lensed images *[Patnaik, Porcas & Browne, 1995*; *Ros et al., 2000]*) and extra modelling constraints [*Trotter et al., 2000]*. However, in some cases even extensive VLBI information does not discriminate between a wide range of models (e.g. 0957+561 *[Barkana et al., 1999]*).

A few lenses do provide good additional constraints. For example, the lens CLASS B1933+503 (Figure 4) has ten images, formed by the 4-image lensing of two components of the background source and the double imaging of a third. *Cohn et al. [2001]* find the best model for B1933+503 is quite close to the simplest mass model, being slightly shallower than a singular isothermal ellipsoid. The pseudo-Jaffe profile (Fig. 3) and cusp models also fit the data, provided that the free parameters are adjusted to be quite close to an isothermal ellipsoid *[Muñoz, Kochanek & Keeton, 2001]*.

A good constraint is the presence of an Einstein ring, since this effectively gives information on the mass distributions along many lines of sight through the lensing galaxy. *Kochanek, Keeton & McLeod [2001]* discuss Einstein rings and derive a constraint for the system PG1115+080. A further constraint comes from the fact that odd (third or fifth) images are not seen in the vast majority of lens systems. *Rusin & Ma [2001]* argue that this implies that the mass profiles of lensing galaxies are not much shallower than the isothermal profile. Alternatively, the data can be used as a constraint on the radius of any mass "core" that is present *[Narasimha, Subrahmanian & Chitre 1986*; *Blandford & Kochanek 1987*; *Wallington & Narayan 1993*; *Norbury et al., 2001]*.

3.4 Spiral galaxies

Turner, Ostriker & Gott [1984] predicted that $\sim 94\%$ of lenses associated with normal galaxies should occur in the range $0''.3$–$6''$, with a peak at $\sim 1''.5$ and that ellipticals should dominate the lensing cross-section, with only 20% being contributed by spirals. These predictions are broadly consistent with the presently observed distribution.

However, spiral galaxies should produce image separations predominantly in the relatively unexplored range $0''.1$–$0''.3$. What are the prospects for detection of a useful number of systems in this separation range? The cross-sections are dominated by edge-on disks and the lensing rate depends on: (i) the balance between the masses of the disks and those of their associated dark matter haloes and on (ii) whether or not there is significant evolution of the spiral population between $z = 1$ and the present. *Keeton & Kochanek [1998]* predict that, when averaged over all inclinations, there should be little change in the contribution of spirals over the predictions of *Turner et al. [1984]*. In contrast the models of *Blain, Möller & Maller [1999]*, *Bartelmann and Loeb [1998]* and *Bartelmann [2000]* which invoke maximal disks and also consider the effect of evolution, predict enhancements of the spiral contribution by factors of two or more compared with *Turner et al. [1984]*. *Bartelmann* (private communication) estimates that between 10% and 20% of *all* galaxy-mass lenses could have separations in the range $0''.1$–$0''.3$. Taken with the 1 : 600 JVAS/CLASS lensing rate for arcsecond-scale separations, these calculations suggest that the lensing rate in this image separation range should be one per few thousand background objects searched.

These recent calculations have been motivated by the prospect of lens searches at 0.1-arcsecond resolution in the sub-mm (with ALMA) and infra-red (with NGST). Radio-based surveys are also well-suited to an unbiased search for spiral galaxy lensing since they are also unaffected by dust obscuration in edge-on systems. A pilot radio search, largely based on JVAS and sensitive to image separations in the range 150–300 milliarcsec, has yielded a null result for a sample of 1665 sources *[Augusto, Wilkinson & Browne, 1998]*. In prospect, however, is a 10000–source survey based on the CLASS catalogue which has a high probability of detecting small–separation lensing and which would place significant constraints on the uncertain disk/halo mass ratio in spiral galaxies at the redshifts ($z \sim 0.5$) appropriate to lensing.

3.5 Dark galaxies

Recently it has been proposed that very massive (10^{12-13} $\mathrm{M}_\odot$) dark objects could give rise to quasar pairs with separations $\sim$ 10 arcseconds by gravitational lensing *[Hawkins, 1997]*. There is, however, no supporting evidence for such a population of massive dark objects. *Kochanek, Falco and Muñoz, [1998]* argue that a comparison of the radio and optical properties of the pairs rules out the massive dark lens hypothesis. And HST imaging of confirmed arcsecond-scale lenses found in the JVAS/CLASS surveys always shows a lensing galaxy with a relatively normal mass–to–light ratio between the images *[Jackson et al., 1998]*.

