

# A Generalised Flux Function for 3D Reconnection

Anthony Yeates

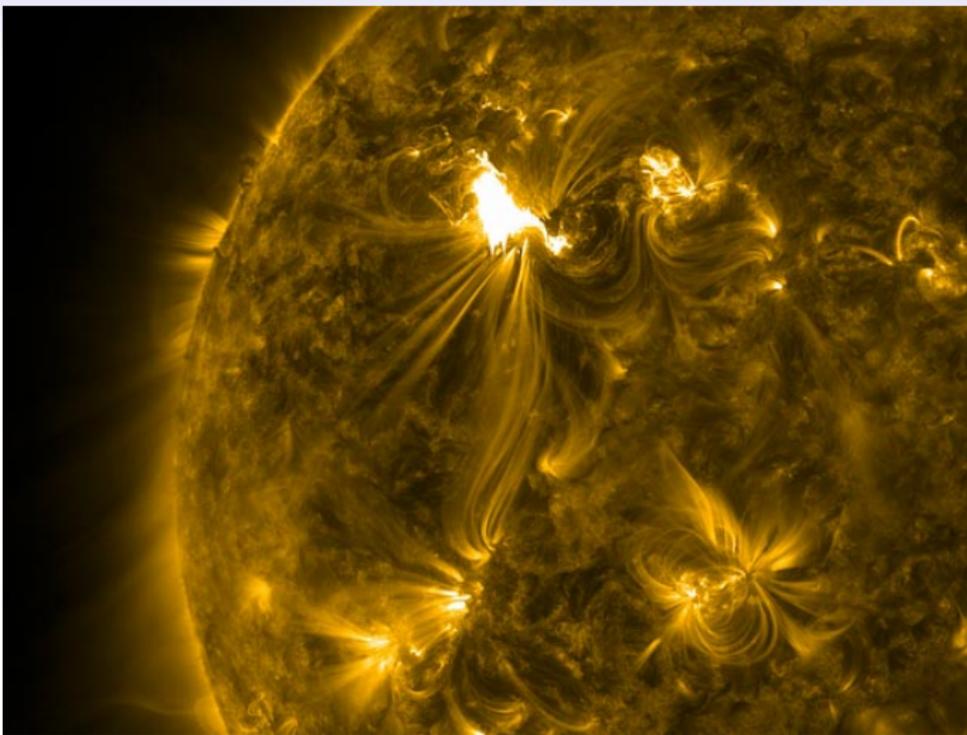
*with*

Gunnar Hornig (Dundee)

27th March 2012

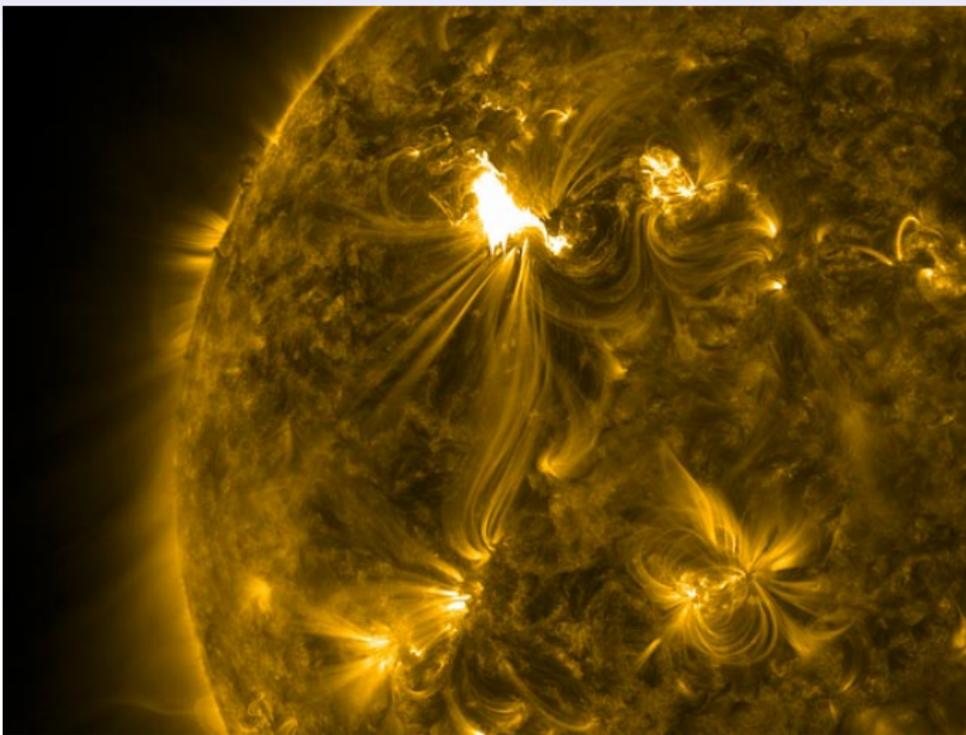
RAS National Astronomy Meeting 2012, Manchester





