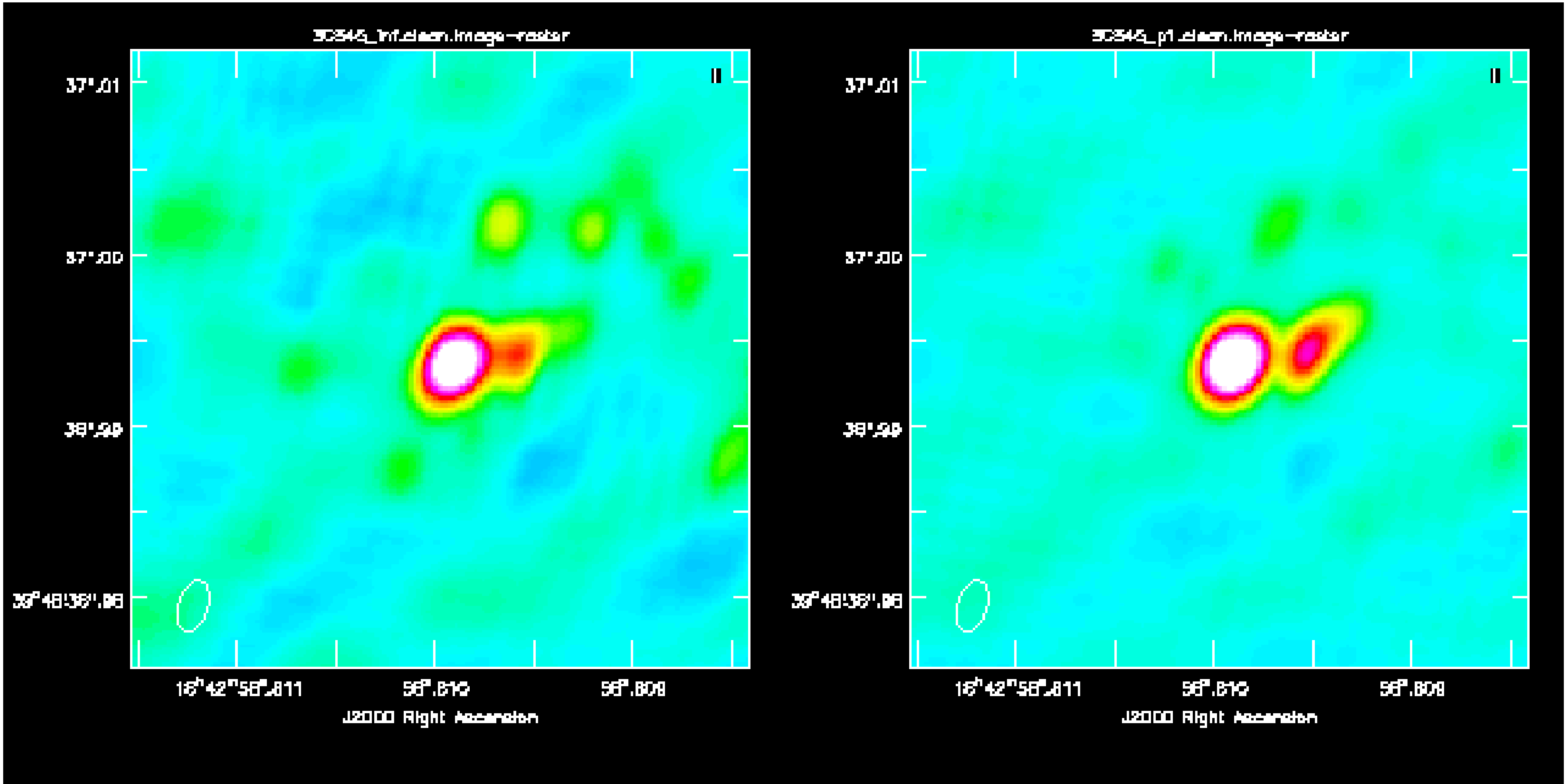
Tailoring calibration

Original slides by Anita Richards

Acknowledgements: Robert Laing (ESO), Rick Perley (NRAO)



Tailoring calibration

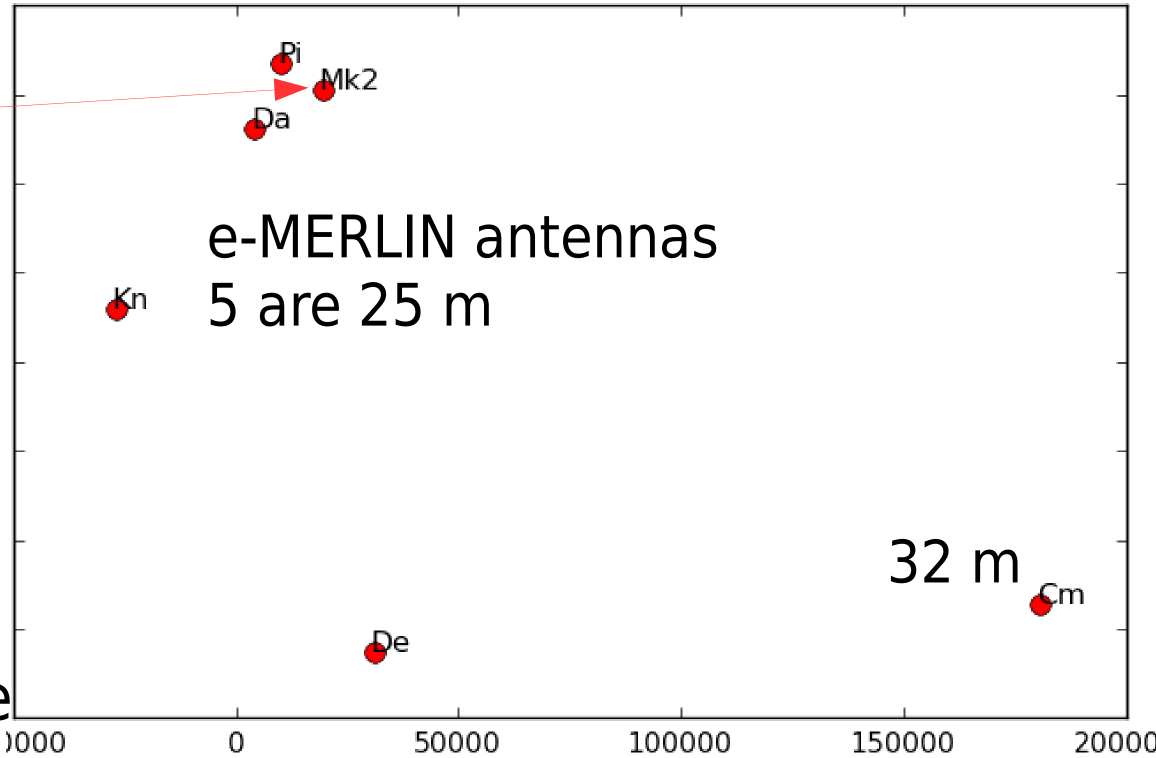
- How do you know what parameters to set?
  - What values?
    - e.g. what solution interval for phase calibration?
- May depend on physical/instrument properties which are fixed for a given observation, for example:
  - T_{sys} tables – replace -ives by interpolated values
    - System temperature cannot be negative!
  - Image pixel size: >3 pixels across synthesised beam θ
    - Use θ = min(λ)/max(baseline), explained in Imaging talk
  - Easy to pipeline

Observation-dependent parameters

- Calibration strategy depends on
  - Observing frequency, baselines, bandwidth etc.
  - Weather and source elevation
  - Calibration source properties
- Imaging depends on all these and on science goals
  - Faint, extended source?
  - Need very accurate astrometry?
  - Very bright, self-calibratable source?
  - Spectral lines?
  - Sources all over the field of view?

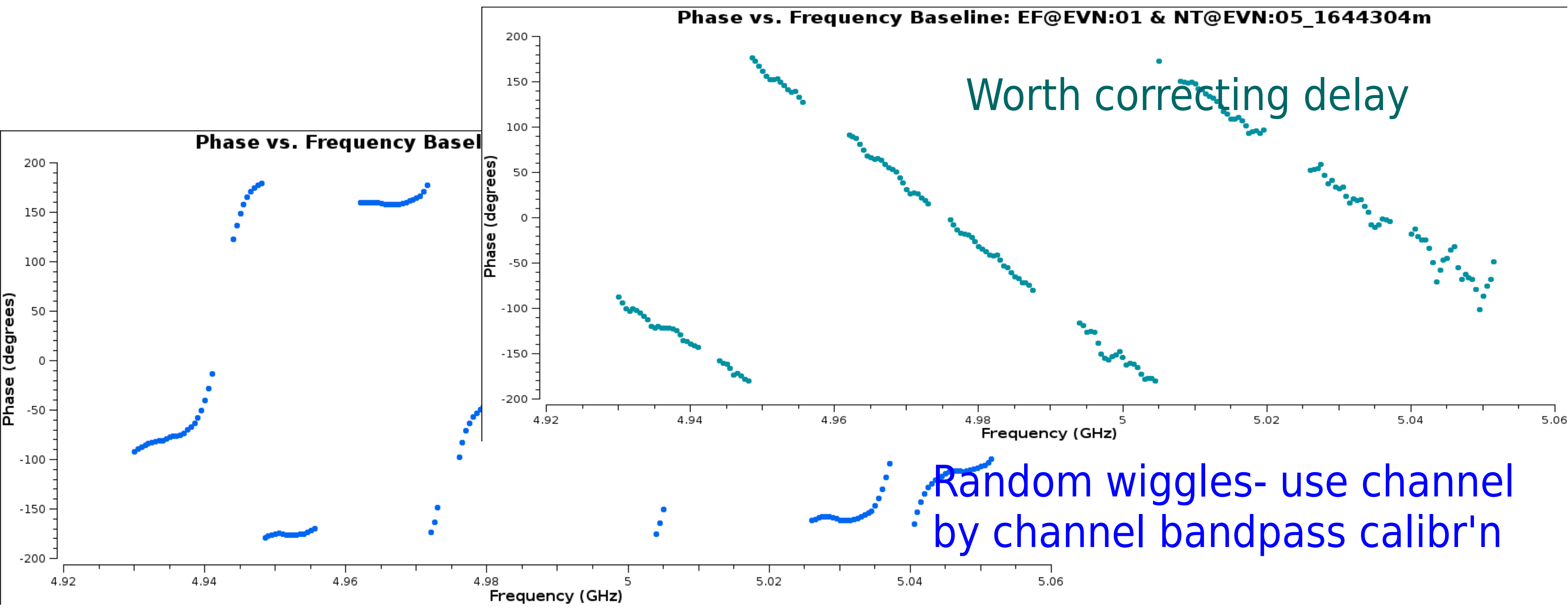
Chosing reference antenna

- The antenna with the best chance of good solutions to all other antennas
  - The one with the most short baselines?
    - Greater atmospheric differences on long baselines
  - Most sensitive?
- e-MERLIN: usually use Mk2 (or Pi or Da)
  - Cm too far away
- EVN: Ef both most sensitive and central
- Refant phase fixed 0^{o}
  - All other phases relative



