ADVANCED RADIO INTERFEROMETRIC IMAGING

Lecturer: James Allison (University of Oxford)

Credits: J. Radcliffe, N. Jackson (ERIS2015), A. Offringa (ERIS 2015), T. Muxlow (ERIS 2013)

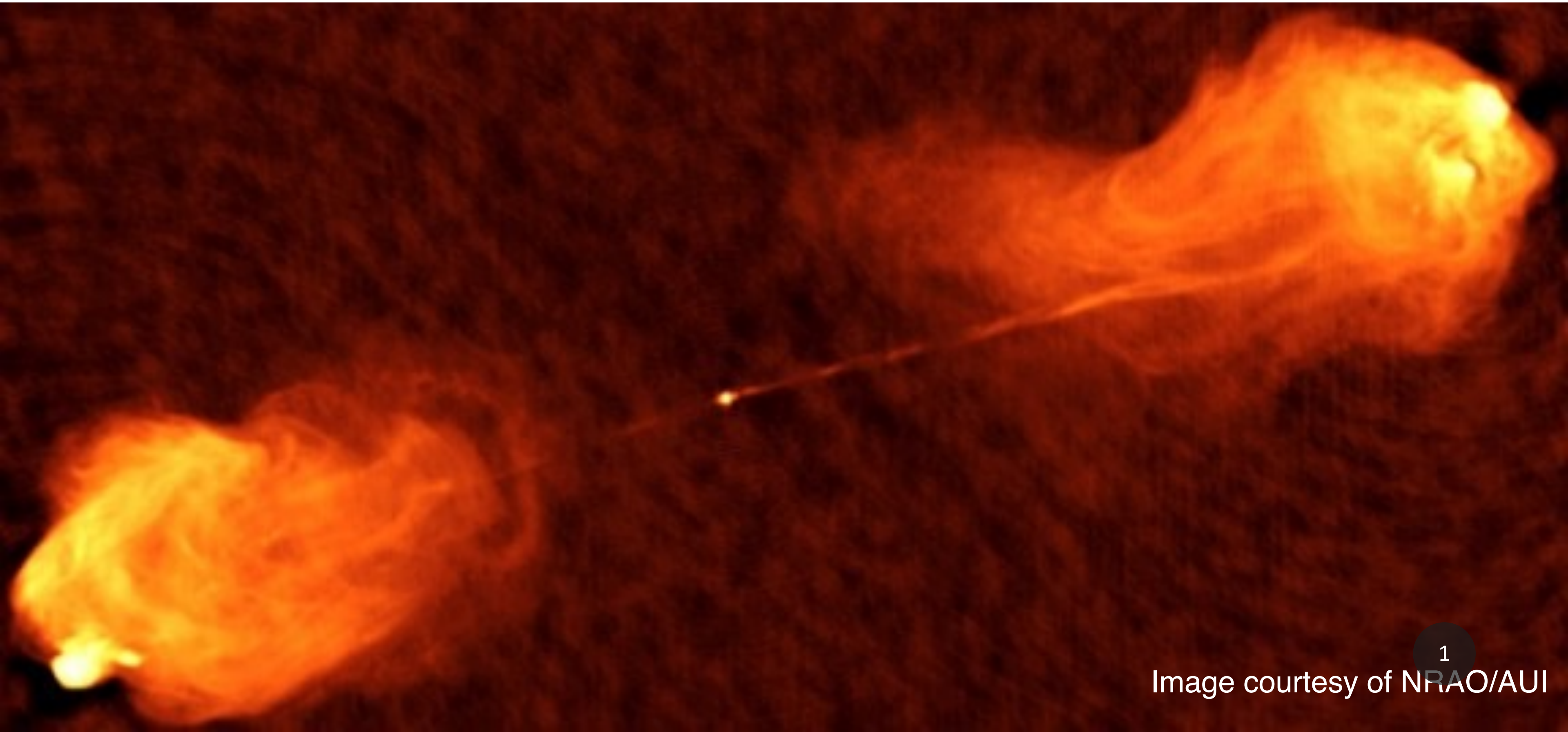


Image courtesy of NRAO/AUI

INTRODUCTION

Topics discussed:

- Recap of CLEAN
- When to use multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution
- Self-calibration using CLEAN components
- Primary beam correction
- Mosaicking
- Direction-dependent effects during imaging

INTRODUCTION

After calibration the visibilities are represented by (+ errors):

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i (ul+vm+w(\sqrt{1-l^2-m^2}-1))} dl dm$$

(u,v,w) interferometer’s geometrical vector

(l,m) sky position

$I(l,m)$ sky brightness (our ‘image’)

Want to calculate $I(l,m)$ from $V(u,v,w)$

Nb: (l,m,n) notation is essentially the same as (x,y,z) coordinates used in the prev. talks

INTRODUCTION

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dl dm$$

If we have a small field of view (l~0, m~0) then w→0:

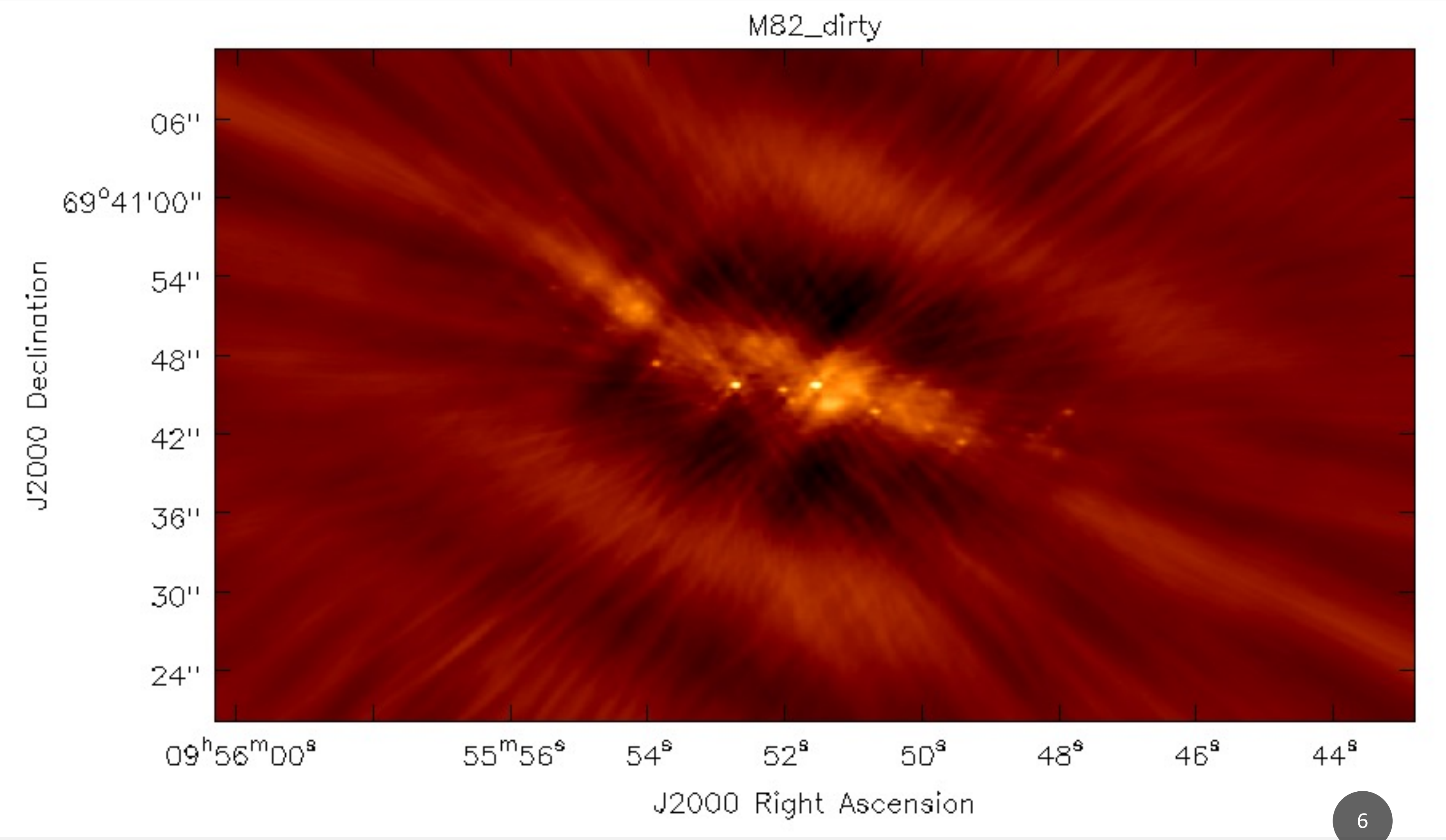
$$V(u,v) \approx \iint I(l,m) e^{-2\pi i(ul+vm)} dl dm$$

The relationship between $V(u,v)$ and $I(l,m)$ is?

OUR EXAMPLE

THE 'DIRTY' IMAGE

Example VLA-A data targeting M82



DECONVOLUTION

The Högbom algorithm (1974)

1. Find the strength and position of the brightest peak.

2. Subtract the dirty beam x peak strength x loop gain/damping factor position of the peak, the dirty beam B multiplied by the peak strength and a damping factor (usually termed the loop gain).

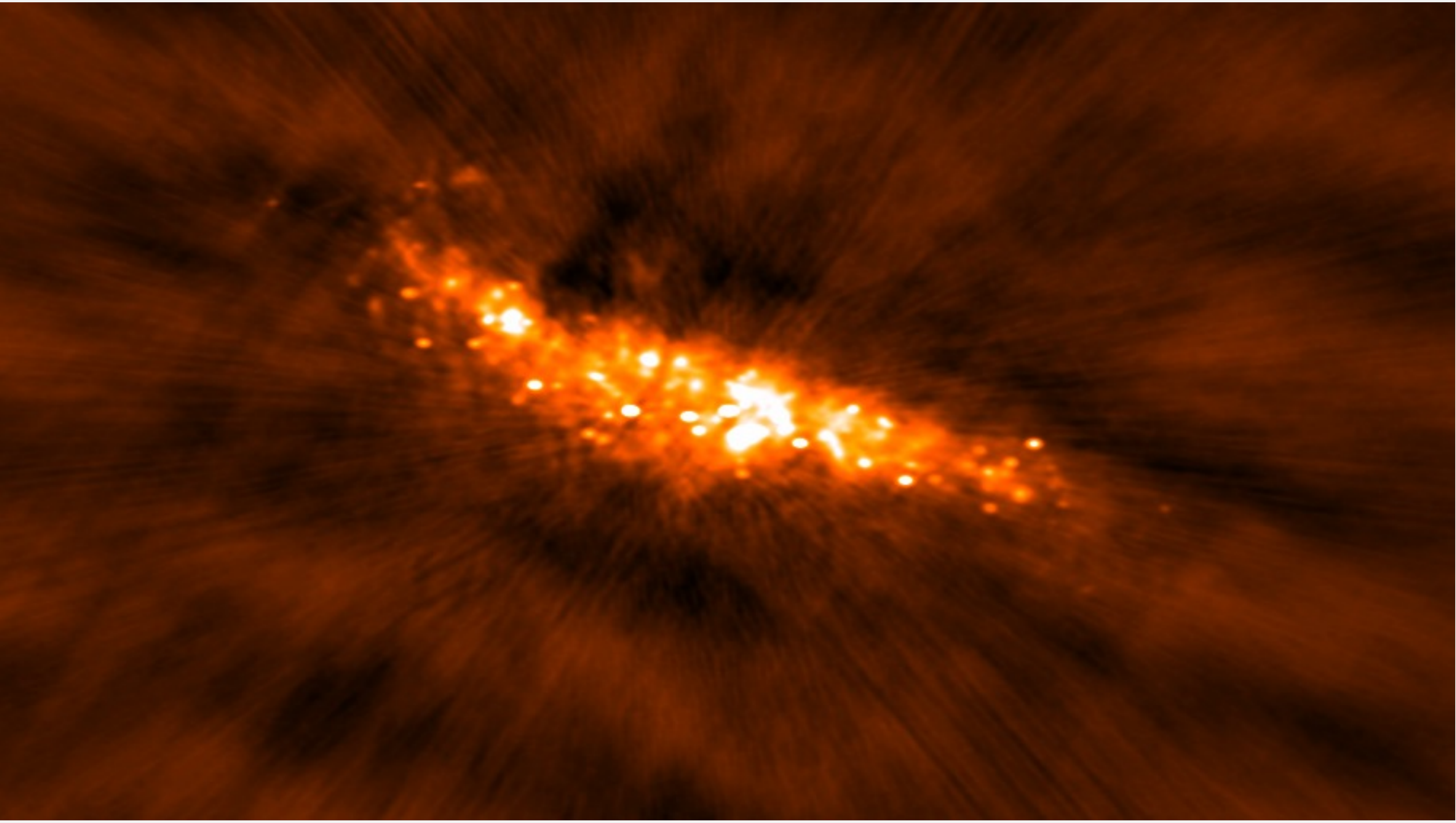
3. Go to 1. unless any remaining peak is below some user-specified level or number of iterations reached.

