from the Cosmic Microwave Background *[de Bernardis et al., 2000]*, supernova studies *[Perlmutter et al., 1999]* and preliminary lensing constraints from JVAS/CLASS *[Helbig, 2000]*. [A colour version of this figure is available on the CD-ROM version of this article.]

Kochanek [1996a] applied the above method to the best available lens statistics at that time, including optical lens surveys and the first phase of the JVAS/CLASS radio survey (section 2). He found that $\Omega_\Lambda < 0.66$ for models of a "flat" ($\Omega_m + \Omega_\Lambda = 1$) universe. In general, lensing gives an effective upper limit on Ω_Λ due to the very rapid increase in the expected number of lenses with increasing cosmological constant. *Kochanek [1996b]* and *Cooray [1999]* both extended the analysis to predict the number of lenses that would be found in the complete JVAS/CLASS survey.

Quast & Helbig [1999], *Helbig et al. [1999]* and *Helbig [1999]* derive constraints from the JVAS survey (2384 sources) only and combine these constraints with supernova and CMB results. They obtain a good fit to all data combined, provided that $\Omega_m + 0.4 \simeq \Omega_\Lambda$, which for a flat universe requires $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, exactly the best-guess values suggested by reviews such as that by *Carroll [2000]*. *Helbig [2000]* (Figure 6) extends the lens analysis for the first time to the full CLASS sample. This work is preliminary in that efforts are still continuing to obtain the number distribution function $n(S, z)$ for the parent population of radio sources and achieve the necessary understanding of evolution of lensing galaxies. The current result is marginally inconsistent (at 2-σ) with the Supernova Cosmology Project results *[Perlmutter et al., 1999]*, although final analysis is not complete. However, it now appears that lensing constraints, as well as the statistics of supernovae, rule out the Einstein-de Sitter model in which the universe is flat and composed entirely of matter ($\Omega_m = 1$, $\Omega_\Lambda = 0$).

4.3 The cosmological density of dark compact objects

The possibility that a first generation of objects with masses comparable with those of globular clusters formed prior to galaxies has long been recognised (see *Carr [1994]* for a comprehensive review). Such Jeans–mass ($\sim 10^{6.5}$ $M_\odot$) objects forming shortly after the decoupling of matter and radiation in the early universe could have evolved to black holes and it is possible that some of the dark matter could be in this, difficult to detect, form *[Carr & Sakellariadou, 1999]*.

Since for a lens at a cosmologically significant distance the image separation is $\sim 2{\times}10^{-6}(M_{CO}/M_{\odot})^{1/2}$ arcseconds, searching for Jeans–mass compact objects (CO) requires the milliarcsec resolution which is only obtainable with VLBI. *Press & Gunn [1973]* developed the idea of detecting supermassive CO, by their gravitational lensing effects on VLBI radio images, well before the discovery of gravitational lenses in 1979. They showed that in a universe filled with a mass density $\Omega_{CO} \sim 1$, the probability of a distant source being multiply imaged by a supermassive CO is of order unity, while for $\Omega_{CO} < 1$, the probability decreases in direct proportion to the mass density. From this they drew the important conclusion that the fraction of distant galaxies that is lensed by CO directly measures Ω_{CO} and is independent of the mass M_{CO} of the lenses. The latter property is simply understood. A given value of Ω_{CO} can be made up of a large number of low–mass objects or a small number of high–mass ones, hence the number density n of CO of a particular mass is proportional to $1/M_{CO}$. For point masses the gravitational lensing cross–section $\sigma \propto M_{CO}$ *[Turner, Ostriker & Gott, 1984]* and hence the path length to lensing $(1/n\sigma)$ is independent of the lens mass. However, the average image separation measures M_{CO} directly and is essentially independent of Ω_{CO}. These ideas were further developed by *Nemiroff [1989]*, *Nemiroff & Bistolas [1990]* and *Kassiola, Kovner & Blandford [1991]*.

Wilkinson et al. [2001] have searched a sample of 300 flat-spectrum radio sources, largely corresponding to the strongest sources in JVAS, for examples of multiple imaging. The sources were drawn from the Pearson-Readhead and Caltech-Jodrell Bank VLBI surveys (see references in *Wilkinson et al. [2001]*) and involved systematic observations with intercontinental VLBI arrays at a resolution ~ 1 mas. Great care has to be taken to achieve completeness by being conservative in the rejection of systems as lens candidates since at this high resolution compact radio sources exhibit a range of intrinsic structures which can mimic the effects of lensing.

