the vacuum – hence its alternative name of "vacuum energy". It is allowed by Einstein's equation of general relativity, although Einstein himself famously repudiated it as an unnecessary complication. Positive values of Λ_0 usually imply a universe which does not recollapse[7] but instead follows a path of accelerating expansion. It can be expressed in the same dimensionless units as Ω_m, in which case the condition for a "flat" universe without curvature can be written as $\Omega_m + \Omega_\Lambda = 1$. Results from the Boomerang and MAXIMA experiments, which measured fluctuations in the cosmic microwave background (CMB) *[Hanany et al., 2000*; *de Bernardis et al., 2000]*, point towards this condition being satisfied. Further constraints have been derived by studies of supernovae *[Perlmutter et al., 1999*; *Riess et al., 1998]*; in the Ω_m vs. Ω_Λ plane the supernovae constraints are approximately orthogonal to those from the CMB. The intersection of the constraints is approximately at $\Omega_m = 0.3$, $\Lambda_0 = 0.7$. In other words, provided that systematic errors affect neither result, we are living in a Universe destined for eternal and accelerating expansion.

In principle, gravitational lensing statistics can also be used to tie down both cosmological parameters *[Fukugita et al., 1992*; *Kochanek, 1996a,b]*. Consider a source at distance l; the optical depth to lensing, and hence the probability of lensing, $P(\text{lens})$, is obtained by integrating along the light path

$$P(\text{lens}) = \int_0^l n(l)\sigma dl$$

where n is the number density of lenses and σ is the cross-sectional area of each lens. This can be written as

$$P(\text{lens}) = \int_0^{z_s} n(z)\sigma c \frac{dt}{dz_l} dz_l$$

where z_l and z_s are the redshifts of the lens and source respectively, c is the speed of light and dt is the time increment along the path. If we measure $P(\text{lens})$ for a complete sample of lenses – that is, one for which all lenses within the sample have been identified – we can then compare this with the right-hand side of the equation. This depends on a number of quantities. First, it is sensitive to the number density $n(z)$ of lensing galaxies and its evolution with redshift z. Second, it depends on the cross-sectional area of individual lenses, σ, which can be calculated if we assume simple models for lens mass distributions. Third, it depends on lengths within the universe, via quantities such as dt/dz_l. Quantities related to length in the universe are in turn sensitive to Ω_m and Ω_Λ, and if all other quantities can be measured by other means, these cosmological parameters can be inferred.

The major problem for radio studies in fact lies with $P(\text{lens})$, as this involves dividing the number of lenses found by the number of sources which are potentially lensed. The former quantity is subject only to Poisson errors, assuming that radio selection allows one to select all the lenses in any sample. The latter, however, requires that one knows the number distribution $n(S, z)$ of radio sources as a function of radio flux density S and redshift z. For faint radio sources, this is difficult to determine as the redshifts of these objects must be measured. A flat-spectrum radio source with $S \sim 25\,\text{mJy}$ is likely to

[7]The only exception to this is for small values of Λ_0 where $\Omega_m > 1$; see e.g. *Peacock, [2000]*.

