Effective field theory for perturbations in dark energy & modified gravity

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The problem...

- General Relativity + FRW + standard model particles + observational data: ***inconsistent***
  - ... invent dark energy ~70%.
- Or perhaps GR is not the right gravitational theory for cosmological scales...
  - c.f. Newton & Mercury/Sun system
- **Models of dark energy:** Λ, quintessence, k-essence, elastic dark energy, ...
- **Modified gravity:** F(R), Horndeski, galileon, Gauss-Bonnet, Aether, TeVeS, ...
  - MG... obtain different gravitational potential for the same matter content
- Lagrangian engineering

Generalized gravitational field equations

- All modified gravity & dark energy theories have gravitational field equations which can be written as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + U_{\mu\nu}$$

- Stems from an action:

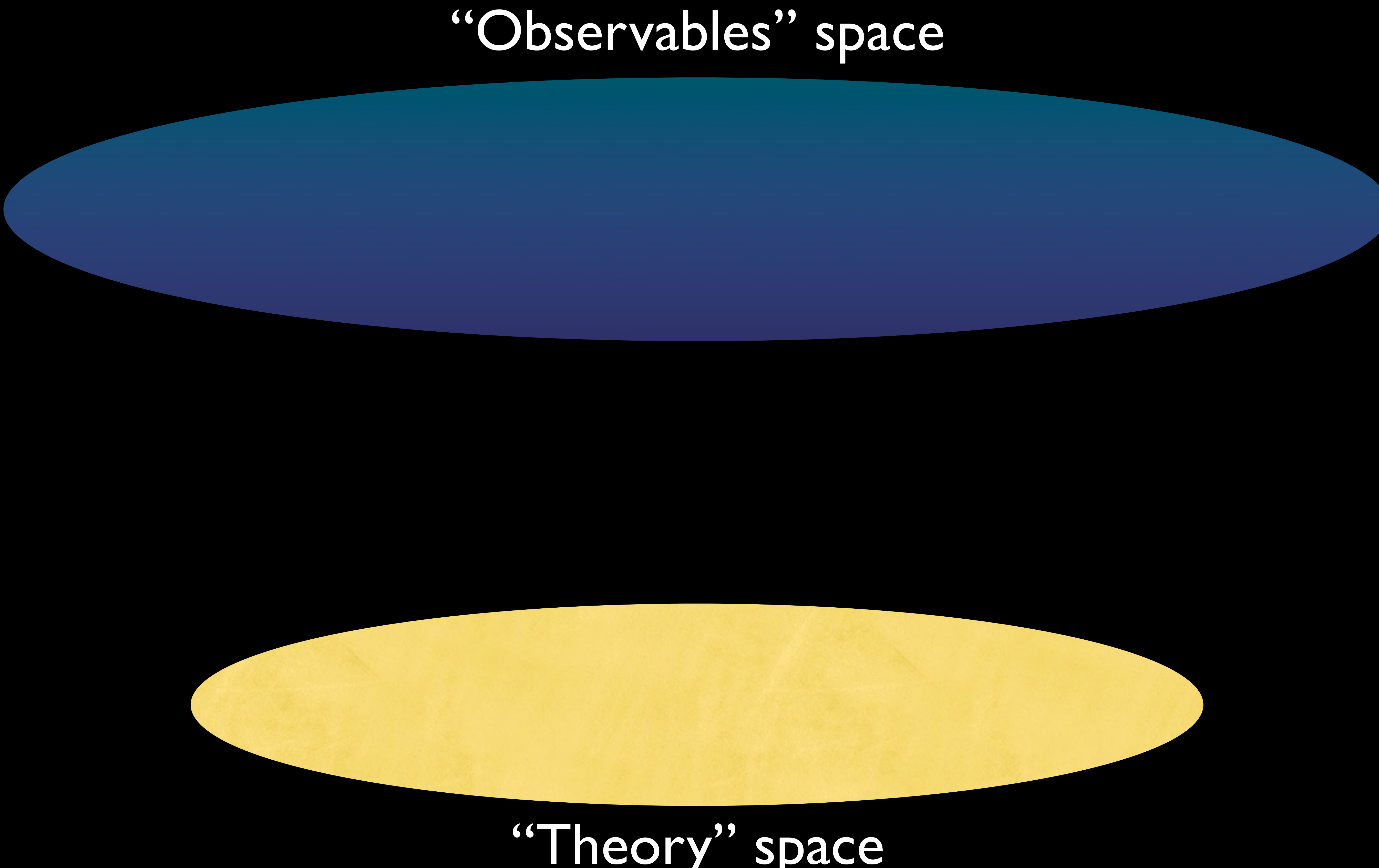
$$\mathcal{L}_{\rm grav} = \mathcal{L}_{\rm GR} + \mathcal{L}_{\rm known} + \mathcal{L}_{\rm dark}$$