No multiple images were found with separations in the angular range 1.5-50 milliarcsec enabling a limit $\Omega_{CO} \leq 0.013$ in the range $\sim 10^6$ to $\sim 10^8$ $\mathrm{M}_{\odot}$ to be placed *[Wilkinson et al., 2001]*. These limits are mildly conservative because lensing increases the observed flux density of a background source and hence lensed sources are drawn from a fainter source population than the unlensed sources; a flux–limited survey will therefore contain more lenses than expected but the "magnification bias" associated with flat spectrum radio sources is only of order unity *[King & Browne, 1996]*. *Garrett et al. [1994]*, using the similarity of the two VLBI images of 0957+561 to constrain the presence of large black holes in the lensing galaxy, have ruled out objects with $M > 3\times10^6 M_{\odot}$ as contributing more than 10% of the halo dark matter.

Uniformly distributed CO in the mass range $\sim 10^6$ to $\sim 10^8$ $\mathrm{M}_{\odot}$ cannot, therefore, comprise $> 1\%$ of the closure density, $(\Omega_{total} = 1)$, which is strongly indicated by the latest measurements of the angular spectrum of the CMB, (*[de Bernardis et al., 2000]*). Similarly such CO do not make up more than $\sim 3\%$ of the Dark Matter density $\Omega_{DM} \sim 0.3$ favoured by current observations. The favoured value of the baryon density from Big–Bang Nucleosynthesis is $\Omega_b h^2 = 0.019 \pm 0.002$ *[Burles, Kirkman & Tytler, 1999]* ($h \equiv H_0/100$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$). Taking a plausible value for h (0.65) implies $\Omega_b = 0.045 \pm 0.005$, thus uniformly–distributed Jeans–mass CO do not make up more than about one third of Ω_b. Perhaps the next interesting limit would be $\Omega_{CO} \leq 0.005$ which would constrain the contribution of supermassive CO to be no more than the baryonic contribution of presently observable stars and galaxies. To reach this limit about 1000 sources would have to be studied with VLBI; this would be a time–consuming, but relatively straightforward, task.

5. PROPAGATION EFFECTS AND PROPERTIES OF LENSING GALAXIES

As radiation propagates from a distant object to the observer the medium leaves an imprint which can give invaluable information about parts of the Universe inaccessible to study by any other means. Gravitational lens systems could have been designed for such studies since, by definition, there exists a distant object whose radiation passes through an intervening galaxy with impact parameter(s) of a few kiloparsecs. Moreover, with multiple images, more than one path is sampled and it then becomes much easier to separate the effects of propagation from the intrinsic properties of the initial signal. This technique has been exploited in the optical and infrared bands to explore the effects of extinction due to dust in the lensing galaxies *[Kochanek et al., 2000b*; *Jackson, Xanthopoulos & Browne, 2000]*. However, in the following section we concentrate on effects on radio signals such as spectral line absorption, Faraday rotation and depolarization and multi-path scatter broadening.

Two lens systems in particular, JVAS B0218+357 and PKS1830−211, display a rich variety of propagation effects; both show neutral hydrogen absorption *[Carilli, Rupen and Yanny 1993*; *Lovell et al., 1996]*, molecular absorption *[Wiklind & Combes, 1995]* and evidence for multi-path scattering *[Biggs, Browne & Wilkinson, 2001*; *Jones et al., 1996*; *Guirado et al., 1999]*. The existence of a rich interstellar medium in these lenses argues in favour of the lens being a spiral galaxy in each case.

In the light of modern astro-particle unified theories, in which the fundamental constants of nature may change with cosmological epoch, high redshift objects in which both molecular and neutral hydrogen absorption occur, assume a new significance. *Drinkwater et al., [1998]* and *Murphy et al., [2001]* make use of data on two objects including the lens system JVAS B0218+357 to put a limit on any change of the fine structure constant (α) as a function of cosmological epoch of $|(\dot{\alpha}/\alpha)| \leq 9\times10^{-16}$ yr^{-1}. Better measurements on JVAS B0218+357 and other lens systems should enable these constraints to be refined.