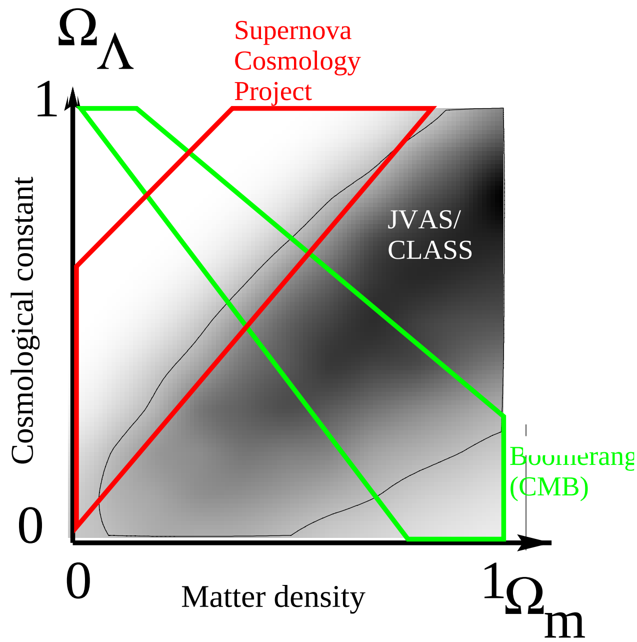

Figure 6: Omega-Lambda plane diagram, showing 2-σ constraints from the Cosmic Microwave Background *[de Bernardis et al., 2000]*, supernova studies *[Perlmutter et al., 1999]* and preliminary lensing constraints from JVAS/CLASS *[Helbig, 2000]*. [A colour version of this figure is available on the CD-ROM version of this article.]

be faint (a visual magnitude $V \sim 25$ is not uncommon), making samples very difficult to study optically, even with the new generation of 8-to-10-m telescopes. Knowing $n(S, z)$ is crucial, as errors as small as 0.1 in the mean redshift, which is typically between 1.0 and 1.5, significantly disturb the best-fit regions for Ω_m and Ω_Λ.

Kochanek [1996a] applied the above method to the best available lens statistics at that time, including optical lens surveys and the first phase of the JVAS/CLASS radio survey (section 2). He found that $\Omega_\Lambda < 0.66$ for models of a "flat" ($\Omega_m + \Omega_\Lambda = 1$) universe. In general, lensing gives an effective upper limit on Ω_Λ due to the very rapid increase in the expected number of lenses with increasing cosmological constant. *Kochanek [1996b]* and *Cooray [1999]* both extended the analysis to predict the number of lenses that would be found in the complete JVAS/CLASS survey.

Quast & Helbig [1999], *Helbig et al. [1999]* and *Helbig [1999]* derive constraints from the JVAS survey (2384 sources) only and combine these constraints with supernova and CMB results. They obtain a good fit to all data combined, provided that $\Omega_m + 0.4 \simeq \Omega_\Lambda$, which for a flat universe requires $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, exactly the best-guess values suggested by reviews such as that by *Carroll [2000]*. *Helbig [2000]* (Figure 6) extends the lens analysis for the first time to the full CLASS sample. This work is preliminary in that efforts are still continuing to obtain the number distribution function $n(S, z)$ for the parent population of radio sources and achieve the necessary understanding of evolution of lensing galaxies. The current result is marginally inconsistent (at 2-σ) with the Supernova Cosmology Project results *[Perlmutter et al., 1999]*, although final analysis is not complete. However, it now appears that lensing constraints, as well as the statistics of supernovae, rule out the Einstein-de Sitter model in which the universe is flat and composed entirely of matter ($\Omega_m = 1$, $\Omega_\Lambda = 0$).

4.3 The cosmological density of dark compact objects

The possibility that a first generation of objects with masses comparable with those of globular clusters formed prior to galaxies has long been recognised (see *Carr [1994]* for a comprehensive review). Such Jeans–mass ($\sim 10^{6.5}$ $M_\odot$) objects forming shortly after the decoupling of matter and radiation in the early universe could have evolved to black holes and it is possible that some of the dark matter could be in this, difficult to detect, form *[Carr & Sakellariadou, 1999]*.

Since for a lens at a cosmologically significant distance the image separation is $\sim 2 \times 10^{-6}(M_{CO}/M_\odot)^{1/2}$ arcseconds, searching for Jeans–mass compact objects (CO) requires the milliarcsec resolution which is only obtainable with VLBI. *Press & Gunn [1973]* developed the idea of detecting supermassive CO, by their gravitational lensing effects on VLBI radio images, well before the discovery of gravitational lenses in 1979. They showed that in a universe filled with a mass density $\Omega_{CO} \sim 1$, the probability of a distant source being multiply imaged by a supermassive CO is of order unity, while for $\Omega_{CO} < 1$, the probability decreases in direct proportion to the mass density. From this they drew the important conclusion that the fraction of distant galaxies that is lensed by CO directly measures Ω_{CO} and is independent of the mass M_{CO} of the lenses. The latter property is simply understood. A given value of Ω_{CO} can be made up of a large number of low–mass objects or a small number of high–mass ones, hence the number density n of CO of a particular mass is proportional to $1/M_{CO}$. For point masses the gravitational lensing cross–section $\sigma \propto M_{CO}$ *[Turner, Ostriker & Gott, 1984]* and hence the path length to lensing ($1/n\sigma$) is independent of the lens mass. However, the average image separation measures M_{CO} directly and is essentially independent of Ω_{CO}. These ideas were further developed by *Nemiroff [1989]*, *Nemiroff & Bistolas [1990]* and *Kassiola, Kovner & Blandford [1991]*.