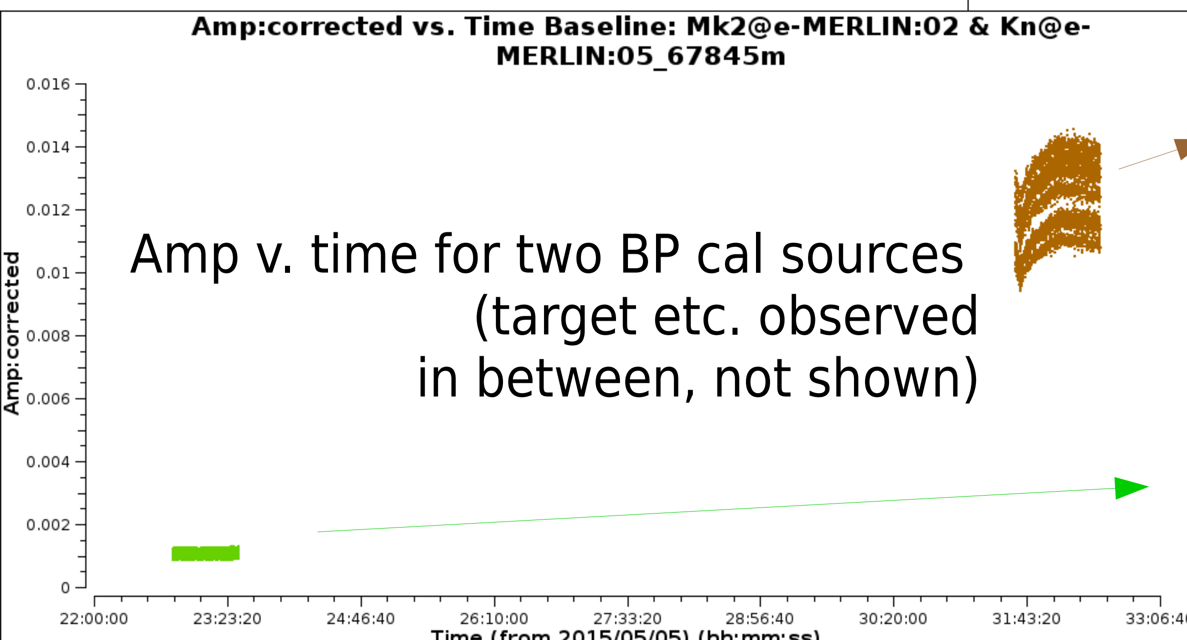
Delay calibration

- Delay corrections for linear phase gradients:
  - Inspect phase v. frequency
  - Only worth correcting delay if you can see it
  - Usually stable for hours but averaging solint limited (~scan) by time-dependent phase stability

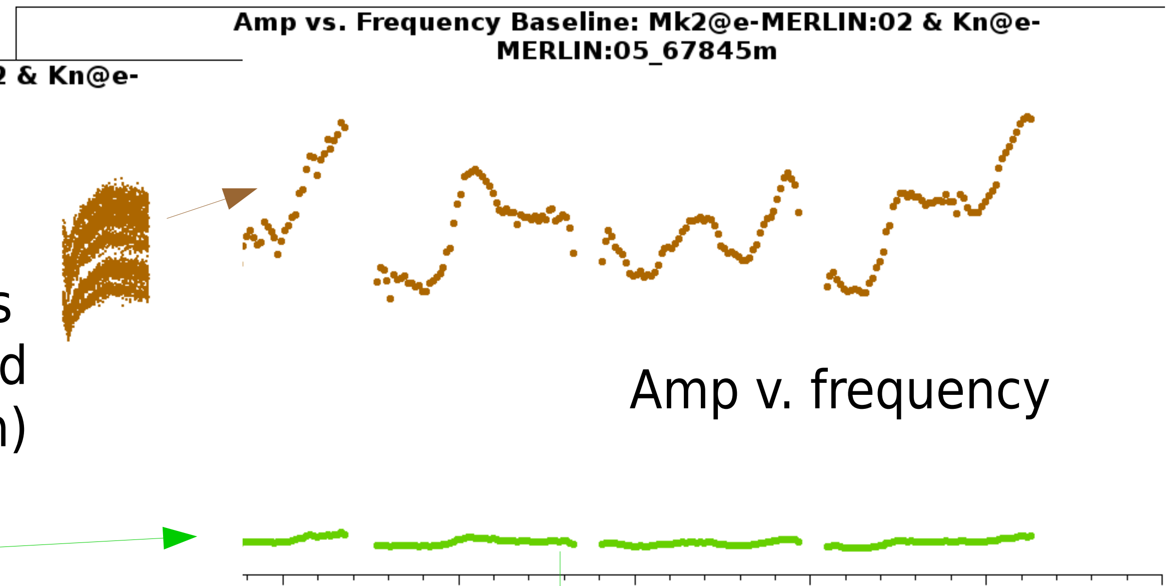


Bandpass calibration

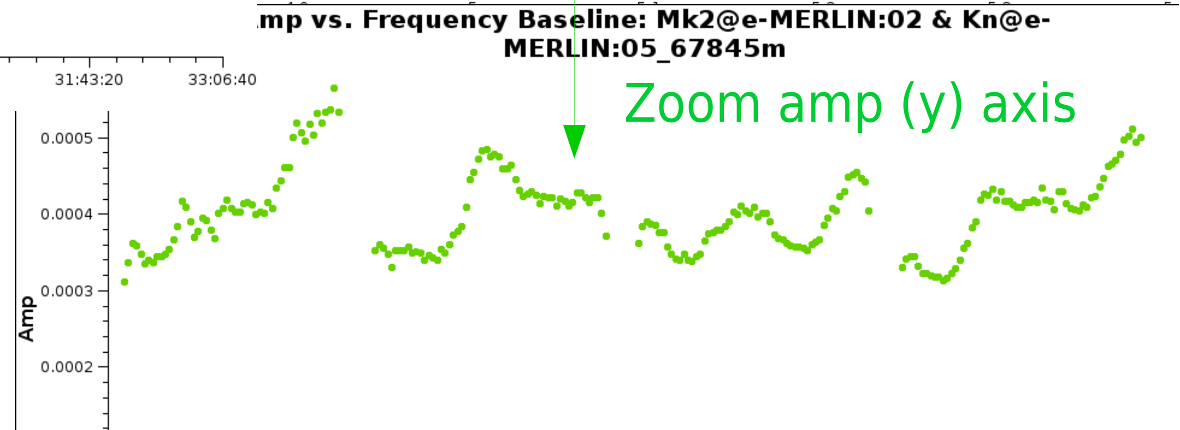
- Correct BP cal phase v. time first (see following slides)
  - In Bandpass, average in time for as long as possible for best S/N per channel



Amp v. time for two BP cal sources (target etc. observed in between, not shown)