An exciting prospect is to be able to map the kinematics of lensing galaxies using high resolution spectral line VLBI. This has been attempted on JVAS B0218+357 using observations at 842 MHz which is the frequency of the redshifted hydrogen line seen in the lensing galaxy (Vermeulen & Hobbs, personal communication). The results of these preliminary observations show absorption affecting both the compact images and part of the Einstein ring. The ultimate promise of such observations is that of being able to determine the velocity dispersion of the lensing galaxy and use it as an additional constraint in the mass modeling process.

Almost nothing is known about the magnetic field structures in high redshift galaxies. Even the origin of magnetic fields in galaxies is obscure *[Fermi 1954*; *Lesch & Chiba 1997]* though one widely explored idea for spiral galaxies is that they are generated by a dynamo process involving differential rotation. Such mechanisms predict an exponential growth of field strength on a characteristic time comparable to the dynamical time-scale of the galaxy; i.e. $\sim 10^8$ yr. Thus a large evolution of magnetic field strength is expected in going from the epoch at which we see most lensing galaxies (a few times 10^9 yr) to the present day. Looking for large-scale Faraday rotation of the background source radio emission produced by the lensing galaxy is one way that galaxy magnetic fields would manifest themselves. A surprising result is that many of the radio-loud lens systems which have detectable polarized radio emission display rotation measures often much larger than those that would be produced by radiation passing through the plane of our Galaxy at a distance of a few kiloparsec from the centre.

Within our own Galaxy radio sources viewed through regions of high electron density have their apparent sizes increased by multi-path scattering, with the size changing $\propto \lambda^2$ *[Rickett, 1977]* where λ is the observing wavelength. In several lens systems there is evidence showing the the angular sizes of the radio components are strongly suggestive of scatter broadening. In these cases the surface brightnesses of well-resolved images are different, a discrepancy that can only arise from propagation effects. One of the most convincing cases for scattering is in PKS1830-211 in which *Guirado et al., [1999]* see evidence for a λ^2 dependence of angular size in one of the images. Scattering has also been claimed in two of the images in CLASS B1933+503 by *Marlow et al.,* *[1999]* and in JVAS B0218+357 by *Biggs, Browne & Wilkinson [2001]*. In the latter object both images show a strong increase in angular size with increasing wavelength over a very wide range of wavelengths but the size variations are not well fitted by a λ^2 law. However, this does not rule out scattering because the projected sizes of the images on the lensing galaxy are hundreds of parsecs and the statistical properties of the ISM of the lens are not necessarily uniform over this kind of length-scale.

With more than 50 (radio) lines of sight through more than 20 lens systems can we reach any general conclusions about the properties of galaxies at redshifts ∼0.5? We think it is too early to be certain but are surprised by the number of lensing galaxies that seem to contain neutral, molecular and ionized gas in amounts that would be regarded as large for a spiral galaxy (based on local knowledge) let alone for an elliptical galaxy which is what most lensing galaxies are believed to be. Perhaps there is more gas in ellipticals than expected; alternatively more of the lensing galaxies are spirals.

It is also possible to use lens systems to help distinguish between the intrinsic and extrinsic models for intraday variability which is a feature of some compact radio sources. In some cases (e.g. *Kedziora-Chudczer et al., [1997]*, *Dennett-Thorpe & de Bruyn [2000]*) it is clear that the variations are produced during propagation from source to observer but in others the origin could be intrinsic implying that the brightness temperatures exceed the 10^{12}-K inverse-Compton limit by orders of magnitude *[Wagner & Witzel, 1995]*. In lens systems the propagation paths are different and thus propagation induced variability should be uncorrelated in the two images whereas intrinsic variability should be perfectly correlated. *Biggs et al., [2001]* have applied this argument to JVAS B0218+357 to show that the variations seen in this object on timescales of days must be *intrinsic* to the source.

An important example of *extrinsic* variability is the discovery of variations in the radio flux of the brighter image in the gravitational lens CLASS B1600+434 *[Koopmans & de Bruyn, 2000]*. Variations of this image, at levels of up to 15 percent, are seen, but after allowing for the lensing time delay, corresponding variations are not seen in the other image. One might then be tempted to ascribe the effect to scattering in our own Galaxy. However, Koopmans & de Bruyn present evidence against this idea, the most persuasive of which is an increase of fractional rms variability with increasing frequency between 1.4 and 5 GHz – completely opposite to that expected from interstellar scintillation. But this dependence is exactly what one would expect if the cause of the variability is microlensing by massive ($\sim 1M_\odot$) objects in the halo of the lensing galaxy. In this model, the radio source contains components on the scale of a few microarcseconds, moving at speeds of $\sim c$, which pass behind the complex magnification pattern of the lens galaxy halo and hence vary strongly in flux density. The line of sight to the brighter image in CLASS B1600+434, along which the compact objects must lie, passes 6 kpc above the plane of the edge-on spiral that constitutes the lensing galaxy!

