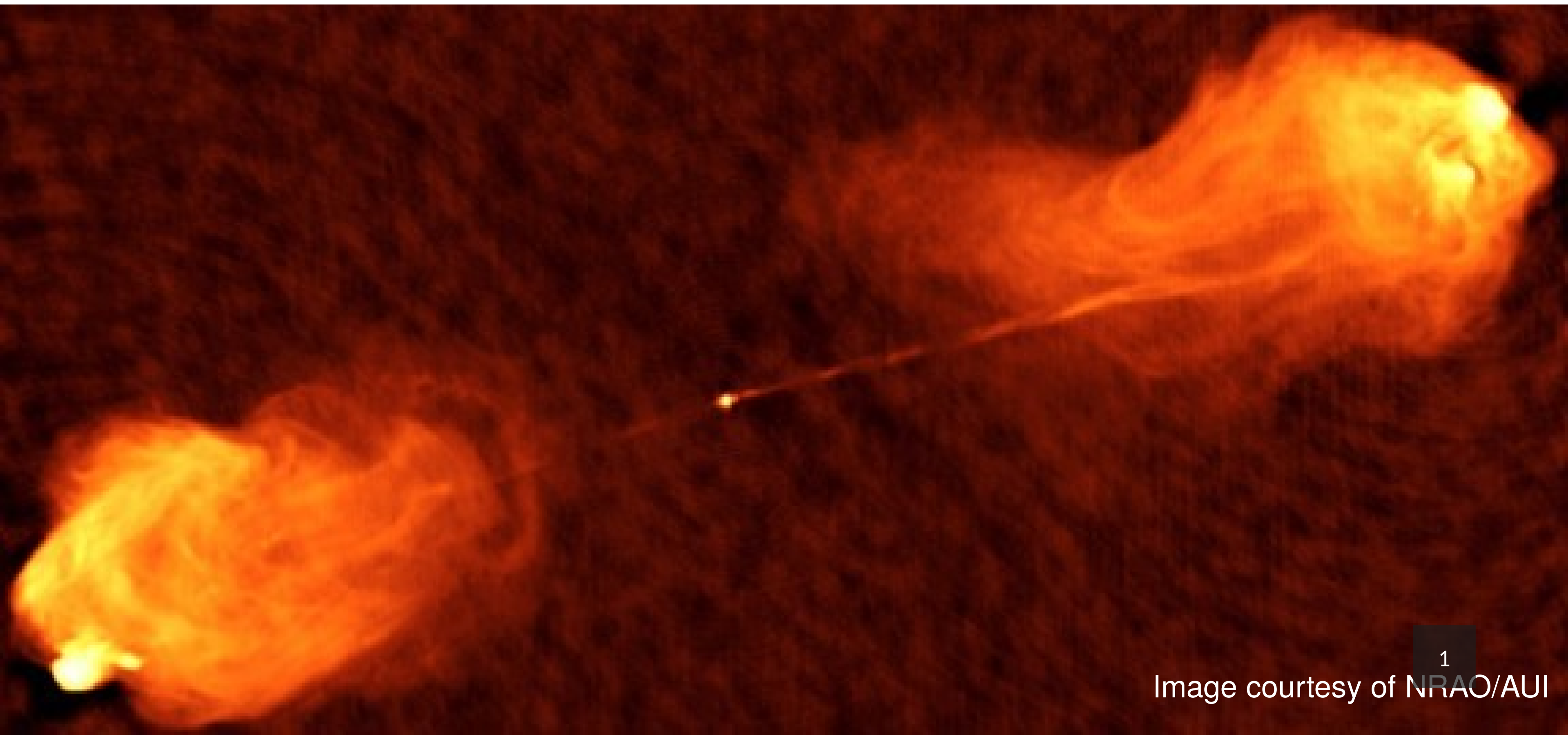
ADVANCED RADIO INTERFEROMETRIC IMAGING

Credits: J. Radcliffe, N. Jackson (ERIS2015), A. Offringa (ERIS 2015), T. Muxlow (ERIS 2013)



INTRODUCTION

Topics discussed:

- Recap of CLEAN
- When to use multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution
- Self-calibration using CLEAN components
- Primary beam correction
- Mosaicing
- Direction-dependent effects during imaging

INTRODUCTION

After calibration the visibilities are represented by (+ errors):

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dl dm$$

(u,v,w) interferometer's geometrical vector

(l,m) sky position

$I(l,m)$ sky brightness (our 'image')

Want to calculate $I(l,m)$ **from** $V(u,v,w)$

Nb: (l,m,n) notation is essentially the same as (x,y,z) coordinates used in the prev. talks

INTRODUCTION

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dl dm$$

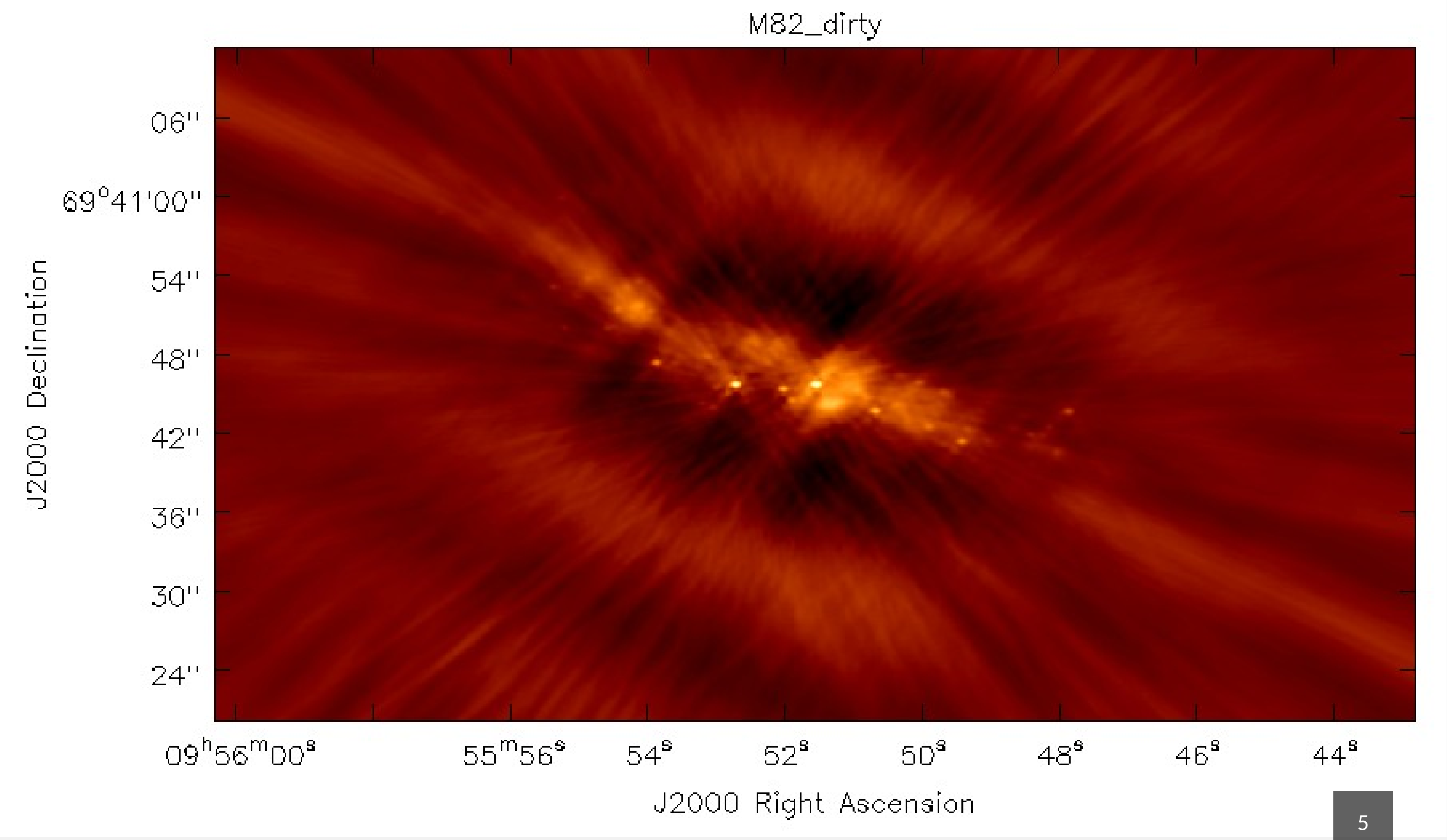
If we have a small field of view ($l \sim 0$, $m \sim 0$) then w term→0:

$$V(u,v) \approx \iint I(l,m) e^{-2\pi i(ul+vm)} dl dm$$

The relationship between $V(u,v)$ **and** $I(l,m)$ **is?**

THE ‘DIRTY’ IMAGE

Example VLA-A data targeting M82



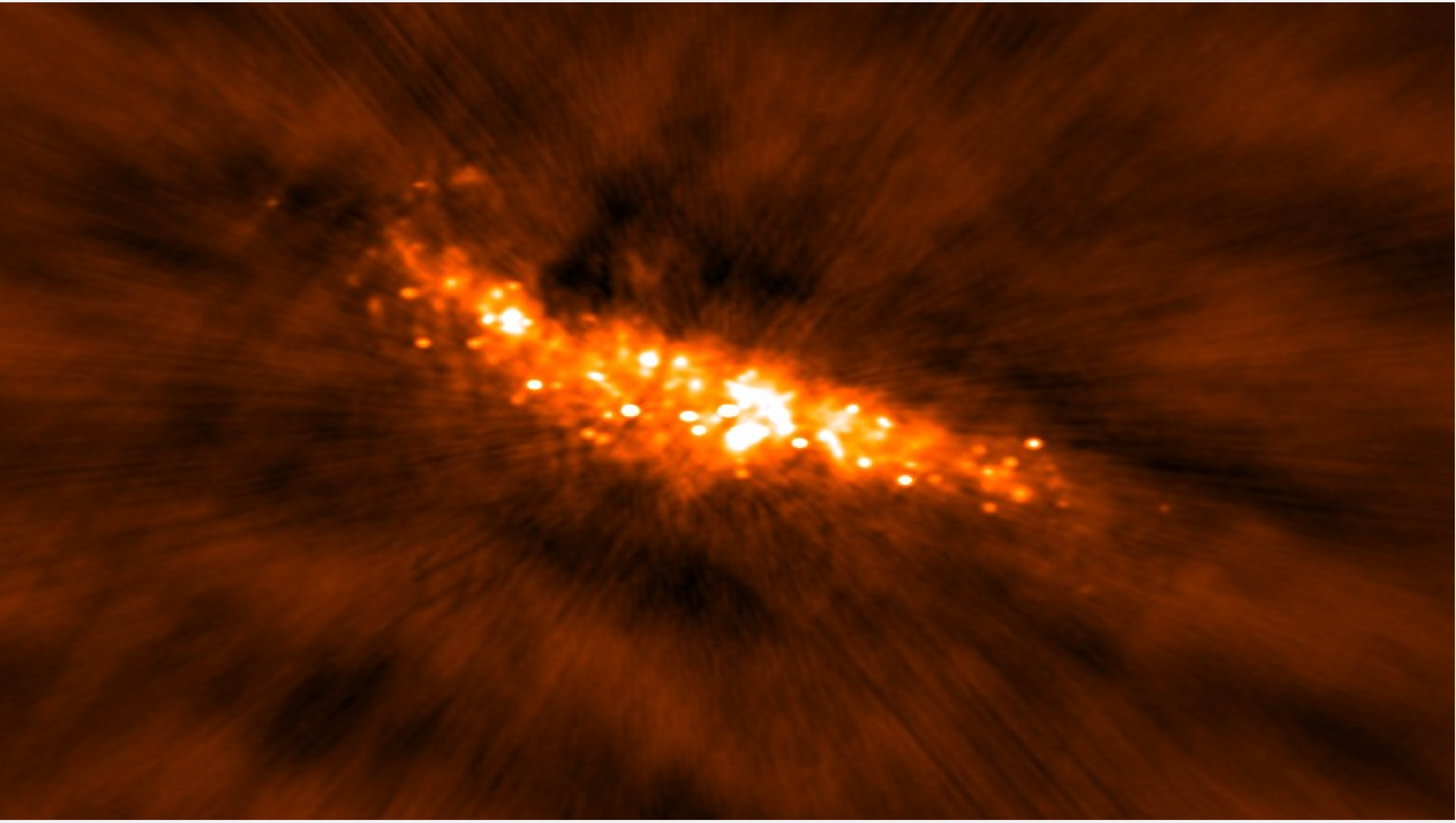
DECONVOLUTION

The Högbom algorithm (1974)

1. Find the strength and position of the brightest peak.
2. Subtract the the dirty beam B multiplied by the peak strength and a damping factor (usually termed the loop gain) at the position of this peak.
3. Go to 1. unless any remaining peak is below some user-specified level or number of interations reached.
4. Convolve the accumulated point source model with an idealized `CLEAN' beam (usually an elliptical Gaussian fitted to the central lobe of the dirty beam).
5. Add the residuals of the dirty image to the `CLEAN' image.