Amp v. frequency

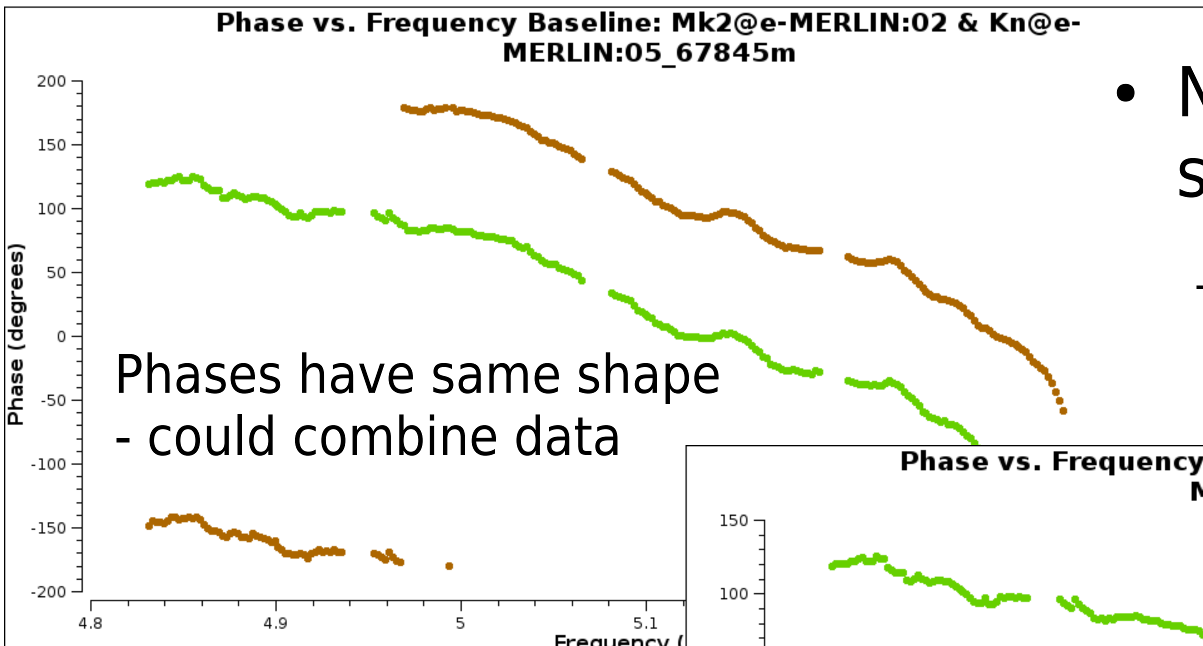


Zoom amp (y) axis

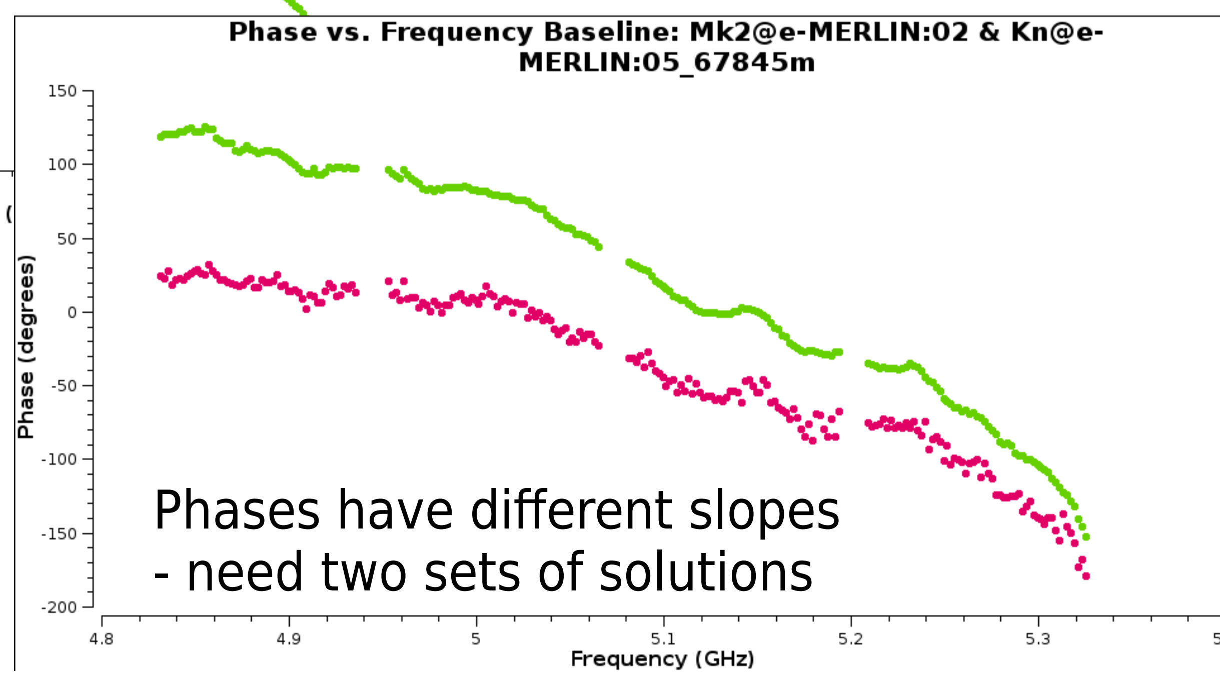
- Both BP cals have same amp wiggles
  - Could combine, interpolate or use just the one with best S/N

Bandpass calibration

- Check BP data phase v. frequency also



- Normalise bandpass solutions
  - Flux scale may differ or be unset



- In applycal, use interp='nearest' to allow extrapolation
- May need to select timeranges

Visibility errors and noise

- Lowest possible noise is 'thermal' limit based on T_{sys}:

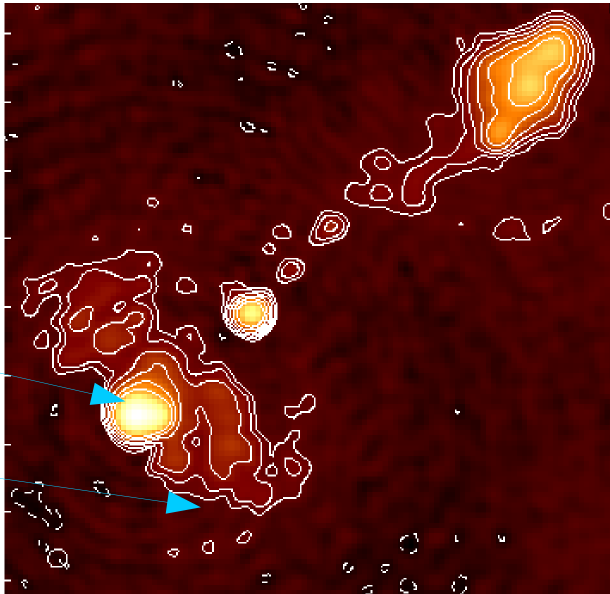
$$\sigma_{sys} = \frac{\langle T_{sys} \rangle}{\eta_A A_{eff} \sqrt{N(N-1)/2 \;\; \Delta\nu \;\; \Delta t \, N_{pol}}}$$

  – Where $$T_{sys} = \frac{1}{\eta_A e^{-\tau_{atm}}}\left[T_{Rx} + \eta_A T_{sky} + (1-\eta_A) T_{amb}\right]$$

- So you can only improve on this by
  – Bigger/more efficient antennas (A_{eff}, η_{A}) or more (N)
  – Lower noise Rx and/or T_{sky} (observing conditions)
  – Or, for given array, observe for longer/wider bandwidth
- But other factors often limit the noise....

What accuracy is needed?

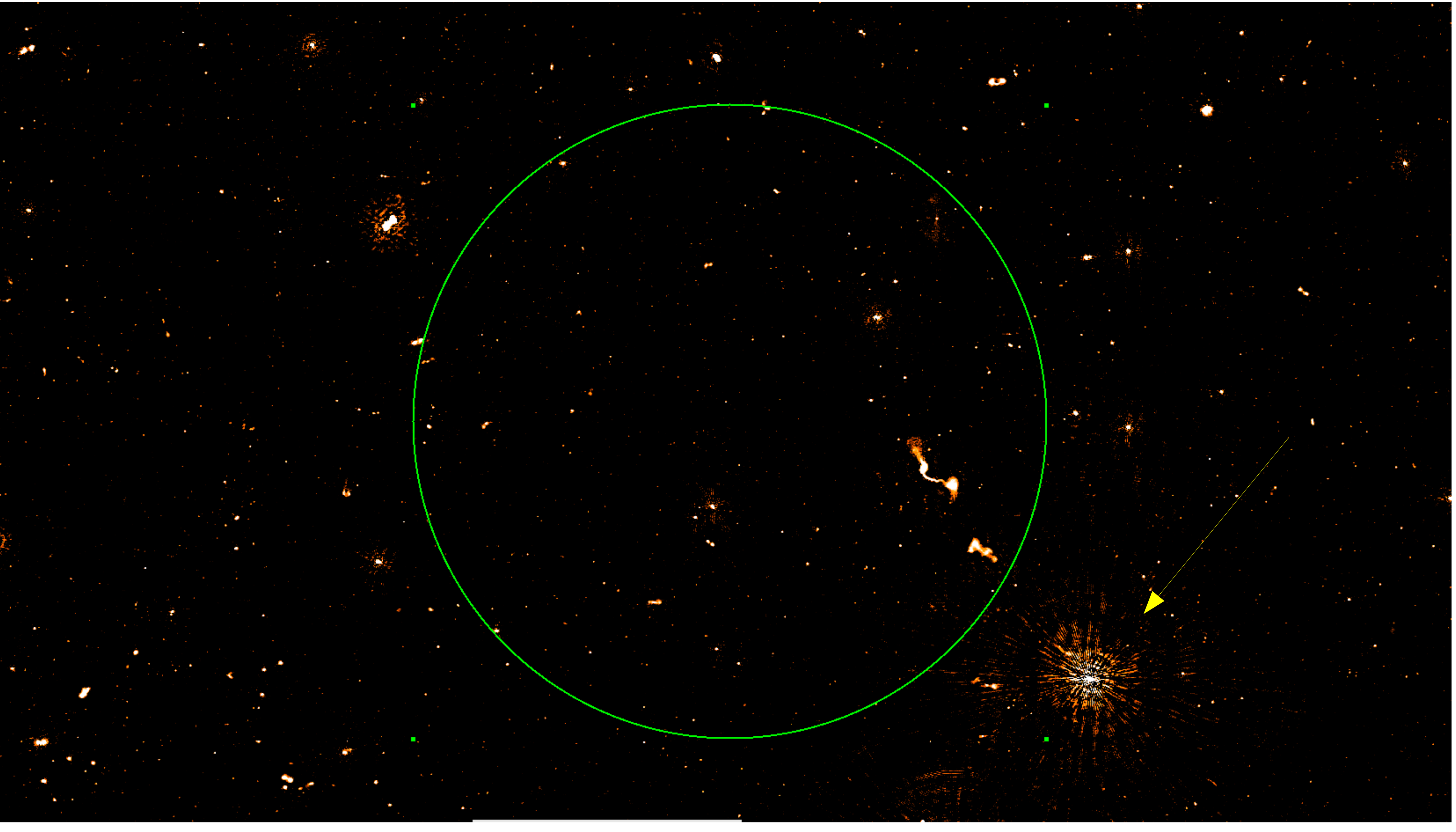
- What is the effect on imaging of visibility errors?
  - How good does the calibration need to be?
    - 'Received wisdom' provides properties and suggested solution intervals, etc. for a given array
  - Good to know that there is a theoretical basis, though
    - Massive data sets: much trial and error takes too long
- How bright is your target?
  - Is the peak bright enough to self-cal?
  - How faint are the weakest features of interest?