4. Convolve the accumulated point source model with an idealized `CLEAN' beam (usually an elliptical Gaussian fitted to the central lobe of the dirty beam).

5. Add the residuals of the dirty image to the `CLEAN' image.

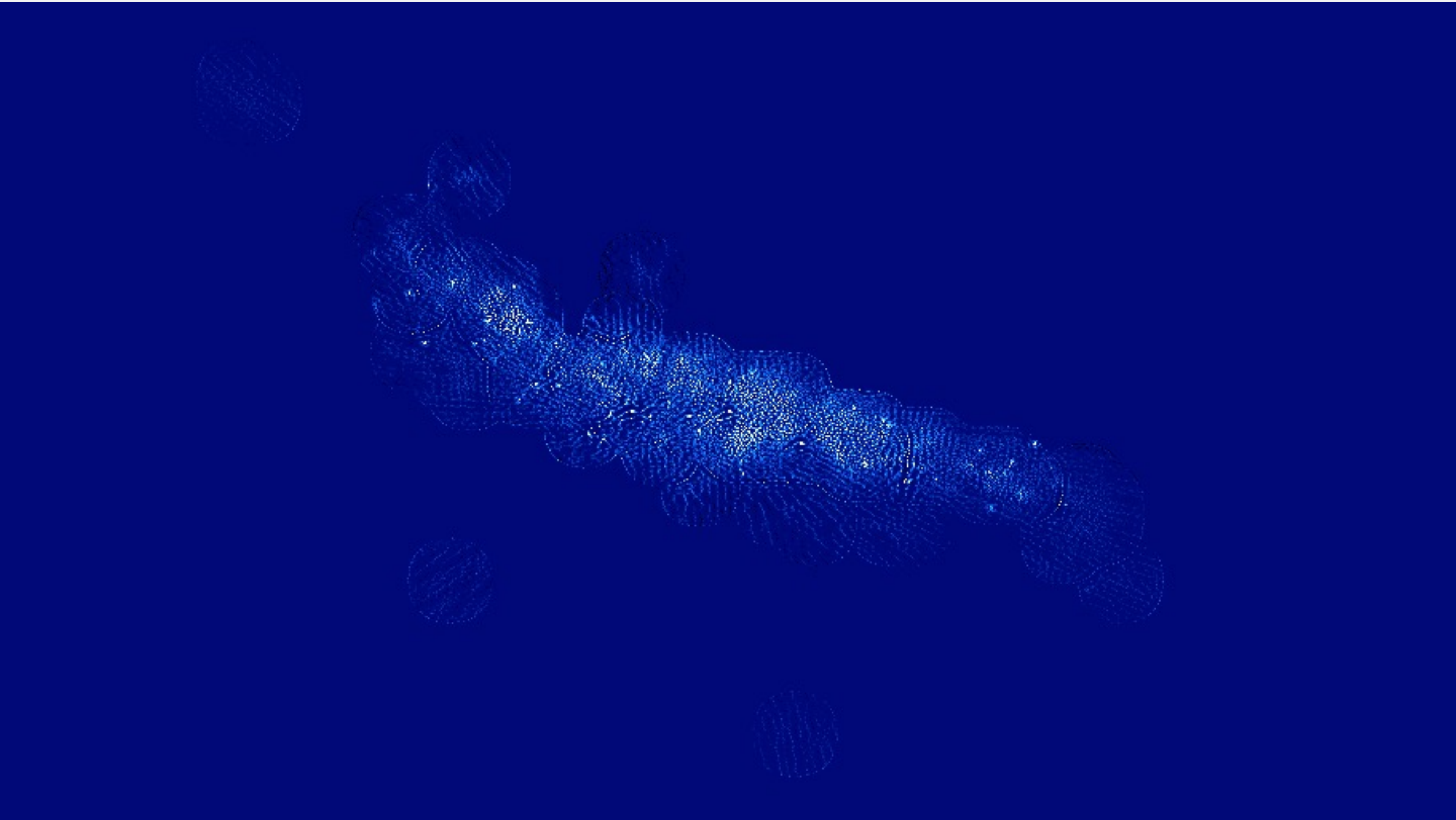
HÖGBOM CLEAN IN ACTION

Hogbom CLEANED image



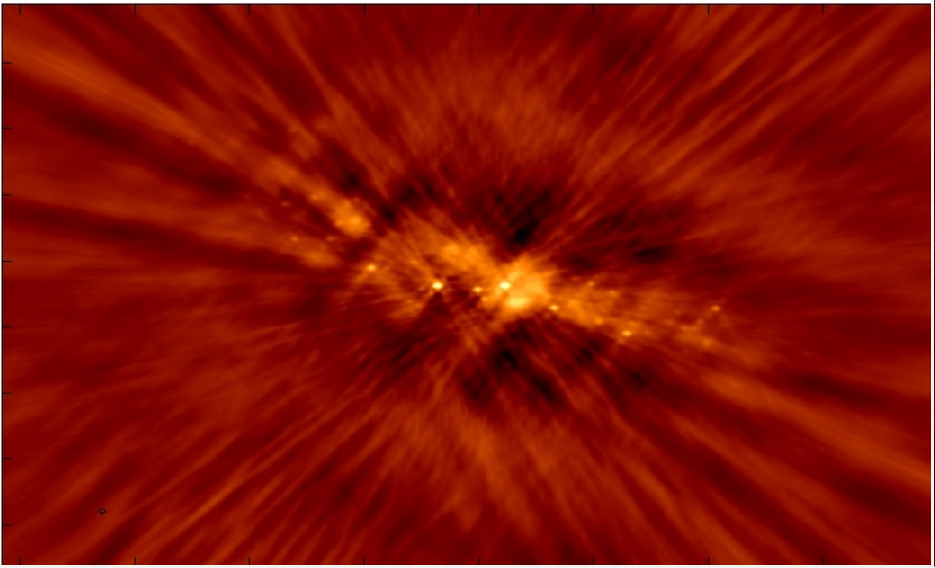
CLEAN IMAGE & MODEL

Hogbom CLEANED model

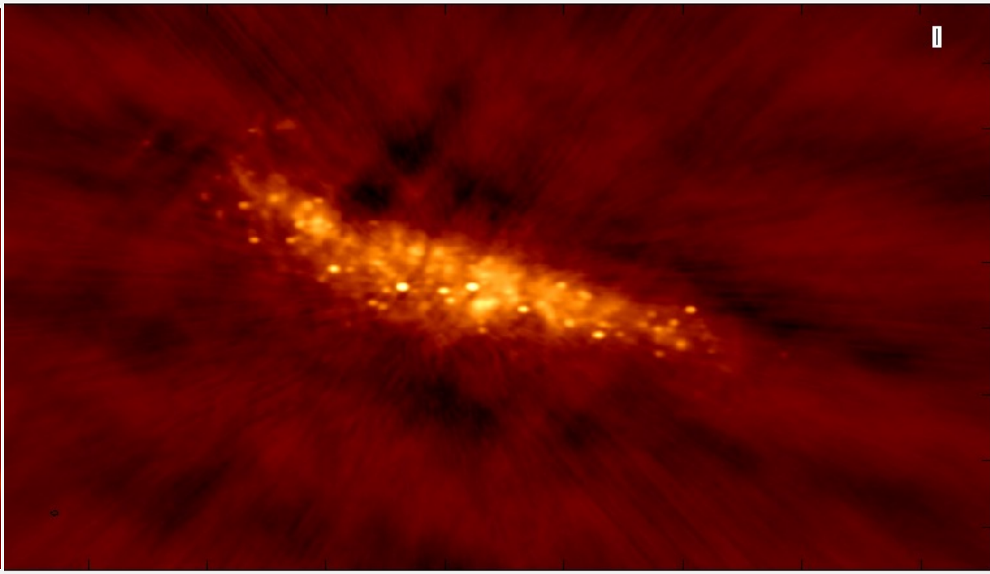


THE MANY FORMS OF CLEAN

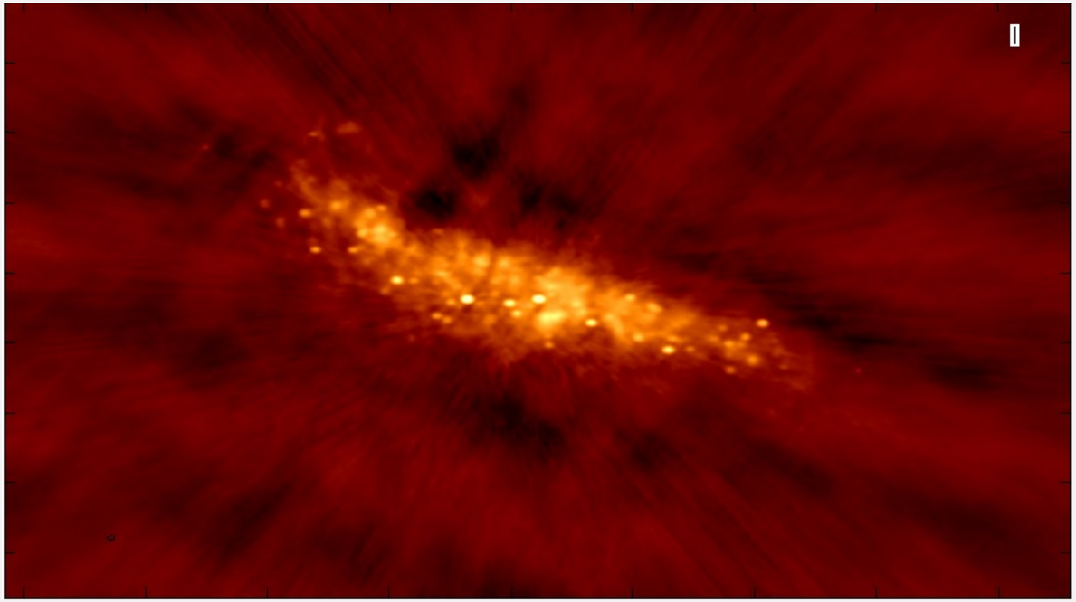
Maximum Entropy Method



Clark



Clark-Stokes



DECONVOLVING DIFFUSE STRUCTURE

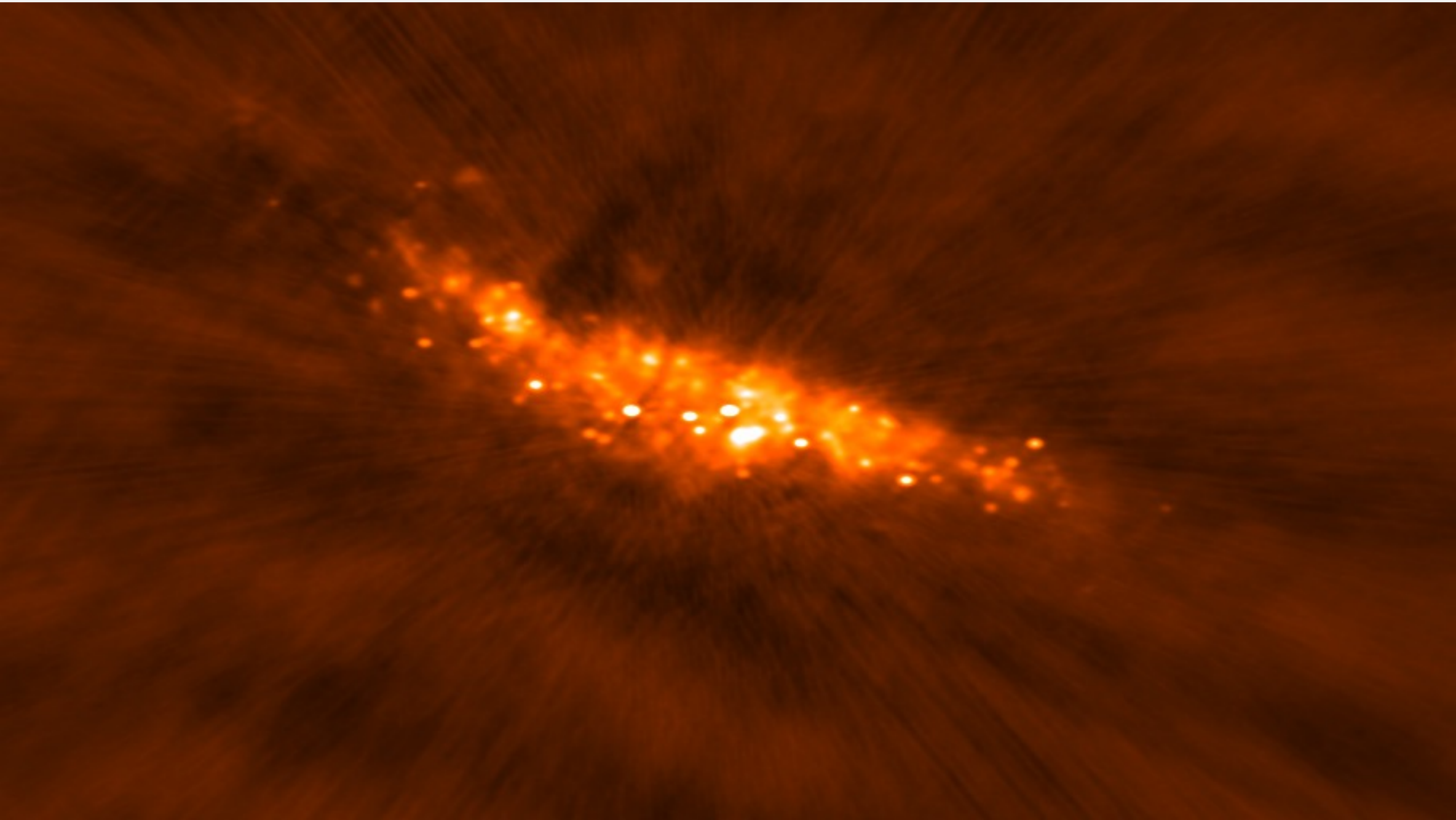
- Improved algorithm by Cornwell (2008) : "multi-scale clean"
- Fits small smooth Gaussian kernels (and delta functions) during a Högbom CLEAN iteration
- Implemented in CASA tclean. Advised to use pixel scales corresponding to orders of the dirty beam size and avoid making scale too large compared to the image width/lowest spatial frequency.
- E.g. For example, if the synthesized beam is 10" FWHM and cell=2", try multiscale = [0,5,15]

```
deconvolver        = 'multiscale'        #  Minor cycle algorithm (hogbom,clark,m
                                         #   ultiscale,mtmfs,mem,clarkstokes)
     scales        = [0, 1, 5, 15]       #  List of scale sizes (in pixels) for
                                         #   multi-scale algorithms
     smallscalebias =        0.6         #  A bias towards smaller scale sizes
     restoringbeam  =         []         #  Restoring beam shape to use. Default
                                         #   is the PSF main lobe
```

CASA tclean

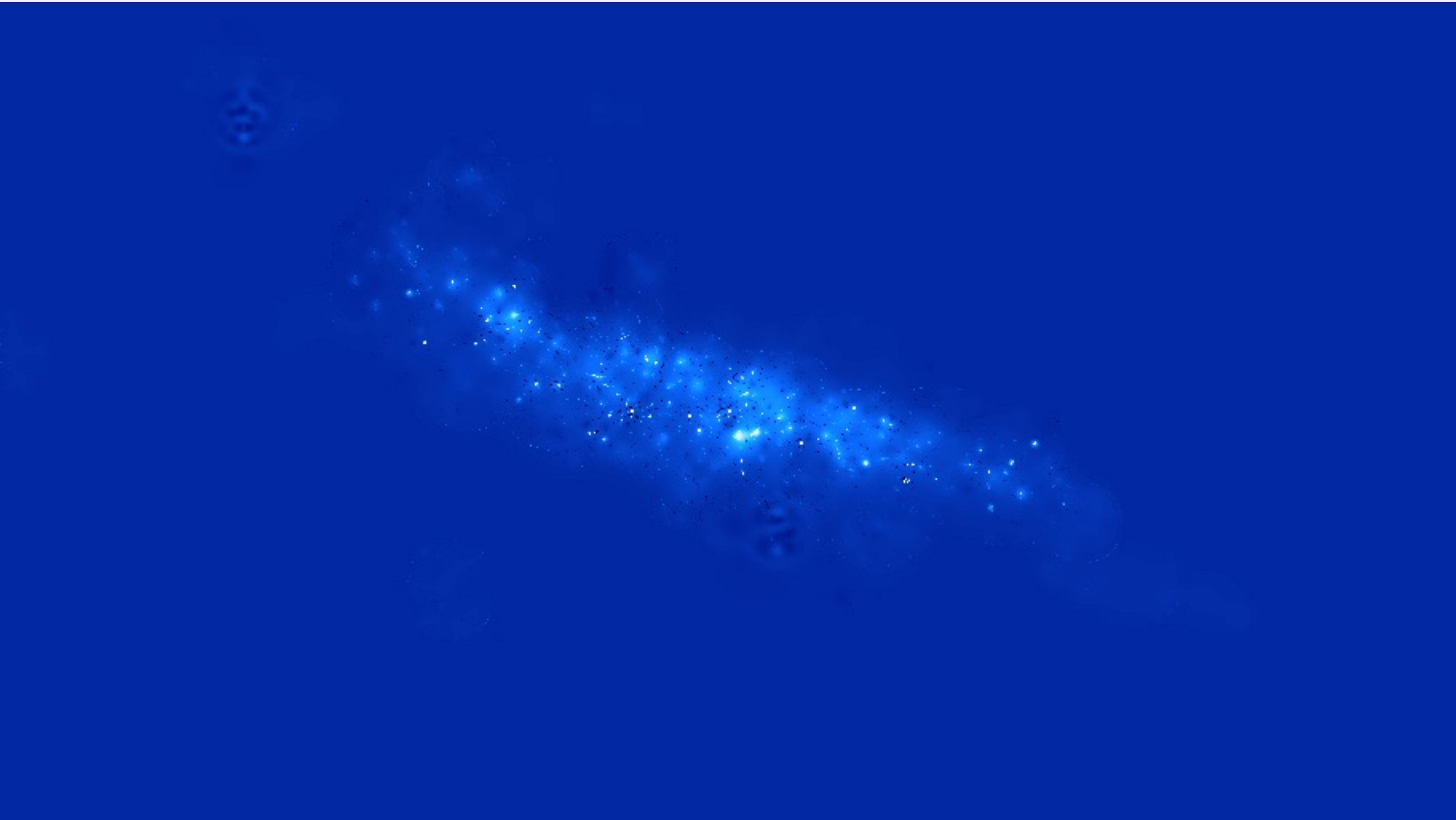
MULTI-SCALE CLEAN

Multi-scale CLEANED image



MULTI-SCALE CLEAN

Multi-scale CLEANED model



WIDE-FIELD IMAGING

2D Fourier Transform does not hold for new sensitive, wide-band, wide-field arrays

Non co-planar baselines becomes a problem i.e. l,m,w >> 0

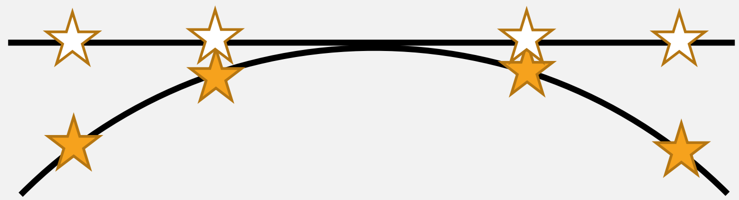
$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dl dm$$

Three-dimensional visibility fuction $V(u,v,w)$ can be transformed to a three-dimensional image volume $I(l,m,n)$ - this is not physical space since l, m, & n are direction cosines.

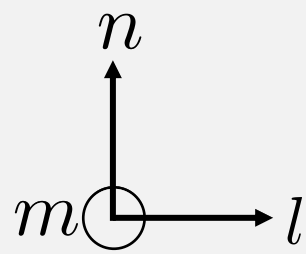
