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Discrimination between cosmological constant, quintessence, and modified gravity

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In what concerns dark energy, the ultimate goal of space and ground based surveys is discriminating between various candidate models and a cosmological constant. Here we report results of a work on finding the best set of parameters and measurables for this purpose. In particular we show that independent measurements of cosmological parameters of homogeneous component - the background cosmology - and anisotropies are necessary, notably for distinguishing between interacting quintessence models and modified gravity. This put in evidence for the advantage of surveys able to observe Large Scale Structures as well as a large number of supernovae. The role of CMB measurements for improving discrimination will be mentioned too. We also propose quantities that determine the discrimination power of a survey independent of observed proxy.

Introduction

- Two categories cover majorities of dark energy models:
  1. Models based on a scalar field such as quintessence and its variants like interacting quintessence models, kessence, varying neutrino mass models, etc.
  2. Modified gravity models in which dark energy is explained as the deviation from Einstein gravity such as scalar-tensor models, in particular $f(R)$ gravity, Chameleon, DGP, etc.
- The main task of cosmologists today is distinguishing between these models and a cosmological constant which is yet the best fit to the data.

Interaction between dark energy and matter

- In majority of these models dark energy can be presented by a scalar field.
- In modified gravity models in Einstein frame, the scalar field interacts with matter.
- In quintessence models addition of an interaction between dark matter and dark energy can solve issues such as coincidence problem and can naturally produce $w_{de} < -1$.
- Phenomenologically, interactions can be described as the violation of energy-momentum conservation of single components.

Phenomenology of interactions

- Interaction in modified gravity: In Einstein frame it is presented by non-minimal interaction of an scalar field and matter with curvature. Thus, the violation of energy-momentum is always proportional to its trace:

$$T^{\mu\nu}_{m\ ;\nu} = -C(\varphi)T^{\nu}_{m\nu}\partial^{\mu}\varphi \quad T^{\mu\nu}_{de;\nu} = C(\varphi)T^{\nu}_{m\nu}\partial^{\mu}\varphi$$

- Interaction in quintessence models: Some authors have used $Q^{\mu} \propto T_m u^{\mu}_m$ for interacting quintessence models (This is Modified Gravity in our classification !)
- In Quintessence interaction has a microscopic origin $\Longrightarrow$ Quantum/particle physics.
- Interactions in Quintessence models can be more diverse:
  - Scattering
  - Production by decay of dark matter

Macroscopic description of interactions

- There are few ways for writing microscopic interaction as a function of macroscopic quantities:
- Fluid description: It depends on the details of the self-interaction potential of the scalar field. This is not very interesting for parametrization of observations
- Field equation and energy-momentum conservation are related and do not include quantum effects such as decay, annihilation, and scattering.
- Boltzmann equation: It provides a connection between microscopic and macroscopic processes and quantities and Quantum effects are included in a collisional term.
- Disadvantage: One needs to measure distributions $f(x, p)$ in spacetime and momentum spaces - the phase space.
- Conclusion: Interactions can be described only approximately as a function of spacetime coordinates.

Approximate description of interaction for quintessence models

- We can use the thermodynamical definition of energy-momentum and number density:

$$n^{\mu}_i = \int d\bar{p}\ p^{\mu}f_i(p,x), \quad d\bar{p} \equiv \frac{g}{(2\pi)^3}d^4p\,\delta(E^2 - \tilde{p}^2 - m_i^2)$$
$$T^{\mu\nu}_i = \int d\bar{p}\ p^{\mu}p^{\nu}f_i(p,x)$$

- Approximate interactions including decay and scattering:

$$T^{\mu\nu}_{m\ ;\nu} \approx -L_m n^{\mu}_m + A_{ms}n^{\mu}_m u_{\varphi\rho}n^{\rho}_{\varphi} \equiv Q^{\mu}_m$$
$$T^{\mu\nu}_{\varphi\ ;\nu} \approx L_{\varphi}n^{\mu}_m + A_{\varphi s}n^{\mu}_{\varphi}u_{m\rho}n^{\rho}_m \equiv Q^{\mu}_{\varphi}$$
$$u^{\mu} \equiv \frac{n^{\mu}}{|n|}, \quad n^{\mu} \approx \frac{u_{\nu}T^{\mu\nu}}{m} = \frac{\rho u^{\mu}}{m}$$

- L_m and A_{ms} are decay and scattering couplings.
- Self-annihilation is only important in nonlinear high density regions and is neglected.