What accuracy is needed?

- Faint target: need to reach thermal noise
- Bright target: may be dynamic range limited
  - Need the best possible calibration and imaging
  - If self-calibration is possible it just needs to be ‘good enough;
- Astrometry:
  - Need high phase accuracy for position accuracy
    - Special strategies
      - Several phase reference sources
        - Can use multiple elevations/frequencies to measure delay and antenna positions with high accuracy
- Photometry
  - High phase and amplitude accuracy
    - Multiple calibration sources

Dynamic range limitation



Phase errors and dynamic range

- Simplified: flat, linear array, N antennas
  - Single integration observation of a point source
    - Direction such that we only need to consider u axis
  - $N(N-1)/2$ visibilities
- Each baseline visibility is a δ spike in the *uv* plane
  - All but one are 'perfect' (unit amplitude, zero phase)
    - These have $V(u) = \delta(u - u_k)$ for the k^{th} baseline
  - Phase error on baseline length u_0 of ϕ_ε radians
    - $V(u) = \delta(u - u_0)\, e^{-i\phi\varepsilon}$

Phase errors and dynamic range

- Image is formed by Fourier transform
  - $I(x) = \int V(u)\, e^{\,i2\pi ux}\, du$
    - Each baseline contributes at position u_k and complex conjugate $-u_k$ in the visibility plane
- Evaluating the term in the integral for each of the $[N(N-1)/2]-1$ good baselines gives $2\cos(2\pi u_k x)$
- Bad baseline gives $2\cos(2\pi u_0 x - \phi_\varepsilon)$
  - $\sim 2[\cos(2\pi u_0 x) + \phi_\varepsilon \sin(2\pi u_0 x)]$ for small ϕ_ε (in radians)
- The image integral thus sums to

$$I(x) = 2\phi_\epsilon \sin(2\pi u_0 x) + 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)$$

Phase errors and dynamic range

- The synthesised beam is given by

$$B(x) = 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x) \quad = N(N\text{-}1) \text{ for } u = 0$$

- Deconvolution is the subtraction of the beam from the image leaving the residual error

$$R(x) = \left[2\phi_\epsilon \sin(2\pi u_0 x) + 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)\right] - 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)$$
$$= 2\phi_\epsilon \sin(2\pi u_0 x)$$

  - an 'odd' sinusoidal function of amplitude $2\phi_\varepsilon$, period $1/u_0$

- To maintain the flux scale, integrals are normalised:

$$\frac{R(x)}{N(N-1)} = \frac{A\,I(x)}{N(N-1)} - \frac{B(x)}{N(N-1)}$$

Here, 'true' amplitude $A = 1$

Calibration errors and dynamic range

- The rms of the residual $R(x) = \frac{2\phi_\epsilon \sin(2\pi u_0 x)}{N(N-1)}$

  over the whole map is $\sqrt{2}\,\phi_\varepsilon / N(N\text{-}1)$
- For small phase error ϕ_ε, large N, the ratio of the peak / noise residual is thus
  - Dynamic range $D_{\mathrm{B}}(\phi_\varepsilon) \sim I(x) / R(x) \sim N^2 / \sqrt{2}\,\phi_\varepsilon$
    - e.g., radians (5°)~0.09
- Amplitude error ε on a single baseline has the effect

  $V(u) = (1+\varepsilon)\delta(u - u_0)\, e^{-i\phi}$ leading (via a cos function) to
  - Dynamic range $D_{\mathrm{B}}(\varepsilon) \sim N^2 / \sqrt{2}\,\varepsilon$
- **A phase error of 5° is as bad as a 10% amp error**
- **Phase errors are sin (odd), amp are cos (even)**

Calibration errors and dynamic range

- So far considered one-baseline error, one integration
- All baselines to one antenna affected by same error:
  - (N-1) bad baselines (~N for large N)
  - $\boldsymbol{D}_{\mathbf{ant}} = D_{\mathrm{B}} / (\mathrm{N}\text{-}1) = [N^2/(N\text{-}1)] / \sqrt{2}\phi_{\varepsilon} \sim \boldsymbol{N} / \sqrt{\mathbf{2}}\boldsymbol{\phi}_{\boldsymbol{\varepsilon}}$
- If all baselines are affected by random noise,
  - $\boldsymbol{D}_{\mathbf{all}} = D_{\mathrm{B}} / \sqrt{[N(N\text{-}1)/2]} = \sqrt{[N(N\text{-}1)/2]}/\phi_{\varepsilon} \sim \boldsymbol{N}/\boldsymbol{\phi}_{\boldsymbol{\varepsilon}}$
- These expressions are valid if errors are correlated in time, e.g. single phase-ref scan, not much change in u (or v)
- For M periods (scans?) between which noise is uncorrelated
  - Dynamic range is increased to $\boldsymbol{D}_{\mathbf{all}} \sim \sqrt{\mathbf{M}}\, \boldsymbol{N}/\boldsymbol{\phi}_{\boldsymbol{\varepsilon}}$

Calibration for good dynamic range

- Implications so far: take a 10-antenna array
  - Twelve independent scans on a target, phase reference and other calibration applied, well edited
    - Residual phase scatter 20° : $\boldsymbol{D_{\mathrm{all}} \sim \sqrt{M}\, N/\phi_\varepsilon}$
    - ~ 100 dynamic range limit
      - Can you improve by self-calibration?
        - No if map noise has reached the T_{sys} limit and remaining errors are pure noise. If not:
    - Maybe, if some antennas are still imperfectly calibrated
      - Calibrate per antenna, per scan (or longer)
        - Need potential S/N per interval high enough to get ϕ_ε <20°
      - See self-cal talk

Phase-referencing dynamic range

- Most correctable errors affect all baselines to an antenna

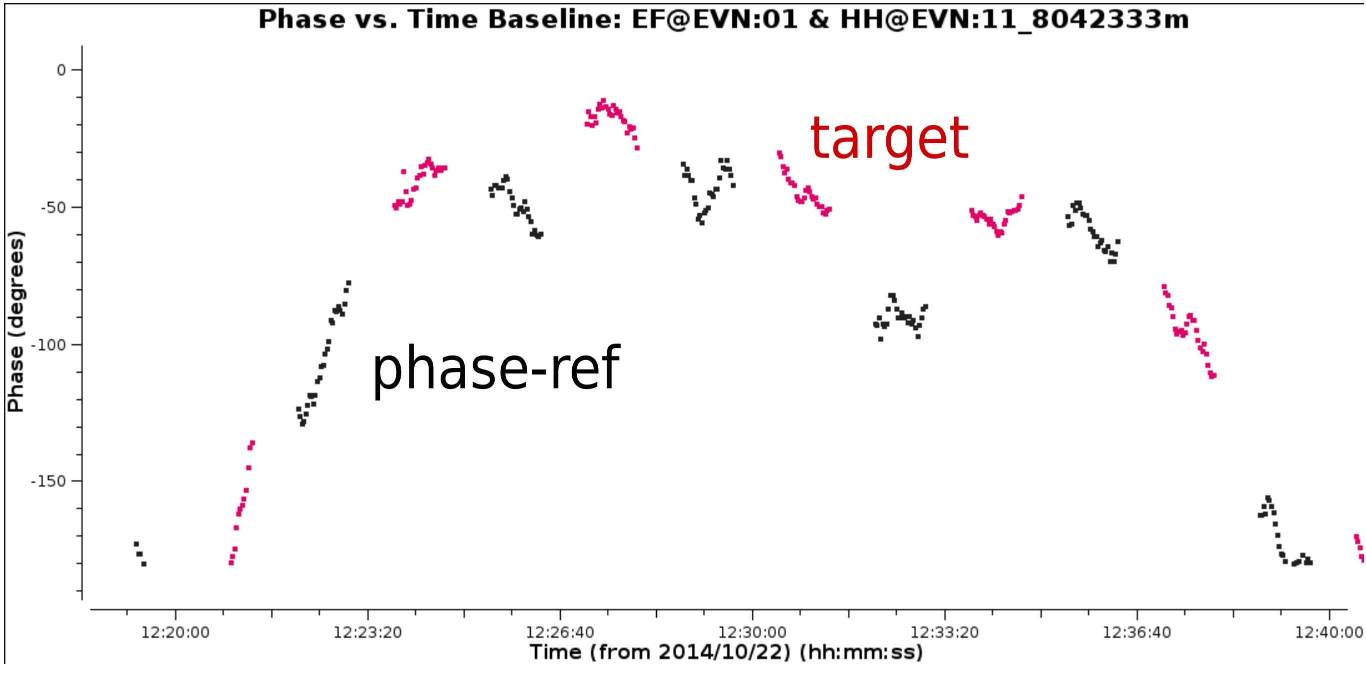
$$\sigma_{ant}(\delta t, \delta \nu) \approx \sigma_{array} \sqrt{\frac{N(N-1)/2}{N-3}} \sqrt{N_{spw} N_{pol}}$$

Solve separately for each spw, pol.