The only non-zero values of I lie on the surface of a sphere of unit radius defined by $\mathrm{n} = \sqrt{1-l^2-m^2}$

WIDE-FIELD IMAGING

The sky brightness consisting of a number of discrete sources are transformed onto the surface of this sphere.



The two-dimensional image is recovered by projection onto the tangent plane at the pointing centre



So how do we achieve this? Two solutions available:

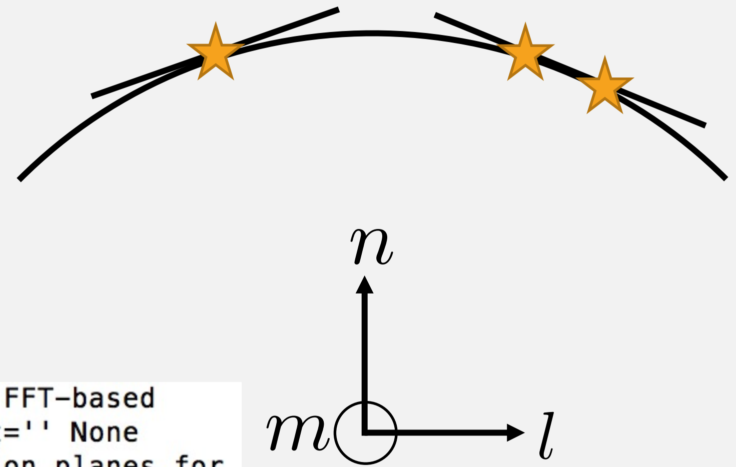
i. Faceting - split the field into multiple images and stitch them together

ii. w-projection - most used solution, effectively performs the above to recover $I(l, m)$

Both available in CASA!

i. FACETING

- Takes advantage of the small field approximation (l,m~0) so the image sphere is approximated by pieces of many smaller tangent planes.
- Within each sub-field, standard two-dimensional FFTs may be used.
- Errors increase quadratically away from the centre of each sub-field, but these are acceptable if enough sub-fields are selected.
- Facets can be selected so as to cover known sources.
- Facets may overlap allowing complete coverage of the primary beam



CASA clean implementation

```
gridmode        = 'widefield'   #  Gridding kernel for FFT-based
                                #   transforms, default='' None
   wprojplanes  =           1   #  Number of w-projection planes for
                                #   convolution; -1 => automatic
                                #   determination
   facets       =           8   #  Number of facets along each axis
                                #   (main image only)
```

ii. w-PROJECTION

Cornwell et al. 2008

$$V(u,v,w)_{=0} * \mathfrak{F}\left(e^{-2\pi i w(\sqrt{1-l^2-m^2}-1)}\right) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dldm$$

- Very dependent on zenith angle, co-planarity of array, field of view and resolution.
- Convolution theorem no longer works when w-terms present.
- CLEAN assumes constant PSF, but PSF changes (slightly) over the image.
- Solved with Cotton-Schwab algorithm (Schwab 1984) (used in CASA automatically).

ii. w-PROJECTION

The Cotton-Schwab + w-projection algorithm:

1) Make initial dirty image & central PSF - Perform minor iterations:

- Find peak
- Subtract scaled PSF at peak with small gain
- Repeat until highest peak ~80-90% decreased

