An efficient iterative method to reduce eccentricity in numerical-relativity simulations of compact binary inspiral

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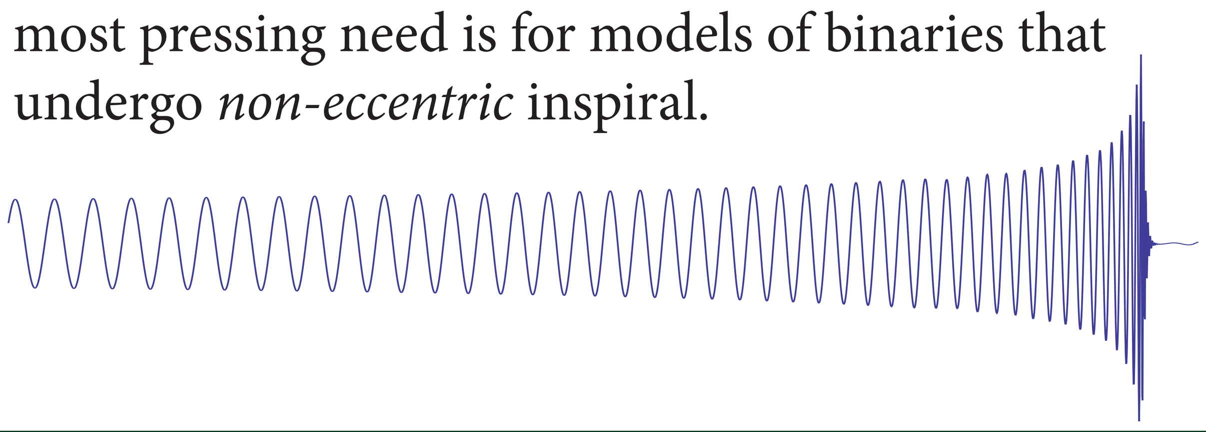
Motivation

Binary black-holes (BBHs) inspiral under the emission of gravitational-waves (GW). A large-scale effort is under way to produce models of these GW signals from the late inspiral, merger and ringdown of BBHs calibrated against large numbers of numerical relativity (NR) simulations. These waveform models will be essential to locate and interpret black-hole-binary GW signals in the data from second-generation laser-interferometric detectors such as Advanced LIGO.

For BBHs formed at separations on the order of astronomical units, even large initial eccentricities will be negligible once the GW enters the frequency band of ground based detectors. Therefore, the most pressing need is for models of binaries that undergo *non-eccentric* inspiral.



Initial parameters for quasi-circular inspiral of BBHs are only known approximately from post-Newtonian (PN) theory. For high mass-ratios and/or high spins PN initial parameters lead to eccentricities ~ 0.01 or higher and need to be decreased.

We present the first systematic procedure to reduce eccentricity for aligned-spin BBH simulations performed using the 'moving-puncture method', which is the most common in the field [1].

Method

The basic idea

Start with a short NR simulation that exhibits eccentricity, and a non-eccentric PN/EOB evolution of the same system. Adjust the initial momenta in the PN/EOB evolution until it exhibits eccentricity oscillations that agree with those in the NR waveforms, in both *amplitude* and *phase*. The inverse adjustment is then applied to the NR initial momenta, and a new NR simulation performed, and the process repeated.

Key quantities

Select an approximate PN/EOB model $\omega_{\mathrm{M}}(p_r, p_t; t)$ of the GW frequency as a function of the initial radial and tangential momenta (p_r, p_t).

Choose initial momenta (p_r^0, p_t^0) for a first NR simulation, such that $e_M(p_r^0, p_t^0) = 0$. Then $e_{\mathrm{NR}}(p_r^0, p_t^0) > 0$.

Define the GW and model frequency residuals relative to the quasi-circular model $\omega_{\mathrm{M}}(t) := \omega_{\mathrm{M}}(p_r^0, p_t^0; t)$

$$\mathscr{R}^i(t) = \omega^i_{\mathrm{NR}}(t) - \omega_{\mathrm{M}}(t)$$

$$\mathscr{R}^\lambda_{\mathrm{M}}(t) := \mathscr{R}_{\mathrm{M}}(\lambda_r, \lambda_t; t) = \omega_M(\lambda_r p_r^i, \lambda_t p_t^i; t) - \omega_{\mathrm{M}}(t)$$

A single iteration step

Choose the momentum scale factors (λ_r, λ_t) so that

$$\mathscr{R}^\lambda_{\mathrm{M}}(t) \approx \mathscr{R}^0(t)$$

with agreement in both the *amplitude* and *phase* of the residuals.

