MANCHESTER 1824

The University of Manchester

Fun with Fourier Transforms

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Outline

- What is a Fourier Transform?
- 2D FT
- Fourier Transforming some household objects
- FTs in interferometry

What is a Fourier Transform?

- A Fourier transform is a mathematical transform which converts a waveform (a function of time or space) to represent it as a function of temporal or spatial frequencies.
- A transformed waveform is returned as a complex sum of sinusoids of differing amplitude and phase.

$$F(v) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i v x} \, dx$$

$$f(x) = \int_{-\infty}^{\infty} F(v) e^{2\pi i v x} \, dx$$

A 1D Fourier Transform of the function *f*(x) and the inverse

The 2D Fourier Transform

- In interferometry, we are considering the spatial distribution of emission in the 2D plane of the sky*, so it is important to consider the 2D Fourier transform. For a 2D function $f(x,y)$ we have:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-2\pi i(ux+vy)}dxdy$$

- If $f(x,y)$ is a function representing the sky brightness then u and v are spatial frequencies.
- $F(u,v)$ is a complex function. The Real and Imaginary parts can be related to amplitudes and phases.

*Under some underlying assumptions and at a give frequency and bandwidth etc... don't worry about this now!

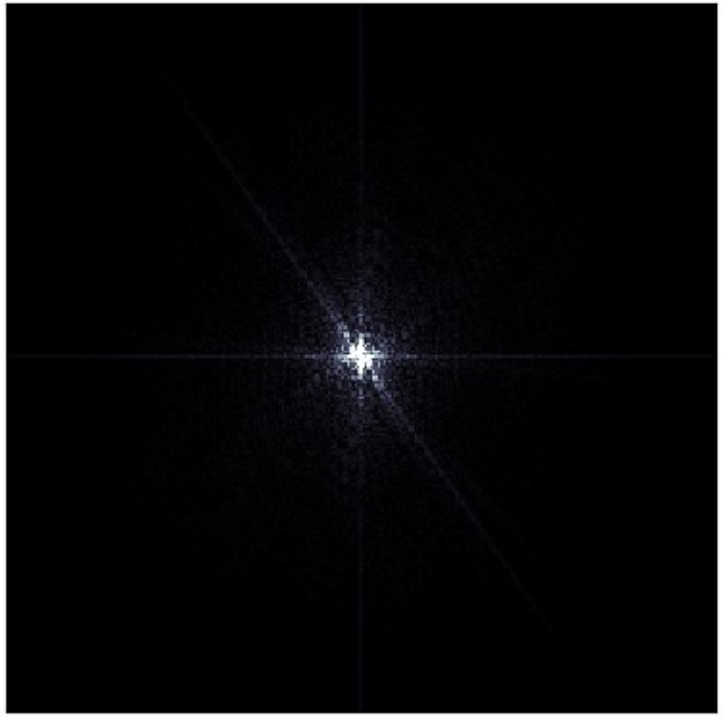
Putting the 2D Fourier transform into practice on everyday household objects...

(well my household).

