

# Lecture 7: Fringe-fitting

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DARA Unit 4

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# Further calibration

Use a skymodel of the **true visibilities** to solve for the rest of the Jones matrices:

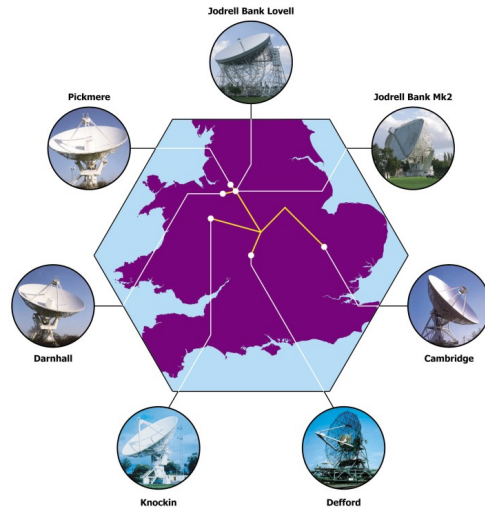
$$\vec{V}_{ij}^{\text{obs}} = M_{ij} B_{ij} F_{ij} G_{ij} D_{ij} \checkmark E_{ij} \checkmark P_{ij} \checkmark T_{ij} \vec{V}_{ij}^{\text{true}}$$

Remember these Jones matrices contain visibility-like information, which is represented as *complex numbers* ( $a+bi$ ) that can be decomposed into **amplitudes** and **phases**

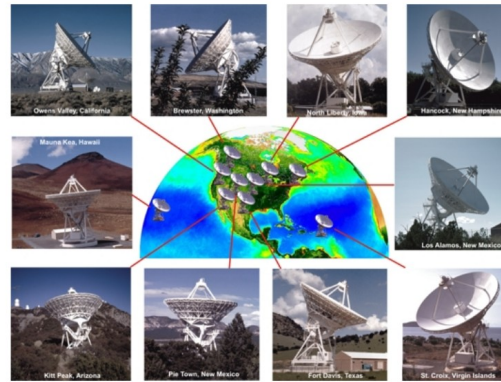
# Outline

- **Recap of Very Long Baseline Interferometry (VLBI)**
- **Fringe-fitting fundamentals**
- **Fringe-fitting in practice**

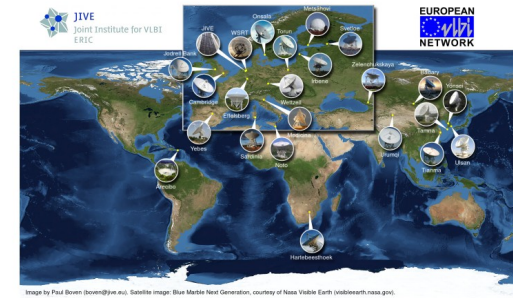
# Recap of VLBI



e-MERLIN



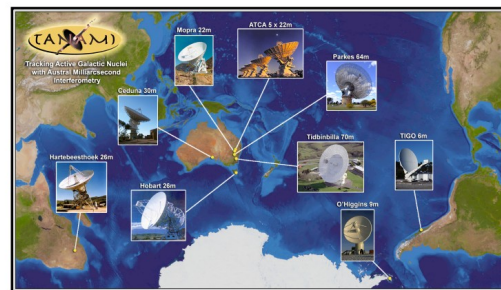
VLBA



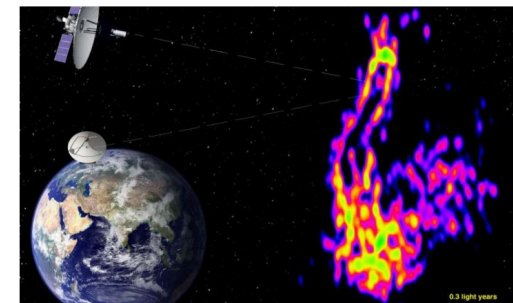
EVN



LBA



Tanami



RadioASTRON

# Recap of VLBI

- **Common element:** stations are so far apart that they require separate (non-distributed) clocks
- **This drives:**
  - Non-(near)-real time correlation
  - Clock / delay correction required
  - Lower tolerance for errors in geometric model of array
  - Having to cope with resolved sources

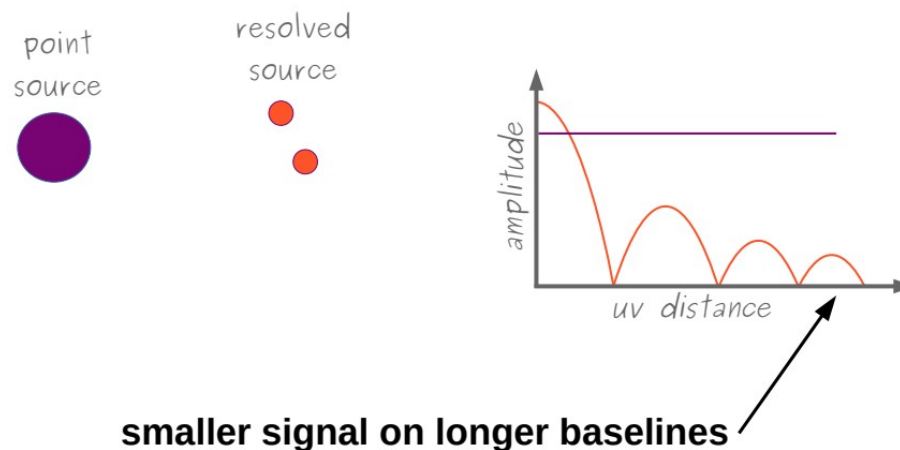
# Recap of VLBI

- **What part does resolution play?**

baseline sensitivity:  $S_{ij} = \sqrt{\frac{S_{sys,i}}{\sqrt{2\Delta\nu\Delta t}}} \sqrt{\frac{S_{sys,j}}{\sqrt{2\Delta\nu\Delta t}}}$

*note there is no dependence on baseline length!*

## what impact does source structure have?



# Fringe-fitting fundamentals

take the relation between phase and delay:

$$\phi_{\nu,t} = 2\pi\nu\tau_t$$



differentiating with respect to time yields:

$$d\phi_{\nu,t} = 2\pi\nu d\tau_t$$

the error in is dependent on both  $\nu$  and  $t$

take the first order expansion:

$$\Delta\phi_{\nu,t} = \phi_0 + \left( \frac{\delta\phi}{\delta\nu}\Delta\nu + \frac{\delta\phi}{\delta t}\Delta t \right)$$

frequency dependence  time dependence 

# Fringe-fitting fundamentals

$$\Delta\phi_{\nu,t} = \phi_0 + \left( \frac{\delta\phi}{\delta\nu} \Delta\nu + \frac{\delta\phi}{\delta t} \Delta t \right)$$