SDO/AIA, 6th March 2012

**Magnetic reconnection:** the change of connectivity of magnetic field lines in a non-ideal plasma.

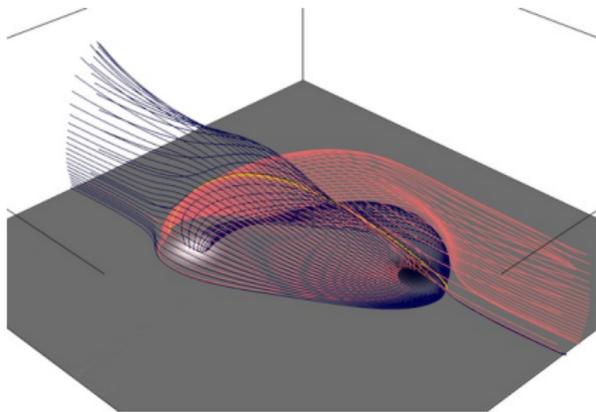


SDO/AIA, 6th March 2012

**Magnetic reconnection:** the change of connectivity of magnetic field lines in a non-ideal plasma. **\*\*Can occur anywhere in 3D\*\***

# Magnetic field partitions

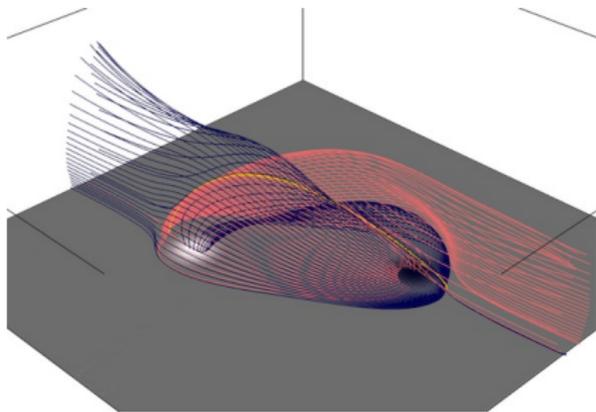
- 3D magnetic skeleton



[Parnell, Haynes & Galsgaard,  
*JGR* 2010]

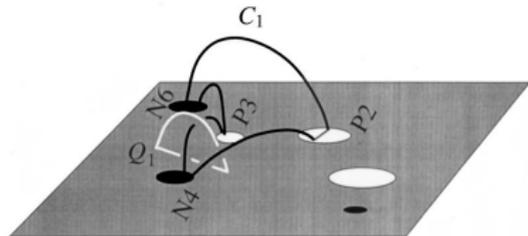
# Magnetic field partitions

- 3D magnetic skeleton



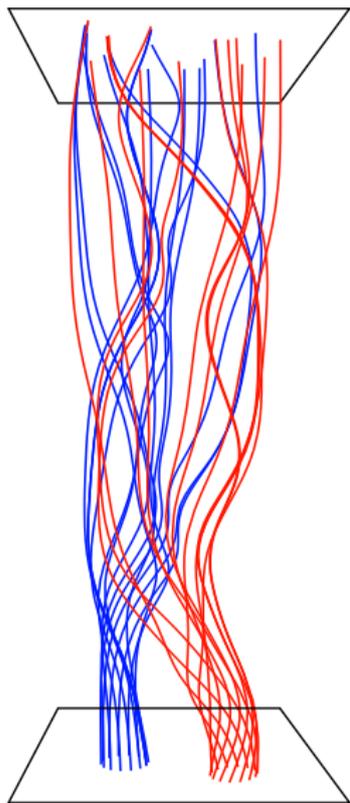
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- Boundary connectivity