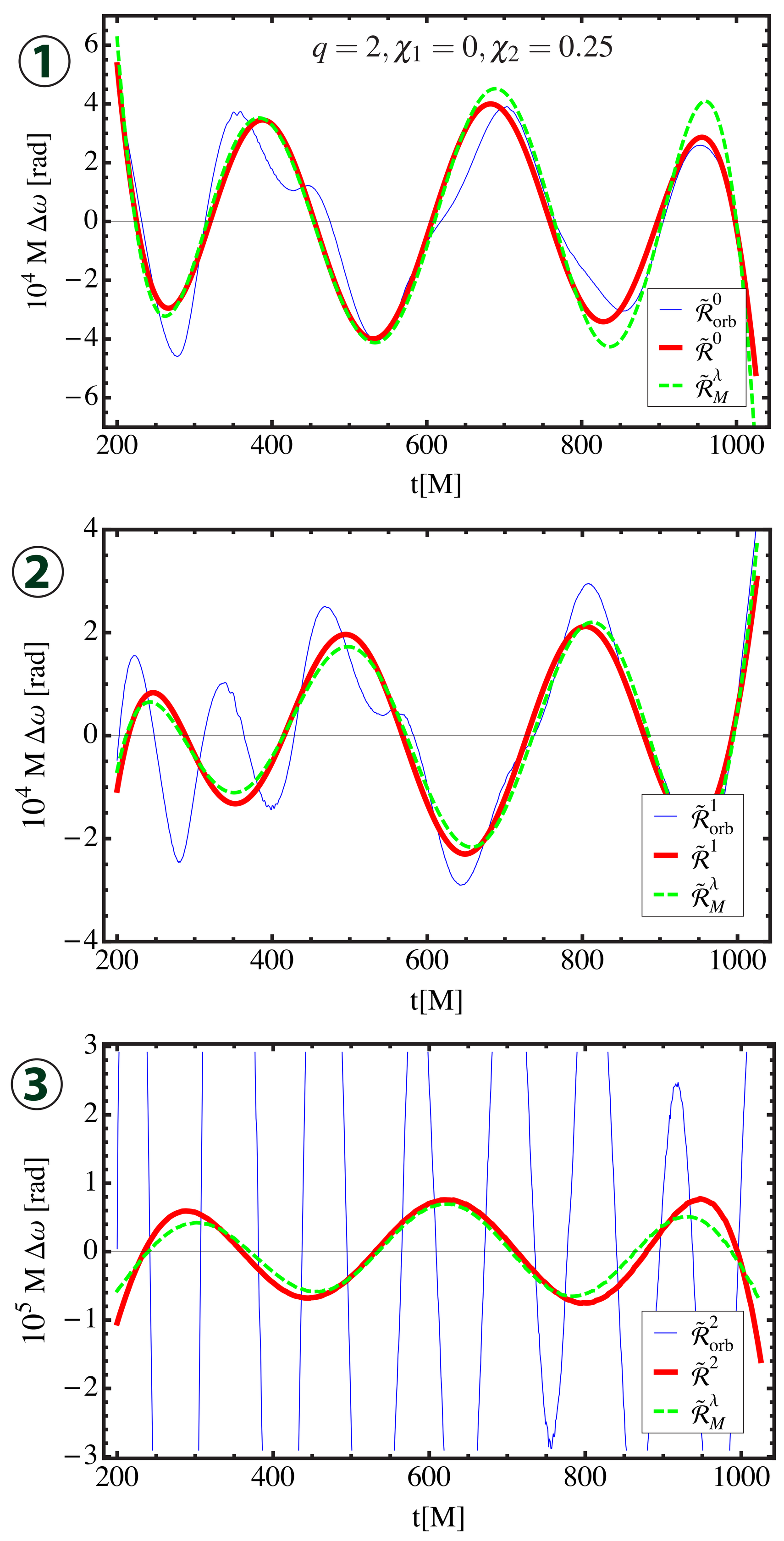
Produce updated initial momenta for the next NR simulation

$$p_r^1 = p_r^0/\lambda_r^0$$
$$p_t^1 = p_t^0/\lambda_t^0$$

with the expectation that $e^1_{\mathrm{NR}} < e^0_{\mathrm{NR}}$.

Iterate until eccentricity is below a desired target.