  – Sensitivity calculators generally give σ per total b/w
    - 8 spw, 2 polarizations, 1 min, 10-ant EVN σ_{array} 0.15 mJy
      – from www.evlbi.org/cgi-bin/EVNcalc.pl
    - Sensitivity limit per antenna σ_{ant} ~1.5mJy for 1 min
  – Use $\boldsymbol{D_{\mathrm{ant}} \sim N / \sqrt{2}\phi_\varepsilon}$, say want 5º phase accuracy
  – $S_{\mathrm{phsref}} / \sigma_{ant} = D_{\mathrm{ant}} \sim N / \sqrt{2}\phi_\varepsilon$ in radians
    - Need phase-ref flux density S_{phsref} > 120 mJy
      – In practice, need more to allow for bandpass etc. errors
        - This is assuming solutions per 1-min scan

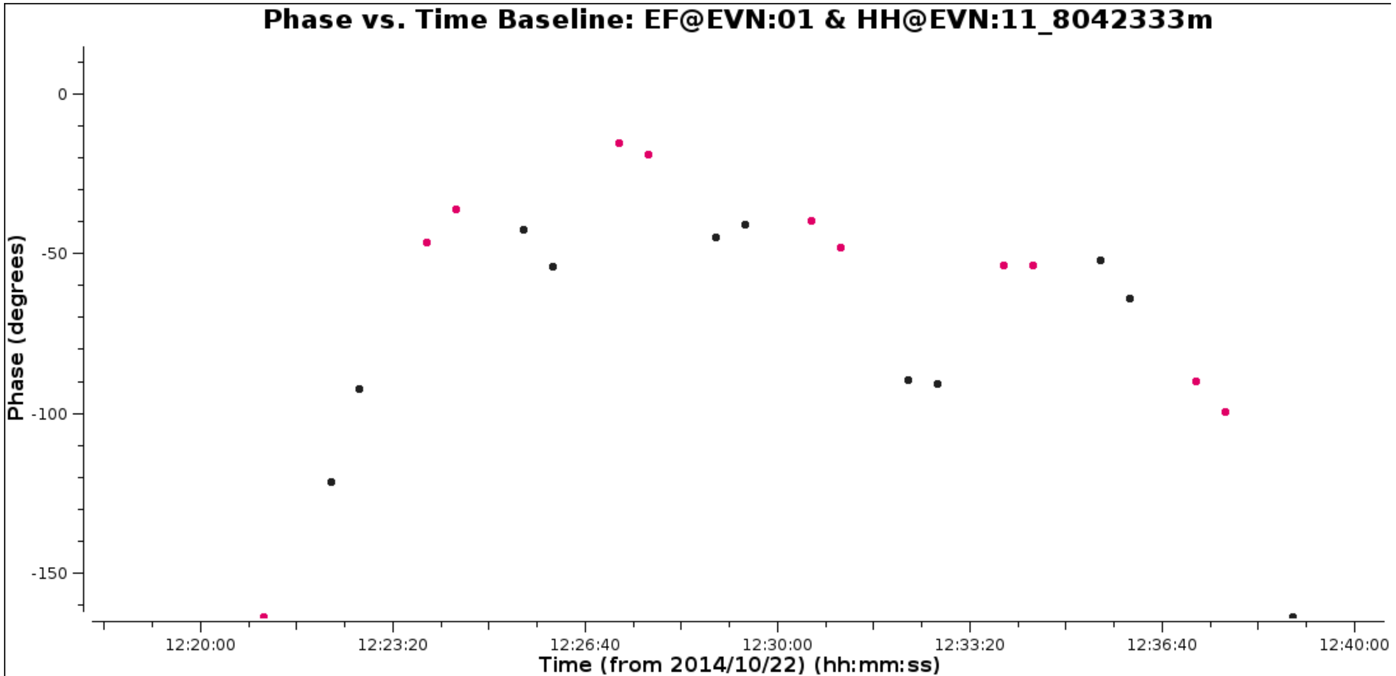
Time-dependent phase cal

- Apply bandpass/delay corrections
- Phase reference source:
  - Need to interpolate solutions to target
- Does the phase-ref phase track the target phase?
- Consistent trend seen here
  - Target wiggles may be structure
  - Some deviations



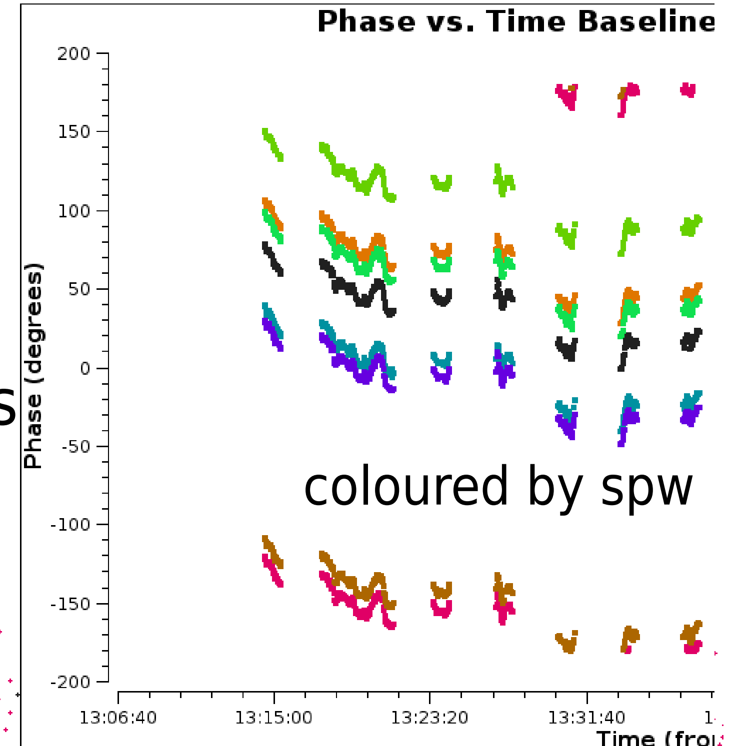
Time-dependent phase cal

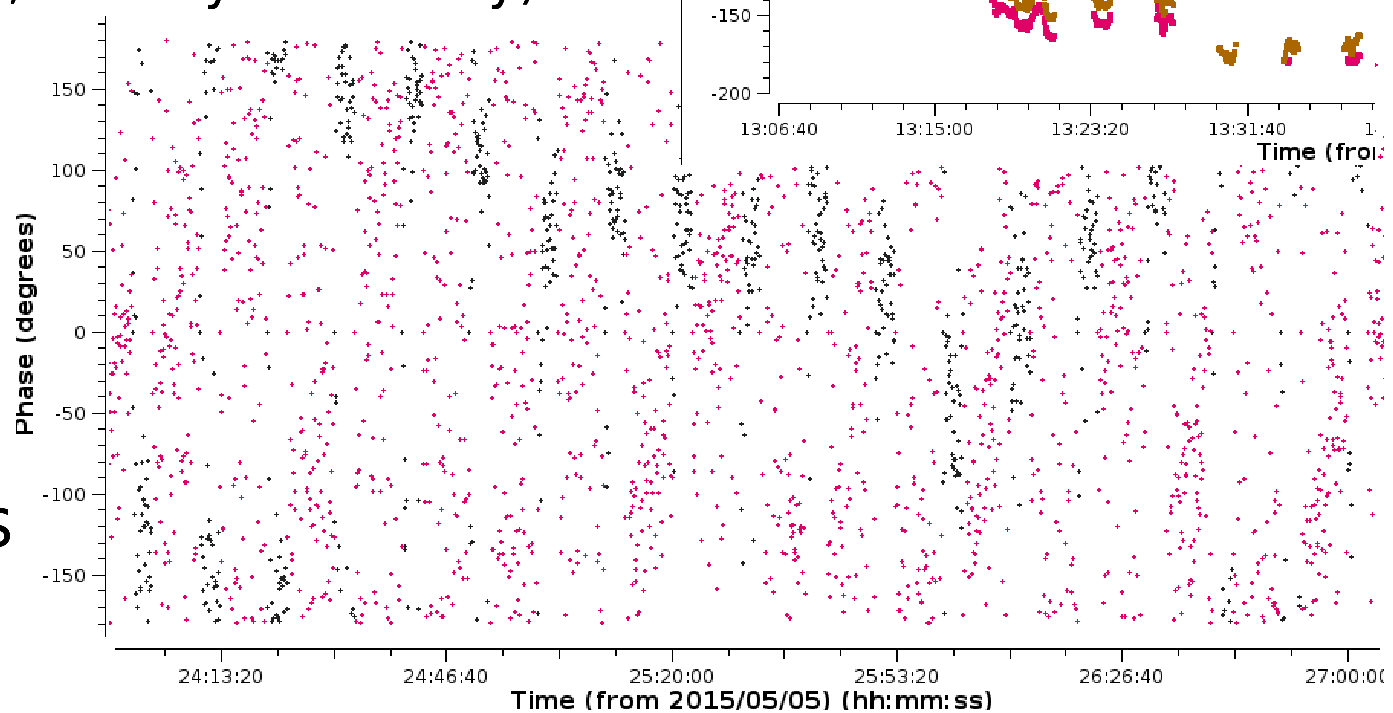
- Need to interpolate phase-ref solutions to target
  - Ideally no more than 2 solutions per phase-ref scan
    - Allows simple linear interpolation
      - Must track phase properly
    - Check enough S/N in e.g. half scan
      - Seeing low scatter by eye is OK!
- Previous plot with 30-s averaging
  - 30-s corrections will track phase better than per-scan