$\phi_0$  = the **phase** error at  $\nu_0, t_0$

$\frac{\delta\phi}{\delta\nu}$  = the **delay** or delay residual

$\frac{\delta\phi}{\delta t}$  = the **rate**, delay rate, or rate residual

normal phase calibration only estimates  $\phi_0$

**Fringe-fitting** is any process that also estimates *delays* and *rates*

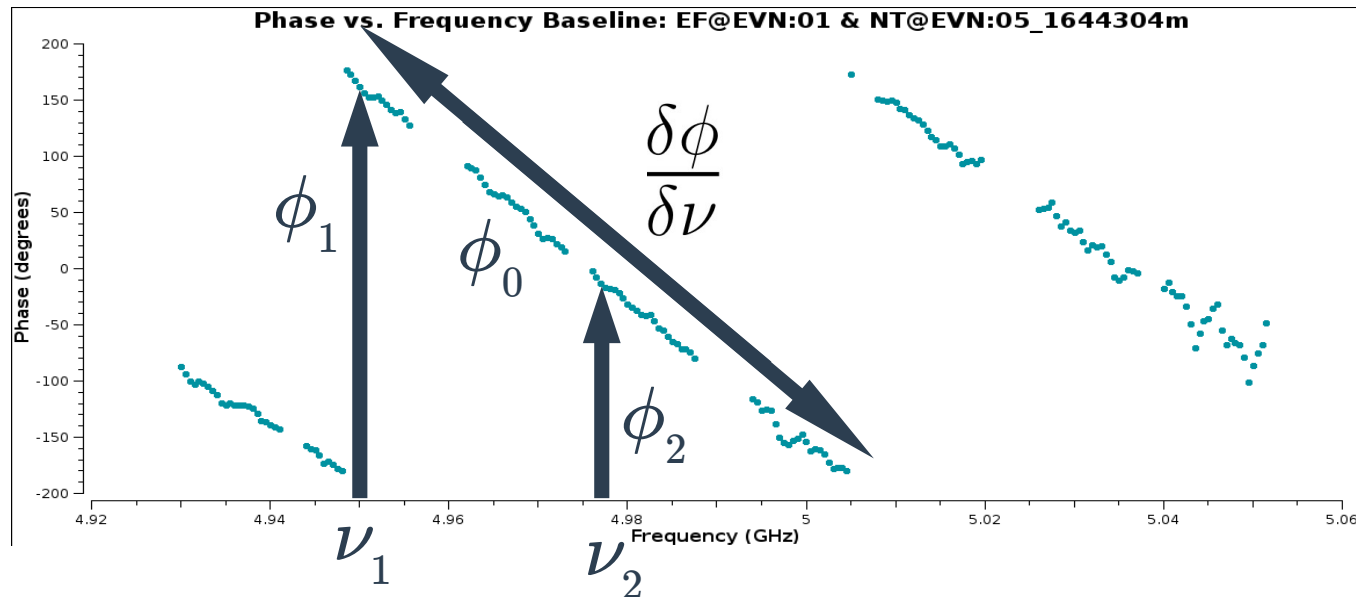
Effectively increases S/N by increasing solution interval



# Fringe-fitting fundamentals

How does solving for delays and rates increase S/N?

a practical example of delays



$\phi_1$  is only valid at  $\nu_1$  and  $\phi_2$  is only valid at  $\nu_2$

$\phi_0 + \frac{\delta\phi}{\delta\nu}$  is valid over a large range of  $\Delta\nu$

# Baseline-based fringe-fitting

The fringe-fitting equation can be written in terms of individual baselines, e.g. with antennas  $i,j$ :

$$\Delta\phi_{ij} = \phi_{i,0} - \phi_{j,0} + \left( \left[ \frac{\delta\phi_i}{\delta\nu} - \frac{\delta\phi_j}{\delta\nu} \right] \Delta\nu + \left[ \frac{\delta\phi_i}{\delta t} - \frac{\delta\phi_j}{\delta t} \right] \Delta t \right)$$

so you can construct this equation for each pair of baselines and solve for phases, delays, and rates on each baseline!

## Disadvantages:

- The source must be detected on all baselines
- Antenna-based quantities are not conserved

# Global fringe-fitting

**Global fringe-fitting** is usually what people mean when they say *fringe-fitting*. Here we use the baseline-based equations:

$$\Delta\phi_{ij} = \phi_{i,0} - \phi_{j,0} + \left( \left[ \frac{\delta\phi_i}{\delta\nu} - \frac{\delta\phi_j}{\delta\nu} \right] \Delta\nu + \left[ \frac{\delta\phi_i}{\delta t} - \frac{\delta\phi_j}{\delta t} \right] \Delta t \right)$$

but now we solve for the entire system of equations *simultaneously* to find antenna-based solutions!

Major advantage:

- Even if an antenna doesn't see the source on every baseline, the antenna-based solution can still be found!