2) Major iteration: ‘Correct’ residual

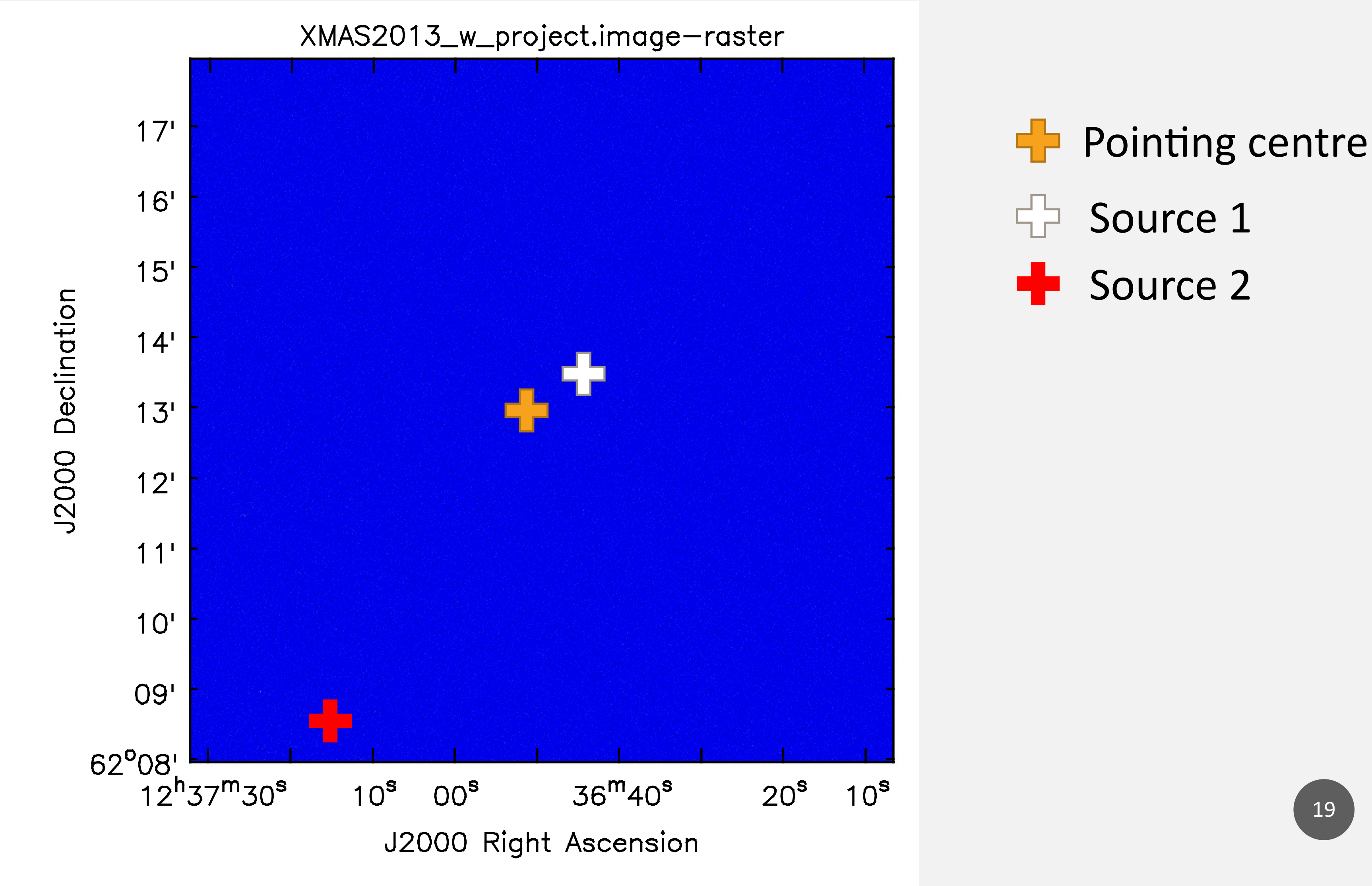
- Predict visibility for current model
- Subtract predicted contribution and re-image

```
gridmode        = 'widefield'     #  Gridding kernel for FFT-based
                                  #   transforms, default='' None
     wprojplanes  =          -1   #  Number of w-projection planes for
                                  #   convolution; -1 => automatic
                                  #   determination
     facets       =           1   #  Number of facets along each axis
                                  #   (main image only)
```

CASA clean implementation

w-PROJECTION

Take the GOODS-N field as observed by 1.4 GHz e-MERLIN

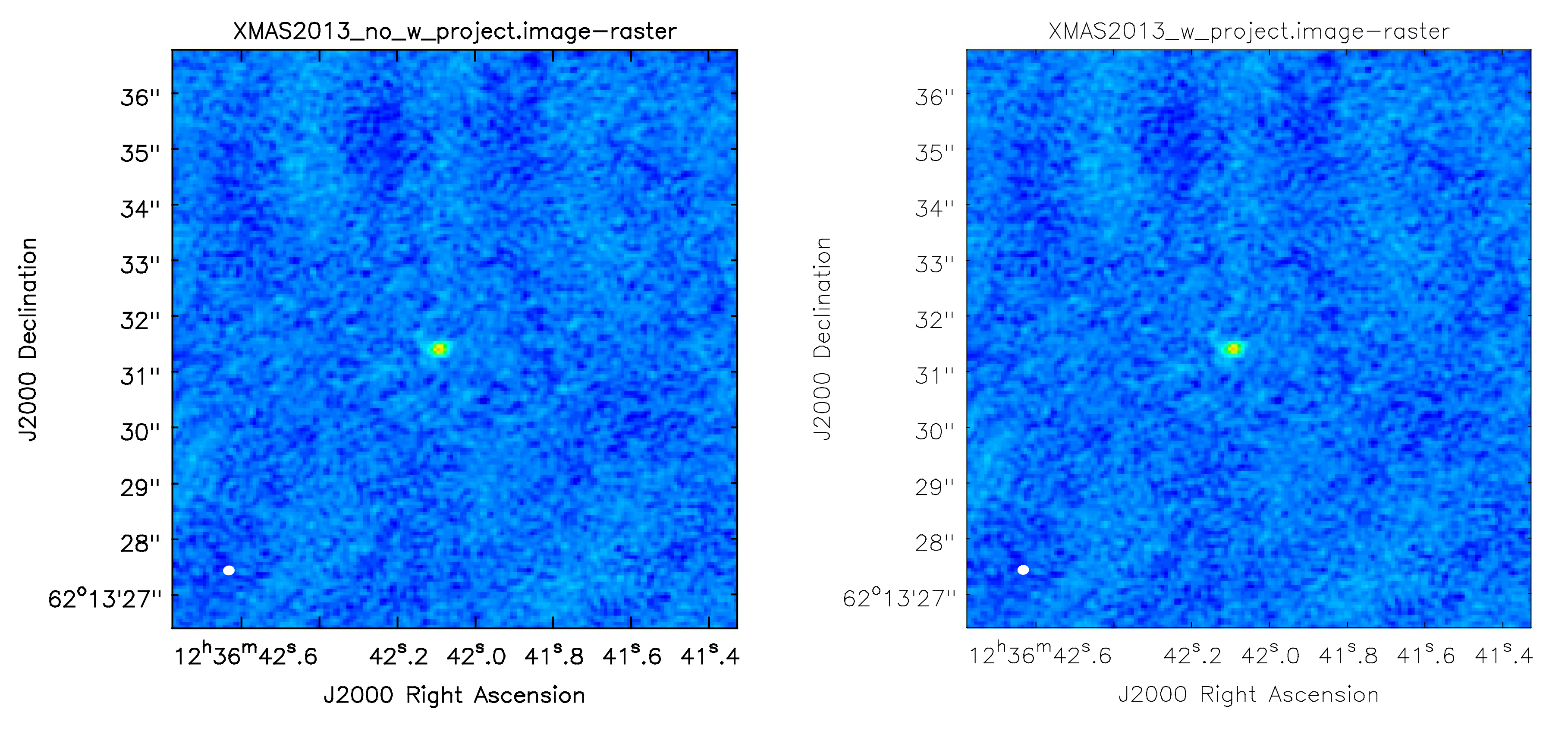


w-PROJECTION

Source 1: Near the pointing centre

No w-projection

w-projection

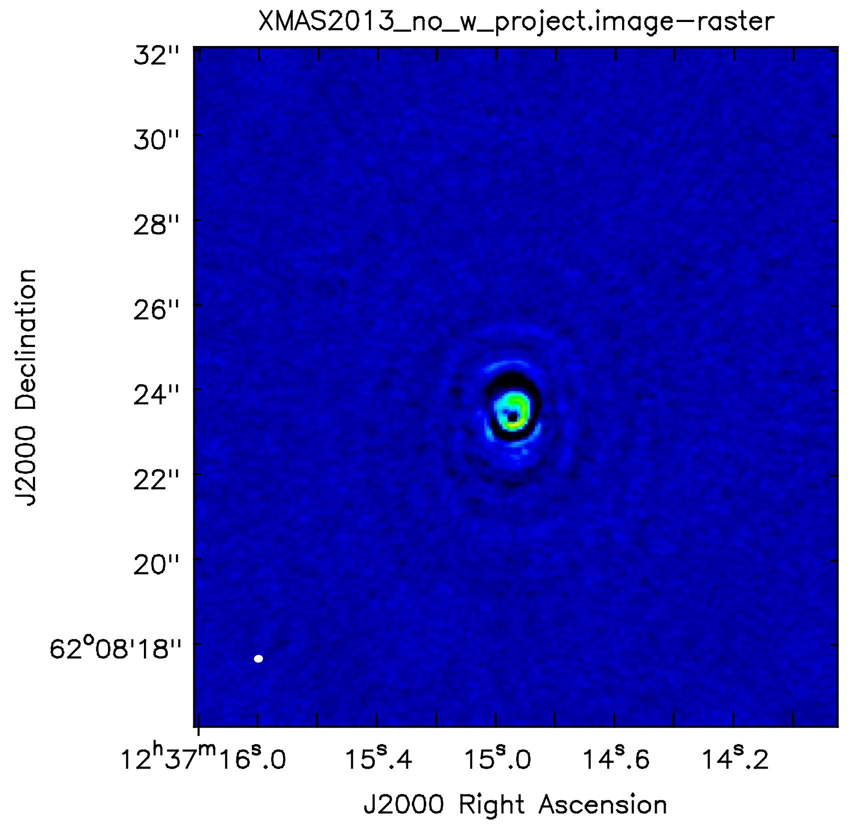


Pretty much identical! Small field approximation holds and 2D FT suffices

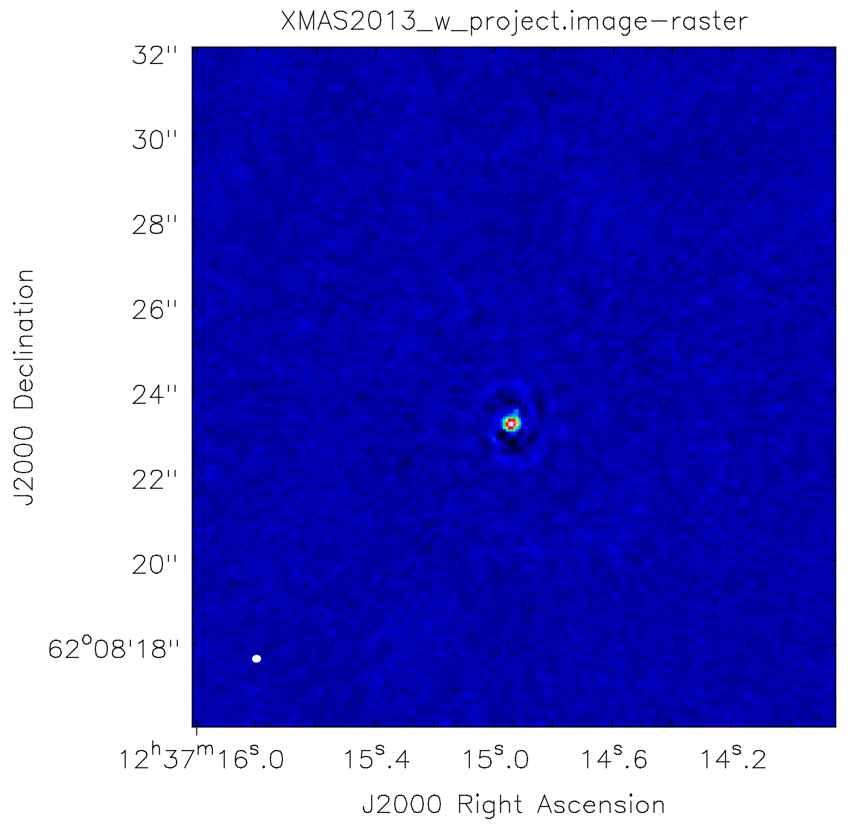
w-PROJECTION

Source 2: Away from the pointing centre

No w-projection



w-projection



Small field approximation breaks and you need w-projection!