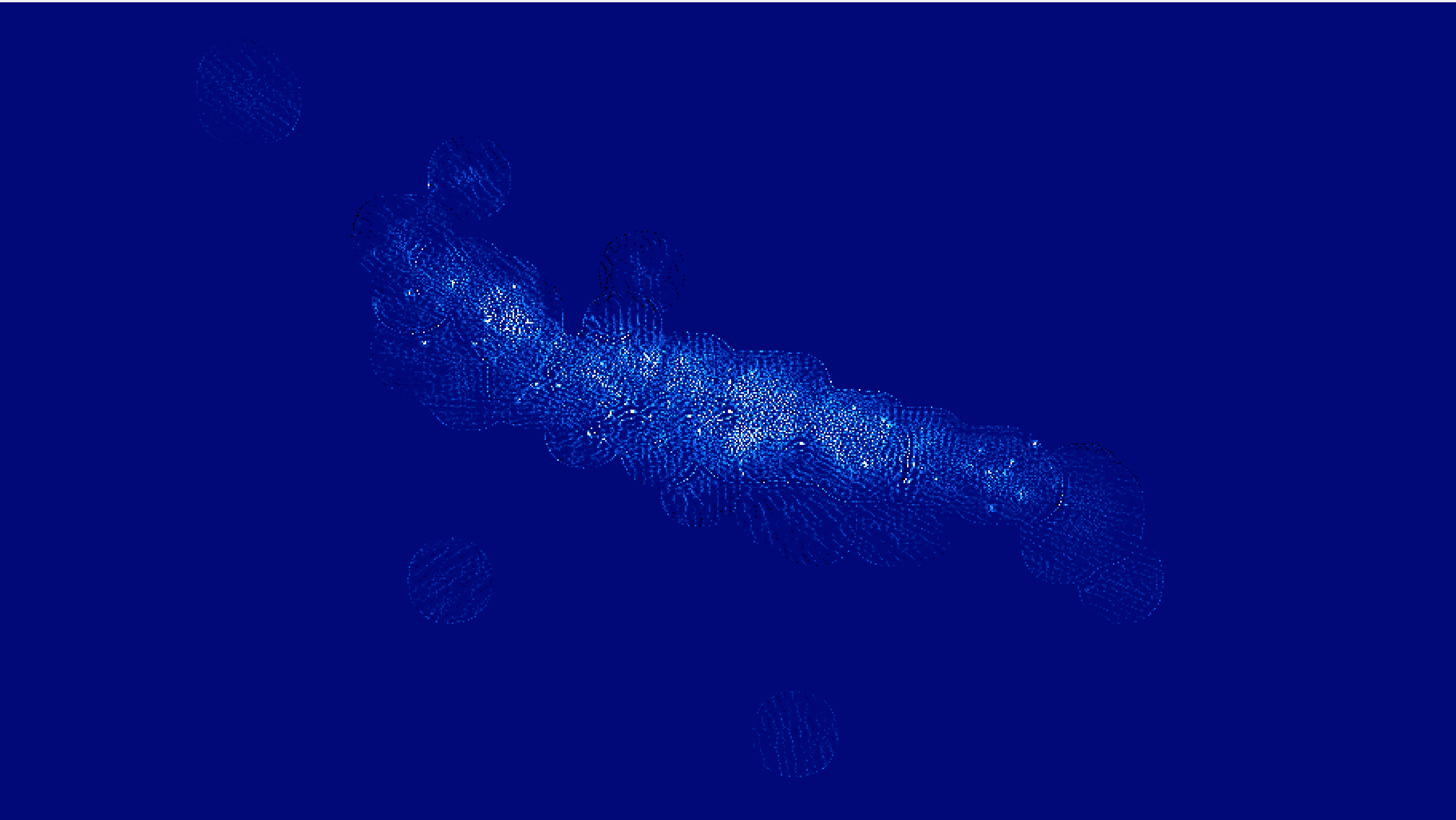
HÖGBOM CLEAN IN ACTION

Hogbom CLEANED image



CLEAN IMAGE & MODEL

Hogbom CLEANED model



DECONVOLVING DIFFUSE STRUCTURE

- Improved algorithm by Cornwell (2008) : “multi-scale clean”
- Fits small smooth Gaussian kernels (and delta functions) during a Högbom CLEAN iteration
- Implemented in CASA clean & tclean. Advised to use pixel scales corresponding to orders of the dirty beam size and avoid making scale too large compared to the image width/lowest spatial frequency.
- E.g. For example, if the synthesized beam is 10" FWHM and cell=2", try multiscale = [0,5,15]

```
deconvolver        = 'multiscale'     #  Minor cycle algorithm (hogbom,clark,m
                                      #   ultiscale,mtmfs,mem,clarkstokes)
    scales         = [0, 1, 5, 15]    #  List of scale sizes (in pixels) for
                                      #   multi-scale algorithms
    smallscalebias =         0.6      #  A bias towards smaller scale sizes
    restoringbeam  =          []      #  Restoring beam shape to use. Default
                                      #   is the PSF main lobe
```

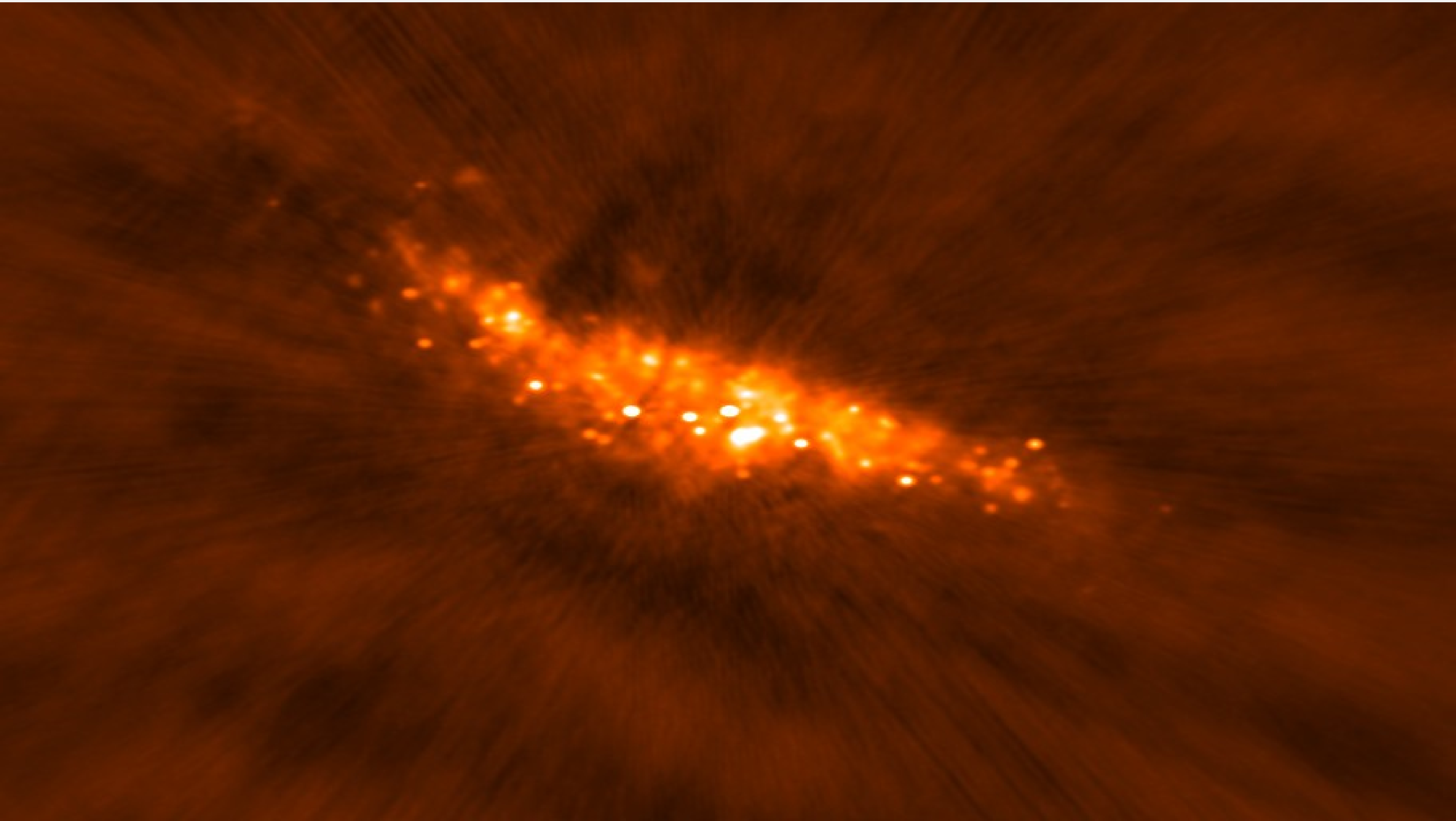
CASA tclean

```
multiscale         = [0, 1, 5, 15]    #  Deconvolution scales (pixels); [] =
                                      #   standard clean
    negcomponent   =          -1      #  Stop cleaning if the largest scale
                                      #   finds this number of neg components
    smallscalebias =         0.6      #  a bias to give more weight toward
                                      #   smaller scales
```

CASA clean

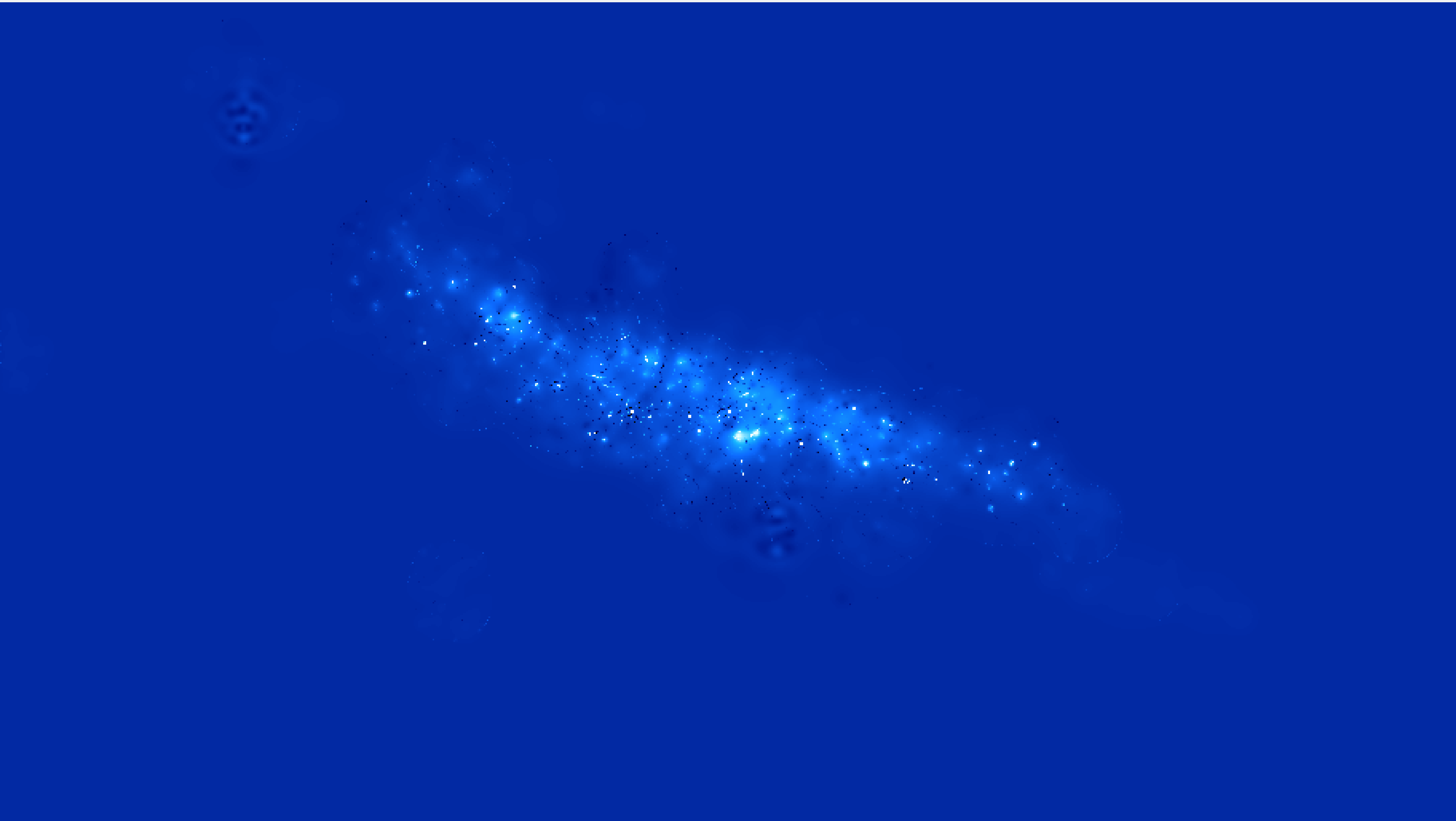
MULTI-SCALE CLEAN

Multi-scale CLEANED image



MULTI-SCALE CLEAN

Multi-scale CLEANED model



WIDE-FIELD IMAGING

2D Fourier Transform does not hold for new sensitive, wide-band, wide-field arrays