- At perturbed order: structures...

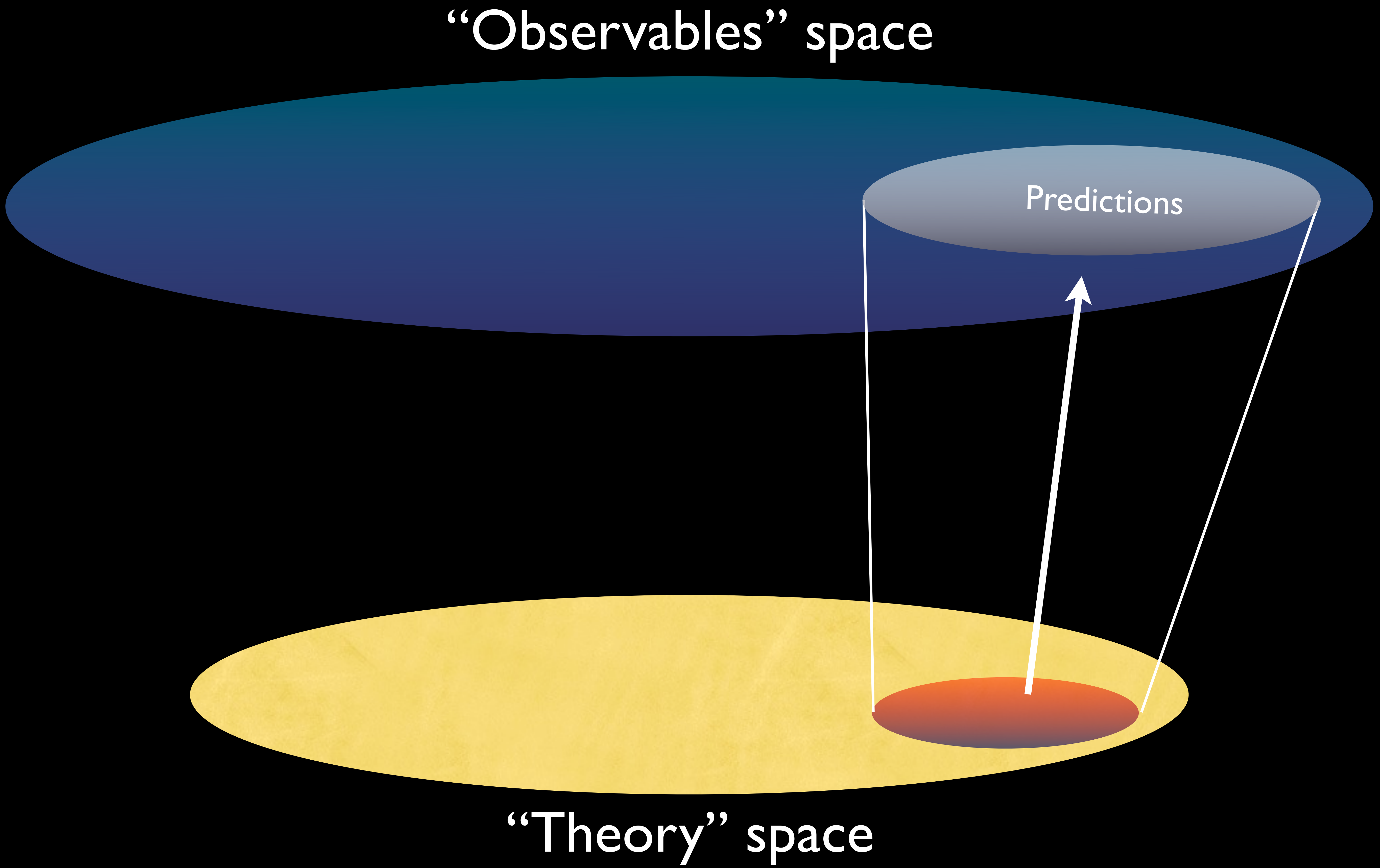
$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}$$