Further work is under way to test whether other lensing galaxies show this effect. It is very important as it allows us for the first time to deduce the presence of massive objects in the halo of a galaxy at redshift ∼0.5. The implications for the nature of dark matter in galaxies of such observations are potentially very important.

6. THE FUTURE

6.1 ALMA and the submillimetre range

Detailed study of the submillimetre wavelength range is relatively new. Major submillimetre telescopes, such as the James Clerk Maxwell Telescope on Mauna Kea and the IRAM telescope on Pico Veleta, have only recently been equipped with bolometer arrays allowing sensitivities of

∼1 mJy to be routinely achieved at 400–800 μm. This has led to the first blank-field surveys and source counts in the submillimetre range *[Smail, Ivison & Blain, 1997]* to these levels; such surveys often examine the regions around clusters to make use of the magnification effect of the weak gravitational lensing by the cluster.

The submillimetre range is physically an interesting region to study galaxies. This is because it contains the steeply falling Rayleigh-Jeans tail of sources at a few tens of Kelvin, such as the dusty envelopes of star-forming galaxies. Much interest has been generated in the star formation history of the universe by recent studies in the optical and UV such as *Madau et al. [1996]*, although much pioneering work at moderate redshift was in fact done using 21-cm line radio surveys *[Condon et al., 1982]*.

Since the effect of distance is to introduce a redshift, the steeply inverted Rayleigh-Jeans tail ($F_\nu \propto \nu^\alpha$, $\alpha > 0$) means that sources can actually appear *brighter* at a given observed wavelength at redshift $z = 5$ than at $z = 1$ (see e.g. *Blain & Longair [1996]*) despite the $(1+z)^4$ falloff in surface brightness with distance. This makes the submillimetre a useful region for detecting very high-redshift star-forming objects – which in turn formed a major part of the science case for the Atacama Large Millimetre Array *[Wootten, 2001]*.

The submillimetre range is also a very promising region for new detections of gravitational lenses, as high-redshift objects traverse a longer path through the universe and are thus more likely to be lensed. The integral source count is likely to be very steep in the submillimetre, giving a large magnification bias and resulting in a much higher lens fraction for even modest lens magnifications *[Blain, 1996, 1997]*. *Blain [1996]* shows that for some models, lensed sources could even be in the majority for some ranges of 850-μm flux. He also discusses optimum strategies for finding large numbers of gravitational lenses with instruments such as ALMA, despite its small ($\sim 15''$) field of view. In principle it should be possible to find ∼100 lenses in an observing time of about 2 weeks, although whether this is better done with a few deep pointings or a wider, more shallow survey depends on how the density and luminosity of star-forming galaxies evolve with redshift.

The likely large number of lenses to be found with next-generation submillimetre instruments has implications for cosmology. *Blain [1998]* has shown that a relatively small lensing survey could distinguish sensitively between cosmological models, as lensing rates for sources of 850-μm fluxes between 1 mJy and 100 mJy differ by factors of 3-5 for plausible cosmologies and the large lensing fraction implies a small Poisson error on the statistics. A major problem is the systematic error involved in understanding the evolution of the lensing galaxies which boost the flux of the background submillimetre sources. A still more difficult problem is likely to be encountered in the determination of the luminosity function of the parent population. For this, redshifts of samples of sources with $S_{850\mu m}$ ∼2 mJy will need to be obtained. This is not easy; many such sources are optically very faint *[Smail, Ivison & Blain, 1997]* and it has proved extraordinarily difficult to obtain complete redshift information for much less extreme sources *[Marlow et al., 2000*; *McKean et al., 2001]*.

6.2 The Square Kilometre Array

The design goals of the SKA call for sensitivities approaching 100 times that of the VLA, operation at frequencies up to at least 5 GHz, and superb imaging capability at resolutions < 0.03 arcsec at 5 GHz (for an up-to-date description of the SKA project see http://www.ras.ucalgary.ca/SKA/). One can assume, therefore, that with a search methodology similar to that adopted for JVAS/CLASS (and outlined in section 2.2) many thousands of galaxy-mass lenses could be identified from their radio structures alone. In the next section we outline how one might use this two-orders-of-magnitude larger sample to carry out new astrophysical investigations on galaxies and their cosmological evolution.