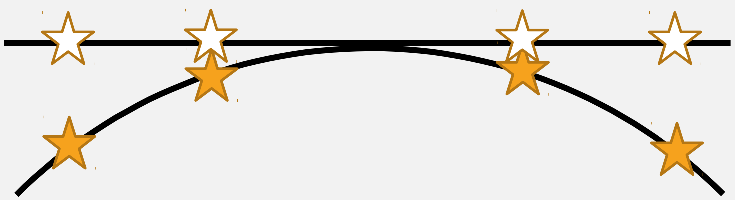
Non co-planar baselines becomes a problem i.e. l,m,w >> 0

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dldm$$

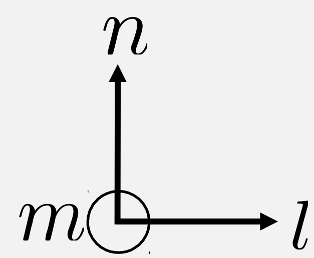
The three-dimensional visibility function $V(u,v,w)$ can be transformed to a three-dimensional image volume $I(l,m,n)$ - this is not physical space since l, m and n are direction cosines.

The only possible values of n lie on the surface of a sphere of unit radius defined by $n = \mathrm{sqrt}(1\text{-}l^2\text{-}m^2)$

WIDE-FIELD IMAGING



The sky brightness consisting of a number of discrete sources ★ are transformed onto the surface of this sphere.



The two-dimensional image ☆ is recovered by projection onto the tangent plane at the pointing centre

So how do we achieve this? Two solutions available:

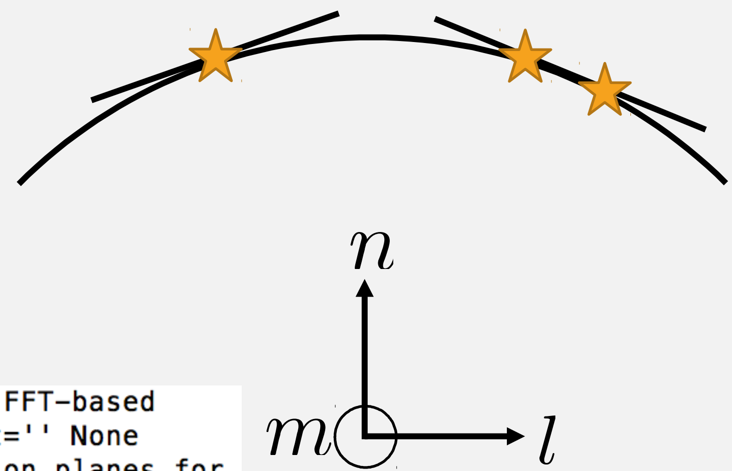
i. Faceting - split the field into multiple images and stitch them together

ii. w-projection - most used solution, effectively performs the above projection to recover $I(l,m)$

Both available in CASA! (or other, faster imagers, e.g. wsclean)

i. FACETING

- Takes advantage of the small field approximation (l,m~0) so the image sphere is approximated by pieces of many smaller tangent planes.
- Within each sub-field, standard two-dimensional FFTs may be used.
- Errors increase quadratically away from the centre of each sub-field, but these are acceptable if enough sub-fields are selected.
- Facets can be selected so as to cover known sources.
- Facets may overlap allowing complete coverage of the primary beam



CASA clean implementation

```
gridmode        = 'widefield'      #  Gridding kernel for FFT-based
                                   #   transforms, default='' None
     wprojplanes =           1     #  Number of w-projection planes for
                                   #   convolution; -1 => automatic
                                   #   determination
     facets      =           8     #  Number of facets along each axis
                                   #   (main image only)
```

ii. w-PROJECTION

Cornwell et al. 2011

$$V(u,v,w) * \mathfrak{F}(e^{-2\pi i w(\sqrt{1-l^2-m^2}-1)}) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dl dm$$

- Very dependent on zenith angle, co-planarity of array, field of view and resolution.
- Convolution theorem no longer works when w-terms present.
- CLEAN assumes constant PSF, but PSF changes (slightly) over the image.
- (Partly) solved with Cotton-Schwab algorithm (Schwab 1984) (used in CASA automatically).

ii. w-PROJECTION

The Cotton-Schwab + w-projection algorithm:

1) Make initial dirty image & central PSF - Perform minor iterations:

    - Find peak
    - Subtract scaled PSF at peak with small gain
    - Repeat until highest peak ~80-90% decreased

2) Major iteration: ‘Correct’ residual

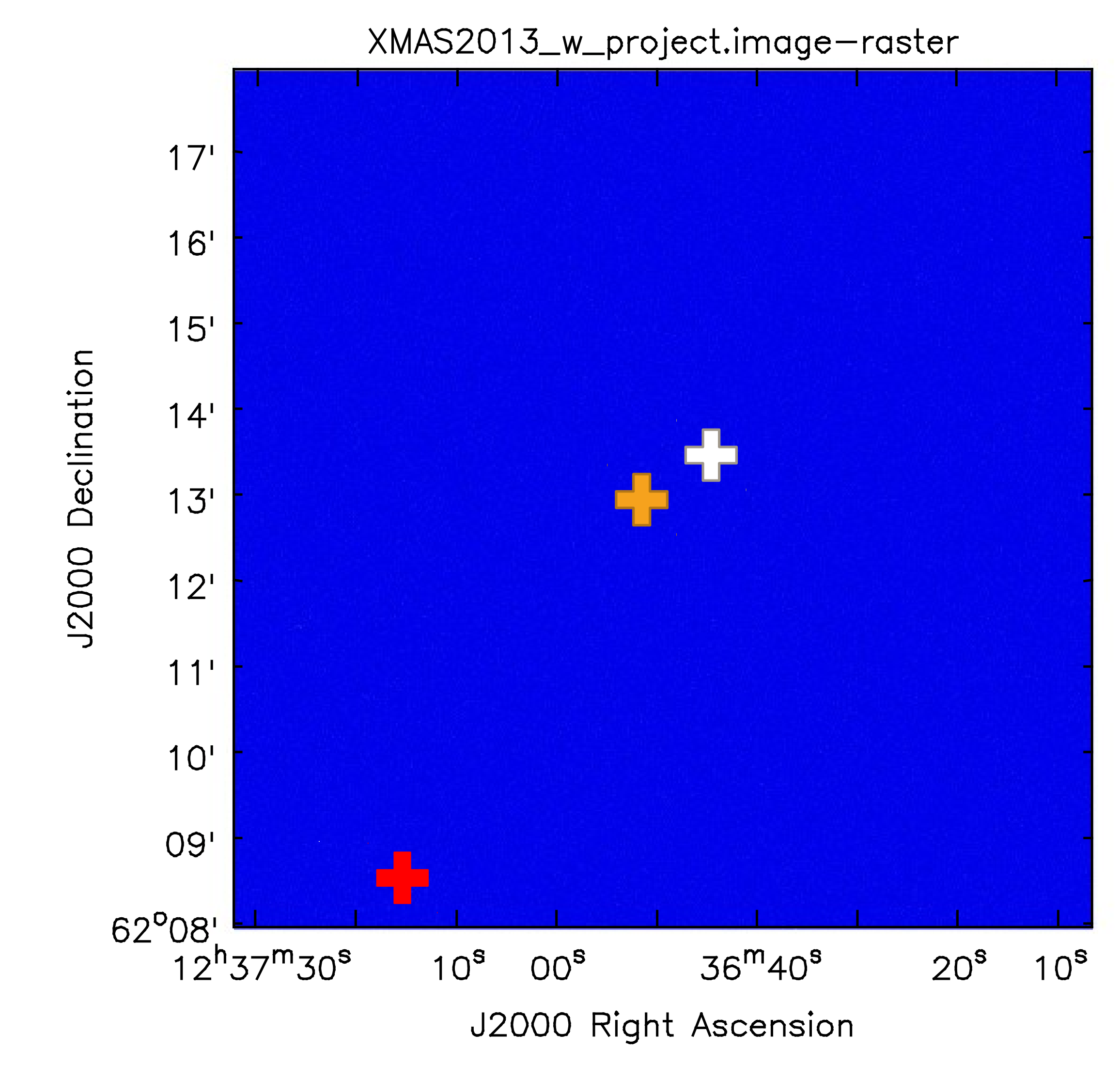
    - Predict visibility for current model
    - Subtract predicted contribution and re-image

```
gridmode        = 'widefield'      #  Gridding kernel for FFT-based
                                   #   transforms, default='' None
     wprojplanes   =        -1     #  Number of w-projection planes for
                                   #   convolution; -1 => automatic
                                   #   determination
     facets        =         1     #  Number of facets along each axis
                                   #   (main image only)
```