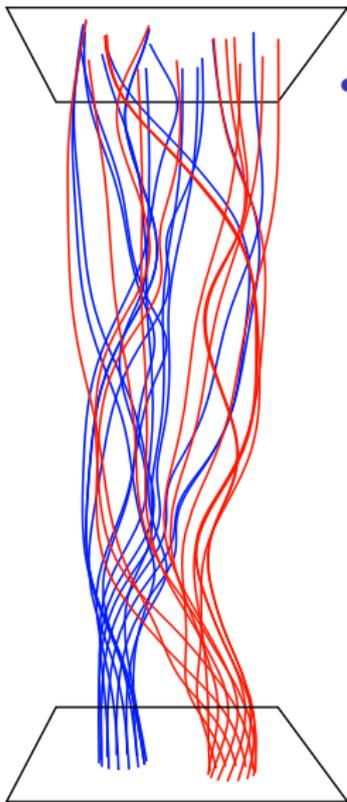


[Longcope, *ApJ* 2001]

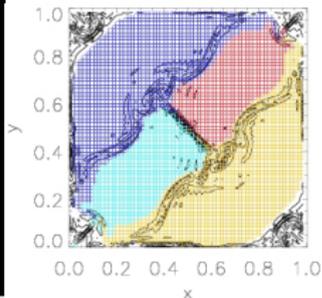
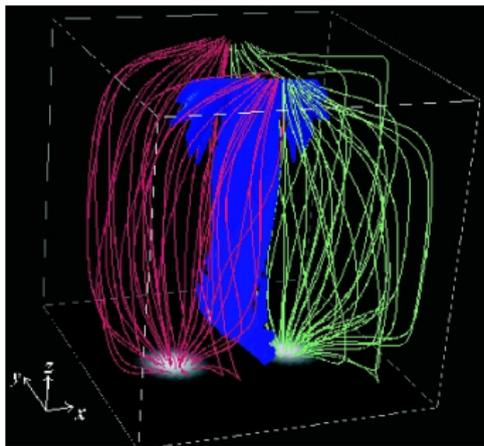
## How to partition a flux tube?



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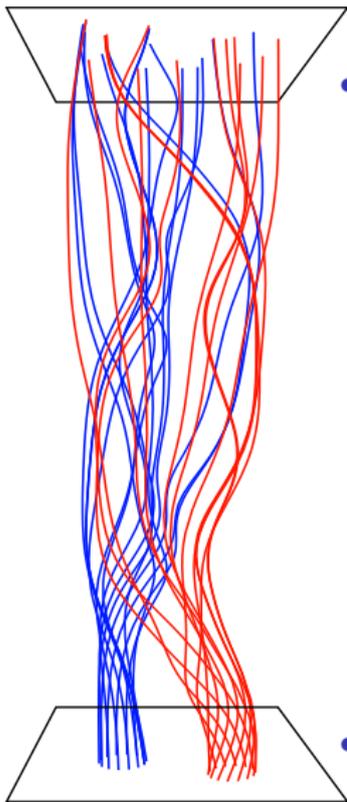


- Sometimes by boundary connectivity (**toroidal fluxes**):

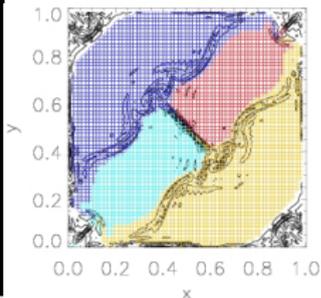
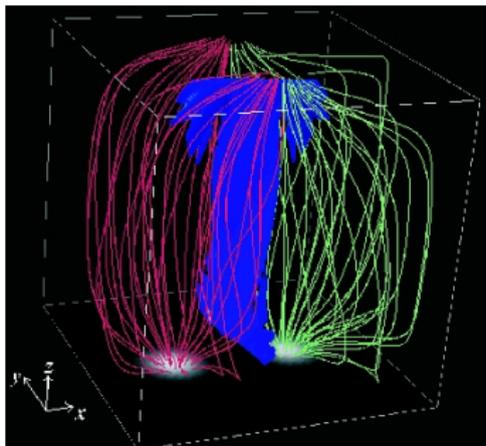


[Wilmot-Smith & De Moortel, *A&A* 2007]