MULTI-FREQUENCY SYNTHESIS

- Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid

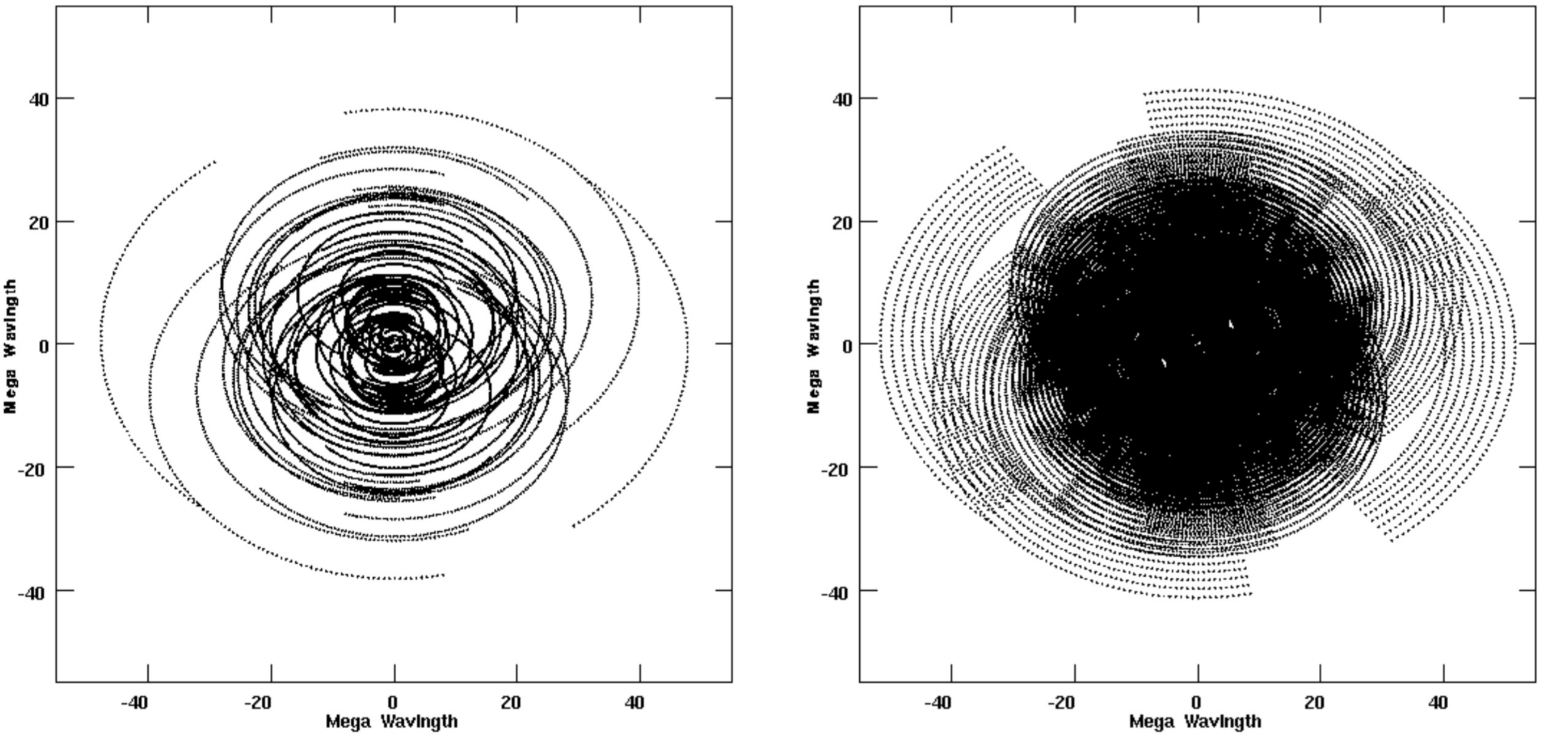


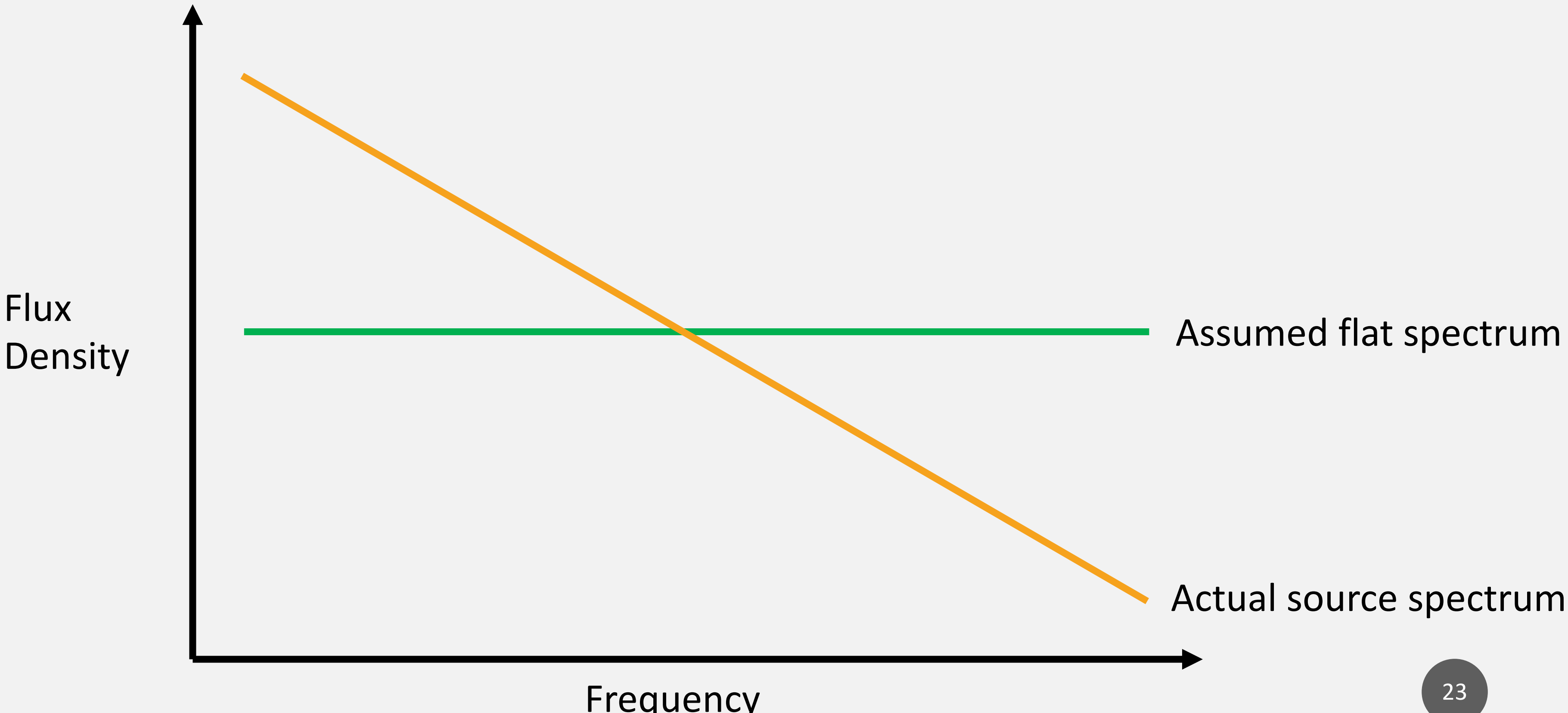
Figure 16.1: *Left (a):* VLBA (u, v) coverage for a full track at $\delta = 50°$. *Right (b):* Using MFS observations with 8 frequencies spread over 25%.

Conway & Sault (1995)

MULTI-FREQUENCY DECONVOLUTION

- Similar but not the same! (same name often used). Also known as multi-term multi-frequency synthesis (MTMFS) imaging.

Takes spectral variation of sky brightness distribution into account during deconvolution using linear Taylor series approximation.



MULTI-FREQUENCY DECONVOLUTION

I_ν^m represents the sky emission in terms of a Taylor series about a reference frequency:

$$I_\nu^m = \sum_{t=0}^{N_t - 1} b_\nu^t I_t^{\text{sky}} \text{ where } b_\nu^t = \left(\frac{\nu - \nu_0}{\nu_0}\right)^t$$

A power model is used to describe the spectral dependence of the sky. One practical choice is a power law with emission.

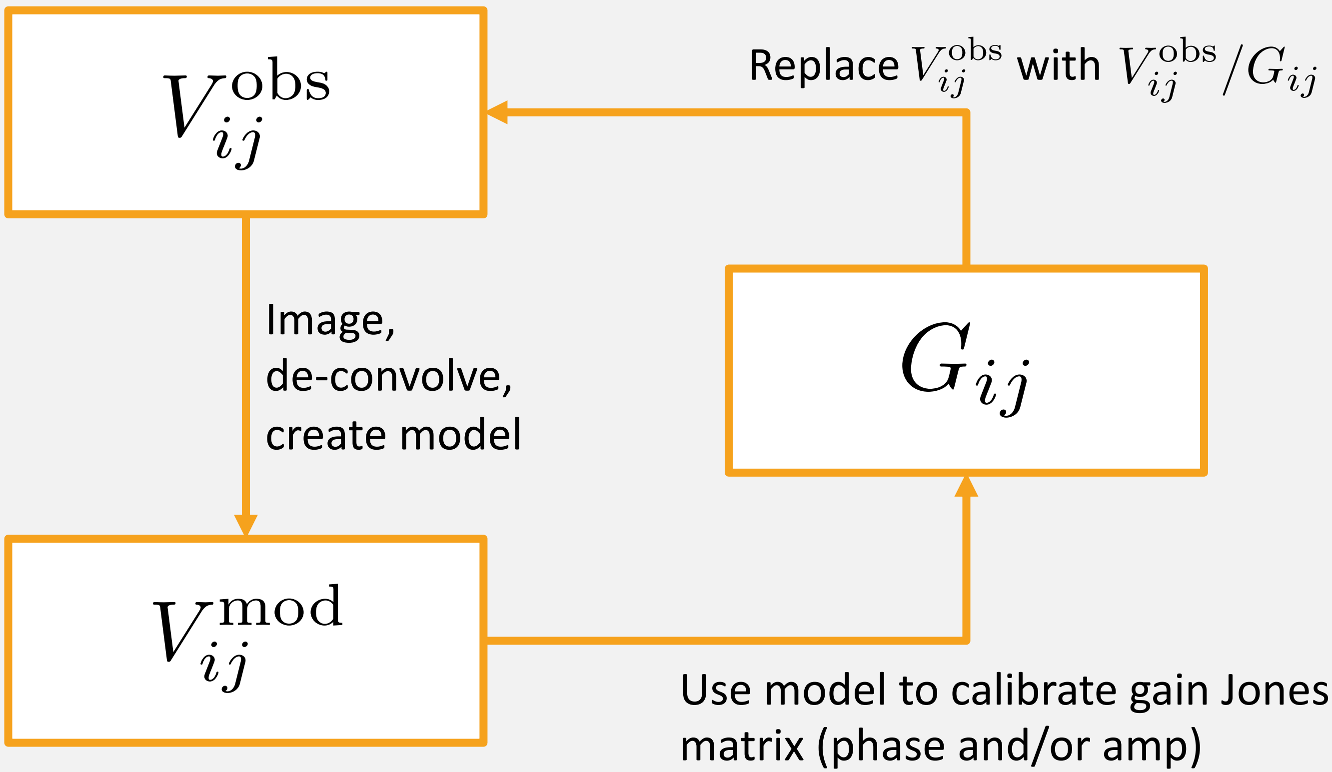
$$I_\nu^{\text{sky}} = I_{\nu_0}^{\text{sky}} \left(\frac{\nu}{\nu_0}\right)^{I_\alpha^{\text{sky}} + I_\beta^{\text{sky}} \log\left(\frac{\nu}{\nu_0}\right)}$$

- Useful for wideband, high dynamic range and sensitive imaging.
- Incorporated in CASA in combination with multi-scale CLEAN as ‘mtmfs’

SELF-CALIBRATION USING CLEAN

Self-calibration recap:

Given: $V_{ij}^{\text{obs}} = G_{ij} V_{ij}^{\text{real}}$



And.. repeat until model/solution converges!