# Fringe-fitting in practice

- **Global fringe-fitting** is implemented in CASA
- Sets one antenna as the reference antenna, and finds the phases, delays, and rates for all other antennas
- The solutions are therefore *relative* to the reference antenna
- **Calibrator source** is needed to find the solutions
  - Assumed to be a bright point source unless model is given
    - A point source has constant phase in the  $u$ - $v$  plane
- Phase corrections only; no amplitudes!

```

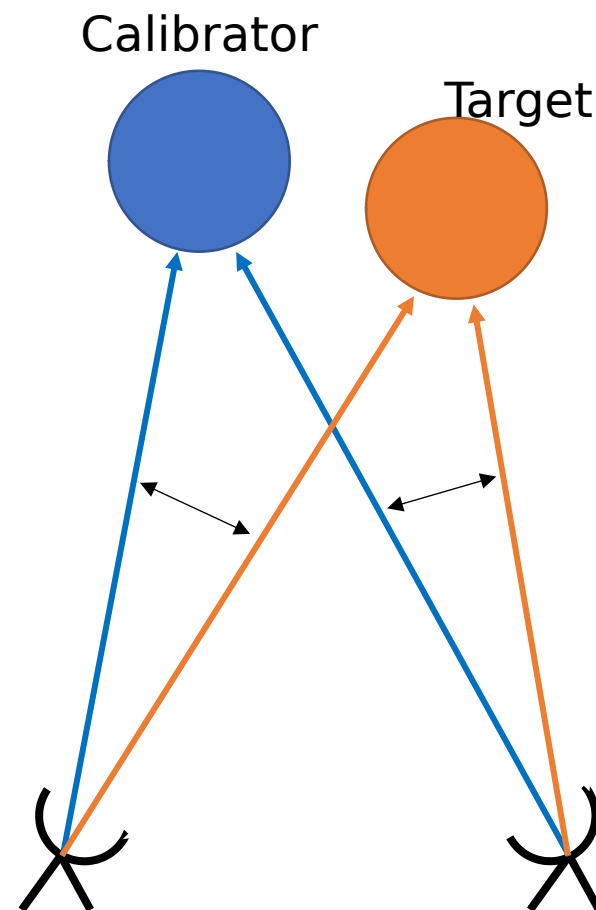
-----> inp(fringefit)
# fringefit :: Fringe fit delay and rates
vis          = ''      # Name of input visibility file
caltable     = ''      # Name of output gain calibration table
field        = ''      # Select field using field id(s) or
                        # field name(s)
spw          = ''      # Select spectral window/channels
intent       = ''      # Select observing intent
selectdata   = True     # Other data selection parameters
timerange    = ''      # Select data based on time range
uvrange      = ''      # Select data based on antenna/baseline
antenna      = ''      # Scan number range
scan         = ''      # Select by observation ID(s)
observation   = ''      # Optional complex data selection
msselect     = ''      # (ignore for now)

solint       = 'inf'    # Solution interval: egs. 'inf', '60s'
                        # (see help)
combine      = ''      # Data axes which to combine for solve
                        # (obs, scan, spw, and/or field)
refant       = ''      # Reference antenna name(s)
minsnr       = 3.0      # Reject solutions below this signal-
                        # to-noise ratio (at the FFT stage)
zerorates    = False    # Zero delay-rates in solution table
globalsolve  = True     # Refine estimates of delay and rate
                        # with global least-squares solver
delaywindow  = []       # Constrain FFT delay search to a
                        # window; a two-element list, units of
                        # nanoseconds
ratewindow   = []       # Constrain FFT rate search to a
                        # window; a two-element list, units of
                        # seconds per second
append       = False    # Append solutions to the (existing)
                        # table
docallib     = False    # Use callib or traditional cal apply
                        # parameters
gaintable    = []       # Gain calibration table(s) to apply on
                        # the fly
gainfield    = []       # Select a subset of calibrators from
                        # gaintable(s)
interp       = []       # Temporal interpolation for each
                        # gaintable (=linear)
spwmap       = []       # Spectral windows combinations to form
                        # for gaintables(s)

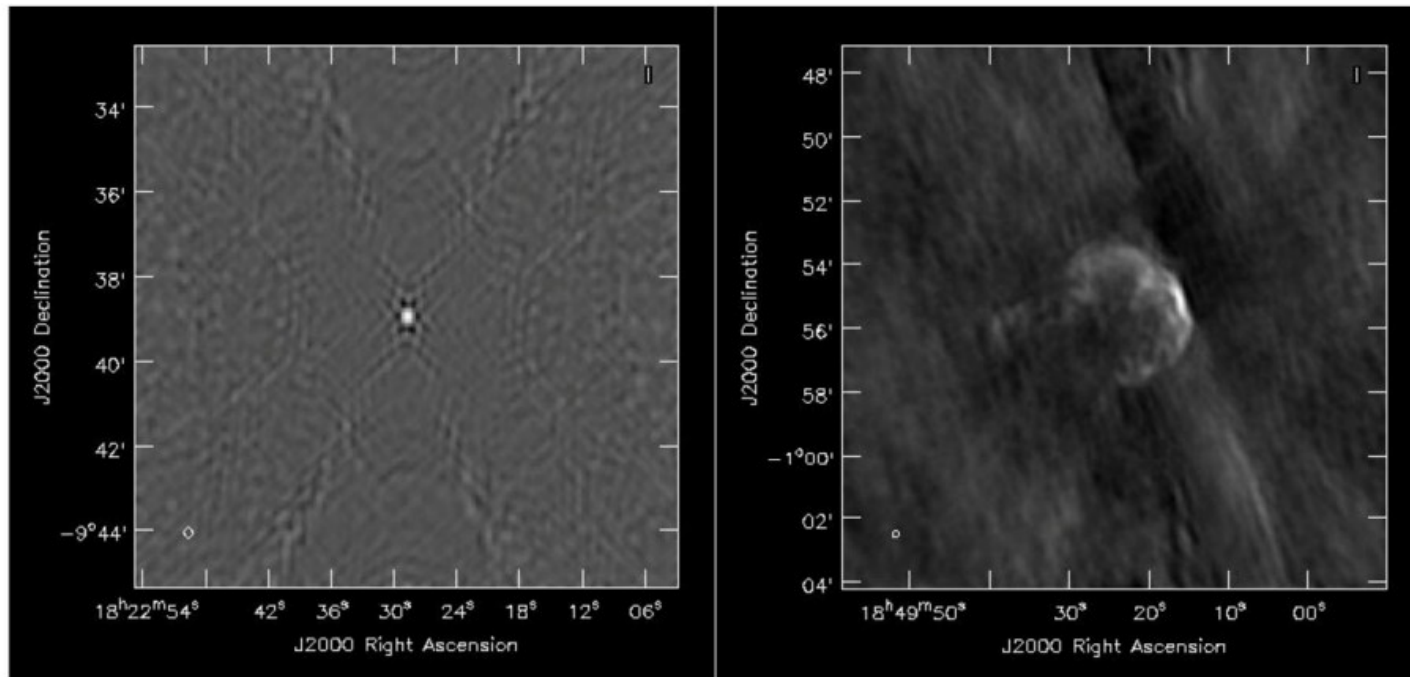
parang       = False    # Apply parallactic angle correction on
                        # the fly
    
```