Phillips, Browne & Wilkinson [2001] use the lack of larger separation ($6'' - 15''$) gravitational lenses to investigate mass distributions on larger ($\sim 10^{13} M_\odot$) scales. In particular, the lack of observed lenses with separations on this angular scale means that groups and clusters of galaxies must have substantially softer central potential wells than would be expected for singular isothermal sphere models. On smaller scales, *Trentham, Möller & Ramirez-Ruiz [2001]* have pointed out that current "cold-dark matter" models of cosmology require numerous condensations of matter on scales smaller than galaxies, possibly 10^7–$10^{10} M_\odot$. Such condensations would be likely to produce little or no starlight; future lensing studies on sub-mas scales could find and count them, but only if they are more centrally concentrated than current theories suggest.

4. COSMOLOGICAL PARAMETERS

4.1 The Hubble constant, H_0

Hubble's discovery of the expansion of the universe, in 1929, resulted from his observation that nearby galaxies were receding from us with a velocity v proportional to their distance d. The velocity was measured by the shift in optical spectral lines from their expected wavelength λ_{lab} to the observed wavelength λ_{obs}, by the usual non-relativistic Doppler formula

$$\frac{v}{c} = \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{lab}}}{\lambda_{\mathrm{lab}}} \equiv z$$

where z is the redshift. Since the empirical relation between recession velocity and redshift is of the form $v = H_0 d$, where H_0 is a constant, we have $d = cz/H_0$ for nearby galaxies. For more distant cases the relation between distance and redshift is more complicated and involves knowledge of the global topology of the universe, determined mainly by the matter density Ω_m and cosmological constant. Nevertheless, the Hubble constant, H_0, is a vital number as it allows distances to be calculated from easily observable quantities in the case of objects at enormous distances. It is also related to the age of the Universe; in most simple cosmological models $t_{\rm univ} \sim H_0^{-1}$.

The Hubble constant is not an easy quantity to determine as it requires the measurement of the distance of far-away objects. The traditional method relies on a number of local "distance indicators" such as moving clusters of stars and a special class of variable stars known as Cepheids whose brightness variations have a well-determined relationship to their absolute luminosity. If the brightness variations in Cepheids are measured and the luminosity inferred, this, together with a measurement of flux density, allows an immediate determination of distance. This in principle allows the Hubble constant to be measured for galaxies in which Cepheid variables can be resolved. Unfortunately, despite the devotion of considerable observational resources including long observations with the Hubble Space Telescope, the current position is still one of controversy, with recent estimates ranging from 53 ± 7 km s^{-1}Mpc^{-1} *[Sandage, 1999]* to the Hubble Space Telescope Key Project value of 71 ± 6 km s^{-1}Mpc^{-1} *[Mould et al., 2000]*. Worse still, "traditional" methods determine H_0 only for relatively short distances by cosmological standards, usually to the nearest clusters of galaxies at distances of a few tens of Mpc.

Gravitational lenses in principle allow a clean determination of H_0 on cosmological scales, as was pointed out by *Refsdal [1964]*. The key requirement is that the lensed object be variable in flux density. If this is the case, the lensed images will of course also vary. However, each lensed image results from light that has taken a slightly different path through the lens; each image will therefore show a variation at a slightly different time, reflecting the different propagation times of light on these different paths. The delay between variations of the image allows us to measure the difference in lengths of light paths – the latter quantity will just be $c\Delta\tau$, where $\Delta\tau$ is the measured time delay (which results from the sum of the geometrical and Shapiro time delays). The delay is proportional to the square of the image separation in the system and ranges from 10 days to just over 400 days in systems examined so far. This result allows us to calculate in parsecs a distance within the system; knowing all the angles within the system, we can then immediately derive the distances to the lens and lensed object. If we know the redshifts of the lens and source, Hubble's constant follows from the definition given above. In principle, if the Hubble constant could be determined accurately from lens systems at different redshifts, this would even allow determination of other cosmological parameters such as Ω_m, because these parameters affect the redshift–distance relation at high redshift.

There are three problems with this approach. The major difficulty is that for all the angles to be determined, we must have a good knowledge of the mass distribution of the lens, since this affects the deflection angle α. For example, an overestimate of the central mass in the lensing galaxy would cause an overestimate of α, which in turn would result in a distance estimate that was too small and consequently an overestimate of H_0. As previously discussed, the correct mass distribution of lenses still has significant uncertainties, although some constraints are available by considering the positions and flux densities of the lensed images.

The other two problems are observational. First, the redshifts of both lens and source must be determined, which requires observations of optical spectral lines and hence an optical identification for both lens and source. In practice, it has proved difficult to measure redshifts for very weak radio sources and our group is currently following up seven "difficult" source redshifts out of a sample of 19 lenses. Second, the time delays must be determined accurately enough to give useful constraints on H_0.