SELF-CALIBRATION USING CLEAN

- Clean components can be used as calibration model
- Often applied as:

Phase calibration

↓

Shallow CLEAN (avoid CLEAN bias)

↓

Phase calibration

↓

Deep CLEAN

↓

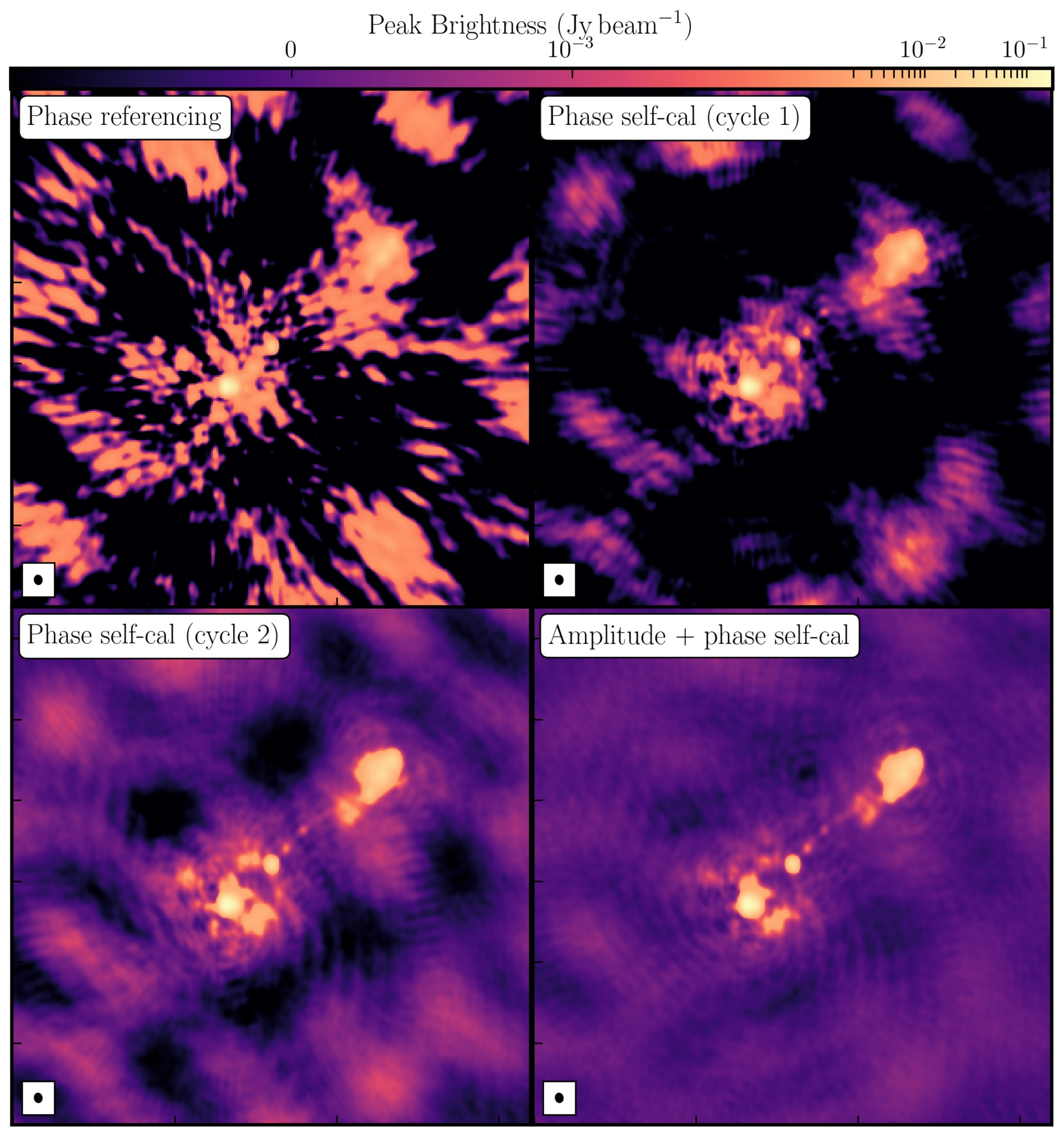
Amplitude & phase calibration

↓

Deep CLEAN

SELF-CALIBRATION USING CLEAN

Self-calibration improvements on C-band e-MERLIN observations of 3C277.1



PRIMARY BEAM CORRECTION

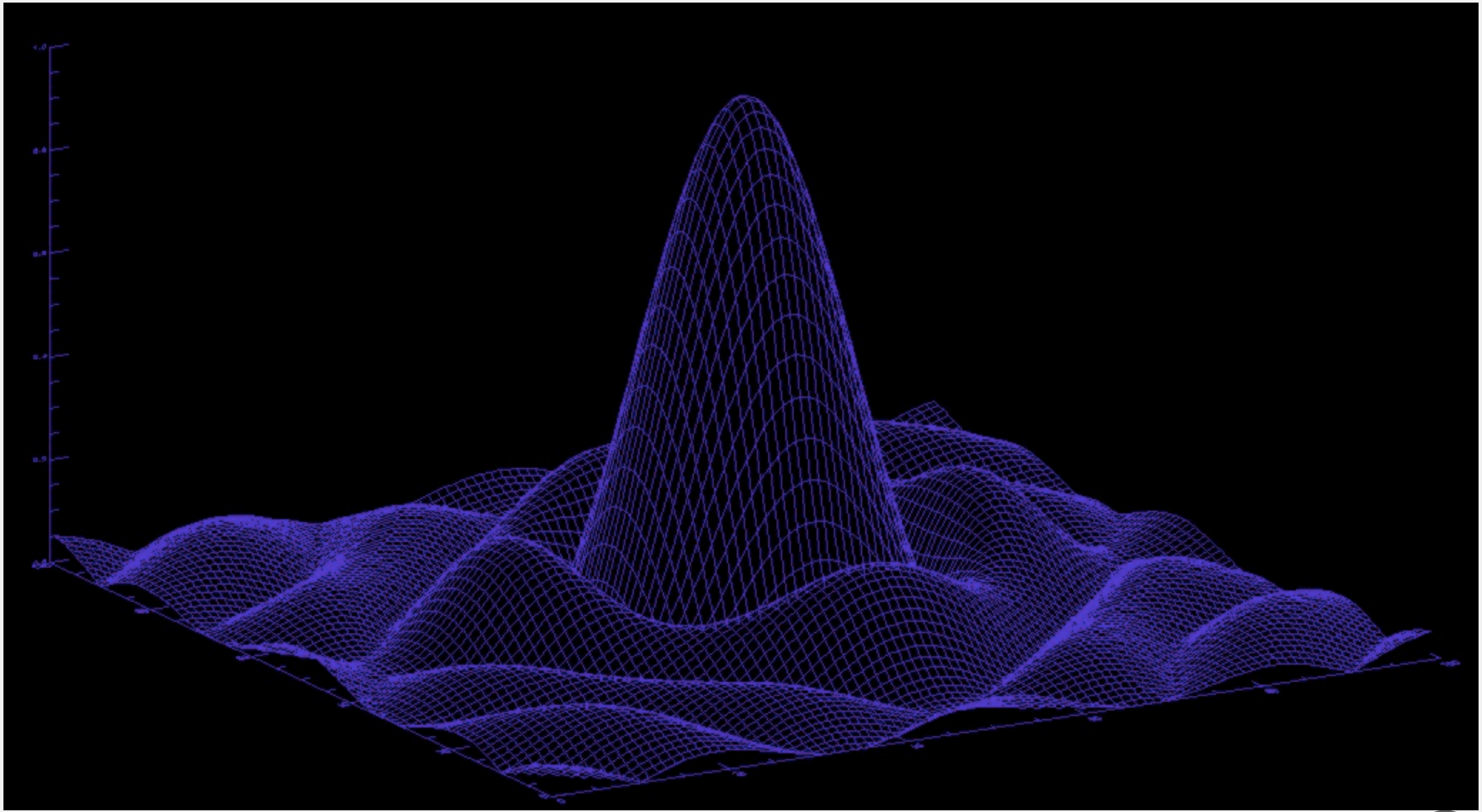
- Correction is required for the antenna response
- This is called "primary beam" correction (as opposed to the synthesized beam / psf)
- For dishes, the primary beam is ~constant but can be very complex away from the FWHM.

To correct for: multiply final image with the inverse beam!

Scalar for total brightness, matrix for polarised

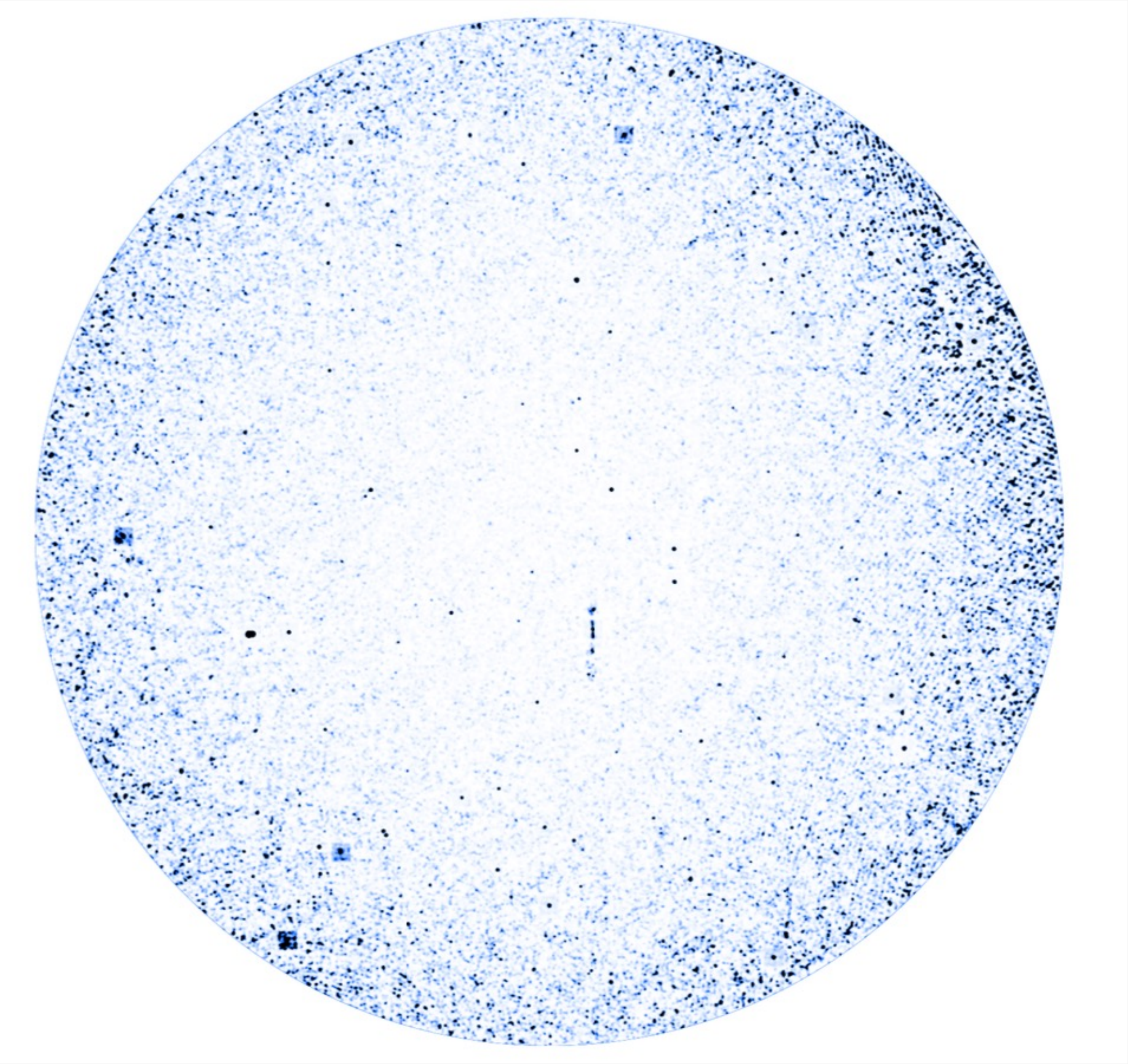
PRIMARY BEAM CORRECTION

Complex sidelobe structure + asymmetries!