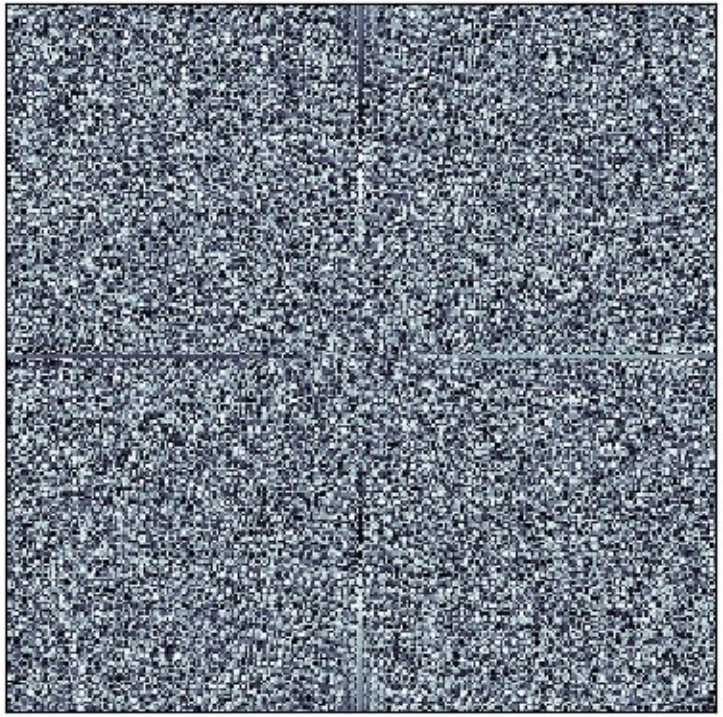
sky

FT

amp



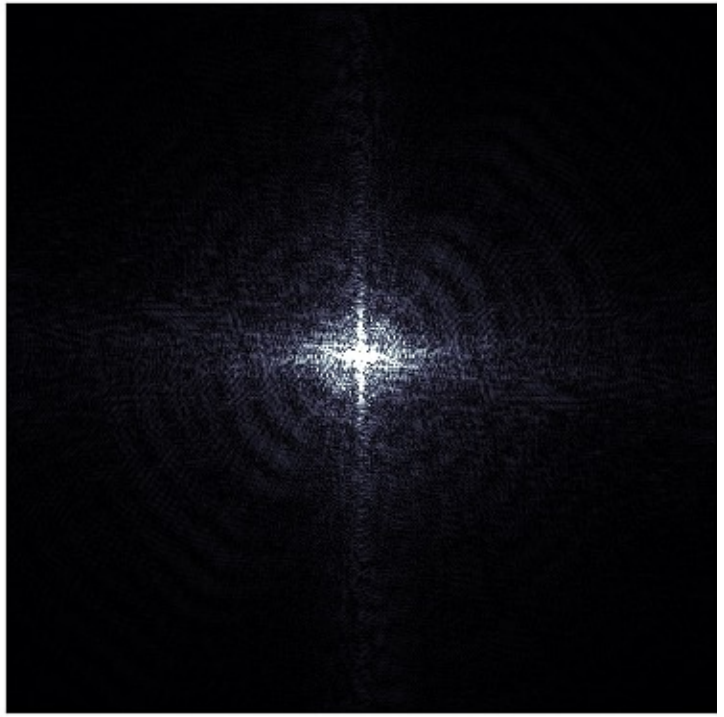
phase



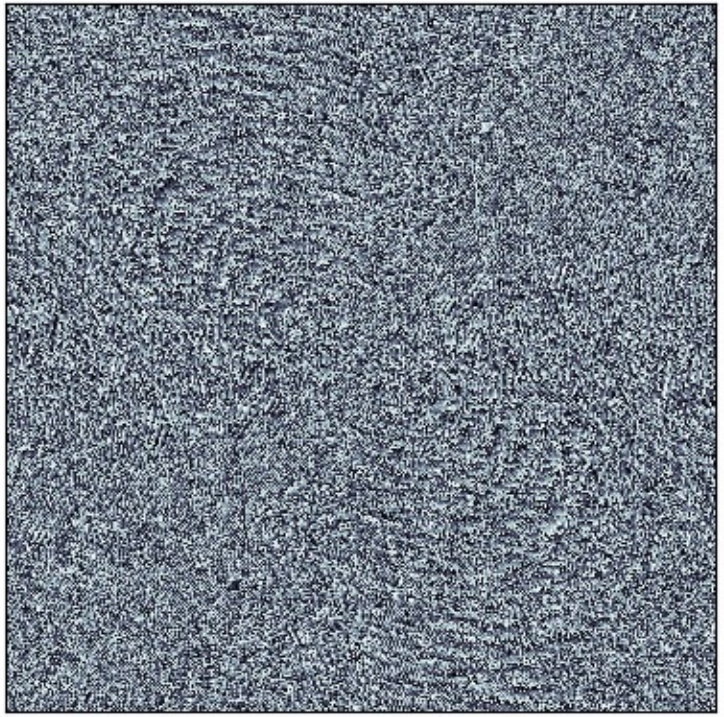
sky

FT

amp

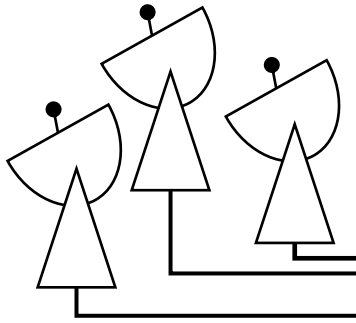


phase



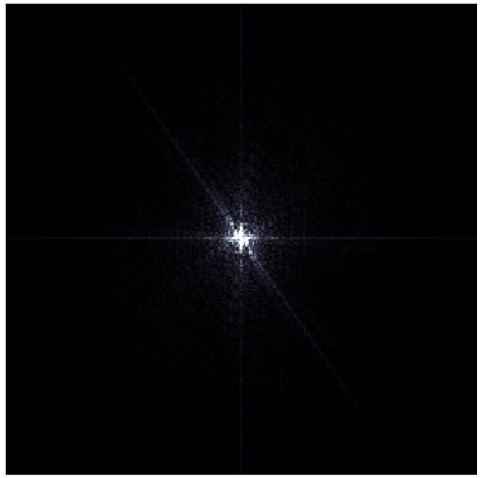
sky

What is our interferometer doing?