CASA clean implementation

w-PROJECTION

Take the GOODS-N field as observed by 1.4GHz e-MERLIN



Pointing centre

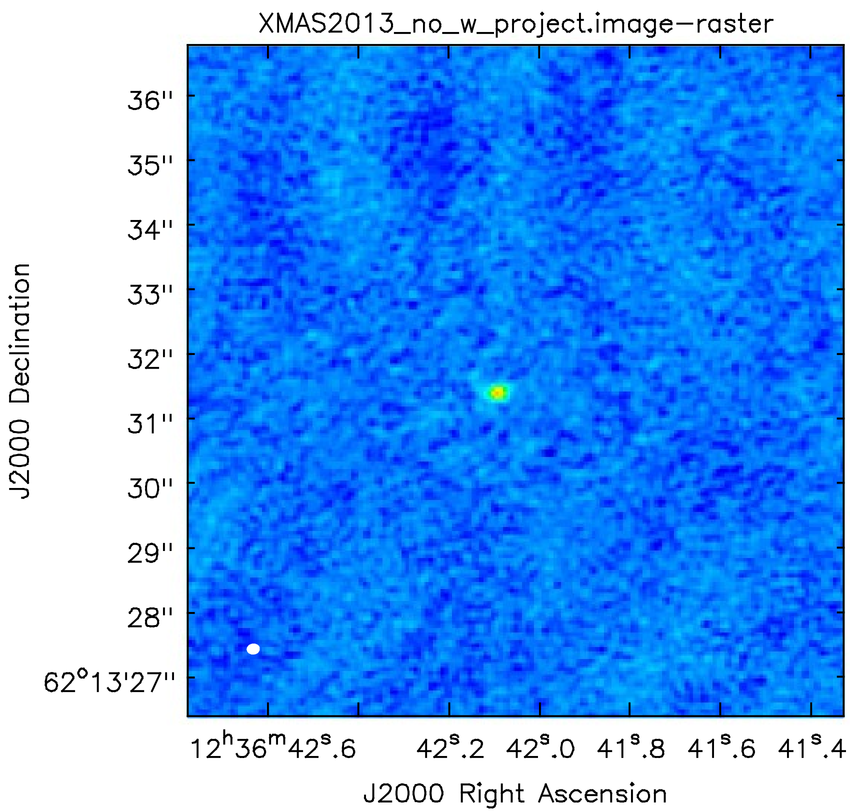
Source 1

Source 2

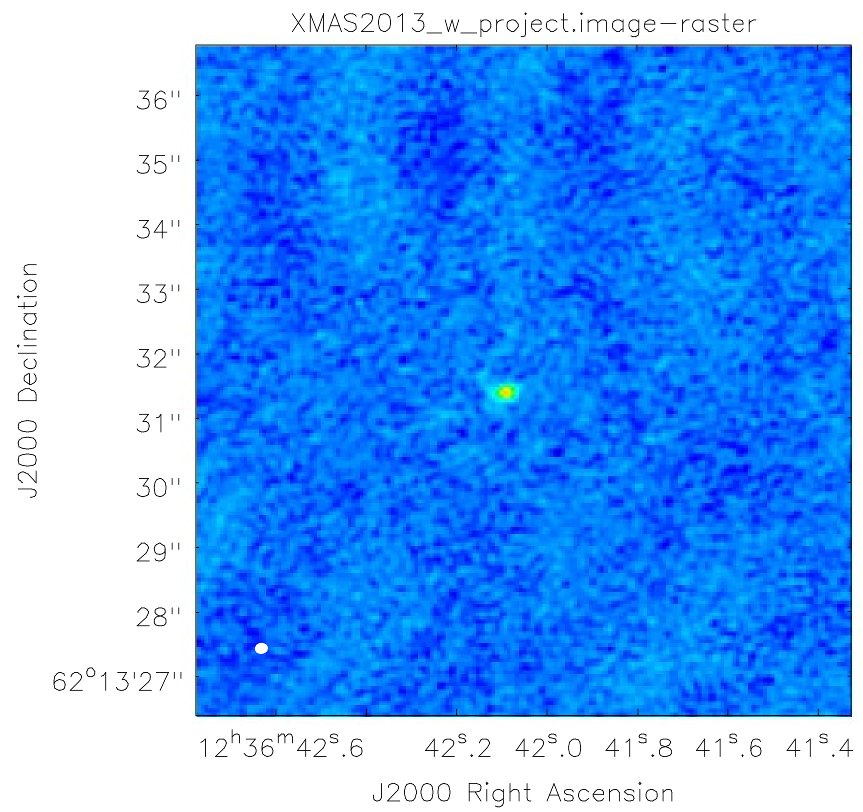
w-PROJECTION

Source 1: Near the pointing centre

No w-projection



w-projection



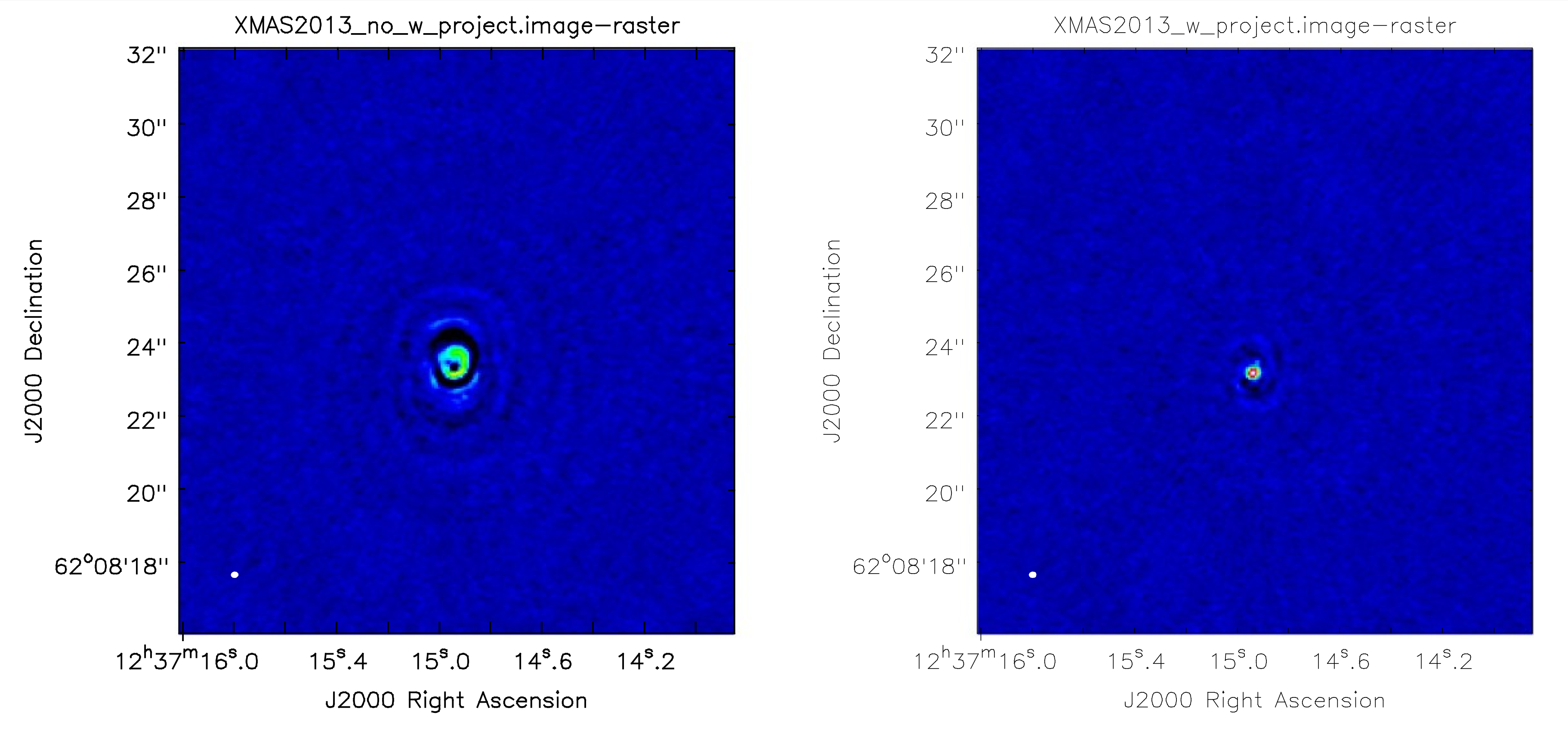
Pretty much identical! Small field approximation holds and 2D FT suffices

w-PROJECTION

Source 2: Away from the pointing centre

No w-projection

w-projection



Small field approximation breaks and you need w-projection!

MULTI-FREQUENCY SYNTHESIS

Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid

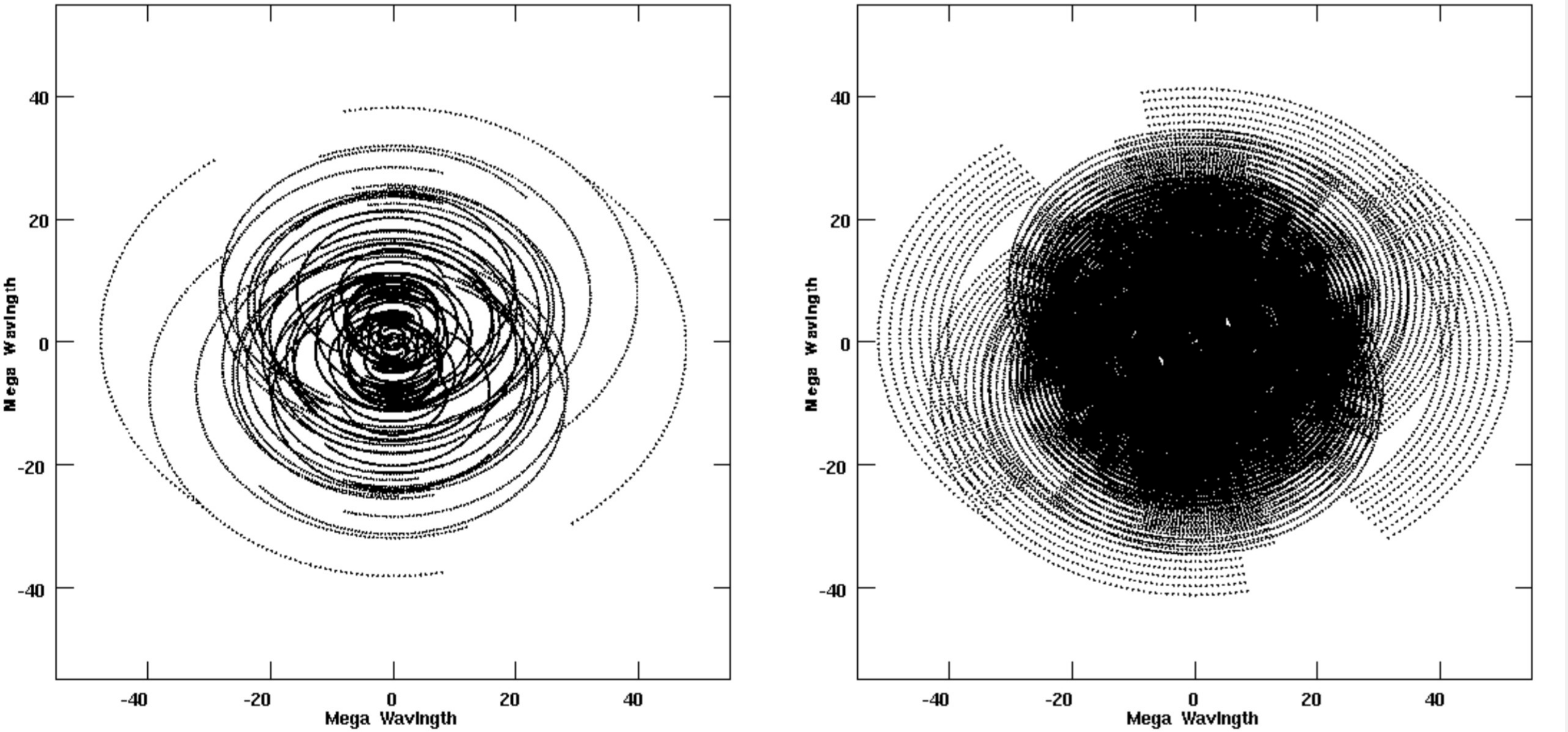


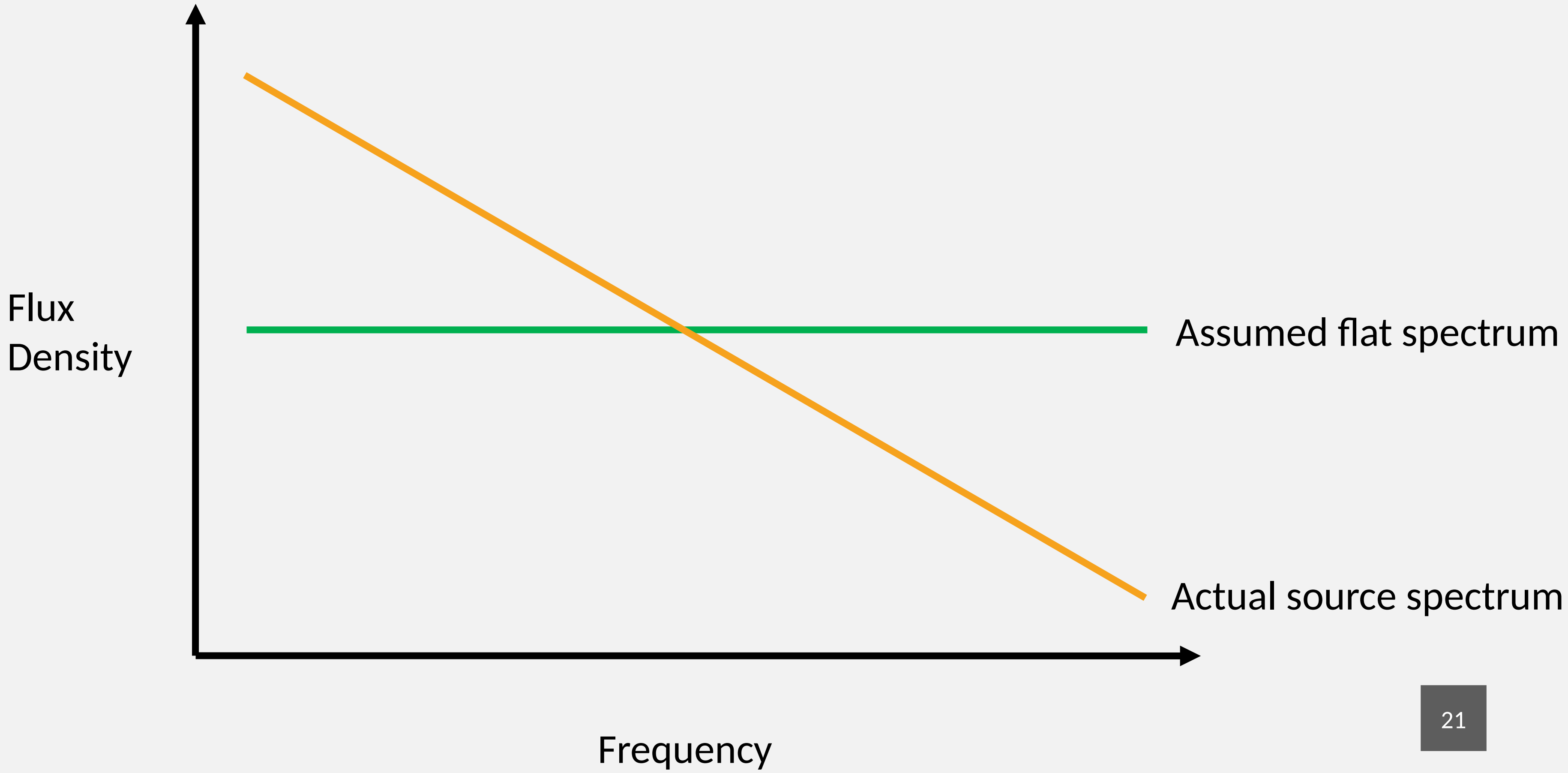
Figure 16.1: *Left (a):* VLBA (u, v) coverage for a full track at $\delta = 50°$. *Right (b):* Using MFS observations with 8 frequencies spread over 25%.