Knockin primary beam holographic scan

PRIMARY BEAM CORRECTION

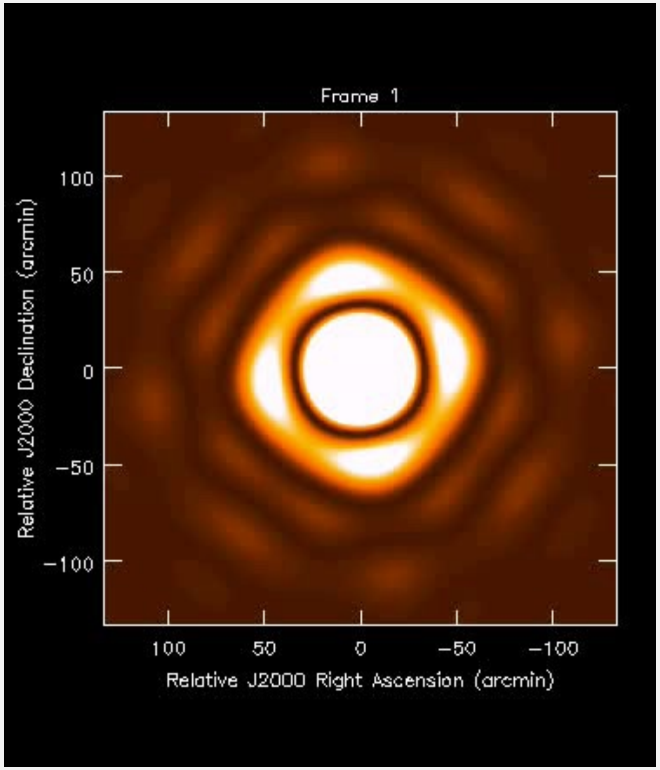


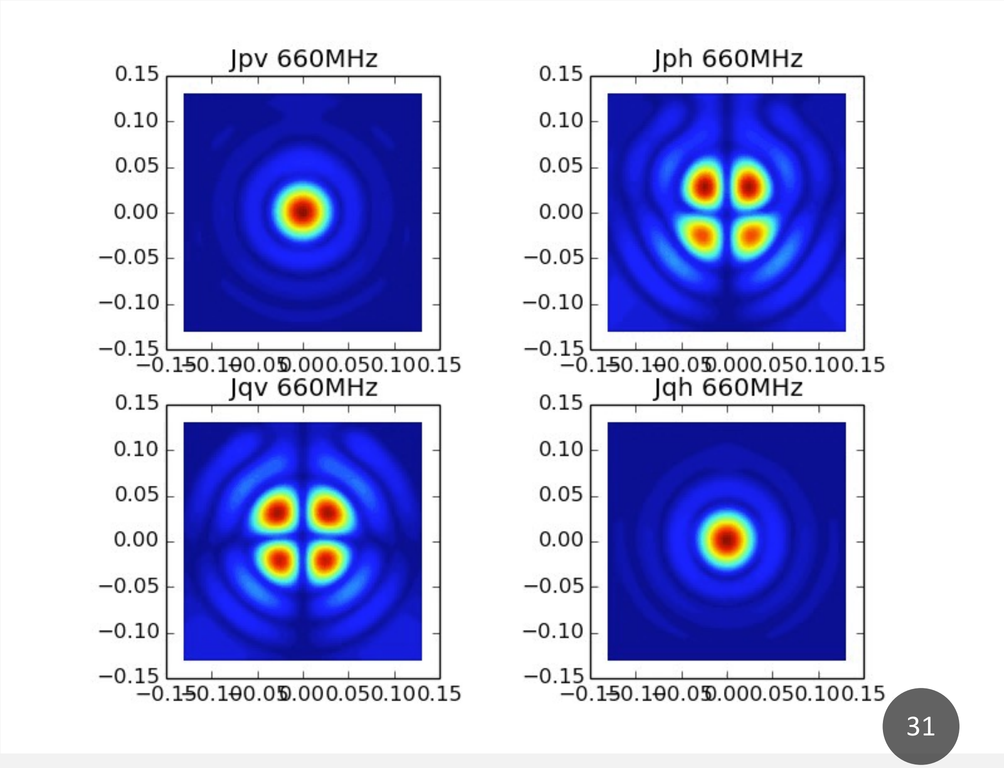
Primary beam corrected JVLA+MERLIN image of GOODS-N

Note the increased noise level towards the edge of the field

VARIABLE PRIMARY BEAMS

- Primary beam of arrays can vary with time and frequency!
- Has to be accounted for during cleaning and primary beam correction if imaging the whole primary beam (CASA has this for the JVLA + ALMA. VLBI arrays don't image the pb often!)





VARIABLE PRIMARY BEAM

Primary beam spectral variation for the UK Lovell Telescope 1.4-1.6GHz

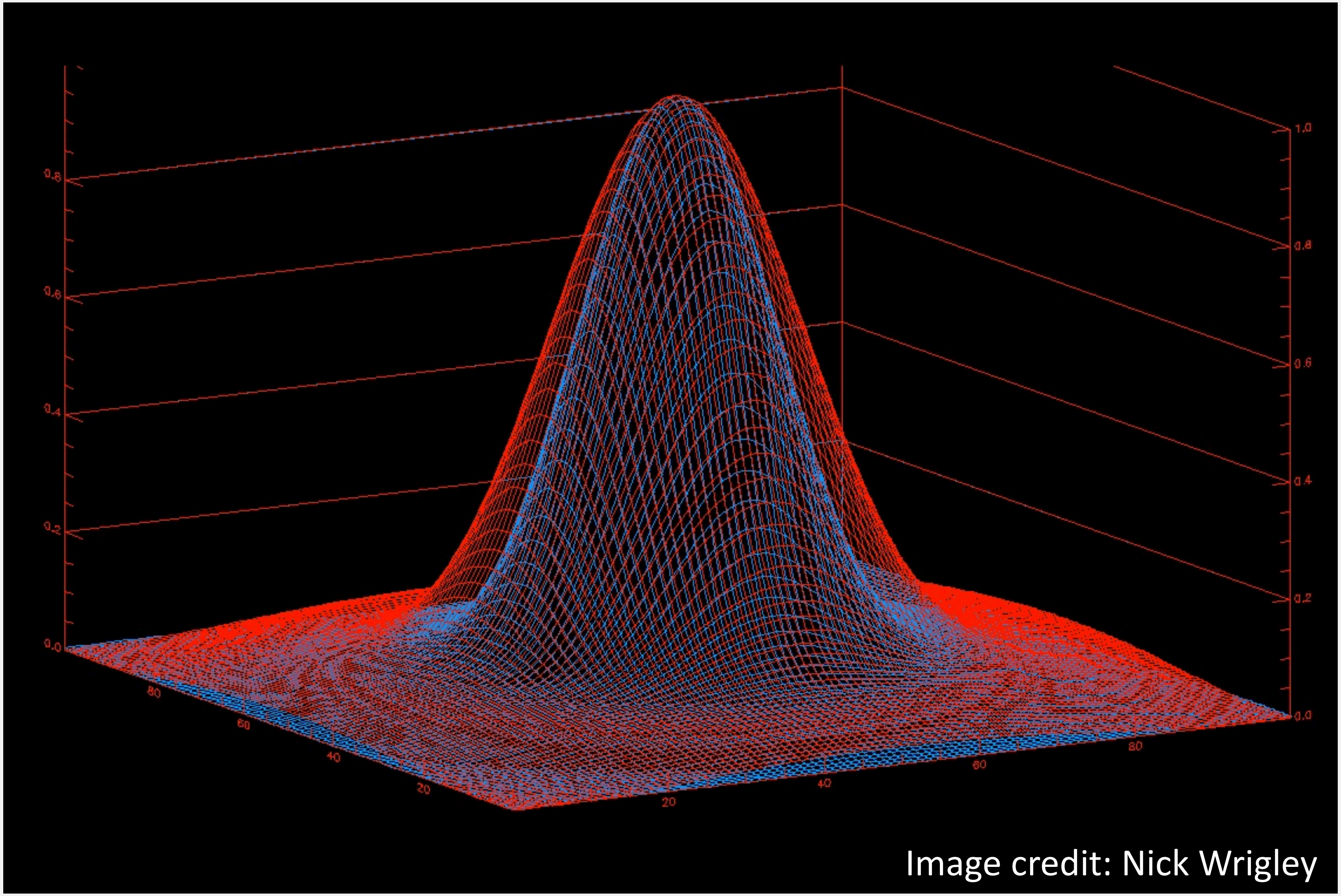
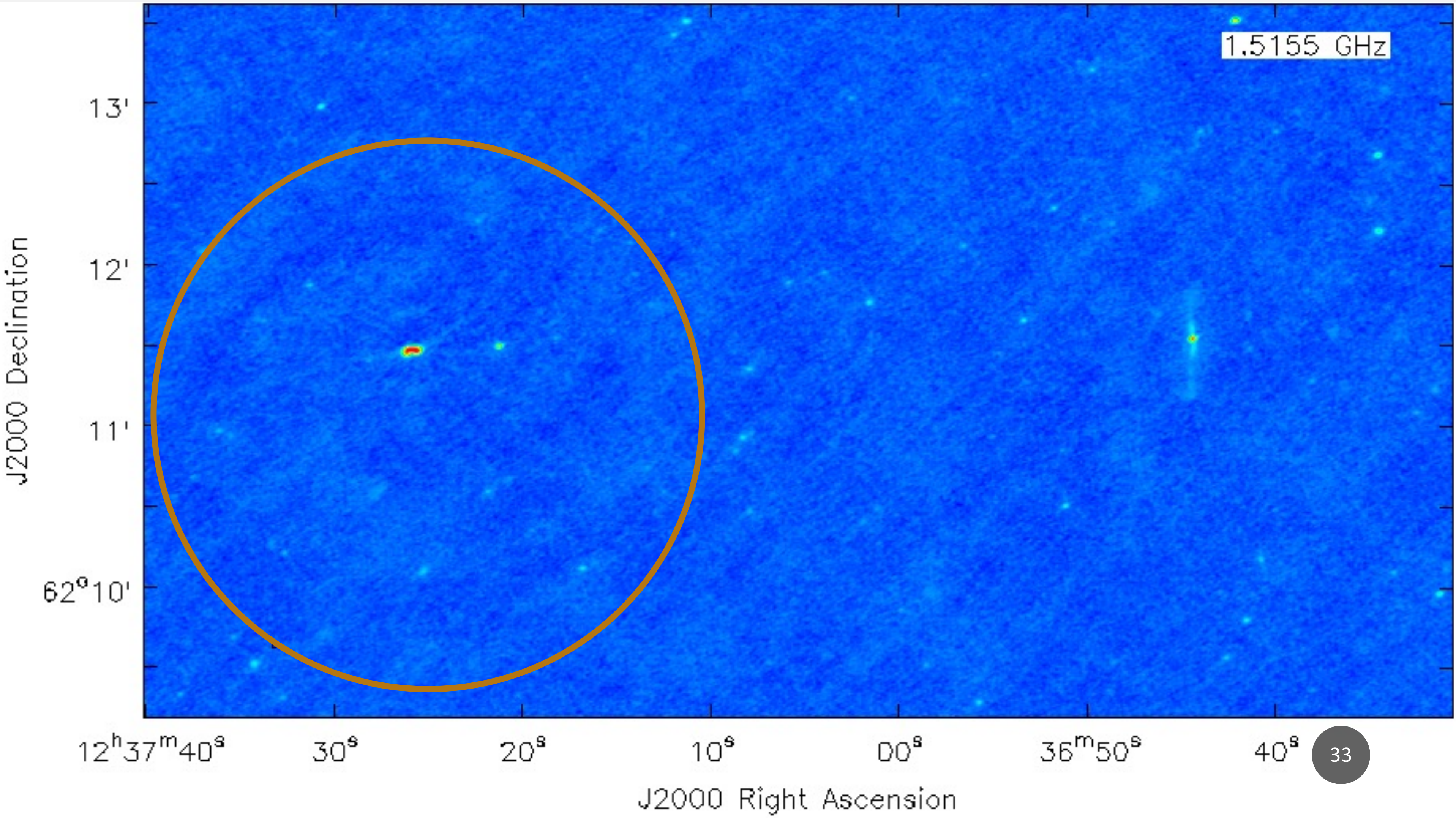