# Phase referencing

- Use **Global fringe-fitting** to find phases on calibrator source near target
  - Calibrator structure is known (target unknown)
- Interpolate phase solutions to target
- Typically observations will “nod” between the calibrator and target, with a cycle time set by the timescale of atmospheric fluctuations
  - $\sim 10$  min at 5 GHz ;  $\sim 5$  min at 1.6 GHz
- Calibrator and target must be close!
  - Typically within  $\sim 1$  deg
- Biggest problem is the atmosphere
  - Troposphere at high freqs, ionosphere at low



# Phase referencing



J1822-0938  
(calibrator)

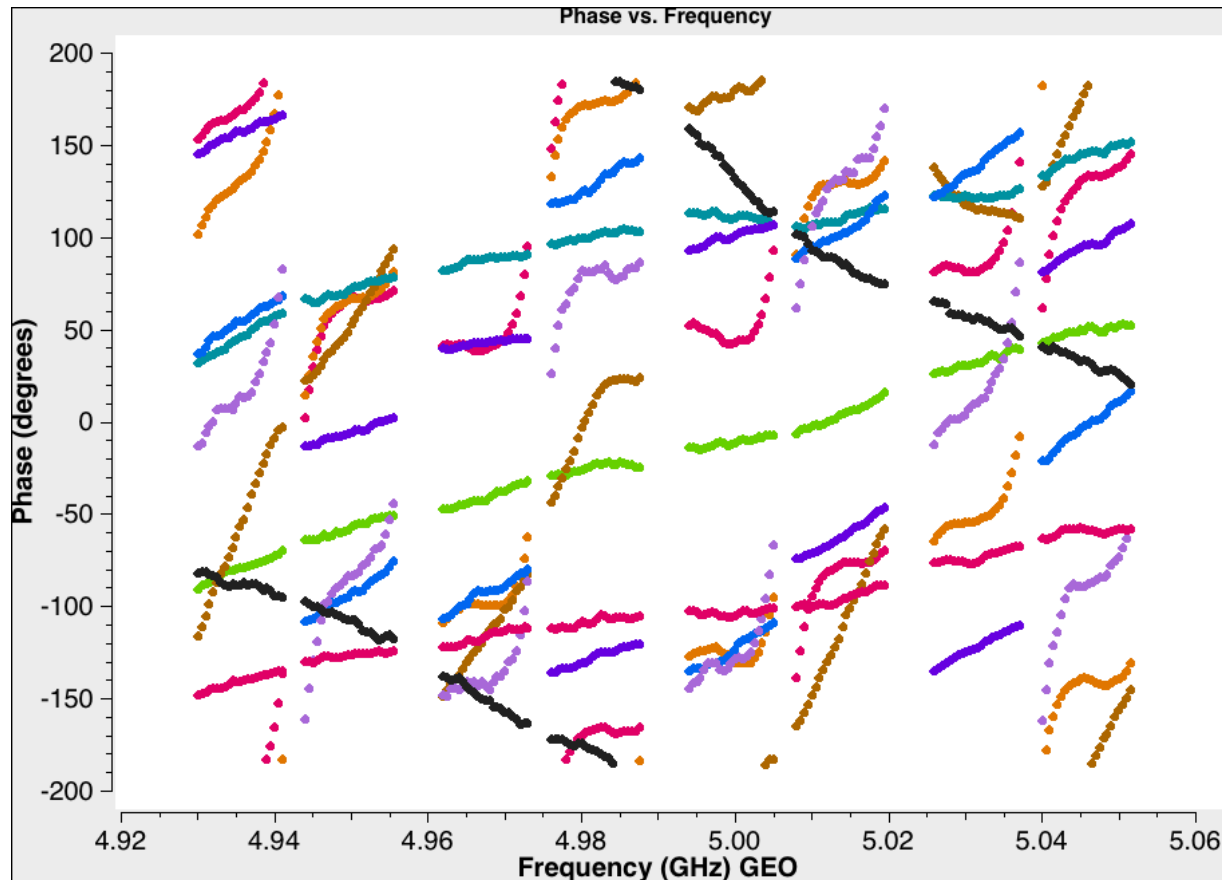
3C391  
(science)