Conway & Sault (1995)

MULTI-FREQUENCY DECONVOLUTION

Similar but not the same! (same name often used). Also known as multi-term deconvolution (as in CASA).

Takes spectral variation into account during deconvolution



MULTI-FREQUENCY DECONVOLUTION

I_ν^m represents the sky emission in terms of a Taylor series about a reference frequency:

$$I_\nu^m = \sum_{t=0}^{N_t-1} b_\nu^t I_t^{\mathrm{sky}} \text{ where } b_\nu^t = \left(\frac{\nu-\nu_0}{\nu_0}\right)^t$$

A power model is used to describe the spectral dependence of the sky. One practical choice is a power law with emission.

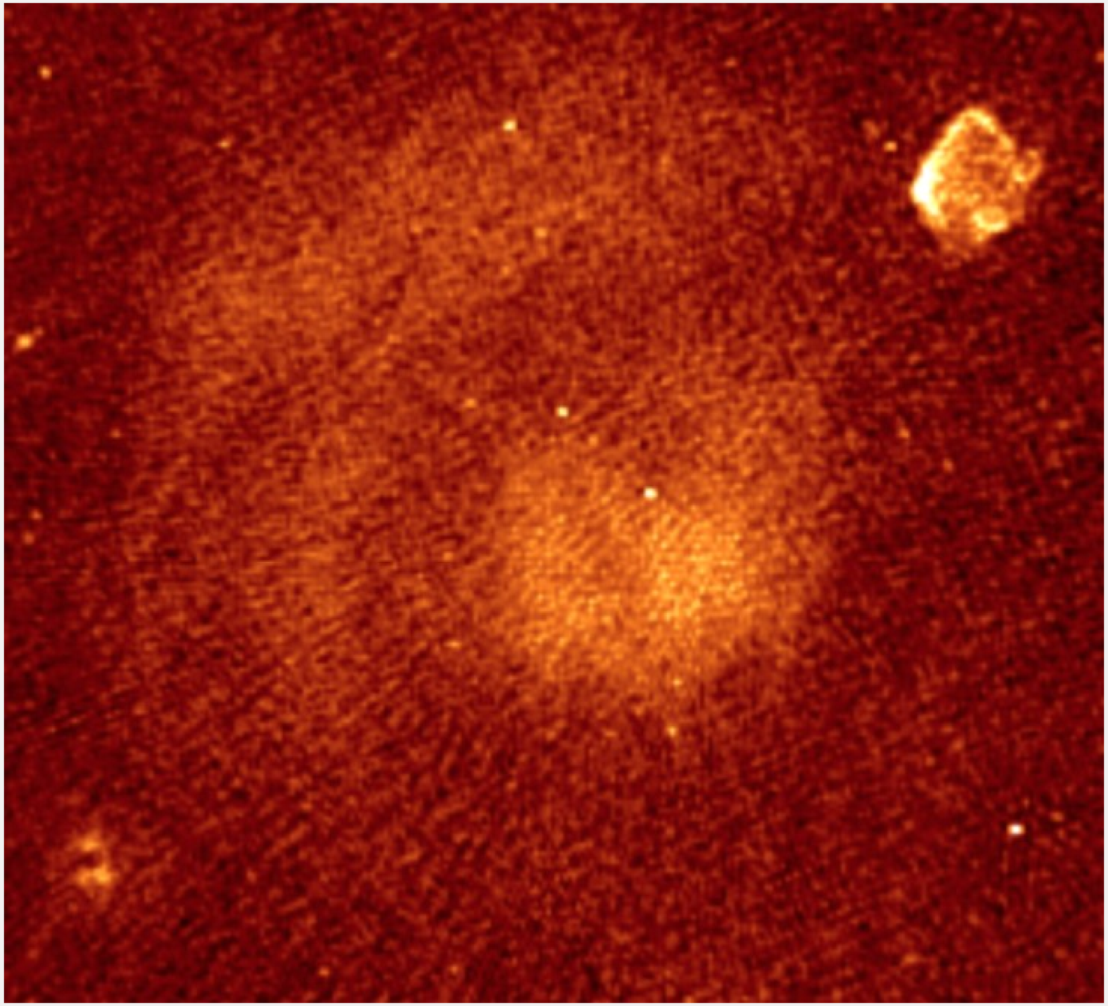
$$I_\nu^{\mathrm{sky}} = I_{\nu_0}^{\mathrm{sky}} \left(\frac{\nu}{\nu_0}\right)^{I_\alpha^{\mathrm{sky}} + I_\beta^{\mathrm{sky}} \log\left(\frac{\nu}{\nu_0}\right)}$$

Useful for wideband, sensitive imaging. Incorporated in CASA in combination with multi-scale CLEAN as 'mtmfs'

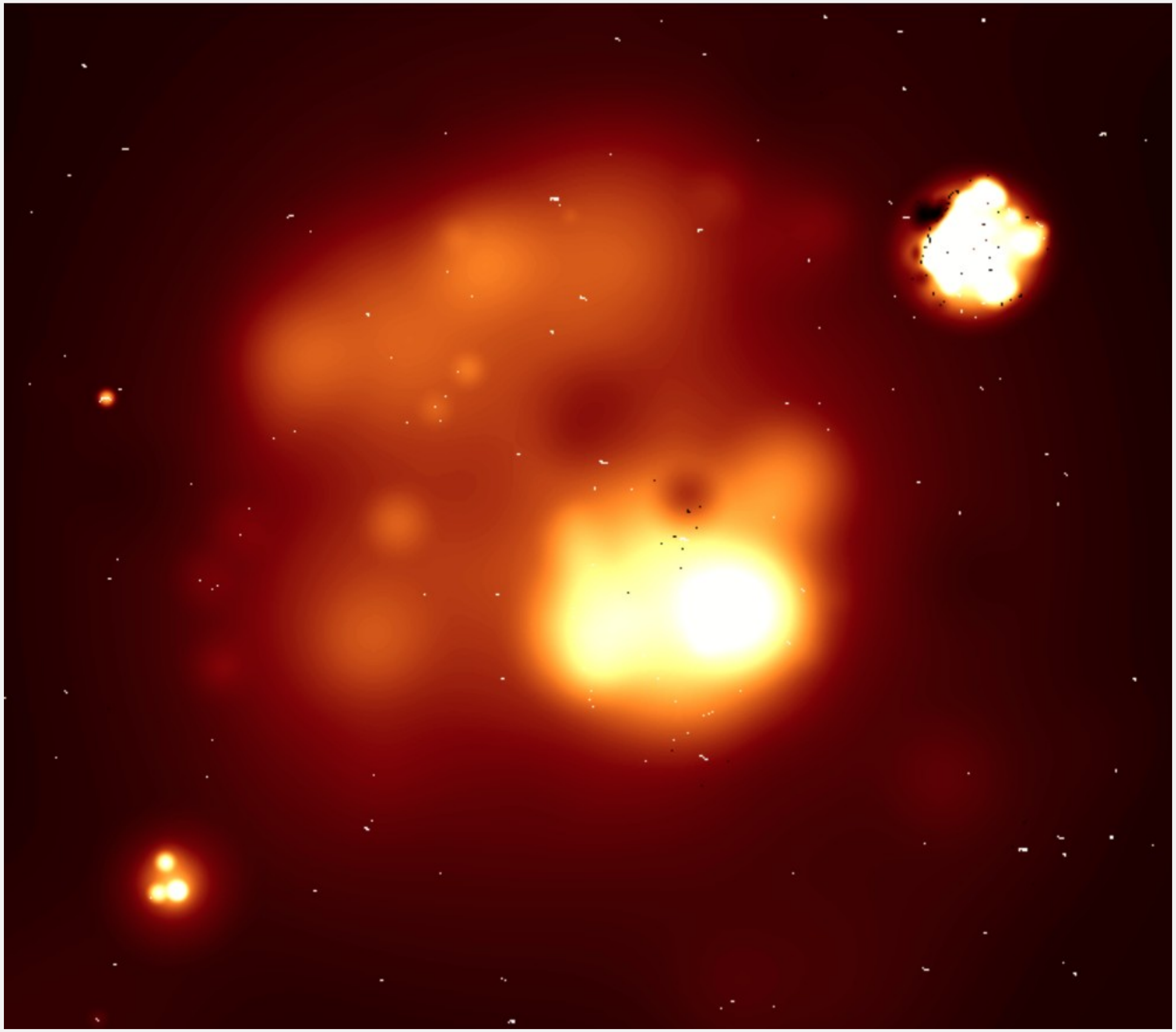
COMPRESSIVE SENSING

- Recent focus on deconvolution using ‘**C**ompressive **S**ensing’ (abbrev. CS – but CS can mean ‘Cotton-Schwab’ too)
- CS methods assume the sky is 'sparse' (“solution matrix is sparse in some basis”)
- Minimizes “L1-norm” (= abs sum of CLEAN components)
- Högbom clean is actually a compressed sensing method which implements a technique known as ‘matching pursuit’ to find the representation of the sky
- So is MS clean
- But other implementations exist that are less closely based on the original CLEAN model (and may be faster)

Source structure looks like (Hogbom cleaned):



Model using multiscale:



Model using generalized CS:

COMPRESSIVE SENSING

- Some compressive sensing implementations do not work well with calibration artefacts (treat them as structure to be modelled)

- Multi-scale is more **robust**

- On well-calibrated data:
  - CS gives more accurate model
  - But residuals don't improve much

- Not implemented in CASA (only available in specialised LOFAR image (AWImagerCS) or stand-alone packages e.g. Purify, DDFacet