Time-dependent phase-cal

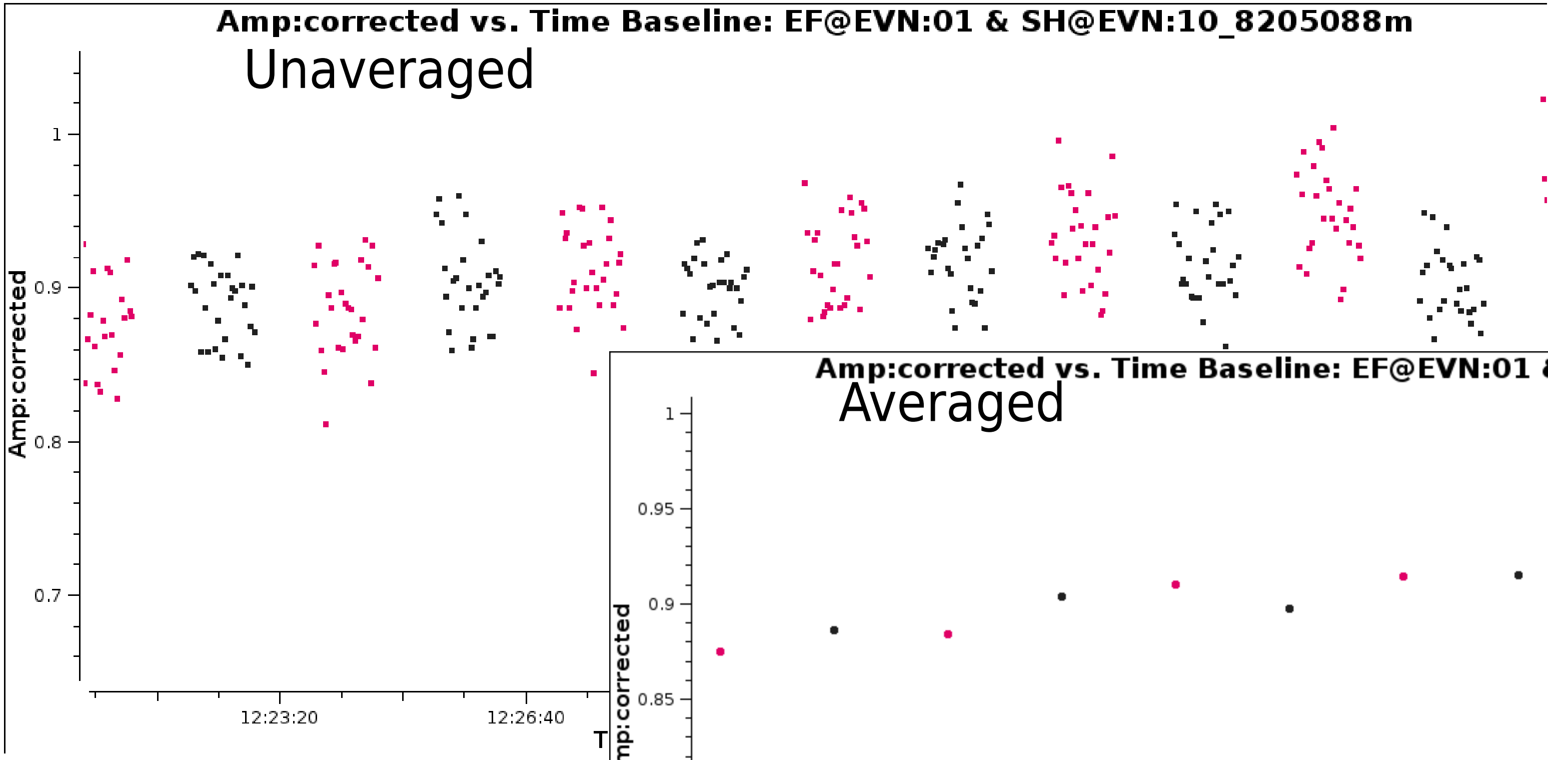
- Phase-ref very faint/fast-changing?
  - Average spw/pol to improve S/N?
    - Check bright cal source – can't average spw or pol. if phase offsets
      - (can use BP cal to align if offsets are stable, if really necessary)
  - Fit spline or polynomial
  - Can be fitted over several scans
  - 1^{st} order term is known as 'rate'





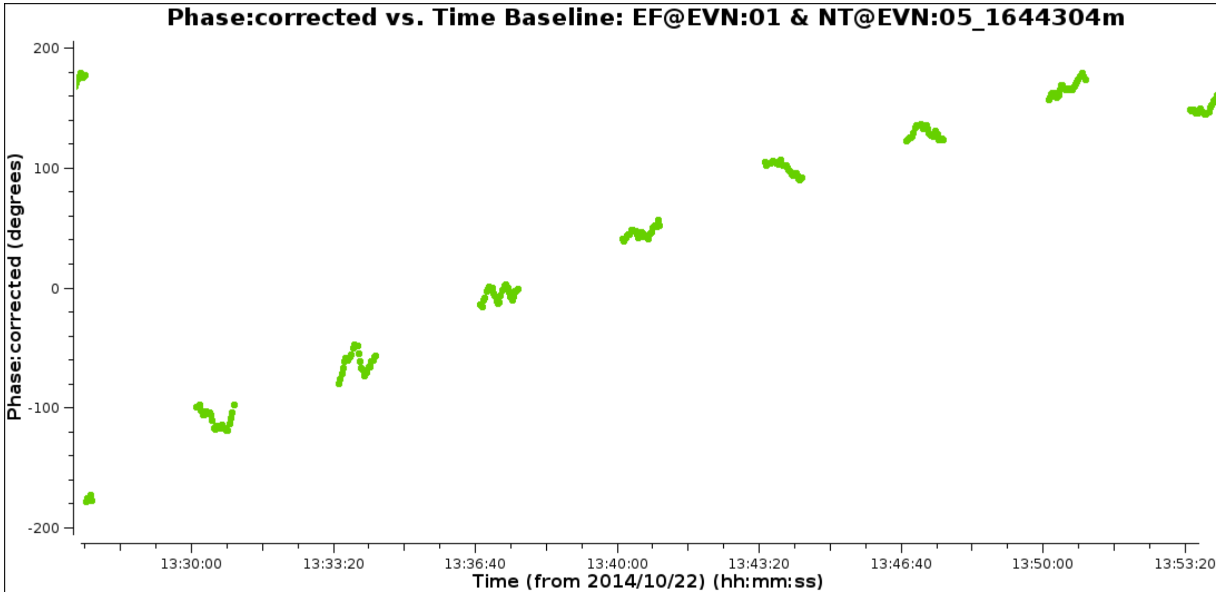
Time-dependent amp cal

- Apply phase solutions first to allow longer solint for amplitude calibration
  - Avoid decorrelation
    - If necessary, use shorter phase-only solint just for this
- Amp scatter per scan usually just noise
  - Average whole scan
  - Solutions will track changes OK



Phase transfer accuracy

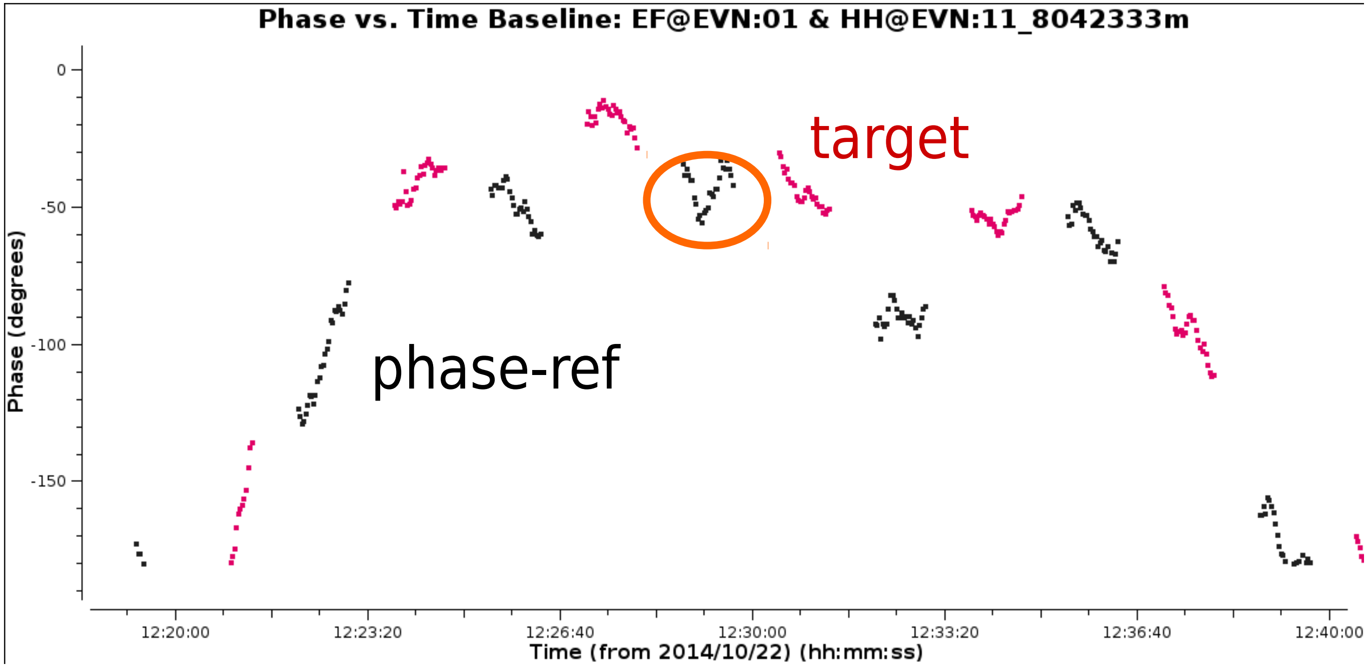
- **Sky separation**
  - Raw BP cal shows phase change $d\phi_{atm}$ is 2π per ~30 min, mainly atmospheric



- Phase-ref: target separation, say $d\theta = 1^{o} = 60$ arcmin
  - Convert θ in degrees to 'R.A.-like' units of time
    - $(d\theta/360^{o})$ x $\cos$(Dec.)x 24hr ~3.75 min at Dec. 20^{o}
- In 3.75 min, $d\phi_{atm}$ gives $\pi/4 = 45^{o}$ phase change
  - Contributes $\theta_{beam}/4$ mas error to astrometric accuracy
    - But if random, only $45^{o}/\sqrt{M} \sim 10^{o}$ to phase noise overall