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Parametrization of homogeneous cosmology

- Friedman equation:

$$\frac{H^2}{H_0^2} = \sum_i \Omega_i(1+z)^{3\gamma_i} + \sum_i \Omega_i(\mathcal{F}_i(z) - 1)(1+z)^{3\gamma_i} + \Omega_{de}(1+z)^{3\gamma_{de}(z)}$$
$i =$ cdm, baryons, hot matter, curvature

- Density variation:

$$B(z) \equiv \frac{1}{3(1+z)^2\rho_0}\frac{d\rho}{dz} = \sum_i \Omega_i\left(\gamma_i\mathcal{F}_i(z) + (1+z)\frac{d\mathcal{F}_i}{dz}\right)(1+z)^{3(\gamma_i-1)} + \Omega_{de}(w(z)+1)(1+z)^{3(\gamma_{de}(z)-1)}$$

- $H(z)$&$B(z)$ can be measured from supernovae and Baryon Acoustic Oscillations (BAO).

Fiducial models and dark energy measurements

- When a fiducial ΛCDM cosmology is considered, measured quantities are functions of physical parameters:

$$\Omega^{(H)}_{eff} = \Omega_{de}, \qquad \gamma^{(H)}_{eff}(z=0) = \gamma_{de}(z=0)$$
$$\gamma^{(H)}_{eff}(z) = \frac{\log\left(\sum_i \frac{\Omega_i}{\Omega_{de}}(\mathcal{F}_i(z) - 1)(1+z)^{3\gamma_i} + (1+z)^{3\gamma_{de}(z)}\right)}{3\log(1+z)}$$
$$\Omega^{(B)}_{eff}(w^{(B)}_{eff}(z)+1)(1+z)^{3\gamma^{(B)}_{eff}(z)} = \sum_i \Omega_i\left(\gamma_i(\mathcal{F}_i(z) - 1) + (1+z)\frac{d\mathcal{F}_i}{dz}\right)$$
$$(1+z)^{3\gamma_i} + \Omega_{de}(w(z)+1)(1+z)^{3\gamma_{de}(z)} = (1+z)A(z)$$
$$A(z) \equiv B(z) - \sum_i \Omega_i\gamma_i(1+z)^{3(\gamma_i-1)}$$

- Superscript (H)&(B) mean measured from Hubble constant $H(z)$ and from density evolution $B(z)$ respectively.

Efficiency of discrimination between interacting and non-interacting models

- When some of coefficients $\mathcal{F}_i(z) \neq 0$, in general: $\Omega^{(H)}_{eff} \neq \Omega^{(B)}_{eff}$, $\gamma^{(H)}_{eff}(z) \neq \gamma^{(B)}_{eff}(z)$.
- Discriminating efficiency Θ is defined as:

$$\Theta \equiv \frac{\Omega^{(A)}_{eff}(w^{(A)}_{eff}(z)+1)(1+z)^{3\gamma^{(A)}_{eff}(z)} - \Omega^{(H)}_{eff}(w^{(H)}_{eff}(z)+1)(1+z)^{3\gamma^{(H)}_{eff}(z)}}{\Omega^{(H)}_{eff}(w^{(H)}_{eff}(z)+1)(1+z)^{3\gamma^{(H)}_{eff}(z)}}$$

- For an estimated error of $\sim 1\%$ in the measurement of $H(z)$ and power spectrum at large scales that can be related to $B(z)$, Euclid is able to discriminate between an interacting dark energy model and ΛCDM up to $\Theta \lesssim 5\%$.