Wilkinson et al. [2001] have searched a sample of 300 flat-spectrum radio sources, largely corresponding to the strongest sources in JVAS, for examples of multiple imaging. The sources were drawn from the Pearson-Readhead and Caltech-Jodrell Bank VLBI surveys (see references in *Wilkinson et al. [2001]*) and involved systematic observations with intercontinental VLBI arrays at a resolution ~ 1 mas. Great care has to be taken to achieve completeness by being conservative in the rejection of systems as lens candidates since at this high resolution compact radio sources exhibit a range of intrinsic structures which can mimic the effects of lensing.

No multiple images were found with separations in the angular range 1.5-50 milliarcsec enabling a limit $\Omega_{CO} \leq 0.013$ in the range $\sim 10^6$ to $\sim 10^8$ $M_\odot$ to be placed *[Wilkinson et al., 2001]*. These limits are mildly conservative because lensing increases the observed flux density of a background source and hence lensed sources are drawn from a fainter source population than the unlensed sources; a flux–limited survey will therefore contain more lenses than expected but the "magnification bias" associated with flat spectrum radio sources is only of order unity *[King & Browne, 1996]*. *Garrett et al. [1994]*, using the similarity of the two VLBI images of 0957+561 to constrain the presence of large black holes in the lensing galaxy, have ruled out objects with $M > 3 \times 10^6 M_\odot$ as contributing more than 10% of the halo dark matter.

Uniformly distributed CO in the mass range $\sim 10^6$ to $\sim 10^8$ $M_\odot$ cannot, therefore, comprise $> 1\%$ of the closure density, ($\Omega_{total} = 1$), which is strongly indicated by the latest measurements of the angular spectrum of the CMB, (*[de Bernardis et al., 2000]*). Similarly such CO do not make up more than $\sim 3\%$ of the Dark Matter density $\Omega_{DM} \sim 0.3$ favoured by current observations. The favoured value of the baryon density from Big–Bang Nucleosynthesis is $\Omega_b h^2 = 0.019 \pm 0.002$ *[Burles, Kirkman & Tytler, 1999]* ($h \equiv H_0/100$ km s^{-1} Mpc^{-1}). Taking a plausible value for h (0.65) implies $\Omega_b = 0.045 \pm 0.005$, thus uniformly–distributed Jeans–mass CO do not make up more than about one third of Ω_b. Perhaps the next interesting limit would be $\Omega_{CO} \leq 0.005$ which would constrain the contribution of supermassive CO to be no more than the baryonic contribution of presently observable stars and galaxies. To reach this limit about 1000 sources would have to be studied with VLBI; this would be a time–consuming, but relatively straightforward, task.

5. PROPAGATION EFFECTS AND PROPERTIES OF LENSING GALAXIES

As radiation propagates from a distant object to the observer the medium leaves an imprint which can give invaluable information about parts of the Universe inaccessible to study by any other means. Gravitational lens systems could have been designed for such studies since, by definition, there exists a distant object whose radiation passes through an intervening galaxy with impact parameter(s) of a few kiloparsecs. Moreover, with multiple images, more than one path is sampled and it then becomes much easier to separate the effects of propagation from the intrinsic properties of the initial signal. This technique has been exploited in the optical and infrared bands to explore the effects of extinction due to dust in the lensing galaxies *[Kochanek et al., 2000b*; *Jackson, Xanthopoulos & Browne, 2000]*. However, in the following section we concentrate on effects on radio signals such as spectral line absorption, Faraday rotation and depolarization and multi-path scatter broadening.