Phase transfer accuracy

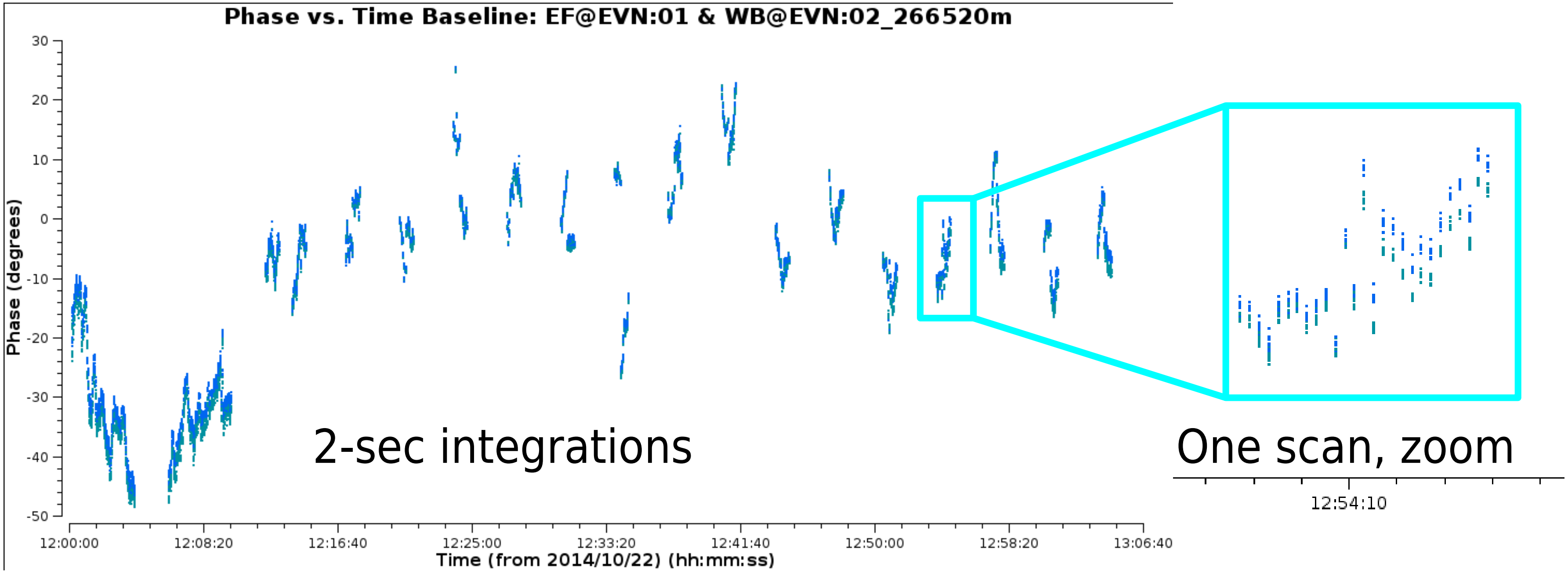
- **Phase jitter**
- ~20^{o} deviations within phsref scans
- Combine in quadrature with $d\phi_{atm}$ error 45^{o}
  - ~50^{o} phase error ϕ_{ε}



- Target M=17 scans, N~10 antennas for 3C345
  - $\boldsymbol{D_{\mathrm{all}} \sim \sqrt{\mathrm{M}}\, N/\phi_{\varepsilon}}$ gives dynamic range limit ~50
    - Might be less due to amp. errors etc. (I got 32 initially)

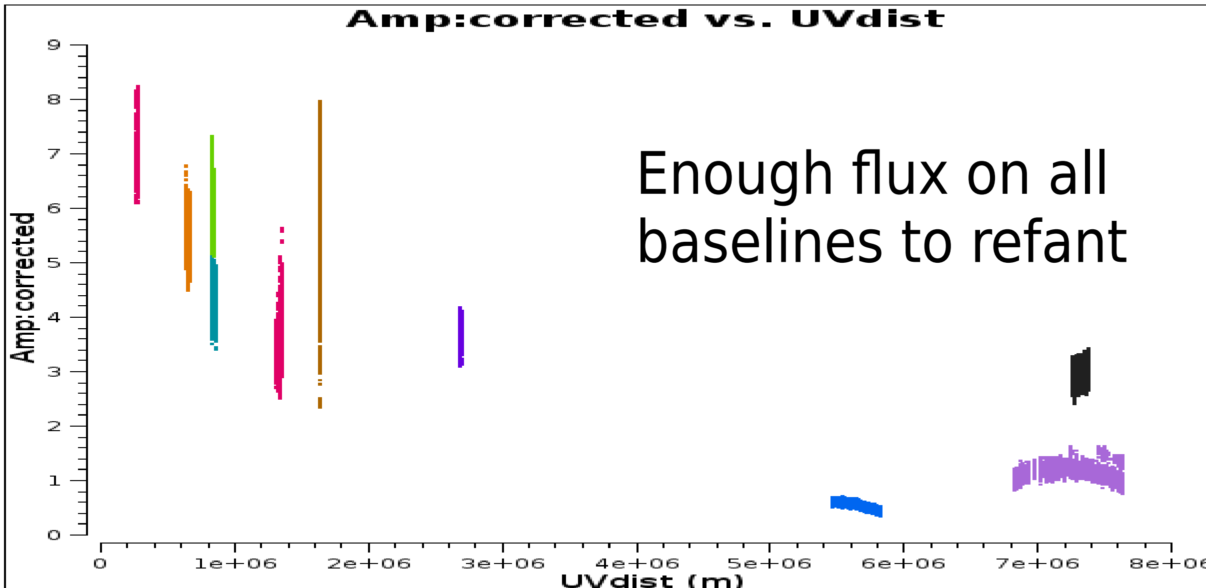
Self-cal timescales

- Target phase (after phs-ref corrections) changes rapidly
  - May be partly source structure, but seen even on short b'lines
    - Not just random noise even on 10-sec timescales

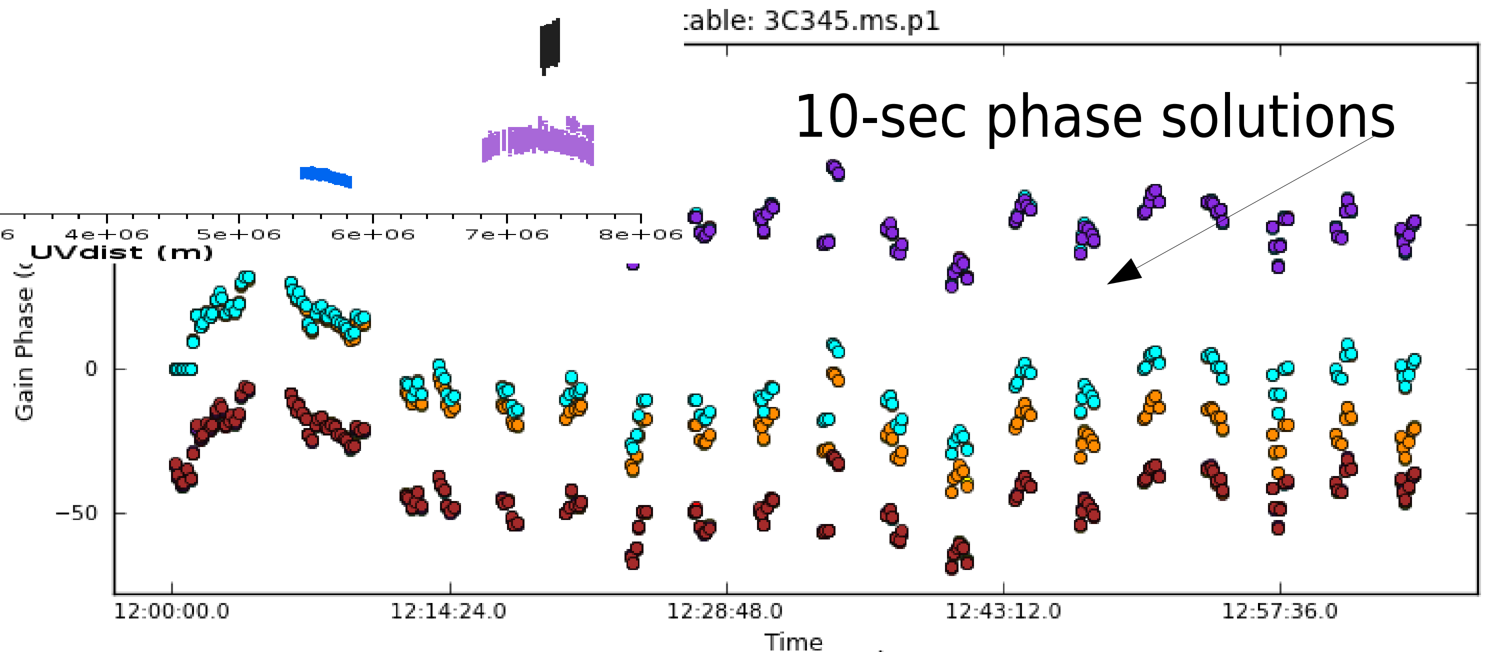


  - Thermal noise 0.3 mJy in 10 sec
    - From previous expression, phsref must be >240 mJy on all baselines to give enough S/N for 10 sec solution interval