Image credit: Nick Wrigley

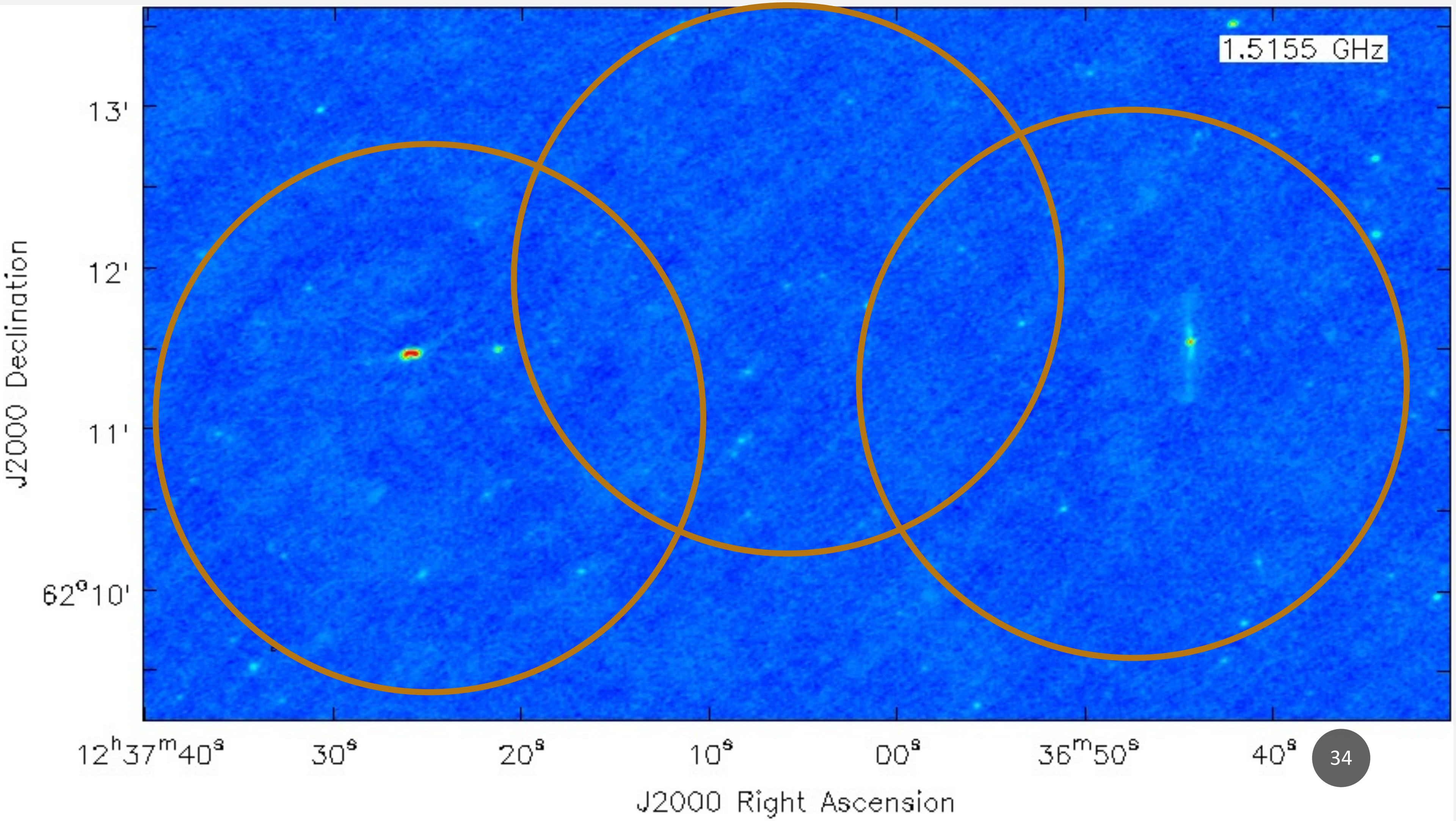
MOSAICKING

What if this is our primary beam and we want to see the FR-I galaxy too?



MOSAICKING

We can use multiple pointings and combine them with correct weighting



MOSAICKING

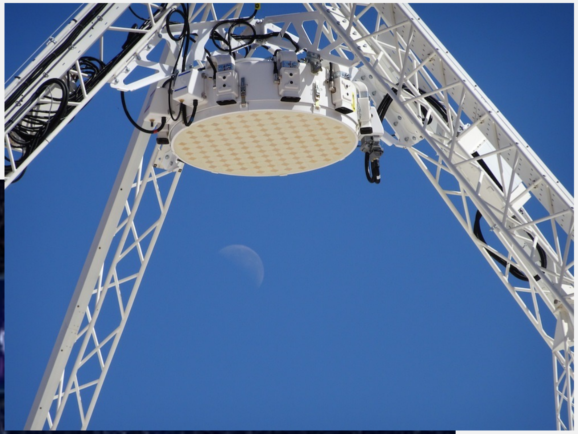
- To create the mosaicked image $M(l,m)$
- Need to weight with 1/σ^2 = (primary beam)² or $B_i^2(l,m)$

$$M(l,m) = \frac{\sum_i B_i^2(l,m)(I_i(l,m)/B_i(l,m))}{\sum_i B_i^2(l,m)}$$

$$= \frac{\sum_i B_i(l,m) I_i(l,m)}{\sum_i B_i^2(l,m)}$$

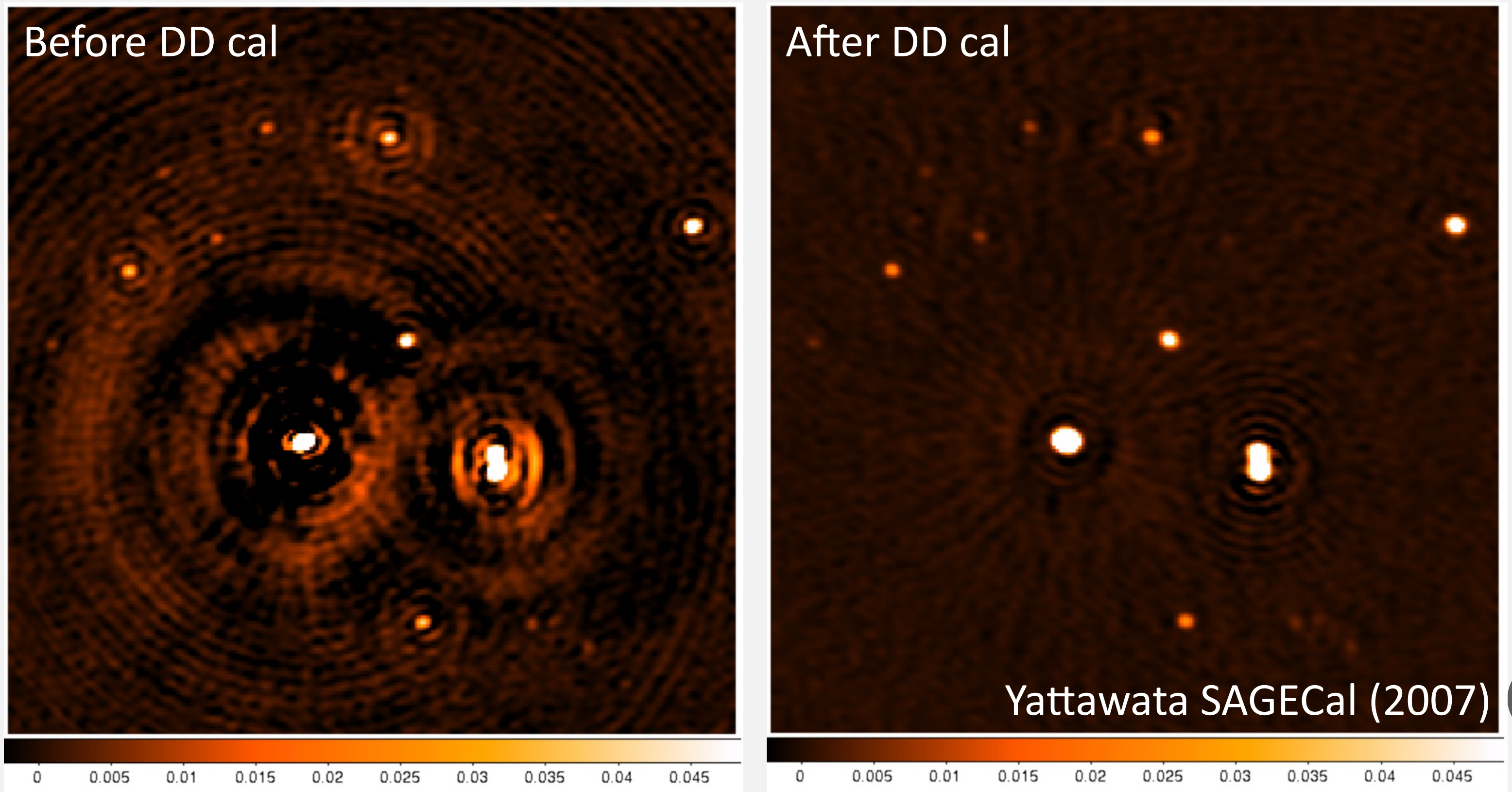
MOSAICKING

Some telescopes like ASKAP are equipped with Phased Array Feeds that enable simultaneous multi-beam imaging of a field




DIRECTION DEPENDENT CALIBRATION

- Direction dependent (DD) effects may need further corrections applied during imaging… not a fully solved problem!
- Can be ionosphere, tropospheric, instrumental (e.g. a -projection)
- Affects position, brightness & polarisation angles!

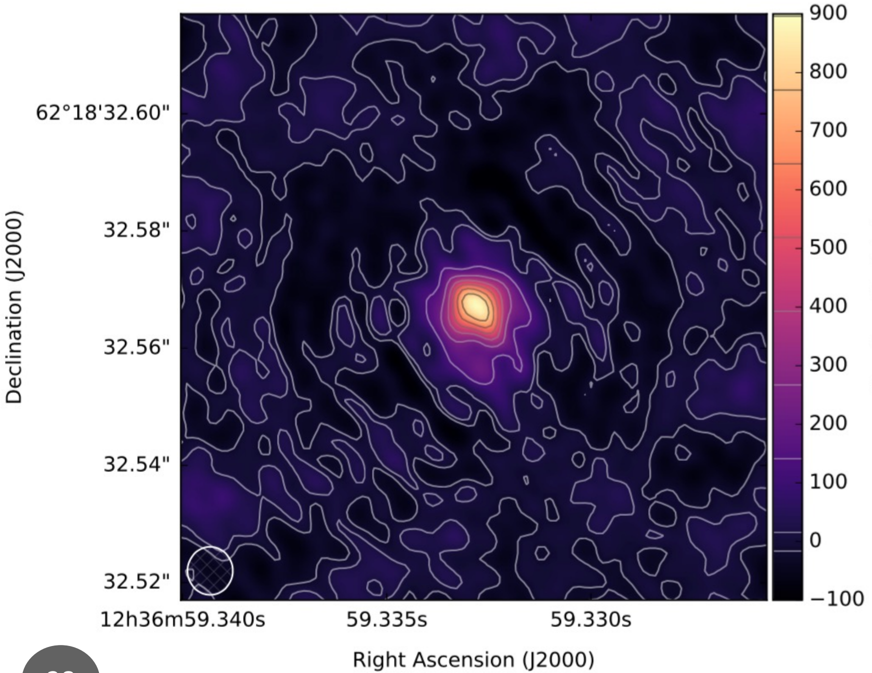


DIRECTION DEPENDENT CALIBRATION

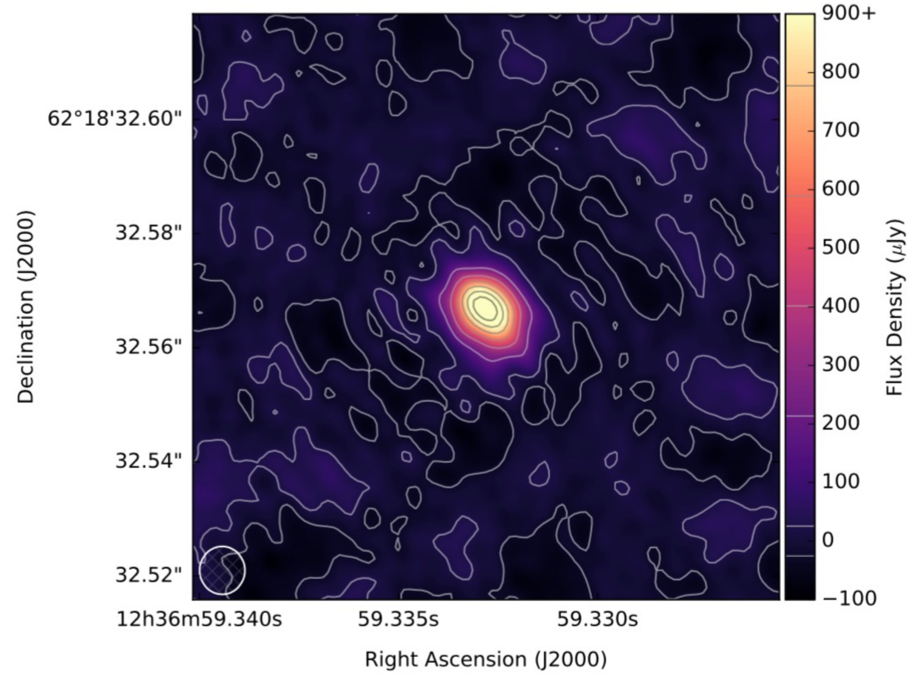
Possible solutions:

- Image in small ‘facets’ where DD's effects are constant
- Peeling
- Direction-dependent calibration during visibility gridding (LOFAR does this)
- For VLBI, multi-source self-calibration (below)

Pre-MSSC



Post-MSSC



Radcliffe et al. 2016

SUMMARY

Topics discussed:

- CLEAN
- When to use Multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution
- Self-calibration using CLEAN components
- Primary beam correction
- Mosaicking
- Direction-dependent effects during imaging