Two lens systems in particular, JVAS B0218+357 and PKS1830−211, display a rich variety of propagation effects; both show neutral hydrogen absorption *[Carilli, Rupen and Yanny 1993*; *Lovell et al., 1996]*, molecular absorption *[Wiklind & Combes, 1995]* and evidence for multi-path scattering *[Biggs, Browne & Wilkinson, 2001*; *Jones et al., 1996*; *Guirado et al., 1999]*. The existence of a rich interstellar medium in these lenses argues in favour of the lens being a spiral galaxy in each case.

In the light of modern astro-particle unified theories, in which the fundamental constants of nature may change with cosmological epoch, high redshift objects in which both molecular and neutral hydrogen absorption occur, assume a new significance. *Drinkwater et al., [1998]* and *Murphy et al., [2001]* make use of data on two objects including the lens system JVAS B0218+357 to put a limit on any change of the fine structure constant (α) as a function of cosmological epoch of $|(\dot{\alpha}/\alpha)| \leq 9 \times 10^{-16}$ yr^{-1}. Better measurements on JVAS B0218+357 and other lens systems should enable these constraints to be refined.

An exciting prospect is to be able to map the kinematics of lensing galaxies using high resolution spectral line VLBI. This has been attempted on JVAS B0218+357 using observations at 842 MHz which is the frequency of the redshifted hydrogen line seen in the lensing galaxy (Vermeulen & Hobbs, personal communication). The results of these preliminary observations show absorption affecting both the compact images and part of the

Einstein ring. The ultimate promise of such observations is that of being able to determine the velocity dispersion of the lensing galaxy and use it as an additional constraint in the mass modeling process.

Almost nothing is known about the magnetic field structures in high redshift galaxies. Even the origin of magnetic fields in galaxies is obscure *[Fermi 1954*; *Lesch & Chiba 1997]* though one widely explored idea for spiral galaxies is that they are generated by a dynamo process involving differential rotation. Such mechanisms predict an exponential growth of field strength on a characteristic time comparable to the dynamical time-scale of the galaxy; i.e. $\sim 10^8$ yr. Thus a large evolution of magnetic field strength is expected in going from the epoch at which we see most lensing galaxies (a few times 10^9 yr) to the present day. Looking for large-scale Faraday rotation of the background source radio emission produced by the lensing galaxy is one way that galaxy magnetic fields would manifest themselves. A surprising result is that many of the radio-loud lens systems which have detectable polarized radio emission display rotation measures often much larger than those that would be produced by radiation passing through the plane of our Galaxy at a distance of a few kiloparsec from the centre.

Within our own Galaxy radio sources viewed through regions of high electron density have their apparent sizes increased by multi-path scattering, with the size changing $\propto \lambda^2$ *[Rickett, 1977]* where λ is the observing wavelength. In several lens systems there is evidence showing the the angular sizes of the radio components are strongly suggestive of scatter broadening. In these cases the surface brightnesses of well-resolved images are different, a discrepancy that can only arise from propagation effects. One of the most convincing cases for scattering is in PKS1830-211 in which *Guirado et al., [1999]* see evidence for a λ^2 dependence of angular size in one of the images. Scattering has also been claimed in two of the images in CLASS B1933+503 by *Marlow et al., [1999]* and in JVAS B0218+357 by *Biggs, Browne & Wilkinson [2001]*. In the latter object both images show a strong increase in angular size with increasing wavelength over a very wide range of wavelengths but the size variations are not well fitted by a λ^2 law. However, this does not rule out scattering because the projected sizes of the images on the lensing galaxy are hundreds of parsecs and the statistical properties of the ISM of the lens are not necessarily uniform over this kind of length-scale.

With more than 50 (radio) lines of sight through more than 20 lens systems can we reach any general conclusions about the properties of galaxies at redshifts $\sim$0.5? We think it is too early to be certain but are surprised by the number of lensing galaxies that seem to contain neutral, molecular and ionized gas in amounts that would be regarded as large for a spiral galaxy (based on local knowledge) let alone for an elliptical galaxy which is what most lensing galaxies are believed to be. Perhaps there is more gas in ellipticals than expected; alternatively more of the lensing galaxies are spirals.