## How to partition a flux tube?



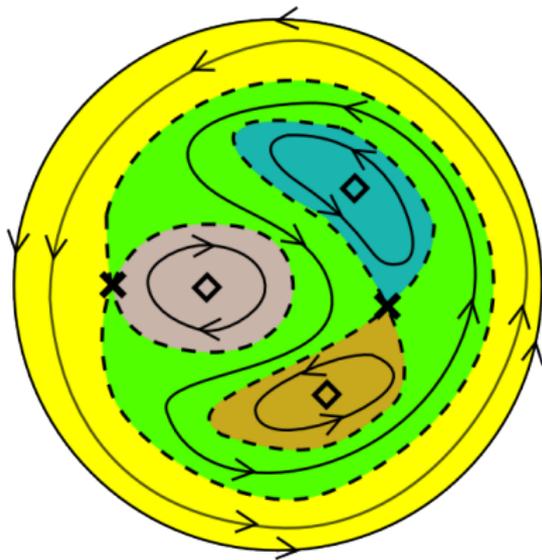
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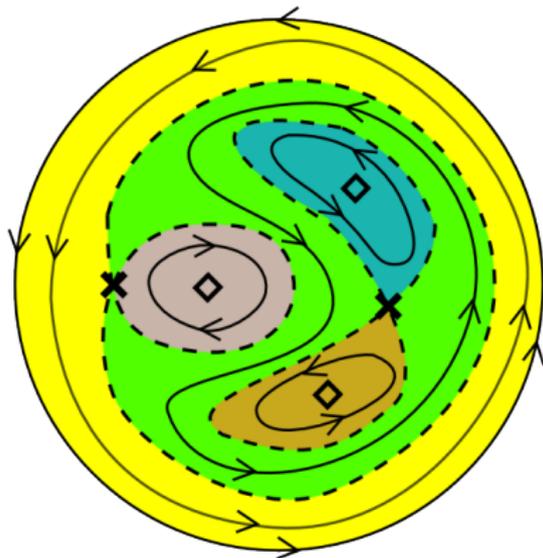
- What about **poloidal** (horizontal) fluxes?

## Poloidal fluxes in 2D



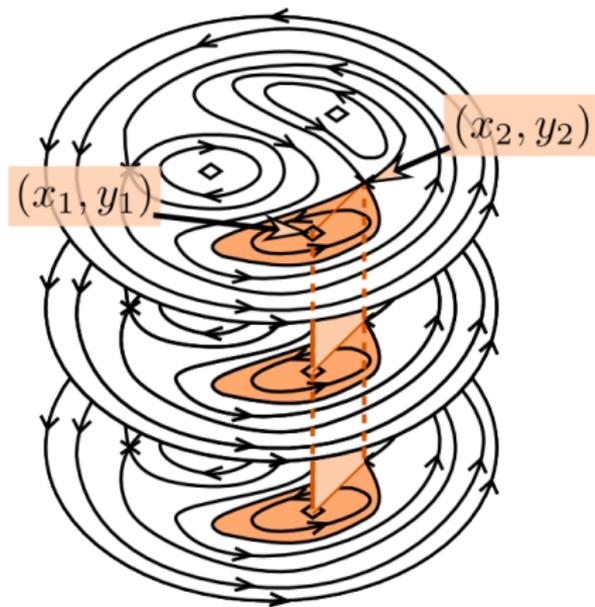
$$\mathbf{B}(x, y) = \nabla \times [A(x, y)\mathbf{e}_z]$$

## Poloidal fluxes in 2D

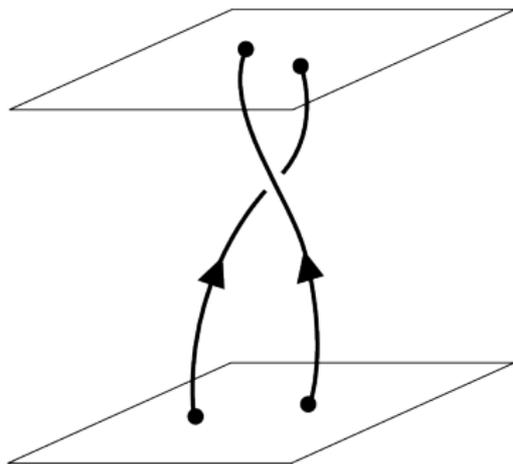


$$\mathbf{B}(x, y) = \nabla \times [A(x, y)\mathbf{e}_z]$$

$$\begin{aligned}\Phi &= \int \mathbf{B} \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l} \\ &= A(x_1, y_1) - A(x_2, y_2)\end{aligned}$$