Eccentricity reduction example



Iteration	p_r	p_t	$e_{\phi,\mathrm{GW}}$	e_Ω	λ_r	λ_t
0	0.000758	0.11710	0.006	0.0045	1	1.0028
1	0.000758	0.11677	0.003	0.0029	1.15	0.999
2	0.000660	0.11689	0.0003	0.0013		

Fig. & Tab. 1: At each step the NR residual $\tilde{\mathscr{R}}^i$ is calculated from the filtered GW signal (red, thick), and for reference we also show $\tilde{\mathscr{R}}^i_{\mathrm{orb}}$ calculated from the orbital frequency (blue). *Dephasing* in the residuals has been removed with a fit. The best-match scale factors lead to the residuals $\tilde{\mathscr{R}}^\lambda_{\mathrm{M}}$ (green, dashed). GW phase eccentricities, $e_{\phi,\mathrm{GW}}(t) := [\phi_{\mathrm{GW}}(t) - \phi_{\mathrm{GW,fit}}(t)]/4$, show our progress while orbital frequency eccentricities $e_\Omega(t) := (\Omega(t) - \Omega_{\mathrm{fit}}(t))/(2\Omega_{\mathrm{fit}}(t))$ are gauge-limited.

Filtering the numerical GW signal

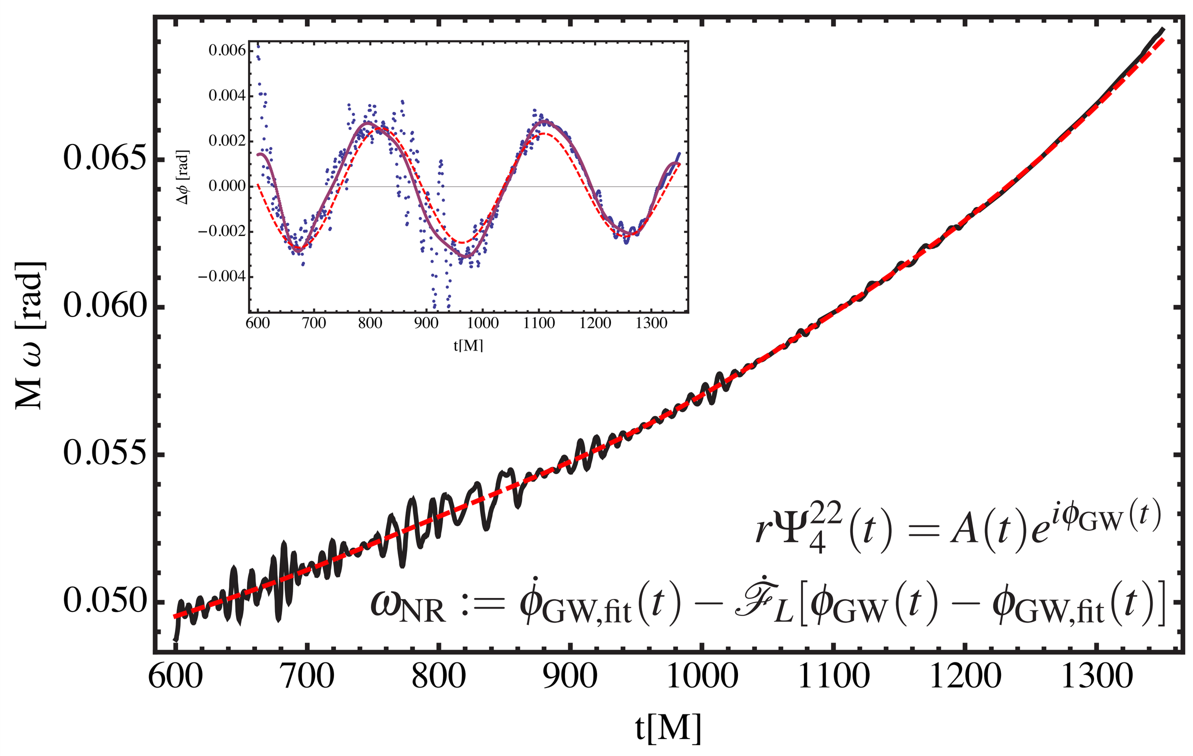


Fig. 2: A combination of fitting and filtering methods is used to produce a cleaned GW frequency while preserving the delicate eccentricity oscillations.