Examples of coefficients $\mathcal{F}_i(z)$

- In general, evolutions of coefficients $\mathcal{F}_i$ in interacting quintessence and modified gravity models are different.
- $f(R)$-Modified Gravity

$$\mathcal{F}_i(z) = \left(\frac{1 + f_R(R(z=0))}{1 + f_R(R(z))}\right)^{\frac{3(1-w_i)}{2}}, \quad w_m = 0, \quad w_h = \frac{1}{3}, \quad w_k = -\frac{1}{3}$$

- A decaying dark matter + ΛCDM:

$$\mathcal{F}_m(t) \approx \exp(\frac{\tau_0 - t}{\tau}) + (1+z)\left(1 - \exp(\frac{t_0 - t}{\tau})\right), \quad \tau \gg \tau_0$$
$$\mathcal{F}_b = \mathcal{F}_h = 1 \quad \gamma(z) = 0 \quad \mathcal{F}_m(z) > \mathcal{F}_m(z=0)$$

- A decaying dark matter producing a quintessence field:

$$\bar{\rho}_i(z) = \bar{\rho}_i(z_0)(1+z)^{3(1+w_i)}\exp\left(L(\tau(z) - \tau(z0)) + A_{si}\int dz\frac{\bar{\rho}_{\varphi}(z)}{(1+z)H(z)}\right)$$
$$\mathcal{F}_i(z) = \exp\left(-L(\tau(z) - \tau(z_0)) + A_{si}\int dz\frac{\bar{\rho}_{\varphi}(z)}{(1+z)H(z)}\right) \approx 1 + L(\tau(z_0) - \tau(z)) + A_{si}\int_{z_0}^{z} dz\frac{\bar{\rho}_{\varphi}(z)}{(1+z)H(z)},$$

All $\mathcal{F}_i(z)$ are expected to be very close 1.

- A combination of measurements of background cosmological parameters and perturbations must be used for discriminating between quintessence and modified gravity models.

How do we measure dark energy parameters ?

- Supernovae are sensitive only to the homogeneous component of matter distribution - the background cosmology.
- Evolution of perturbations depends on dark energy dominantly through its dependence on the homogeneous component:

$$\delta_m(\bar{\rho}_{de}(z) + \delta\rho_{de}(z,k), \cdots) \approx \delta_m(\bar{\rho}_{de}(z), \cdots) + \partial\delta_m/\partial\bar{\rho}_{de}\,\delta\rho_{de}(z,k) + \ldots$$

- Nonetheless, for discrimination between models we need to measure both homogeneous parameters and evolution of fluctuations independently.

Interaction currents in linear perturbations

- Metric:

$$ds^2 = -a^2(1+2\psi)d\eta^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- Energy-momentum conservation for each component:

$$\delta\rho'_m + 3\mathcal{H}(\delta\rho_m + \delta P_m) - (3\phi' + ik_jv^j)(\bar{\rho}_m + \bar{P}_m) = -\delta Q^m_0$$
$$[(\bar{\rho}_m + \bar{P}_m)v_i]' + 4\frac{a'}{a}(\bar{\rho}_m + \bar{P}_m)v_i + ik_i\delta P_m + ik_j\Pi^j_i + ik_i\psi(\bar{\rho}_m + \bar{P}_m) = \delta Q^m_i$$

- Interacting quintessence models (we neglect annihilation and production):

$$\delta Q^0_m = (\frac{L}{a} + \frac{A_s}{a^2}\varphi')\delta\rho_m + (\frac{A_s}{a^2}\delta\varphi' + \frac{L}{a}\psi)\bar{\rho}_m$$
$$\delta Q^i_m = (\bar{\rho}_m + \bar{P}_m)(L - \frac{A_s}{a}\varphi')\frac{v^i}{a} - \bar{P}_m(L\frac{v^j}{a} + A_s\frac{\bar{\varphi}'}{a}v^i)\delta^i_j$$

- Modified gravity:

$$\delta Q^0 = C\left[(\bar{\rho}_m - 3\bar{P}_m)\frac{\psi}{a} - \frac{1}{a}(\delta\rho_m - 3\delta P_m)\right]$$
$$\delta Q^i = -\frac{Cv^i}{a}(\bar{\rho}_m - 3\bar{P}_m)$$

Potentials in Einstein frame

- For discriminating between quintessence and modified gravity models its better to use Einstein frame because both models can be formulated in the same manner. Only their interaction terms would be different.