PRIMARY BEAM CORRECTION

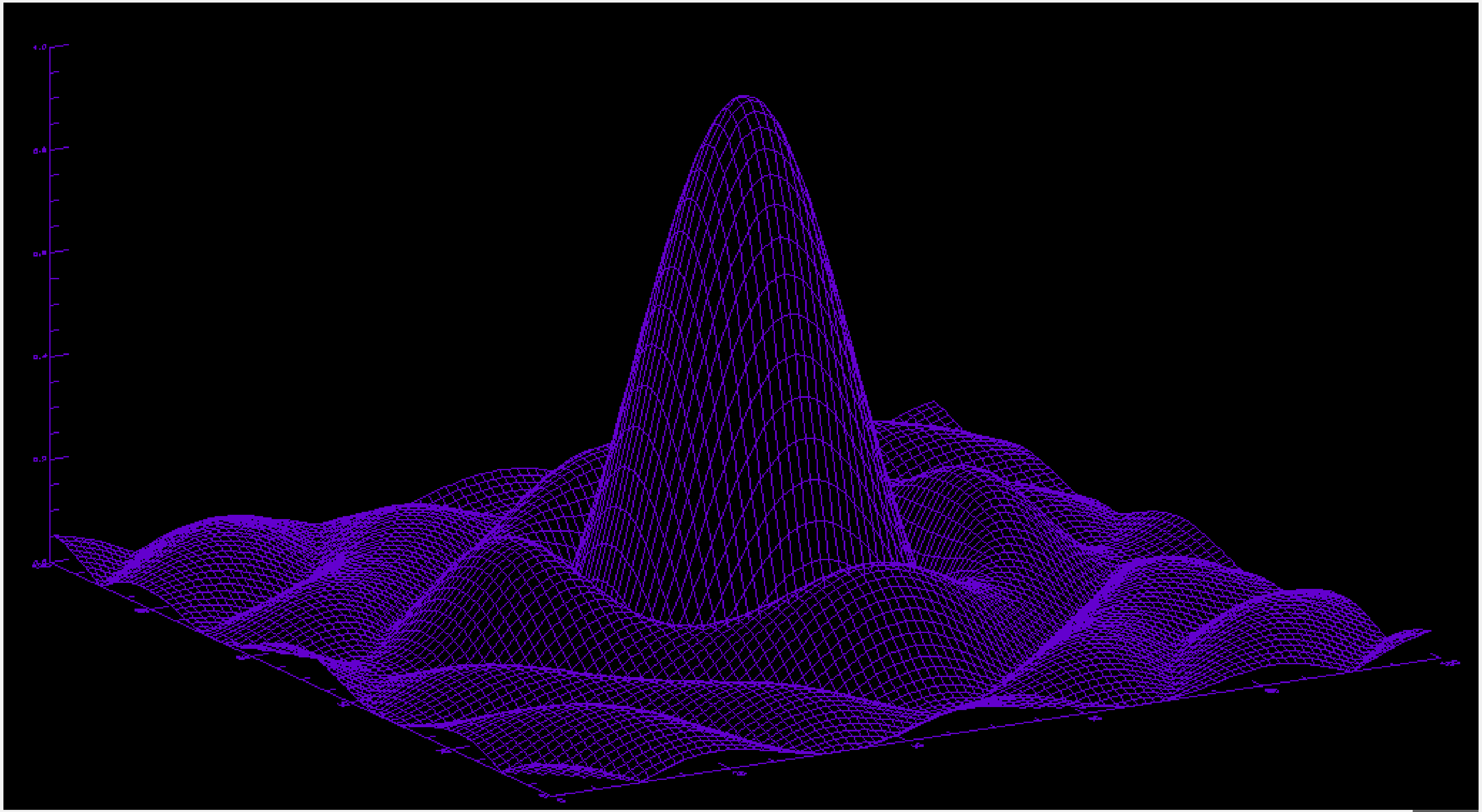
- Correction is required for the antenna response
- This is called "primary beam" correction (as opposed to the synthesized beam / psf)
- For dishes, the primary beam is ~constant but can be very complex away from the FWHM.

To correct for: multiply final image with the inverse beam!

Scalar for total brightness, matrix for polarized

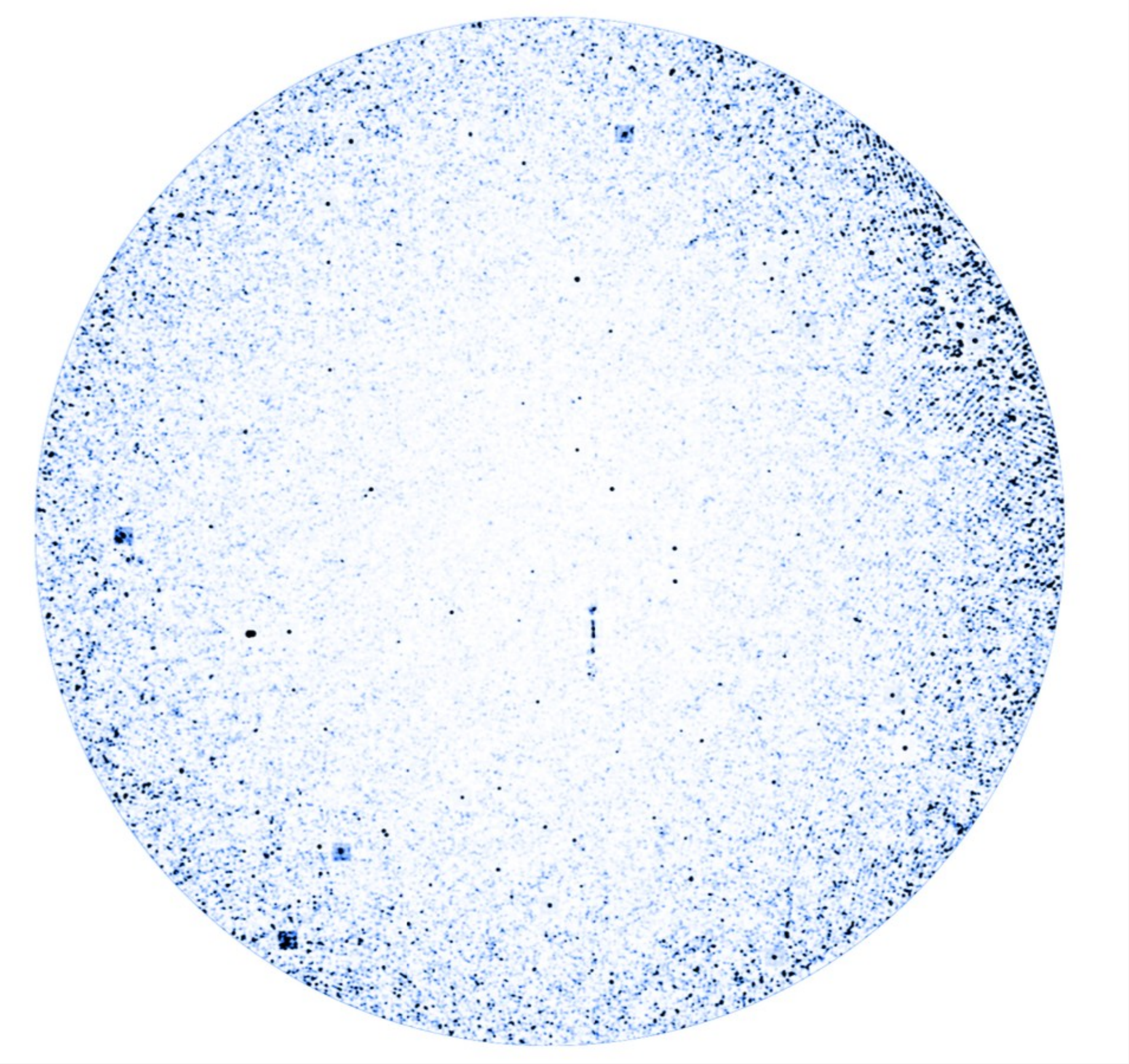
PRIMARY BEAM CORRECTION

Complex sidelobe structure + asymmetries!



Knockin primary beam holographic scan

PRIMARY BEAM CORRECTION

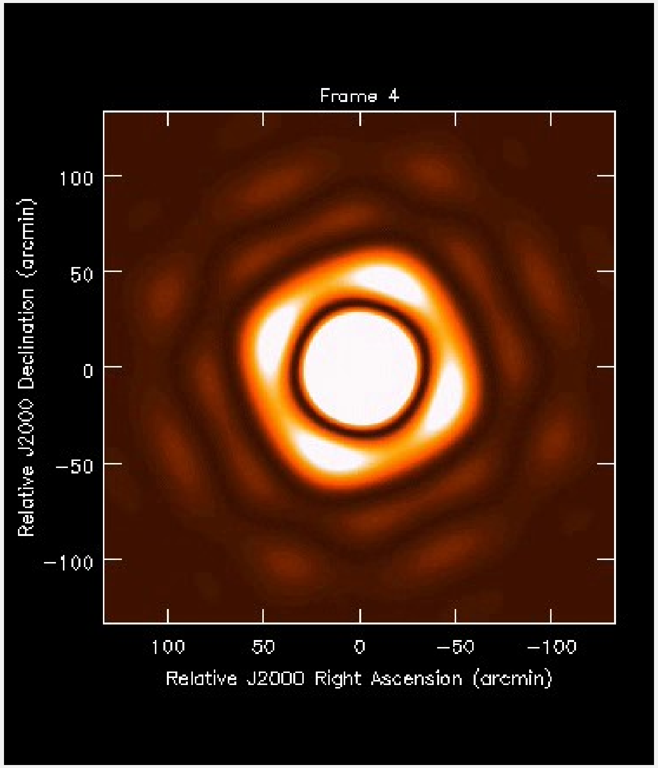


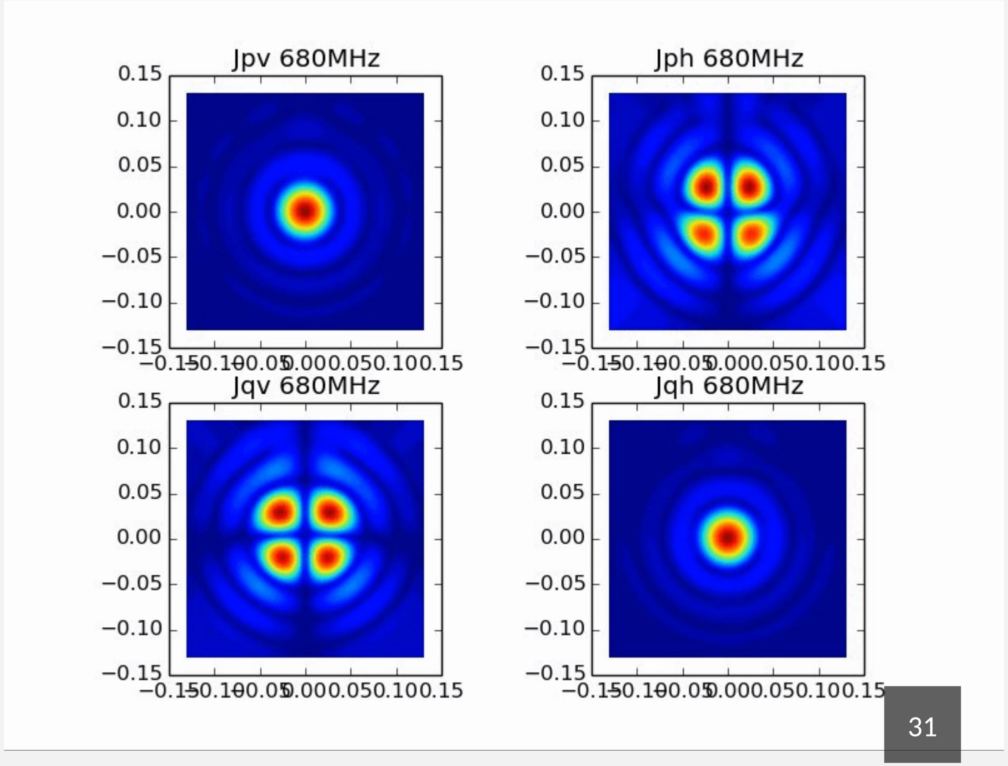
Primary beam corrected JVLA+MERLIN image of GOODS-N

Note the increased noise level towards the edge of the field

VARIABLE PRIMARY BEAMS

- Primary beam of arrays can vary with time and frequency!
- Has to be accounted for during cleaning and primary beam correction if imaging the whole primary beam (CASA has this for the JVLA + ALMA - VLBI arrays don't image the pb often!)