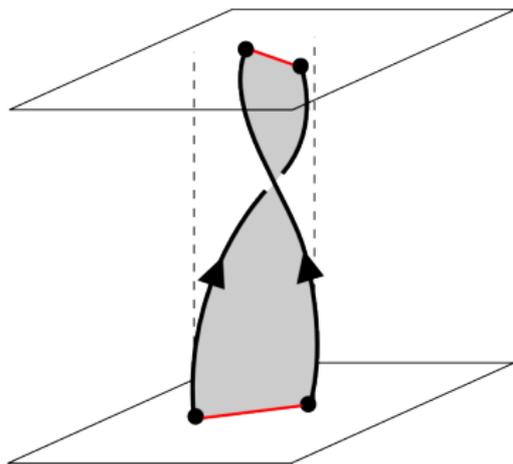


## Generalised flux function



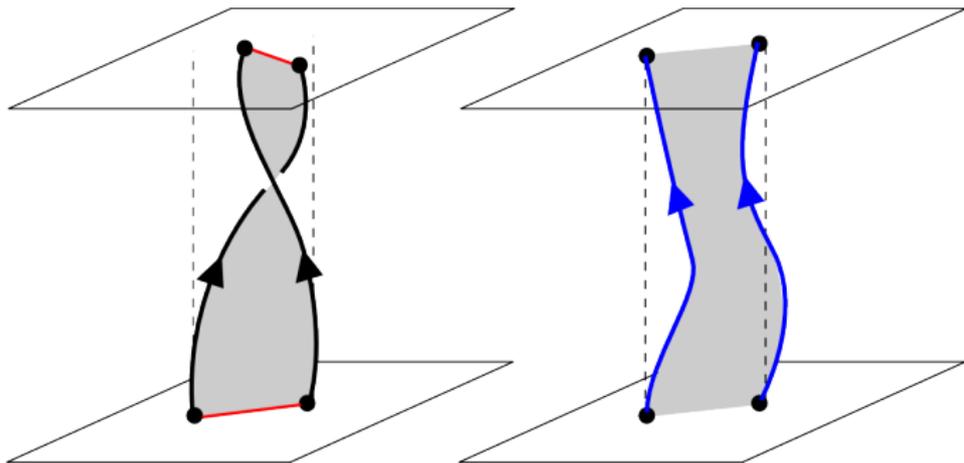
## Generalised flux function

$$\mathcal{A}(x, y) = \int_{(x, y)}^{\mathbf{F}_1(x, y)} \mathbf{A} \cdot d\mathbf{l}$$



## Generalised flux function

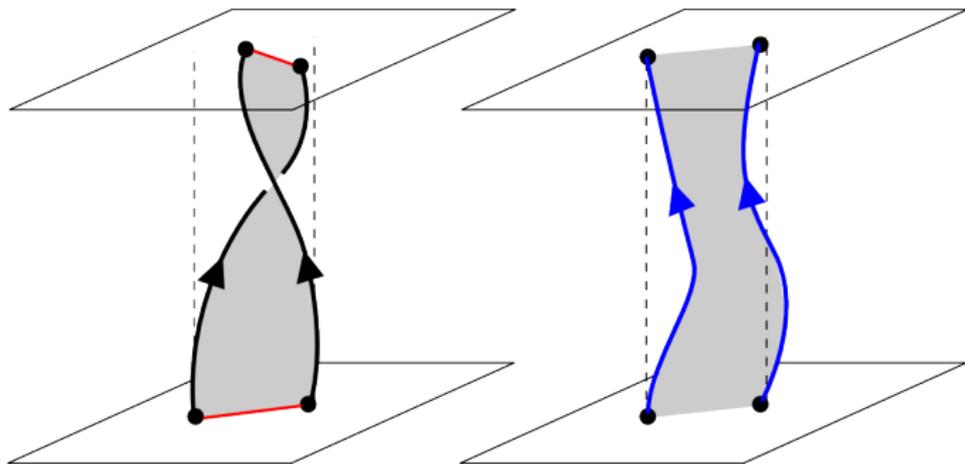
$$\mathcal{A}(x, y) = \int_{(x,y)}^{\mathbf{F}_1(x,y)} \mathbf{A} \cdot d\mathbf{l}$$



- $\mathcal{A}$  values at periodic points are **topological fluxes**.