It is also possible to use lens systems to help distinguish between the intrinsic and extrinsic models for intraday variability which is a feature of some compact radio sources. In some cases (e.g. *Kedziora-Chudczer et al., [1997]*, *Dennett-Thorpe & de Bruyn [2000]*) it is clear that the variations are produced during propagation from source to observer but in others the origin could be intrinsic implying that the brightness temperatures exceed the 10^{12}-K inverse-Compton limit by orders of magnitude *[Wagner & Witzel, 1995]*. In lens systems the propagation paths are different and thus propagation induced variability should be uncorrelated in the two images whereas intrinsic variability should be perfectly correlated. *Biggs et al., [2001]* have applied this argument to JVAS B0218+357 to show that the variations seen in this object on timescales of days must be *intrinsic* to the source.

An important example of *extrinsic* variability is the discovery of variations in the radio flux of the brighter image in the gravitational lens CLASS B1600+434 *[Koopmans & de Bruyn, 2000]*. Variations of this image, at levels of up to 15 percent, are seen, but after allowing for the lensing time delay, corresponding variations are not seen in the other image. One might then be tempted to ascribe the effect to scattering in our own Galaxy. However, Koopmans & de Bruyn present evidence against this idea, the most persuasive of which is an increase of fractional rms variability with increasing frequency between 1.4 and 5 GHz – completely opposite to that expected from interstellar scintillation. But this dependence is exactly what one would expect if the cause of the variability is microlensing by massive ($\sim 1 M_\odot$) objects in the halo of the lensing galaxy. In this model, the radio source contains components on the scale of a few microarcseconds, moving at speeds of $\sim c$, which pass behind the complex magnification pattern of the lens galaxy halo and hence vary strongly in flux density. The line of sight to the brighter image in CLASS B1600+434, along which the compact objects must lie, passes 6 kpc above the plane of the edge-on spiral that constitutes the lensing galaxy!

Further work is under way to test whether other lensing galaxies show this effect. It is very important as it allows us for the first time to deduce the presence of massive objects in the halo of a galaxy at redshift ~0.5. The implications for the nature of dark matter in galaxies of such observations are potentially very important.

6. THE FUTURE

6.1 ALMA and the submillimetre range

Detailed study of the submillimetre wavelength range is relatively new. Major submillimetre telescopes, such as the James Clerk Maxwell Telescope on Mauna Kea and the IRAM telescope on Pico Veleta, have only recently been equipped with bolometer arrays allowing sensitivities of ~1 mJy to be routinely achieved at 400–800 μm. This has led to the first blank-field surveys and source counts in the submillimetre range *[Smail, Ivison & Blain, 1997]* to these levels; such surveys often examine the regions around clusters to make use of the magnification effect of the weak gravitational lensing by the cluster.

The submillimetre range is physically an interesting region to study galaxies. This is because it contains the steeply falling Rayleigh-Jeans tail of sources at a few tens of Kelvin, such as the dusty envelopes of star-forming galaxies. Much interest has been generated in the star formation history of the universe by recent studies in the optical and UV such as *Madau et al. [1996]*, although much pioneering work at moderate redshift was in fact done using 21-cm line radio surveys *[Condon et al., 1982]*.

Since the effect of distance is to introduce a redshift, the steeply inverted Rayleigh-Jeans tail ($F_\nu \propto \nu^\alpha$, $\alpha > 0$) means that sources can actually appear *brighter* at a given observed wavelength at redshift $z = 5$ than at $z = 1$ (see e.g. *Blain & Longair [1996]*) despite the $(1+z)^4$ falloff in surface brightness with distance. This makes the submillimetre a useful region for detecting very high-redshift star-forming objects – which in turn formed a major part of the science case for the Atacama Large Millimetre Array *[Wootten, 2001]*.

The submillimetre range is also a very promising region for new detections of gravitational