Transfer calibration solutions to target, 3C391

# Fringe-fitting in practice

- **Instrumental delays**

- Differing “instrument” responses across spectral windows
- Time independent



**before**  
instrumental delay  
correction

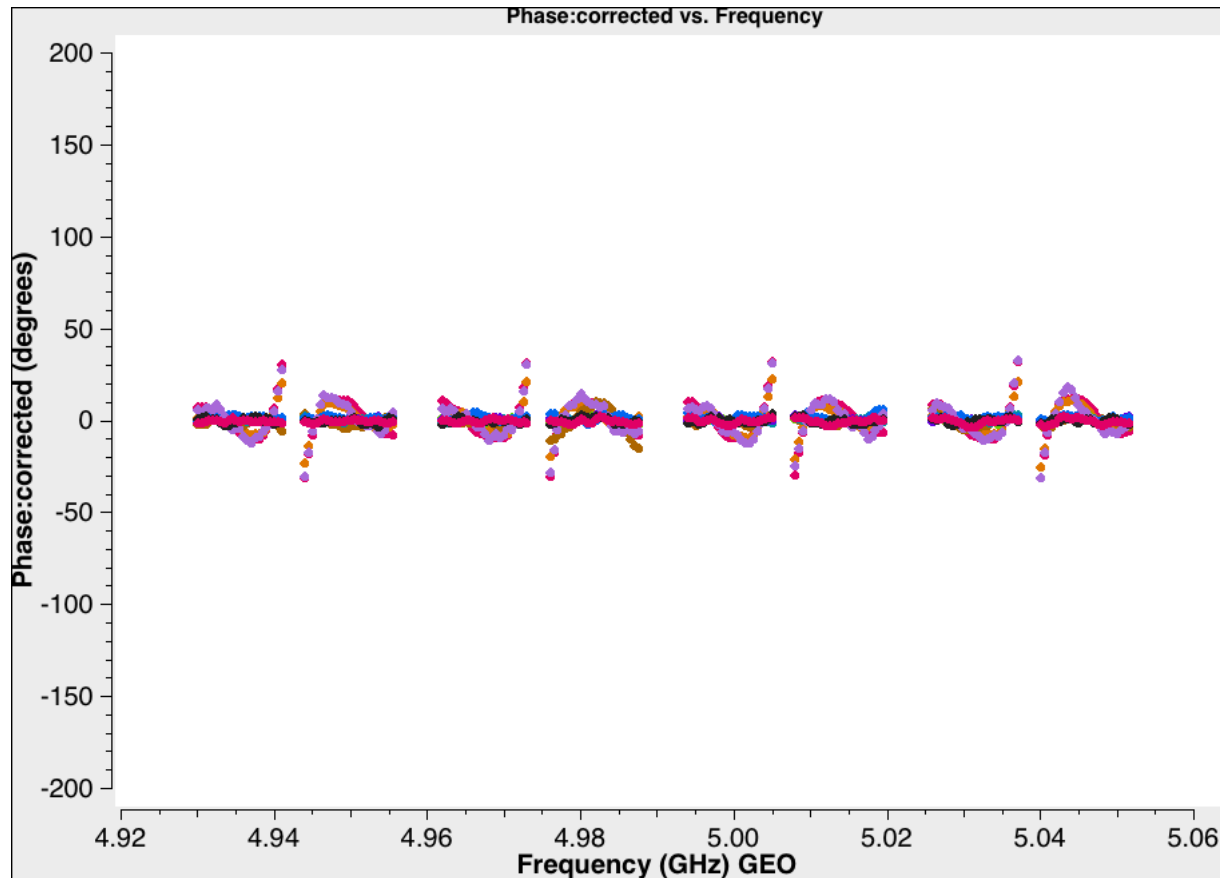
colours are baselines  
to Effelsberg

single short scan

# Fringe-fitting in practice

- **Instrumental delays**

- Differing “instrument” responses across spectral windows
- Time independent



**after**  
instrumental delay  
correction

colours are baselines  
to Effelsberg

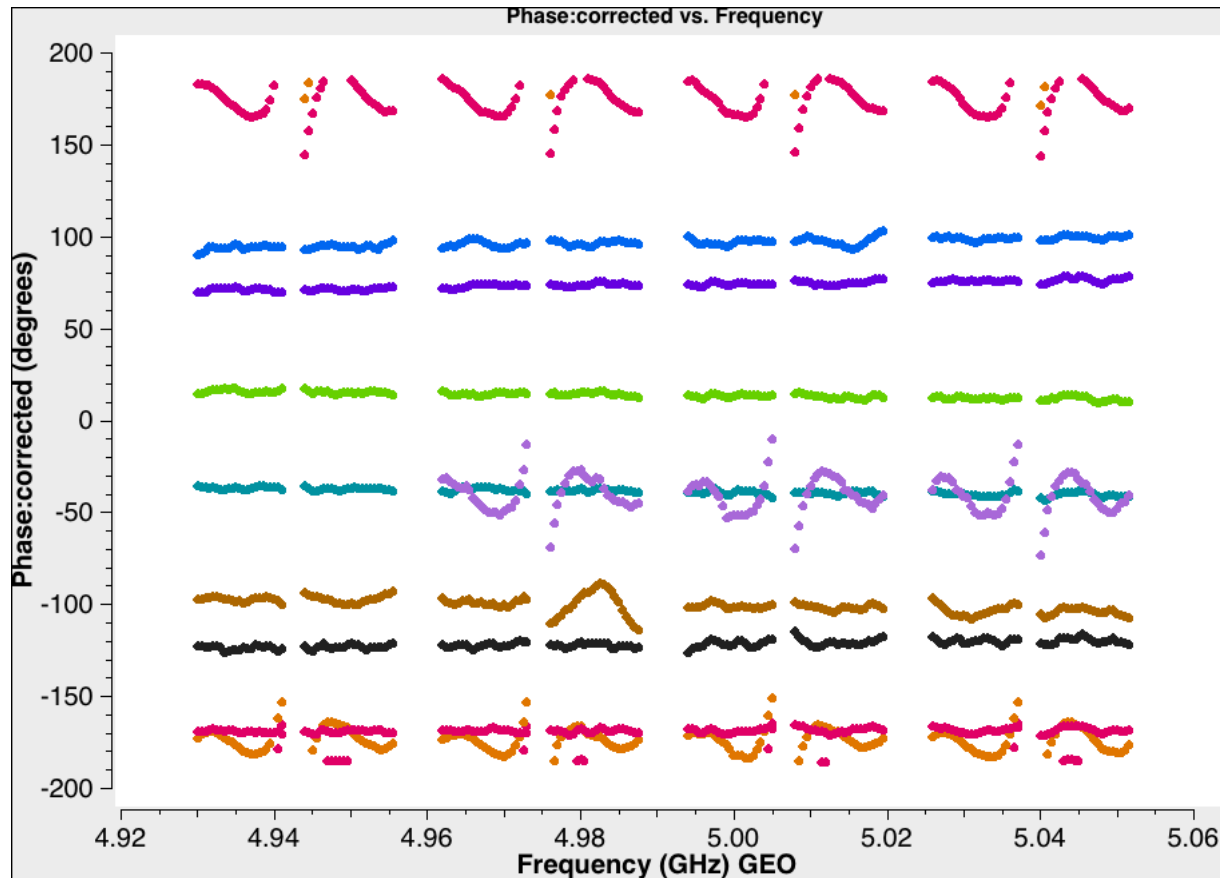
single short scan