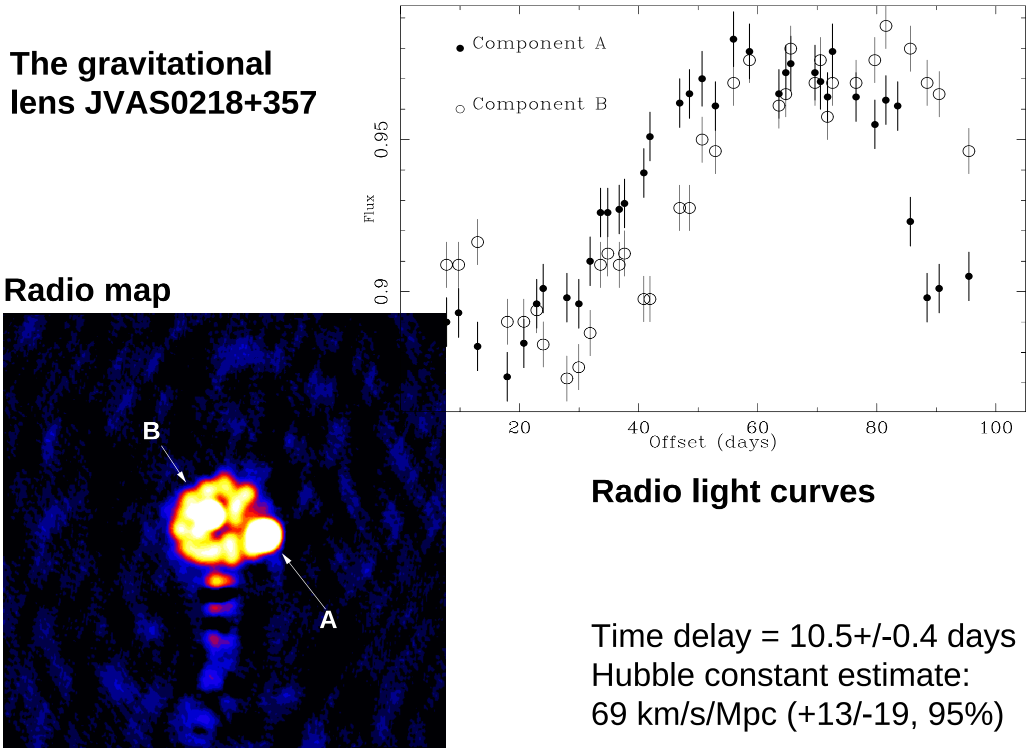
The ideal lens for H_0 determination, known in the business as a “golden lens”, would have a number of characteristics. The main requirements are a highly variable source; a single lensing galaxy with no nearby field galaxies for ease of modelling; an Einstein ring in addition to the lensed images, for best constraints on the model; many lensed images; and a relatively long time delay, for accurate $\Delta\tau$ determination given typical experimental errors $\sigma_{\Delta\tau}$ ~1 day. Unfortunately, long time delays imply large image separations, which in turn tend to be produced by multiple galaxies which are difficult to model; in such cases simpler, smaller lenses are to be preferred as the random error in delay is easier to tackle than the systematic errors introduced by problems with the mass model.

Although no lens so far is perfect, radio-selected gravitational lenses approximate much better to golden lenses. Many contain images of flat-spectrum, intrinsically variable radio sources, some of which are variable both in total intensity and polarization, and radio interferometry achieves the resolution necessary to separate variations in individual images with the necessary accuracy. Radio observations also allow much better sampling of the variability, as they can often be done 24 hours per day from a given site and are less subject to censorship by bad weather or (in the case of longer-timescale variability) by closeness of objects to the Sun for several months per year. It is no coincidence that the majority of the existing H_0 determinations have been made for radio-selected gravitational lenses.

Useful time delay and H_0 determinations have been made for seven lenses so far. Table 1 shows the current best estimates for each of these lenses, using standard SIEs as the mass model. It also attempts to show the major systematic problems and to give a qualitative idea of the likely uncertainties each will introduce.

	Time delay (see refs[1])	Galaxy position	Cluster or nearby gals.	Single lens?	Micro-lensing?	Constraints available?	H_0 (see refs[1])
Golden lens	**<1%**	**yes**	**no**	**yes**	**no**	**YES**	**???**
JVAS B0218+357	3%	no	no	yes	no	YES	$69^{+13}_{-19}(2\sigma)$
0957+561	<1%	yes	YES	yes	yes	yes	$64\pm13(2\sigma)$
HE1104−1805	34%	yes?	(no)	yes	yes	no	
PG1115+080	6%	yes	YES	yes	no	YES	42 ± 6
B1600+434	4%	no	yes	yes	yes	no	$57^{+14}_{-11}(2\sigma)$
B1608+656	4%	no	yes	no	no	yes	$59^{+8}_{-7}(2\sigma)$
PKS1830−211	17%	no	yes	yes	no	YES	65^{+15}_{-9}