VARIABLE PRIMARY BEAM

Primary beam frequency variation for the UK Lovell Telescope 1.4-1.6GHz

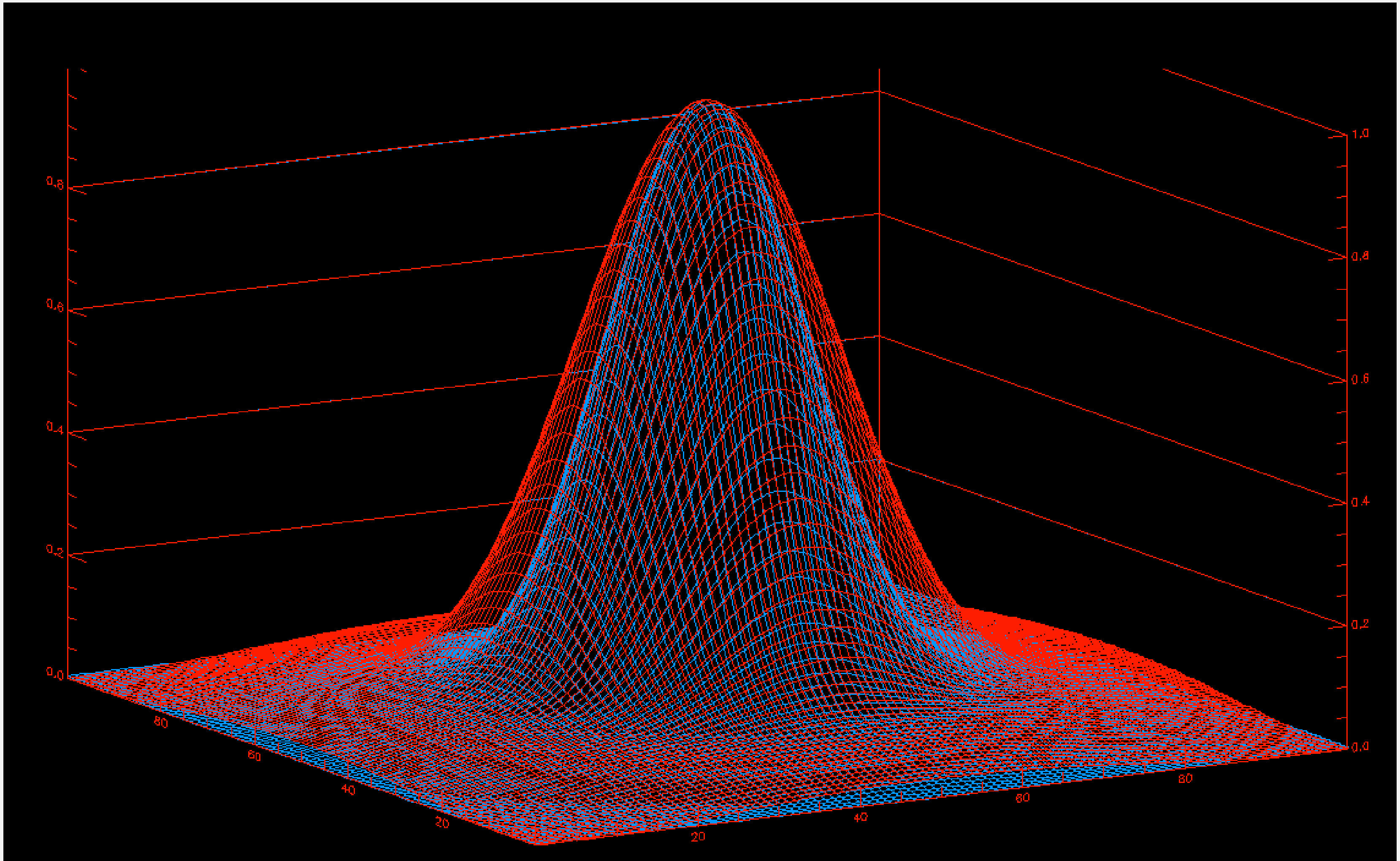
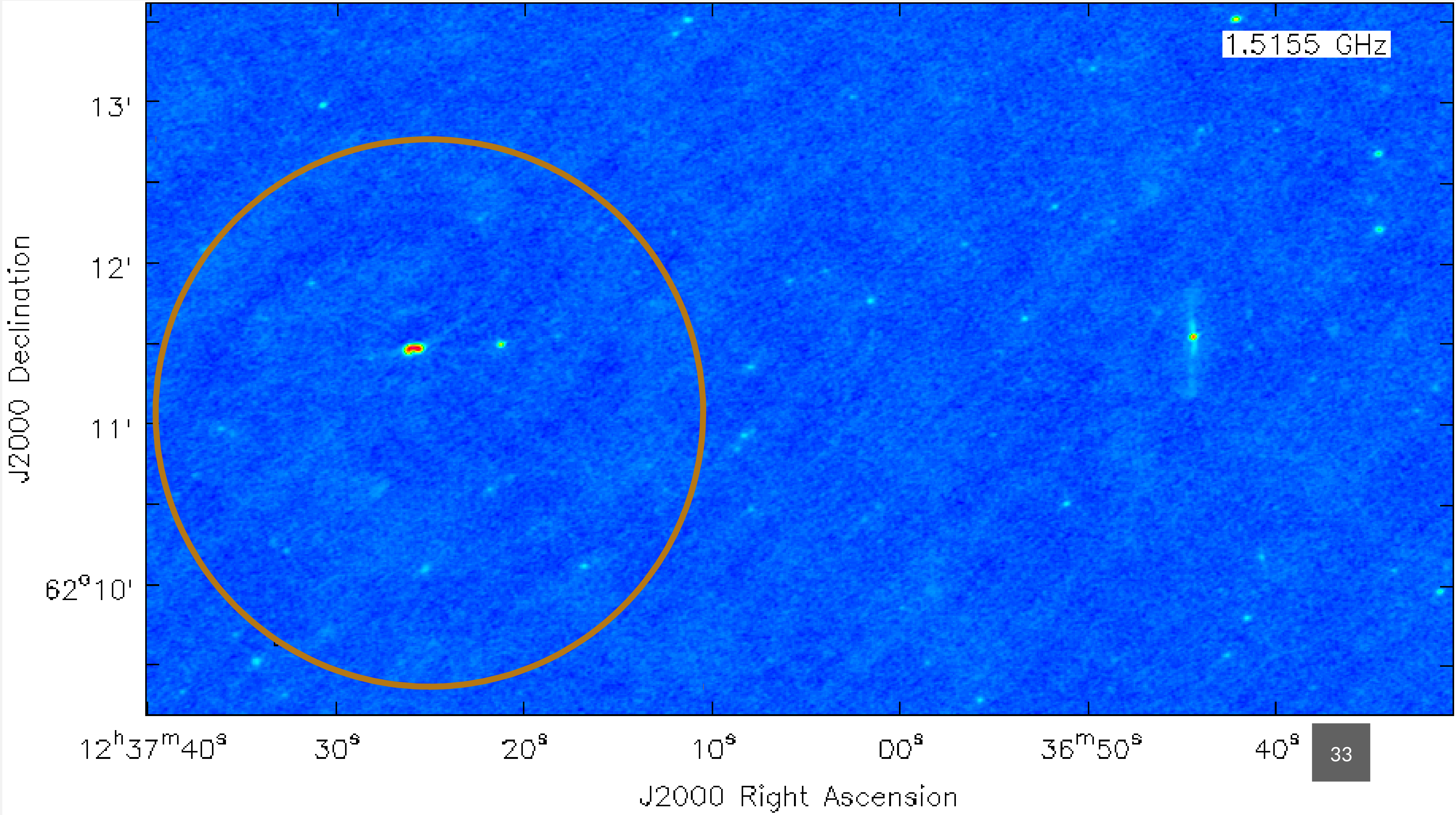


Image credit: Nick Wrigley

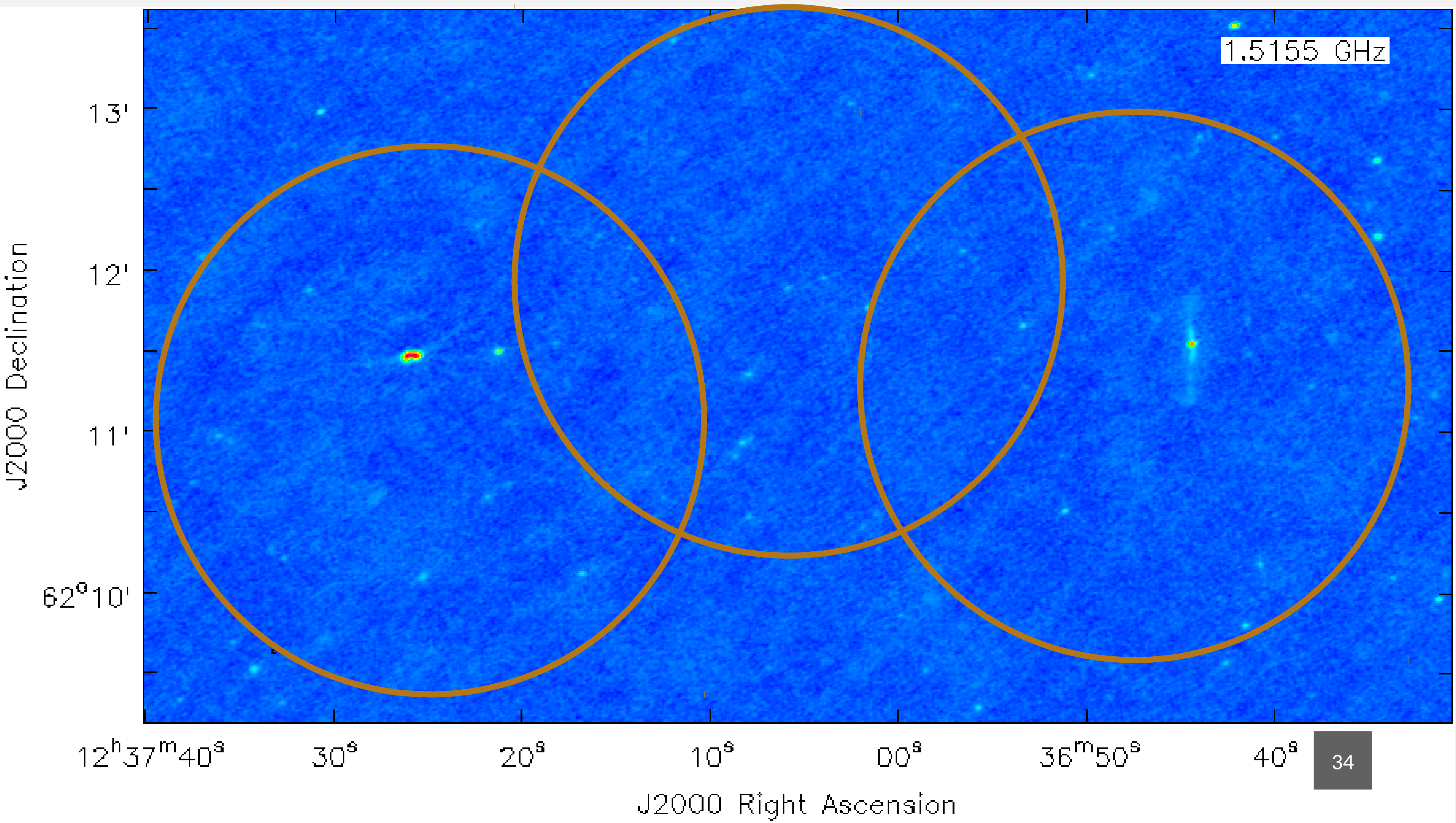
MOSAICING

What if this is our primary beam and we want to see the FR-I galaxy too?



MOSAICING

We can use multiple pointings and combine them with correct weighting



MOSAICING

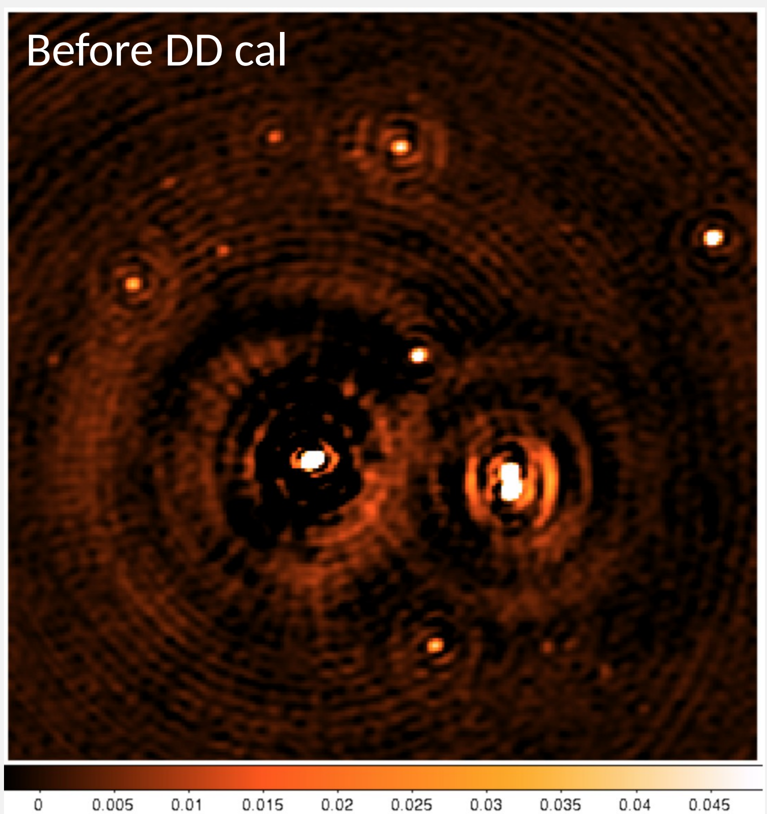
- To create the mosaiced image $M(l,m)$
- Need to weight with 1/noise^2 = (primary beam)^2 or $B_i^2(l,m)$