- Q: How do we write down the allowed, consistent modifications to the gravity field equations? ***A: Lagrangian for the dark sector perturbations***

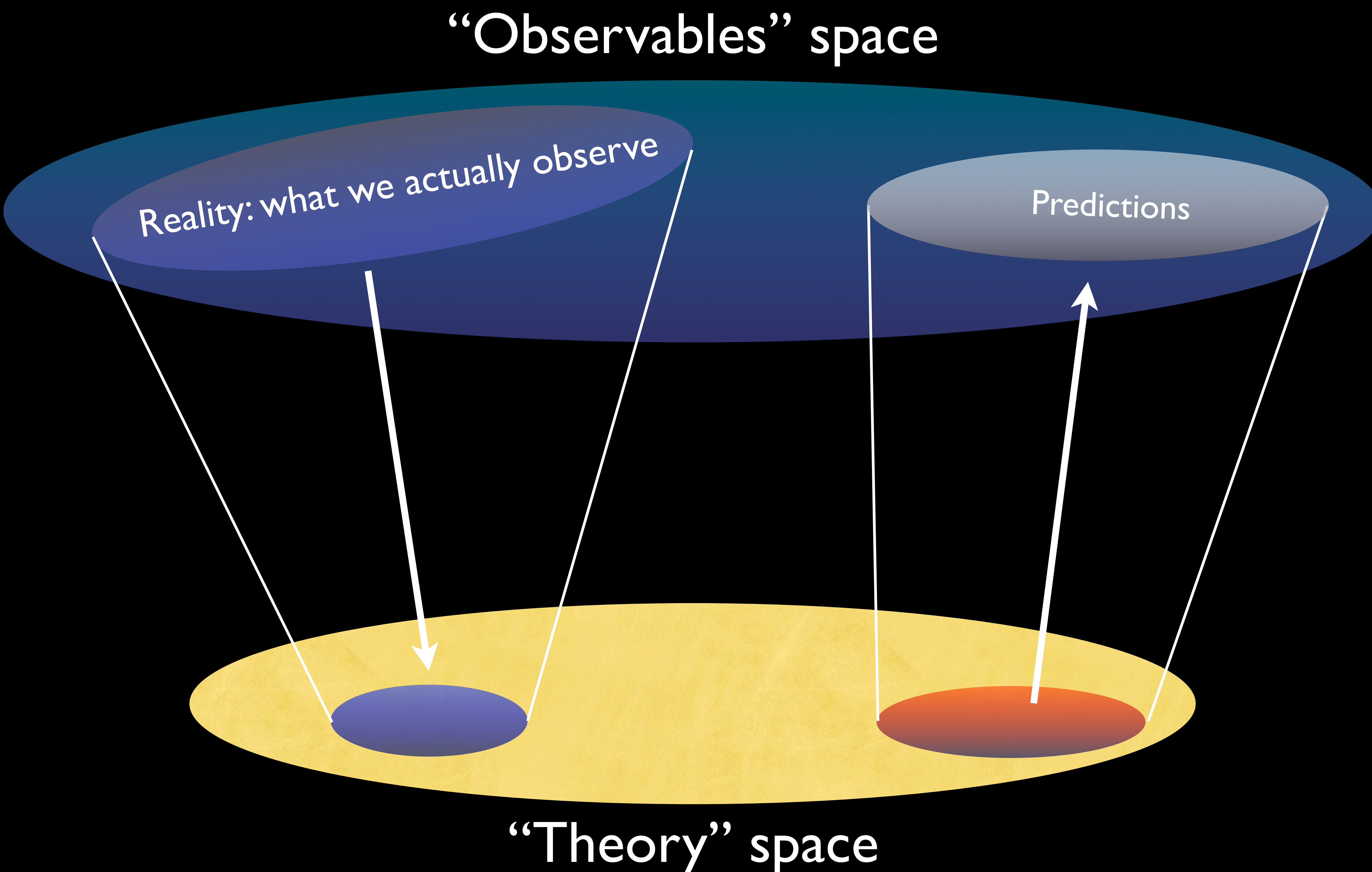
Effective theories: philosophy



Effective theories: philosophy



Effective theories: philosophy



Effective field theory

- Some theories may be probing “theory space” which is already ruled out by observations… but you don’t know that untill you write down the theory & do the calculations
- Want to write down “***everything***” with all possible “free parameters” and find out what values they can take to be ***consistent*** with observational data
    - ***Particle physicists*** do this… field content + symmetry: work out how to measure the free parameters (interaction terms, masses, etc…)
- Doing this with a Lagrangian enables ***clear physical interpretation*** of constraints on parameters

Lagrangian for perturbations

The Lagrangian for perturbations is a ***quadratic functional*** in perturbations of fields...

$$\mathcal{L}_{\{2\}} = \sum_{A=1}^{N} \sum_{B=1}^{N} \mathsf{G}^{\{0\}}_{AB}\, \delta X^{(A)}\, \delta X^{(B)}$$

The Lagrangian for perturbations is equivalent to the ***second measure-weighted variation*** of the action...

$$\delta^2 S = \int \mathrm{d}^4x\sqrt{-g}\underbrace{\left[\frac{1}{\sqrt{-g}}\delta^2(\sqrt{-g}\mathcal{L})\right]}_{\lozenge^2\mathcal{L}} = \int \mathrm{d}^4x\sqrt{-g}\,\mathcal{L}_{\{2\}}$$

This allows us to

(1) "dream up" a Lagrangian for perturbations,