$$\phi = \psi = \frac{4\pi G\bar{\rho}_m}{k^2}\left(\delta_m + 3(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right) + \Delta\psi$$
$$\Delta\psi = \frac{4\pi G}{k^2}\left(\delta\rho_{\varphi} - 3\mathcal{H}\delta\varphi(\bar{\rho}_{\varphi} + \bar{P}_{\varphi})^{\frac{1}{2}}\right)$$
$$\phi' = -\frac{4\pi G\bar{\rho}_m\mathcal{H}}{k^2}\left(\delta_m + (3 + \frac{k^2}{\mathcal{H}^2})(1+w_m)\frac{\mathcal{H}\theta_m}{k^2}\right) + \Delta\phi'$$
$$\Delta\phi' = -\mathcal{H}\Delta\psi + 4\pi Ga^2\delta\varphi(\bar{\rho}_{\varphi} + \bar{P}_{\varphi})^{\frac{1}{2}}$$

- Anisotropic shear is neglected.

Parametrization of Deviation from ΛCDM

- Both modified gravity and quintessence modify ϕ and ψ. Thus, a deviation of ϕ and ψ from ΛCDM does not necessarily mean a Modified Gravity.
- Deviation of potentials from ΛCDM:

$$\Delta\psi = \frac{4\pi G\bar{\rho}_m}{k^2}(\epsilon_0 - 3\epsilon_1) \qquad \Delta\phi' = -\frac{4\pi G\bar{\rho}_m\mathcal{H}}{k^2}\left(\epsilon_0 - (3 + \frac{k^2}{\mathcal{H}^2})\epsilon_1\right)$$
$$\epsilon_0 \equiv \frac{\delta\rho_{\varphi}}{\bar{\rho}_m}, \quad \epsilon_1 \equiv \frac{\mathcal{H}(\bar{\rho}_{\varphi} + \bar{P}_{\varphi})^{\frac{1}{2}}\delta\varphi}{\bar{\rho}_m}$$

- ϵ_0 depends only on the dynamics and ϵ_1 on the kinematics of the scalar field and their measurement gives a direct insight on the physics of dark energy.
- In Einstein frame $\Phi \equiv \phi + \psi = 2\phi = 2\psi$ and can be measured directly by lensing.

Parametrization of Growth Rate

- Evolution equation of growth rate can be parametrize as the following:

$$f'\mathcal{H} + f(\mathcal{H}' + \mathcal{H}^2) + f^2\mathcal{H}^2 + 3(C^2_{sm} - w_m)(\mathcal{H}' + f\mathcal{H}^2) + 3(C^2_{sm} - w_m)\mathcal{H}^2 + \frac{3}{2}\Omega_m(1+w_m)^2\mathcal{H}^2 + k^2C^2_{sm} + E_0f\mathcal{H} + E_1k^2 + E_2\mathcal{H} + E_3\mathcal{H}^2 + E_4 = 0$$

- Coefficients $E_i = 0 \quad i = 0,\ 4$ are functions of ϵ_0 and ϵ_1. For ΛCDM they are all null.
- In quintessence models $E_1 \equiv 0$. Other parameters depend differently on ϵ_0 and ϵ_1 in quintessence and modified gravity.
- For ΛCDM $f \approx \Omega^{\gamma}_m$. Because deviation from ΛCDM is small, an approximate solution for interacting dark energy models can be obtained from linearization of evolution equation of f and can be used for comparing with observations.
- Euclid can measure E_i coefficients with $\sim 10\%$ accuracy.

Outline

- It would be very difficult to distinguish between modified gravity models and a quintessence model with only elastic scattering with dark matter.
- In this case, if quintessence field interacts only with CDM and not with baryon, separate investigation of the two component i.e. separate correlation functions and power spectrum can be useful.
- Euclid is able to observe both SN and LSS with unprecedented accuracies.
- Interacting dark energy models have also implications for processes and quantities not directly observed by Euclid such as the density and nature of hot matter, high energy neutrinos, etc. Combination of Euclid data with observations of other facilities such as Air shower detectors and data from high energy particle accelerators should improve our discrimination power between various dark energy models and help to understand particle physics of dark matter and neutrino physics.

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