## Generalised flux function

$$\mathcal{A}(x, y) = \int_{(x,y)}^{\mathbf{F}_1(x,y)} \mathbf{A} \cdot d\mathbf{l}$$

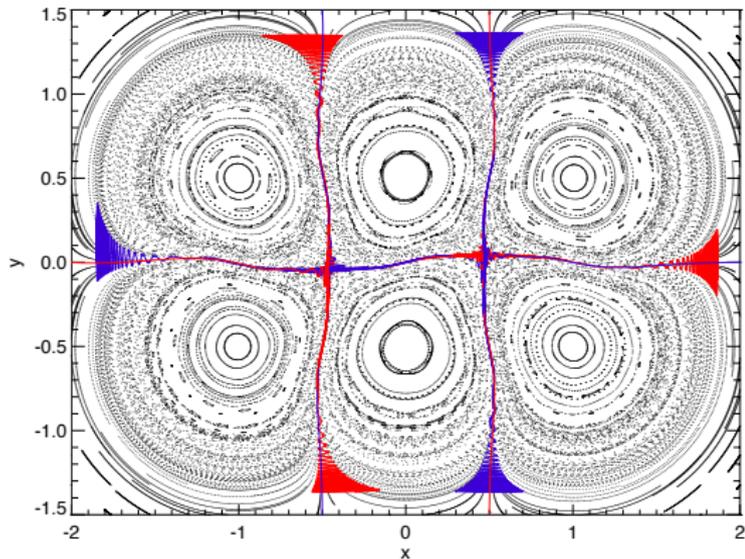
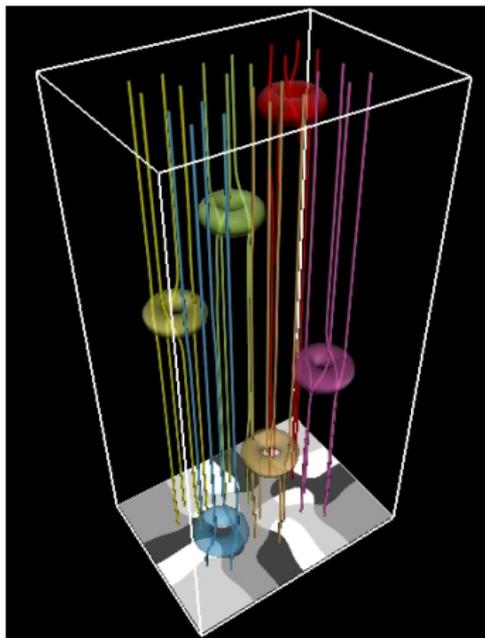


- $\mathcal{A}$  values at periodic points are **topological fluxes**.
- Gauge transformation  $\mathbf{A} \rightarrow \mathbf{A} + \nabla\psi$  gives

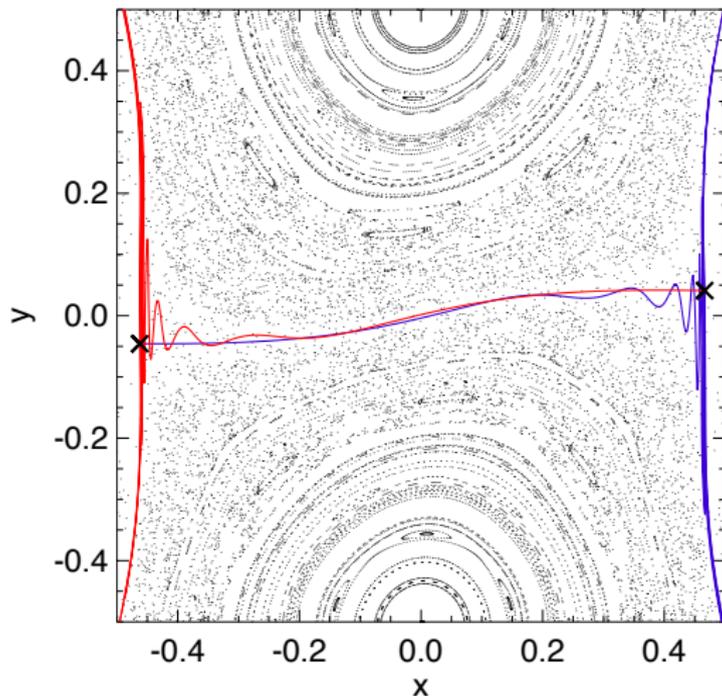
$$\mathcal{A}(x, y) \rightarrow \mathcal{A}(x, y) + \psi \Big|_{(x,y)}^{\mathbf{F}_1(x,y)} .$$