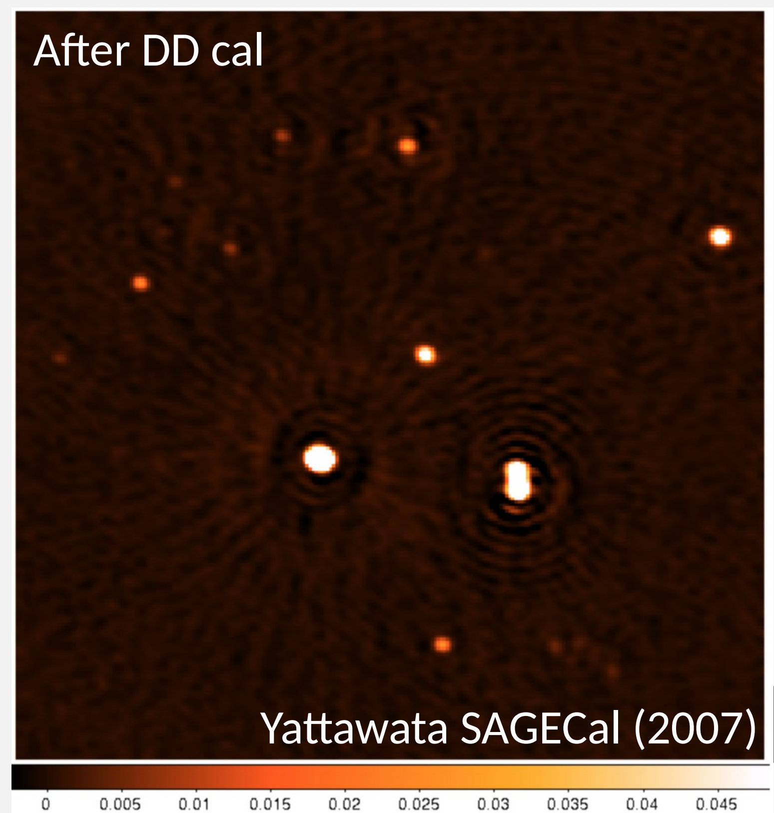
$$M(l,m) = \frac{\sum_i B_i^2(l,m)(I_i(l,m)/B_i(l,m))}{\sum_i B_i^2(l,m)}$$

$$= \frac{\sum_i B_i(l,m)I_i(l,m)}{\sum_i B_i^2(l,m)}$$

DIRECTION DEPENDENT IMAGING & CALIBRATION

- Direction dependent (DD) effects may need further corrections after imaging… not a fully solved problem!
- Can be ionosphere, troposphere, instrumental
- Affects position, brightness & polarisation angles!





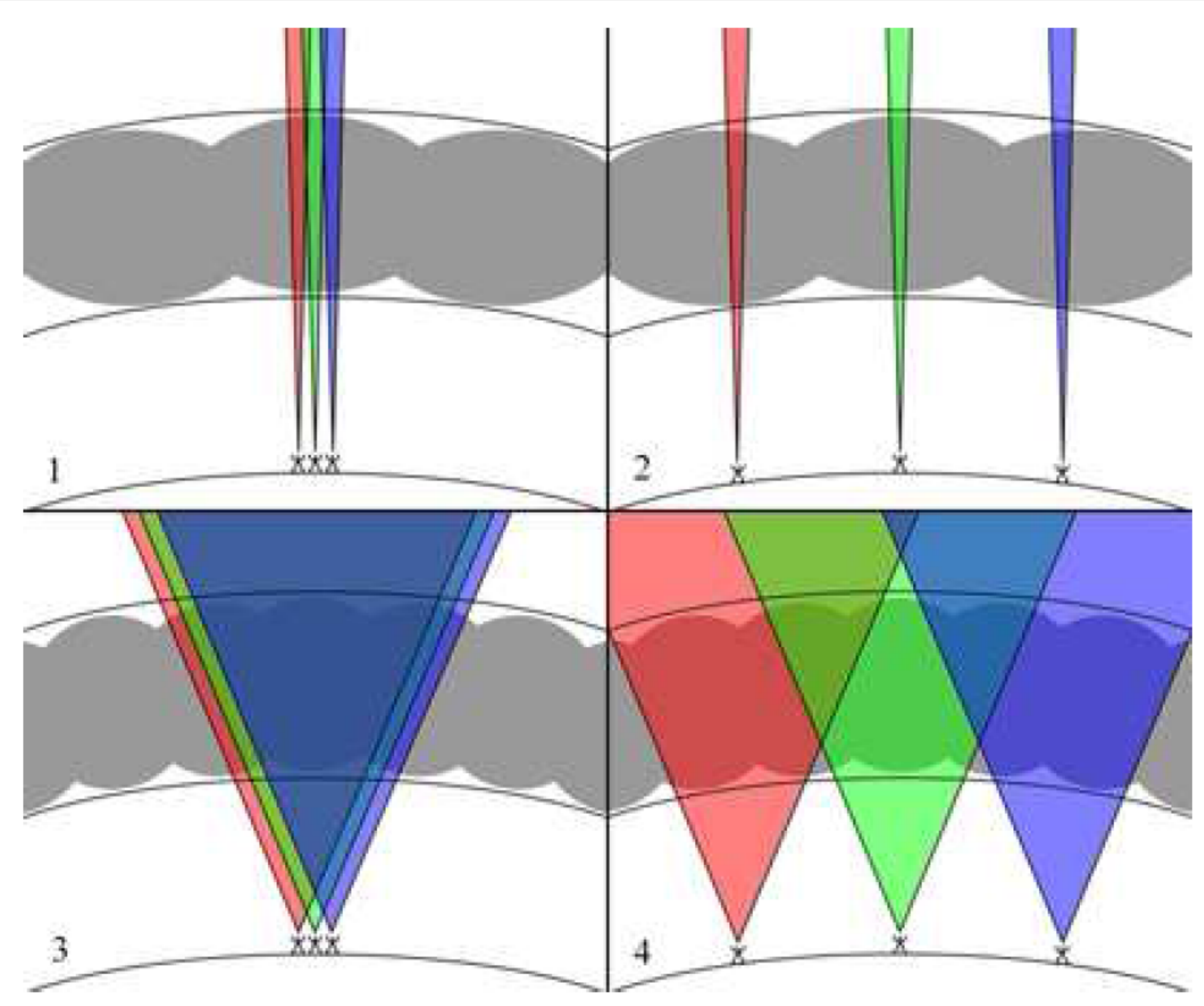
Yattawata SAGECal (2007)

DIRECTION DEPENDENT CALIBRATION

From Intema+08

1. Small array, narrow field of view (e.g. compact JVLA)
2. Large array, narrow field of view (e.g. EVN, MERLIN)
3. Small array, wide field of view (e.g. MWA)
4. Large array, wide field of view (e.g. LOFAR)

Bottom row have DD effects, worst in case 4.



DIRECTION DEPENDENT CALIBRATION

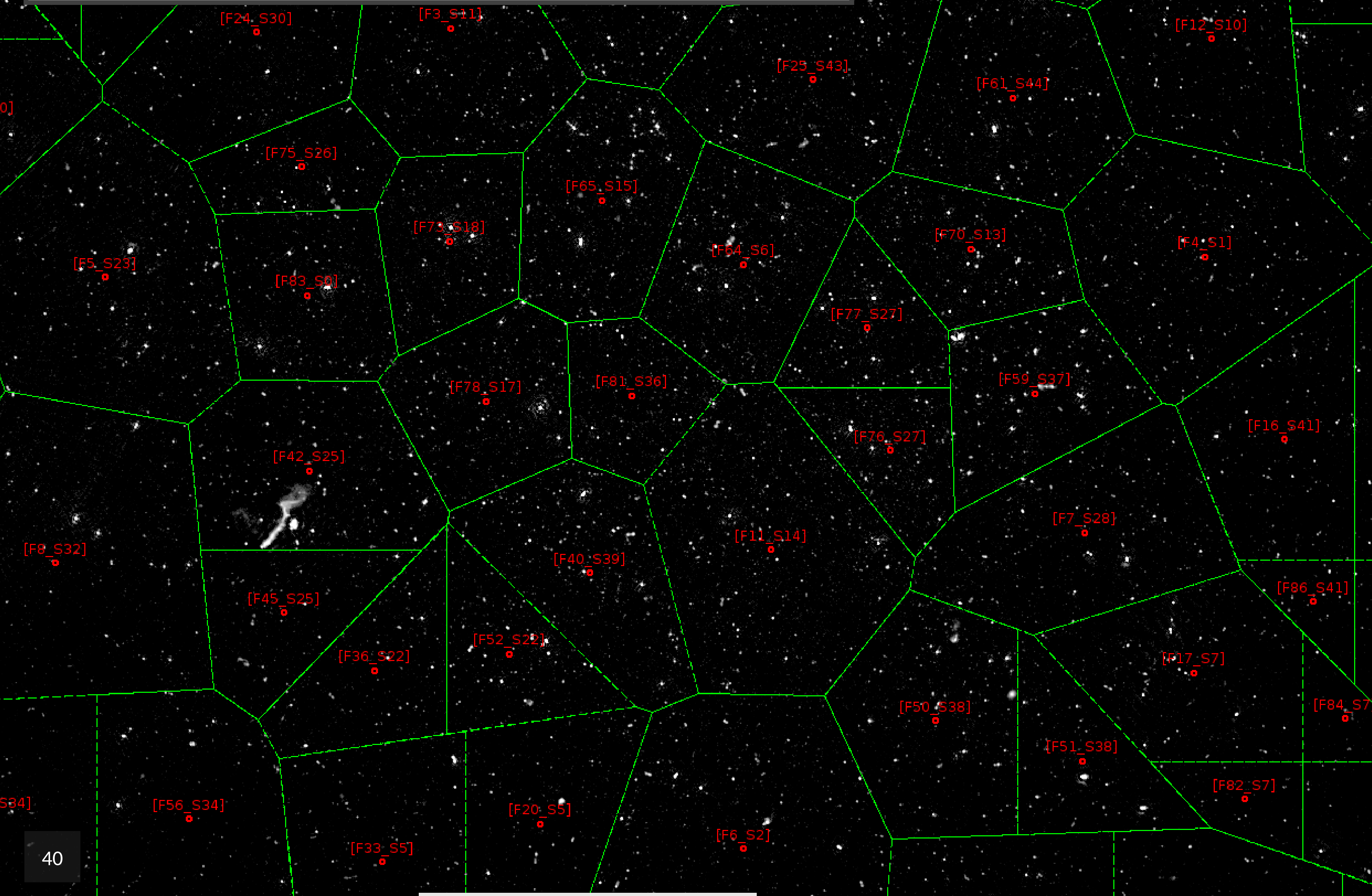
- First need to determine the corrections that are needed, then apply them
- In principle these are separate problems – e.g. if we knew what the ionosphere was doing as a function of direction we could simply apply the correction and image
- But in practice we probably need to determine a model sky and use the differences between the model and the data to figure out direction-dependent effects
- Direction-dependent calibration is usually therefore some form of *self-calibration* since the best information about the sky is the dataset itself
- Then we need some way of applying these corrections to the data…

DIRECTION DEPENDENT IMAGING

Possible solutions:

- Peeling (Noordam+04, Intema+08): self-calibrate in the direction of bright sources, then subtract them off
  - Advantage: easy
  - Disadvantage: does not necessarily lead to good calibration of the faint sources! (but Intema+08 uses peeled sources to estimate ionospheric properties)
- Figure out DD corrections in the u,v plane (A-projection: Bhatnagar+08)
  - Advantage: very accurate
  - Disadvantage: need smoothly varying knowledge of direction-dependent terms. OK for instrumental, less good for ionosphere
- Break the image up into small ‘facets’ in which the DD corrections can be applied in each direction (van Weeren+ 16, Tasse+18)
  - Advantage: only working model for e.g. ionospheric DD corrections
  - Disadvantage: can be very slow if each facet has to be dealt with separately

DIRECTION DEPENDENT IMAGING



SUMMARY

Topics discussed:

- CLEAN
- When to use Multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution and other deconvolution methods
- Primary beam correction
- Mosaicing
- Direction-dependent effects during imaging