Caption to table 1: Summary of the authors' assessment of suitability of existing gravitational lens systems (column 1) with determined time delays (column 2) for the determination of H_0. References for the time delays in column 2 are as follows: JVAS B0218+357 *Biggs et al., [1999]*; B0957+561 *Kundic et al., [1997]*; HE1104-180 *Wisotzki et al., [1998]*; PG1115+080 *Schechter et al., [1997]*; *Barkana, [1997]*; CLASS B1600+434 *Koopmans et al., [2000], Burud et al., [2000]*; CLASS B1608+656 *Fassnacht et al., [1999]*; PKS1830−211 *Lovell et al., [1998], Lidman et al., [1999].* In column 3 we give our assessment of whether the galaxy position is sufficiently well known to avoid major errors in H_0 determination, and in column 4 we assess whether nearby galaxies or clusters produce systematic errors in the mass model. Column 5 indicates whether the lens consists of one



The gravitational lens JVAS0218+357

Radio map

Radio light curves

Time delay = 10.5+/-0.4 days
Hubble constant estimate:
69 km/s/Mpc (+13/-19, 95%)

Figure 5: Hubble Constant determination *[Biggs et al., 1999]* from time delays between variations in the lensed images in JVAS B0218+357, shown here in a MERLIN/European VLBI Network radio map. Residual errors in the Hubble Constant are mainly due to uncertainty about the exact position of the lensing galaxy; the uncertainty without this is 5% (1σ). [A colour version of this figure appears in the CD-ROM version of this chapter].

or more galaxies, column 6 shows whether evidence is available for microlensing affecting the observed fluxes and column 8 indicates whether good observational constraints are available for the mass model, from >2 images or VLBI structure or both. Systems with an Einstein ring are marked YES in this column. Finally, column 9 gives an H_0 estimate. Many lenses have been investigated by a number of authors, and the table generally gives the value quoted by the paper which contains the first measurement of the time delay. Authors have generally derived values assuming (Ω_m, Ω_Λ) = (1,0). For the currently favoured (0.3,0.7) universe, H_0 estimates increase by factors of 5-10%, and by 20% in the higher-redshift system PKS1830−211.

Overall, most lenses have major systematic problems; in JVAS B0218+357 the galaxy position is not well enough known *[Lehár et al., 2000]*, in 0957+561 the cluster mass profile is probably not well enough understood *[Barkana et al., 1999*, but see also *Keeton et al., 2000]*; HE1104−180 is a double with not enough modelling constraints for complete security and as yet no spectroscopic lens redshift, although improved determination of the lens galaxy centre has recently been made *[Courbin, Lidman & Magain 1998]*; in PG1115+080 more constraints are needed on the galaxy and surrounding group [*Impey et al., 1998]* to tie down the mass model *[Keeton & Kochanek 1997]*; *[Courbin et al. 1997]*; CLASS B1600+434 has a spiral lens galaxy whose centre is not well constrained, few modelling constraints in the absence of significant VLBI structure in both images and additionally suffers from the problem of a bright nearby galaxy *[Koopmans, de Bruyn & Jackson 1998]*. In CLASS B1608+656 the principal lensing galaxy is a double whose centre-of-mass position is highly uncertain *[Jackson, Nair & Browne, 1997]*; in PKS1830−211 the galaxy position is uncertain enough to introduce a substantial degeneracy into the mass model *[Lehár et al., 2000]* and the lens galaxy lies in a small cluster. In the authors' opinion, based on the entries in this table, JVAS B0218+357 is the nearest approximation to a "golden lens" with the single caveat that its small size makes the centre of the lensing galaxy difficult to pin down.

The way forward obviously involves the removal of systematic uncertainties in the lensed systems. Unfortunately, the systems with the best-constrained mass models are often not the ones with the measured time delays and the search for the "golden lens" continues. In the authors' opinion one of the best prospects is JVAS B0218+357 (Figure 5), which has the single disadvantage, due to its small size, that the centre of the lensing galaxy, even with current HST imaging is not accurately fixed with respect to the radio components.

4.2 The Cosmological Constant and matter density parameter

There are two very important cosmological parameters which control the overall topology and future development of the universe: Ω_m, the matter density parameter and Λ_0, the "cosmological constant". Ignoring Λ_0, Ω_m expresses the density of matter as a fraction of the density needed to "close" the universe – that is, eventually to halt the universal expansion and bring the universe back to a Big Crunch. In terms of luminous matter $\Omega_m \ll 1$, but there is ample evidence from dynamical studies of galaxies and clusters for a great deal of "dark matter" which contributes to Ω_m but does not emit significant electromagnetic radiation.

The cosmological constant Λ_0 (see *Carroll, [2000]* for a comprehensive review) represents an energy density which is not due to matter, but which can instead be associated with