# Fringe-fitting in practice

- **Instrumental delays**

- Differing “instrument” responses across spectral windows
- Time independent



**applied to a  
different scan**

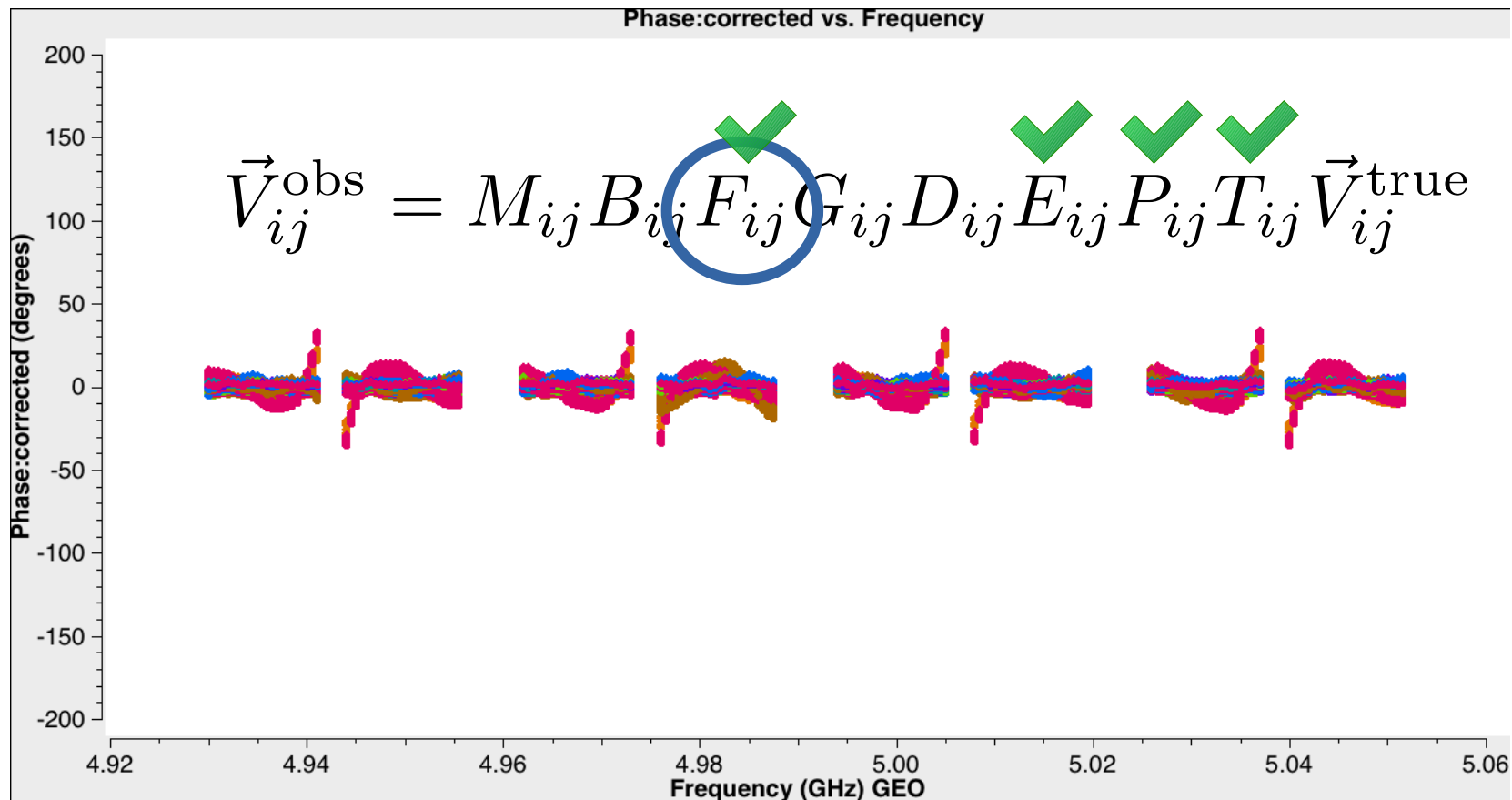
colours are baselines  
to Effelsberg

single short scan

# Fringe-fitting in practice

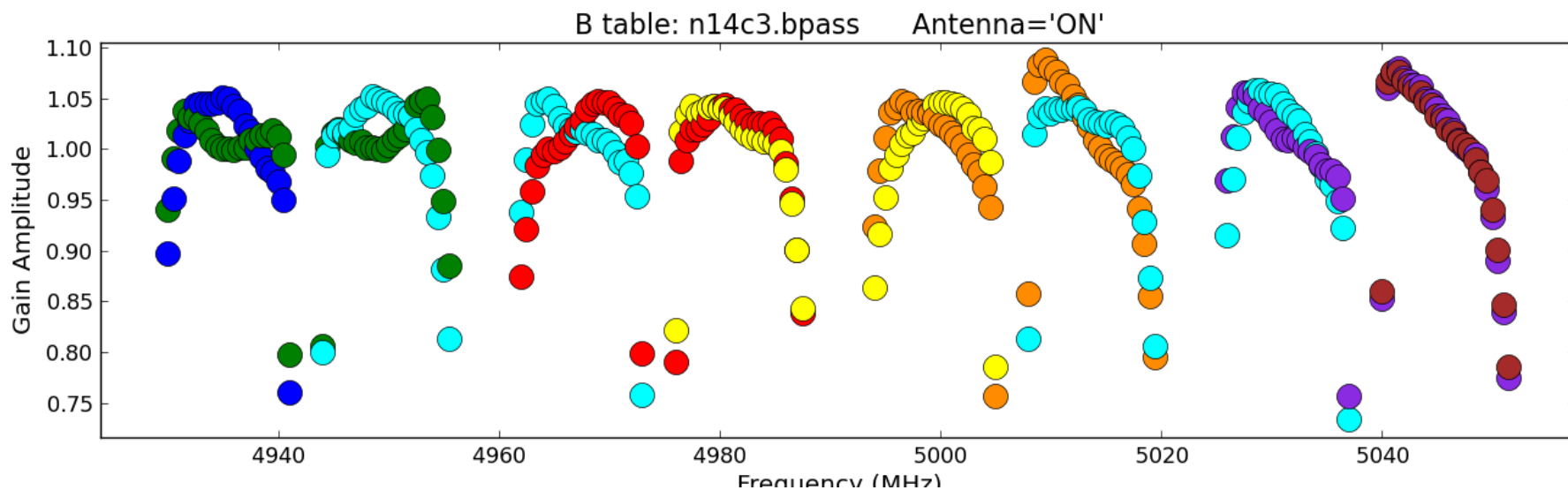
- **Multi-band fringe-fitting**

- Have removed the instrumental effects, now get rid of atmospheric effects



# And now for the amplitudes ...

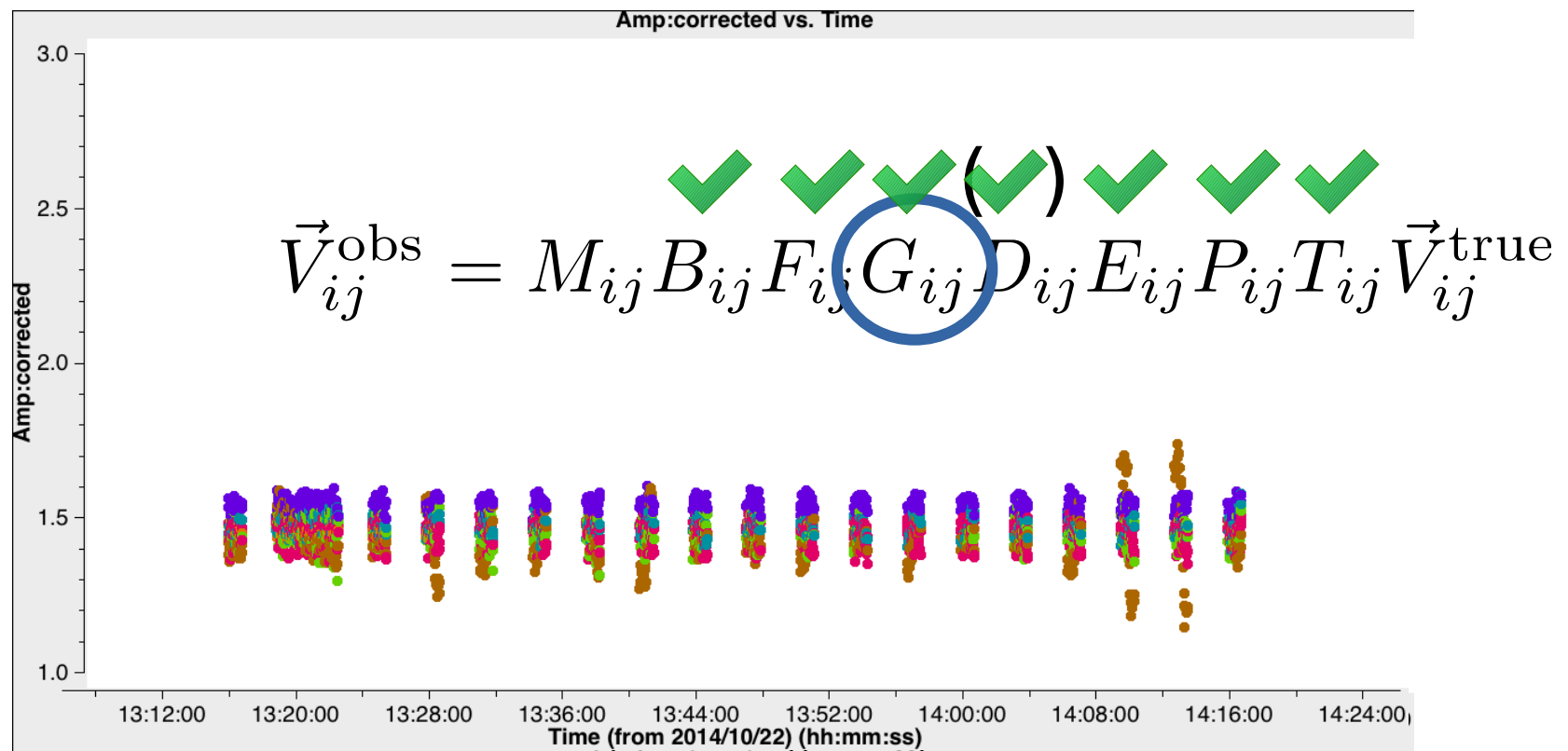
- **Fringe-fitting** only solves for phases, so the amplitudes still need to be corrected.