# Example

- Flux tube with six twist regions:

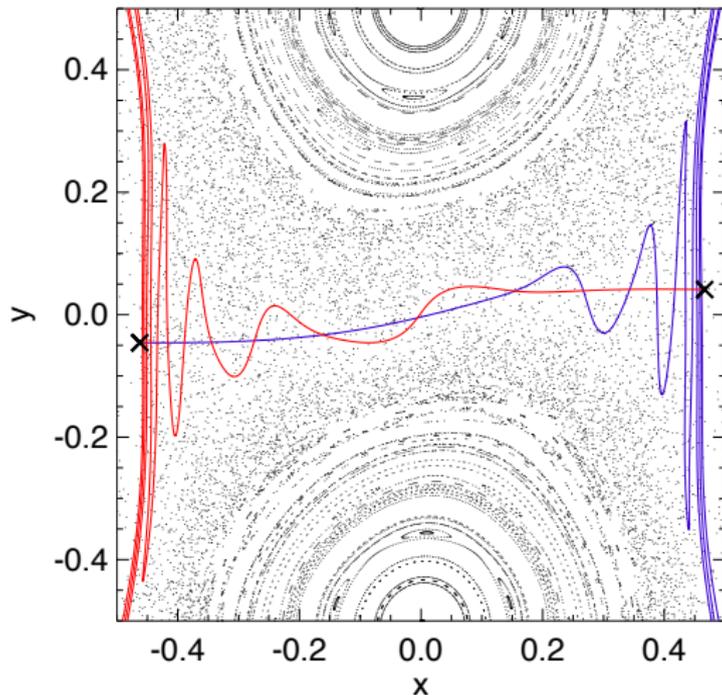


## Example



- (Un)stable manifolds calculated with method of [Krauskopf & Osinga, *J Comp Phys* 1998].

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# Conclusion

- **Partition** of poloidal fluxes in a non-zero flux tube.
- **Measured** by generalised flux function  $\mathcal{A}(x, y)$  at periodic points.
- **Well-defined** measure of global reconnection.

## Further details

- Yeates & Hornig, *Phys Plasmas* **18**, 102118 (2011).

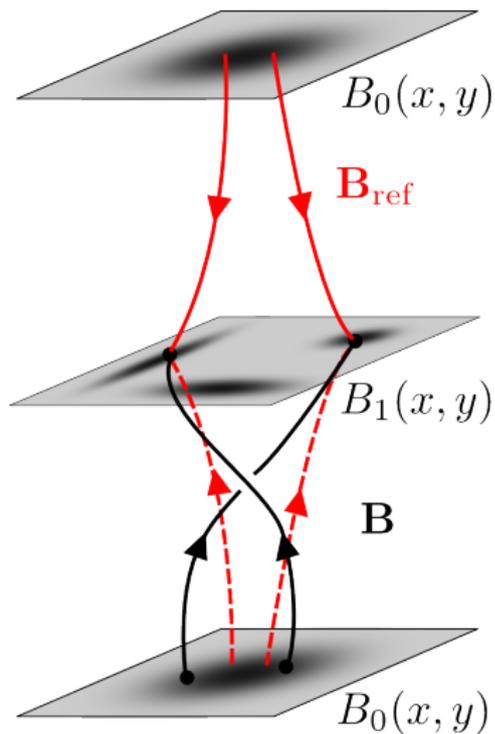
## Future:

- Measure reconnection in numerical relaxation simulations.
- Partition could be refined using higher period orbits.

<http://www.maths.dur.ac.uk/~bmjg46/>

# APPENDICES

## Non-periodic flux tubes



- Define topological fluxes with respect to a reference field.
- Suggest to choose the unique potential field.