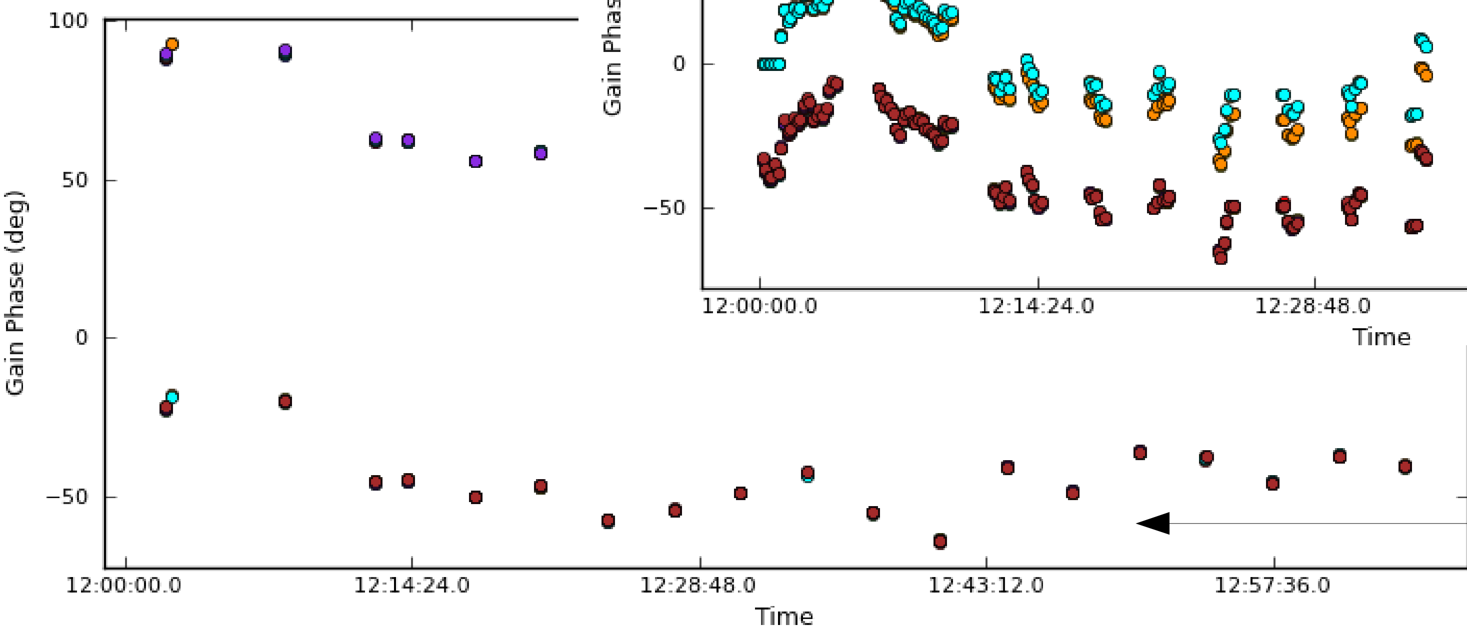
Calibration timescales



Enough flux on all baselines to refant

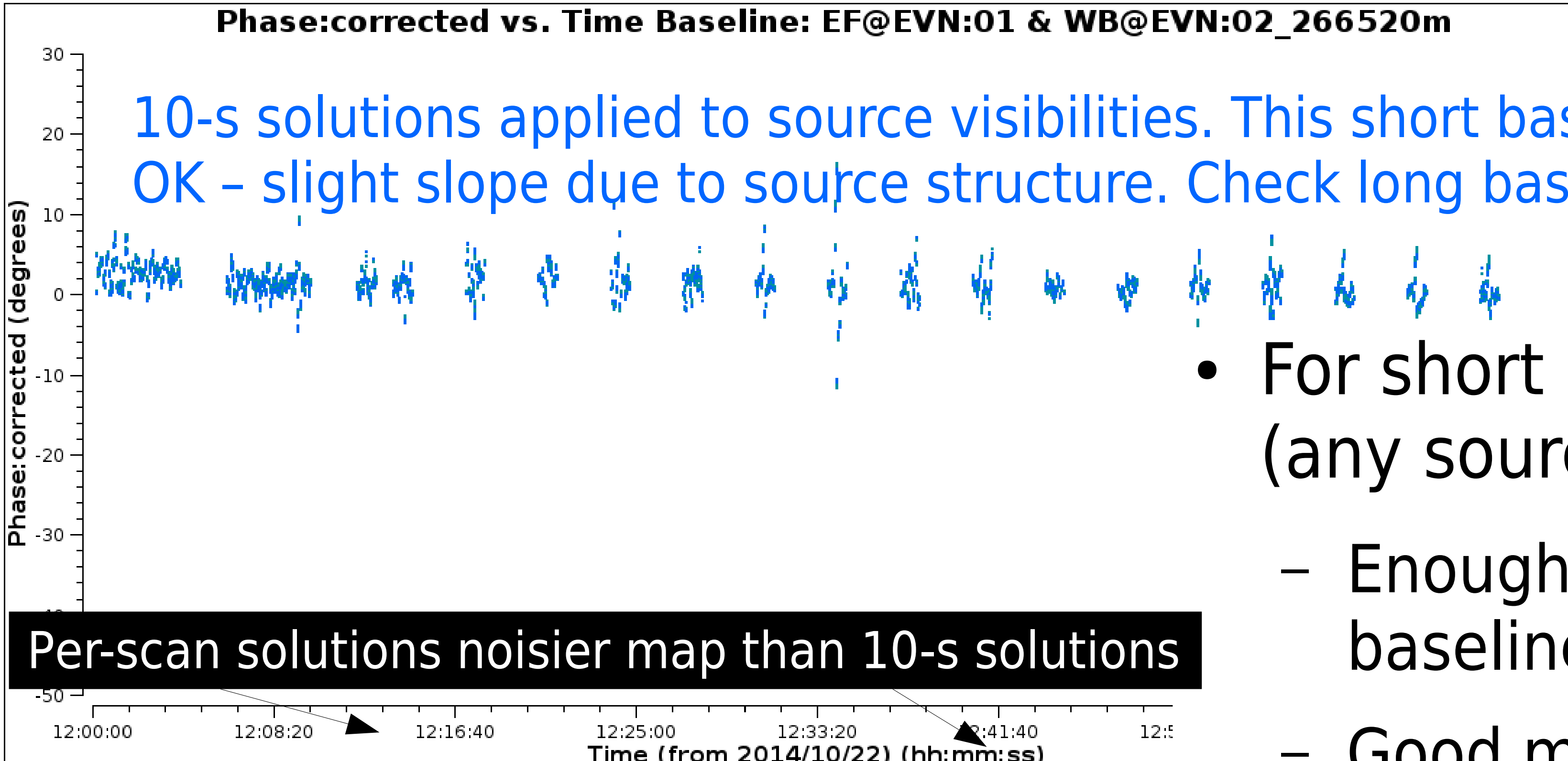


10-sec phase solutions



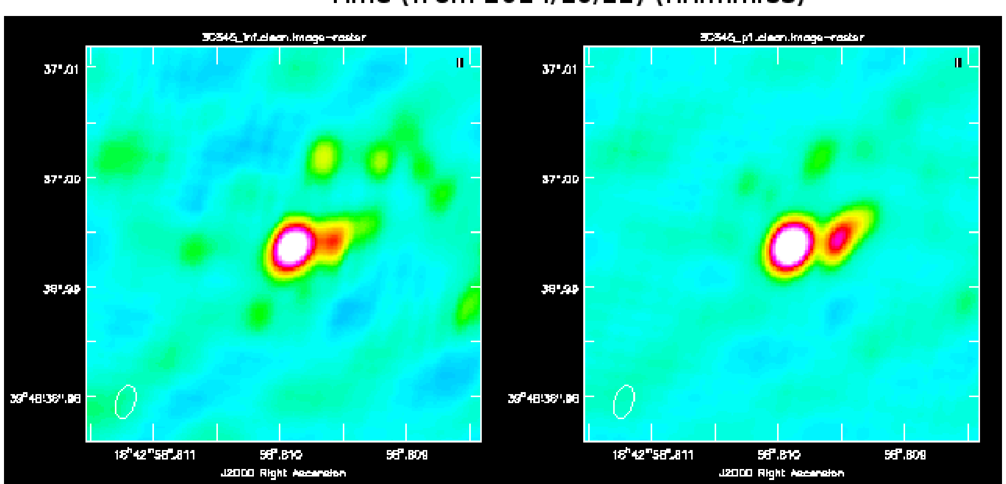
Track structured phase errors better than per-scan solutions

Short phase solutions OK?



10-s solutions applied to source visibilities. This short baseline looks OK – slight slope due to source structure. Check long baselines too!

Per-scan solutions noisier map than 10-s solutions



- For short solutions (any source) need:
  - Enough flux on all baselines to refant
  - Good model
  - Errors structured on short timescale
    - Can't correct random noise

Astrometric accuracy

- In the sort of observations used here, determined by:
  - Phase-ref position accuracy (check in catalogues)
    - May be shifted at different frequencies &/or resolved
      - Typically milliarcsec for VLBI calibrators
  - Antenna position accuracy (ask)
    - 1 cm error at λ 6cm is (1/6)θ_{beam} error
  - Phase transfer accuracy
    - see slide 22, <θ_{beam} error for good phase referencing
  - Position fitting (image analysis sessions)
    - Fit 2D Gaussian to compact source, error ~θ_{beam} /(S/N)
      - NB For target, fit to first image *before* self-calibration
- Add errors in quadrature

Pitfalls

- In CASA, calibration tables are *divided* into the data
  - e.g. apparent visibility amp. 1.5, phase 30^{o} = $\pi/6$ rad
    - Model is amp. 0.5. phase 0^{o}
      - Correction is $3e^{\pi/6}$
      - so ($1.5e^{\pi/6}$ / $3e^{\pi/6}$)=(1.5/3) $e^{(\pi/6-\pi/6)}$ =0.5 e^{0}
- In AIPS, the data are *multiplied* by the corrections
  - In this example, the correction would be $0.333e^{-\pi/6}$
- Beware small CASA amp corrections (large in AIPS)
  - Noise will be greatly increased, may be bad data
- If data look like noise, before you despair:
  - Check correct calibration applied, tweak parameters?
  - Make sure you are plotting RR,LL (or XX,YY) (cross-hands fainter)
    - Don't ever average || hands in uncalibrated data