- **Bandpass calibration** derives amplitude as a function of frequency for each antenna
  - using a well-known, very *bright* calibrator source – need good S/N per channel!



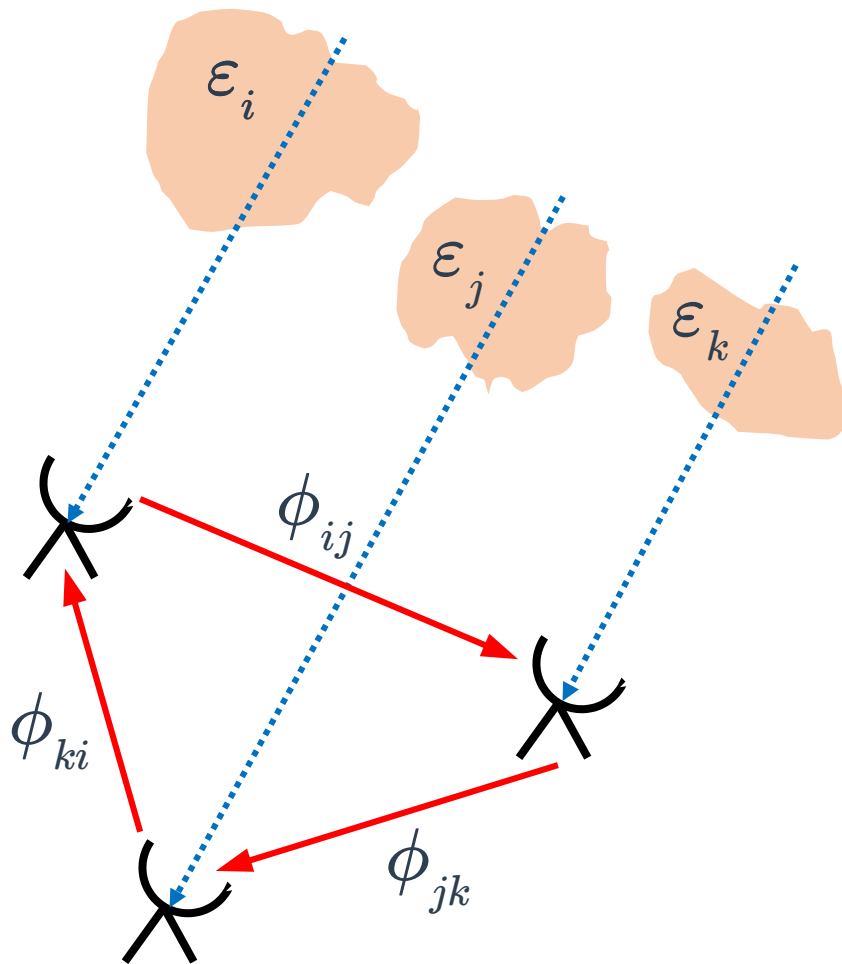
$$\vec{V}_{ij}^{\text{obs}} = M_{ij} \overset{\checkmark}{B_{ij}} \overset{\checkmark}{F_{ij}} \overset{(\checkmark)}{G_{ij}} \overset{\checkmark}{D_{ij}} \overset{\checkmark}{E_{ij}} \overset{\checkmark}{P_{ij}} \overset{\checkmark}{T_{ij}} \vec{V}_{ij}^{\text{true}}$$

# And now for the amplitudes ...

- **Gain calibration** will take care of time-dependent amplitudes
  - due mainly to variable gain in the antenna amplifiers