The enormous sensitivity and superlative imaging capability of the SKA also allows other important lensing investigations on individual systems, which have been touched on in previous sections, to come to full maturity. For example faint extended structures, currently unseen, will almost certainly be detected by the SKA in most lensed images and these will provide powerful constraints on mass distributions on kpc-scales. In addition it is highly likely that the SKA will detect the central ("odd") images in most systems providing a direct measure of the mass concentration within the central few parsecs. Other fascinating possibilities open to an instrument with the SKA's sensitivity involve observations in redshifted HI. As mentioned in section 5 the velocity distribution in the lensing galaxy can be directly studied using the lensed continuum emission from the background object simply as a distributed light source; the hydrogen in front of the extended lensed images can then be probed by imaging in absorbed HI. If, on the other hand, an HI-rich, and by implication extended, background galaxy is itself lensed then the additional spatially-dependent velocity structure of the background galaxy provides a large amount of additional information about the system. If the system is imaged in narrow spectral channels a detailed picture of both the mass distribution in the lens and the HI distribution in the background galaxy can be reconstructed.

The SKA will also have a major impact on cosmology via the technique of weak gravitational lensing. For large impact parameters the projected mass distribution only weakly distorts the faint background galaxies but from the observed distortion pattern one can directly calculate the projected mass surface density, up to some additive constant, (e.g. *Squires & Kaiser [1996]*). The technique depends on having a high surface number density of background objects and so far, most weak lensing studies have been undertaken with ground–based optical telescopes (see e.g. *Bacon et al. [2000]* for a recent example) although studies have been painstakingly made using multiple pointings with HST (e.g. of the cluster CL 1358+62; *Hoekstra et al. [1998]*). The HST's small point spread function (psf) is a major advantage over ground based observations. To recover the lensing signal one needs to correct for the effect of seeing and for objects with sizes comparable to the seeing psf (most faint galaxies), these corrections become very large, amplifying the uncertainty in the ellipticity due to photon noise. The SKA will offer many advantages for weak lensing studies: first, radio source densities similar to those in the Hubble Deep Field can easily be reached; most of these sources will be the same normal galaxies visible to the HST; secondly its psf will be <0.1 arcsec and extremely well-defined; thirdly the SKA's field-of-view will be large, about one square degree; this is sufficient to probe scales of some 20 Mpc on a side (at $z = 0.3$) per pointing. The SKA will therefore provide clean measurements of cluster mass surface densities as well as routine detection of the weak lensing signature due to large-scale structure. A review of the potential impact of the SKA on weak lensing is given by *Schneider [1999]*.

6.3 The future, or: what do we do with a thousand lenses?

There seems little doubt that the next decade will bring surveys which discover lenses not singly, but in battalions of a thousand or more. The obvious advantage of such an advance is an improvement in the random errors in determinations which rely on lens statistics, such as the measurement of Ω_Λ. Although by then both CMB measurements, such as that envisaged for the Planck satellite, and supernova searches should have yielded values for most cosmological parameters, experience has shown that an independent determination is always of value. The CMB experiments will also provide powerful constraints on H_0, but even here a second line of attack is highly desirable.

After the cosmological parameters have been determined, a sample of 1000 gravitational lenses will come into its own as a unique probe of mass distributions of galaxies across the range of luminosity, redshift and Hubble type. The classical method for determining the mass distribution of a galaxy is to find a kinematic tracer such as globular clusters and planetary nebulae and perform spectroscopy and detailed modelling to determine the velocity field and hence the distribution of mass. This is not a trivial process; only a handful of galaxies such as the Milky Way, M31 and M33 have been seriously tackled in this way. To do this at redshifts of 0.5 is beyond the reach even of 40-m class optical telescopes. By contrast, gravitational lensing is already beginning to give some indications of mass distributions at this redshift, though even with 63 galaxy-mass lenses, problems of small-sample statistics bedevil the subject. The major problem is that only very few lenses (about 5-10%) have sufficient observational constraints to allow very detailed model fitting. A large number of lenses will give a significant number of well-constrained mass models for each Hubble type and for a range of luminosities. And the ultimate prize is to be able to trace galaxy evolution; how galaxies form, merge and accrete matter during the entire history of the observable universe.

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