(2) explicitly calculate for some known theory... compare with established theories

> Map from general parameterization to established theories, but we are not limited by them!

Nothing extra

Field content: just the metric

Lagrangian for perturbations

$$\mathcal{L}_{\{2\}} = \frac{1}{8}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}$$

Gravitational effects...

$$\delta U^{\mu\nu} = -\frac{1}{2}\left[\mathcal{W}^{\mu\nu\alpha\beta} + U^{\mu\nu}g^{\alpha\beta}\right]\delta g_{\alpha\beta}$$

Elastic dark energy, Feirz-Pauli & massive gravity in GR,...

Scalar fields

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi)$$

$$\mathcal{L}_{\{2\}} = \mathcal{L}_{\{2\}}(\delta g_{\mu\nu}, \delta\phi, \nabla_\mu \delta\phi)$$

Lagrangian for perturbations

$$\mathcal{L}_{\{2\}} = \mathcal{A}\delta\phi\delta\phi + \mathcal{B}^\mu \delta\phi\nabla_\mu\delta\phi + \frac{1}{2}\mathcal{C}^{\mu\nu}\nabla_\mu\delta\phi\nabla_\nu\delta\phi + \frac{1}{4}\left[\mathcal{V}^{\mu\nu}\delta\phi\delta g_{\mu\nu} + \mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi\delta g_{\mu\nu} + \frac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}\delta g_{\mu\nu}\right]$$

Gravitational effects...

$$\delta U^{\mu\nu} = -\frac{1}{2}\left[\mathcal{V}^{\mu\nu}\delta\phi + \mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi + \mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta} + U^{\mu\nu}g^{\alpha\beta}\delta g_{\alpha\beta}\right]$$

Quintessence, k-essence, Lorentz violating theories,...