# Non-closing errors



On each baseline, the observed phase is related to the real phase + antenna errors:

$$\phi_{ij,obs} = \phi_{ij} + \epsilon_i - \epsilon_j$$

$$\phi_{jk,obs} = \phi_{jk} + \epsilon_j - \epsilon_k$$

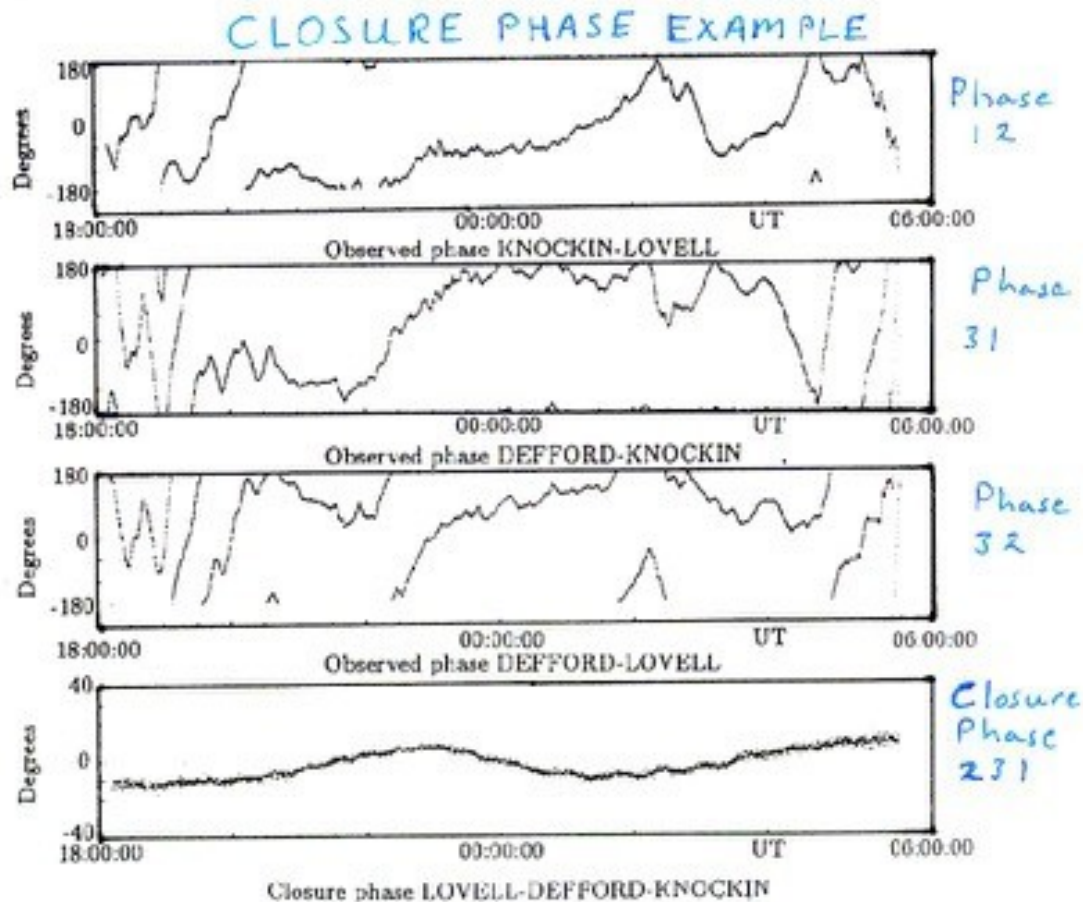
$$\phi_{ki,obs} = \phi_{ki} + \epsilon_k - \epsilon_i$$

Summing these around a triangle of antennas gives us the **closure phase**:

$$\begin{aligned} \phi_{ij,obs} + \phi_{jk,obs} + \phi_{ki,obs} \\ = \phi_{ij} + \phi_{jk} + \phi_{ki} \end{aligned}$$

note that the errors all cancel if you go around the triangle in one direction!

# Non-closing errors



If everything is calibrated perfectly, the closure phases should be flat!

Residuals are *non-closing errors*, which can come from baseline-based errors

Three main sources:

- Source structure not correct in model **(self-calibration)**
- Time averaging
  - Atmospheric conditions can change on very short timescales **(don't average)**
- Frequency averaging
  - Bandpass response varies per telescope as function of frequency **(don't average)**

$$\vec{V}_{ij}^{\text{obs}} = \overset{(\checkmark)}{\underset{\text{blue circle}}{M_{ij}}} \overset{(\checkmark)}{B_{ij}} \overset{(\checkmark)}{F_{ij}} \overset{(\checkmark)}{G_{ij}} \overset{(\checkmark)}{D_{ij}} \overset{(\checkmark)}{E_{ij}} \overset{(\checkmark)}{P_{ij}} \overset{(\checkmark)}{T_{ij}} \vec{V}_{ij}^{\text{true}}$$

# Summary

$$\vec{V}_{ij}^{\text{obs}} = M_{ij} B_{ij} F_{ij} G_{ij} D_{ij} E_{ij} P_{ij} T_{ij} \vec{V}_{ij}^{\text{true}}$$

Diagram illustrating the components of the observed vector  $\vec{V}_{ij}^{\text{obs}}$  relative to the true vector  $\vec{V}_{ij}^{\text{true}}$ . The components are represented by matrices  $M_{ij}, B_{ij}, F_{ij}, G_{ij}, D_{ij}, E_{ij}, P_{ij}, T_{ij}$ . Above the matrices are green checkmarks indicating their status:  $M_{ij}$  has a checkmark in parentheses,  $B_{ij}$  has a checkmark,  $F_{ij}$  has a checkmark,  $G_{ij}$  has a checkmark,  $D_{ij}$  has a checkmark in parentheses,  $E_{ij}$  has a checkmark,  $P_{ij}$  has a checkmark, and  $T_{ij}$  has a checkmark. Below the matrices, blue lines connect them to descriptive text:

- $M_{ij}$  is connected to "non-closing errors" (dotted line).
- $B_{ij}$  is connected to "bandpass" (solid line).
- $F_{ij}$  is connected to "fringe-fitting" (solid line).
- $G_{ij}$  is connected to "gains" (solid line).
- $D_{ij}$  is connected to "polarisation (use polarised calibrator)" (dotted line).

- **Recap of Very Long Baseline Interferometry (VLBI)**
- **Fringe-fitting fundamentals**
- **Fringe-fitting in practice**