Inspiral phase differences

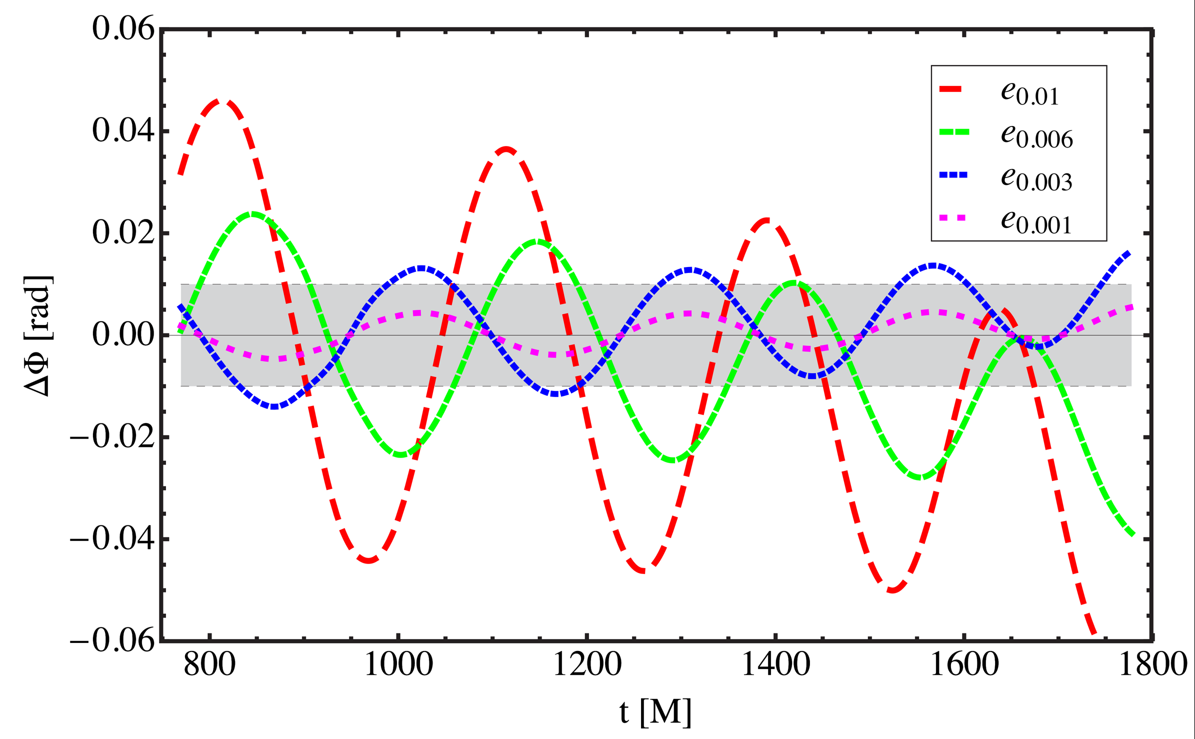


Fig. 3: Phase differences from Ψ_4 with respect to the $e_{\phi,\mathrm{GW}} = 0.0003$ simulation. A stringent NR phase error requirement of ±0.01 rad is indicated by the shaded region.

Target Eccentricity

In studies of the *mismatch* between hybrid PN+NR waveforms, we have seen no evidence that eccentricities as high as 0.01 in the final ~10 orbits will have any noticeable influence on GW searches or parameter estimation in the Advanced detector era. This is somewhat surprising, since at this level the eccentricity is visible by eye in the waveform.

The dominant numerical error that accrues during the inspiral of a BH binary is the phase error. To be conservative we prefer to lower the eccentricity to a level where the *eccentricity-induced oscillations and secular drifts in the GW phase* are well below the *numerical phase errors* in our simulations. We choose a tolerance of $e \sim 10^{-3}$, which produces oscillations in the GW phase with an amplitude of $\Delta\phi \sim 0.01$ rad during inspiral (see Fig. 3), and an accumulated phase offset through merger and ringdown of less than 0.2 rad. This is well within our NR phase errors.

Conclusion

A subset of our method was already presented in previous work [2, 3]. Because our method is applied to the GW signal, it can be adapted to any evolution method, and is not limited to moving-puncture simulations. It could also be adapted to other compact binary simulations, for example neutron-star (NS-NS) binaries, and black-hole--neutron-star (BH-NS) binaries.

Our method can typically reduce eccentricity below 0.001 in one or two iteration steps using a semi-automated procedure to obtain the scale factors.
We will consider an extension of our method to precessing binaries in future work.

References

[1] M. Pürrer, et al, arXiv:1203.4258, (2012)
[2] S. Husa, et al, PRD, 77, 044037 (2008)
[3] M. Hannam, et al, PRD, 82, 124008 (2010)