Total number of free functions

Scalar field case before imposing anything

Tensors in $L_{\{2\}}$	$\Big\{\mathcal{A},$	$\mathcal{B}^{\mu},$	$\mathcal{C}^{\mu\nu},$	$\mathcal{V}^{\mu\nu},$	$\mathcal{Y}^{\alpha\mu\nu},$	$\mathcal{W}^{\mu\nu\alpha\beta}\Big\}$
# components	1	4	10	10	40	24

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi)$$

$$\begin{aligned} \mathcal{X} &\equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \\ &= \frac{1}{2a^2} \dot{\phi}^2 \end{aligned}$$

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$$= \frac{1}{2a^2}\dot{\phi}^2$$

Total: 90

Impose *isotropy*: 90 -> 14

Impose *linking*: 14 -> 11

Impose $\mathcal{L} = \mathcal{L}(\phi, \mathcal{X})$: 11 -> 3

Imposing symmetry on the background

e.g. isotropy of spatial sections...

everything becomes compatible with FRW

(3+1) decomposition $g_{\mu\nu} = \gamma_{\mu\nu} - u_\mu u_\nu \quad \gamma_{\mu\nu} u^\nu = 0 \quad u_\mu u^\mu = -1$

Coefficients in Lagrangian become:

$$\mathcal{W}_{\mu\nu\alpha\beta} = A_\mathcal{W} u_\mu u_\nu u_\alpha u_\beta + B_\mathcal{W}\left(\gamma_{\mu\nu} u_\alpha u_\beta + \gamma_{\alpha\beta} u_\mu u_\nu\right)$$
$$+2C_\mathcal{W}\left(\gamma_{\mu(\alpha} u_{\beta)} u_\nu + \gamma_{\nu(\alpha} u_{\beta)} u_\mu\right) + \mathcal{E}_{\mu\nu\alpha\beta},$$

$$\mathcal{E}_{\mu\nu\alpha\beta} = D_\mathcal{W} \gamma_{\mu\nu}\gamma_{\alpha\beta} + 2E_\mathcal{W} \gamma_{\mu(\alpha}\gamma_{\beta)\nu}.$$

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$$\mathcal{A} = A_{\mathcal{A}},$$

$$\mathcal{B}^\mu = A_{\mathcal{B}} u^\mu,$$

$$\mathcal{C}_{\mu\nu} = A_{\mathcal{C}} u_\mu u_\nu + B_{\mathcal{C}} \gamma_{\mu\nu},$$

$$\mathcal{V}_{\mu\nu} = A_{\mathcal{V}} u_\mu u_\nu + B_{\mathcal{V}} \gamma_{\mu\nu},$$

$$\mathcal{Y}_{\alpha\mu\nu} = A_{\mathcal{Y}} u_\alpha u_\mu u_\nu + B_{\mathcal{Y}} u_\alpha \gamma_{\mu\nu} + 2C_{\mathcal{Y}} \gamma_{\alpha(\mu} u_{\nu)}.$$

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$$\mathcal{C}_{\mu\nu} = A_{\mathcal{C}} u_\mu u_\nu + B_{\mathcal{C}} \gamma_{\mu\nu},$$

$$\mathcal{V}_{\mu\nu} = A_{\mathcal{V}} u_\mu u_\nu + B_{\mathcal{V}} \gamma_{\mu\nu},$$

$$\mathcal{Y}_{\alpha\mu\nu} = A_{\mathcal{Y}} u_\alpha u_\mu u_\nu + B_{\mathcal{Y}} u_\alpha \gamma_{\mu\nu} + 2C_{\mathcal{Y}} \gamma_{\alpha(\mu} u_{\nu)}.$$

Impose some "theory" structure:

$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X})$ reduce 14 -> 3

Function	(a) EDE	(b) $\mathcal{L} = \mathcal{L}(\phi, \mathcal{X})$	(c) $\mathcal{L} = F(\mathcal{X})$	(d) $\mathcal{L} = \mathcal{X} - V(\phi)$
$A_{\mathcal{V}}$	0	$-2(\mathcal{L}_{,\mathcal{X}\phi}\dot{\phi}^2 - \mathcal{L}_{,\phi})$	0	$-2V'$
$B_{\mathcal{V}}$	0	$-2\mathcal{L}_{,\phi}$	0	$2V'$
$A_{\mathcal{Y}}$	0	$-2(\mathcal{L}_{,\mathcal{X}\mathcal{X}}\dot{\phi}^2 + \mathcal{L}_{,\mathcal{X}}\dot{\phi})$	$-2(F''\dot{\phi}^2 + F'\dot{\phi})$	$-2\dot{\phi}$
$B_{\mathcal{Y}}$	0	$-2\mathcal{L}_{,\mathcal{X}}\dot{\phi}$	$-2F'\dot{\phi}$	$-2\dot{\phi}$
$C_{\mathcal{Y}}$	0	$2\mathcal{L}_{,\mathcal{X}}\dot{\phi}$	$2F'\dot{\phi}$	$2\dot{\phi}$
$A_{\mathcal{W}}$	$-\rho$	$-(\mathcal{L}_{,\mathcal{X}\mathcal{X}}\dot{\phi}^4 + 2\rho + P)$	$-(F''\dot{\phi}^4 + 2\rho + P)$	$-(2\rho + P)$
$B_{\mathcal{W}}$	P	$-\rho$	$-\rho$	$-\rho$
$C_{\mathcal{W}}$	$-P$	ρ	ρ	ρ
$D_{\mathcal{W}}$	$\beta - P - \frac{2}{3}\mu$	$-P$	$-P$	$-P$
$E_{\mathcal{W}}$	$\mu + P$	P	P	P

Parameterizing entropy

$$\delta P = w\delta\rho + P\Gamma$$

$$w\Gamma = (\alpha - w)\left[\delta - 3\mathcal{H}\beta(1+w)\theta\right]$$

Standard: Weller & Lewis: $\alpha = c_s^2$ $\beta = 1$

Nothing extra		$\beta = 0$
Quintessence	$\alpha = 1$	$\beta = 1$
Pure k-essence		$\beta = 0$

Physically, what does $\alpha \neq 1$ mean?

Is α always a sound speed?

α is neither group nor phase velocity of waves

$$\alpha = \left(1 + 2\mathcal{X}\frac{\mathcal{L}_{,\mathcal{X}\mathcal{X}}}{\mathcal{L}_{,\mathcal{X}}}\right)^{-1},$$

$$\beta = \frac{2a\mathcal{L}_{,\phi}}{3\mathcal{H}\mathcal{L}_{,\mathcal{X}}\sqrt{2\mathcal{X}}}\left[1 + \mathcal{X}\left(\frac{\mathcal{L}_{,\mathcal{X}\mathcal{X}}}{\mathcal{L}_{,\mathcal{X}}} - \frac{\mathcal{L}_{,\mathcal{X}\phi}}{\mathcal{L}_{,\phi}}\right)\right]\frac{\alpha}{\alpha - w}.$$

Effective metric that scalar field perturbations "feel"...

$$\mathcal{C}^{\mu\nu} = \mathcal{L}_{,\mathcal{X}}g^{\mu\nu} + \mathcal{L}_{,\mathcal{X}}\alpha^{-1}(\alpha - 1)u^\mu u^\nu$$

Scope of using the Lagrangian for perturbations

- ***Formalism***
  - The Lagrangian for perturbations
  - The perturbed dark energy-momentum tensor
  - Lagrangian & Eulerian perturbations (Stuckleberg completion/deformation vector)
- ***Examples***
  - Nothing extra
  - Scalar & vector fields *(aether, TeVeS, ...)*
  - "high order" derivative theories *(galileon, F(R), Gauss-Bonnet, ...)*
- ***Applications***
  - Cosmological perturbations
  - Entropy & anisotropic stresses
  - Massive gravity
  - Modified gravity

Summary

- Construct coherent consistent modifications to the gravitational field equations at perturbed order
- All freedom inside "background" tensors
- Encompass theories never before considered: $\mathcal{L}_{\{2\}}$ needs scalars in background ***and*** perturbed field variables: more freedom!
- Encompass massive gravity & "high derivative" theories: e.g. galileon, Horndeski, Brans-Dicke.
- In a model independent way compute cosmological observables (CMB, lensing, P(k), ...): rule in/out before requiring the actual theory!
- Impose symmetry on background (e.g. isotropy... compatible with FRW)... allows split of BG tensors

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