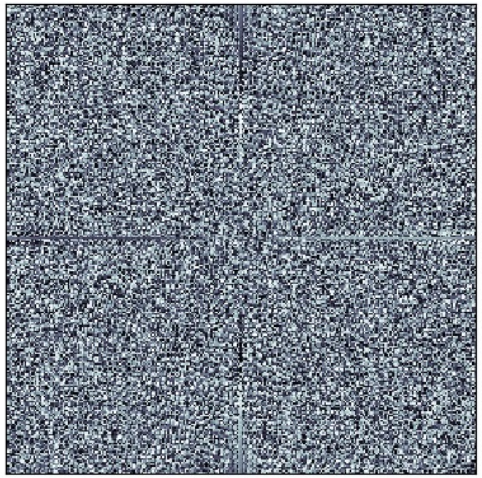


CORRELATOR

amp



phase



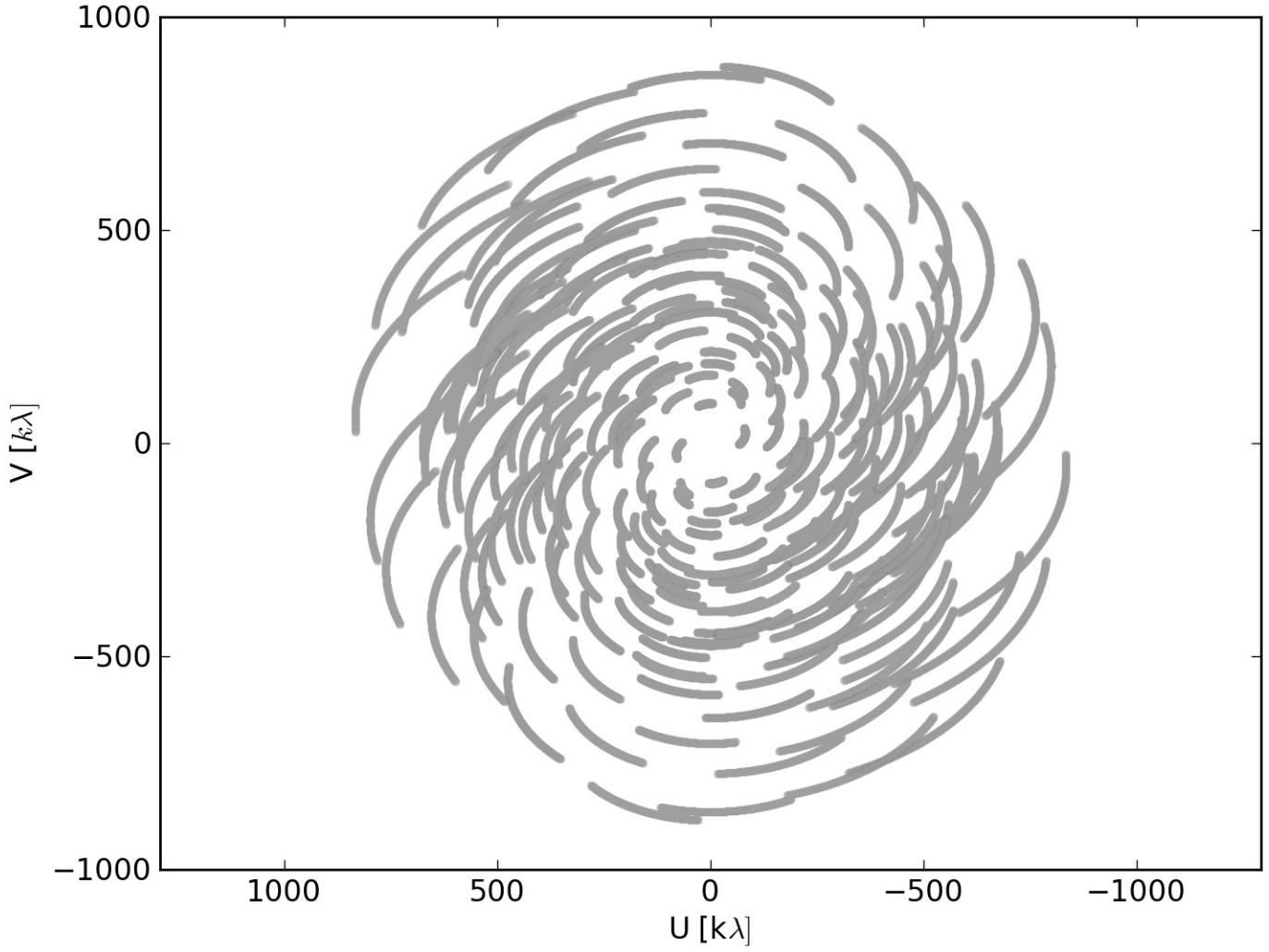
SOFTWARE

?

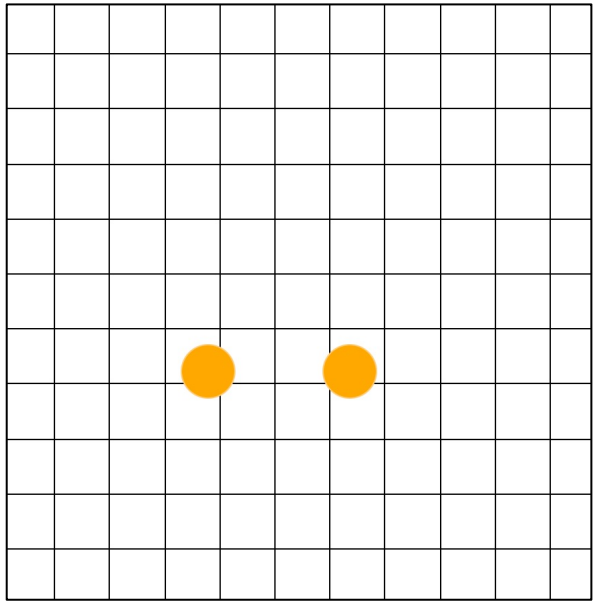
As we observe with an interferometer, the signals detected are processed by a correlator system.

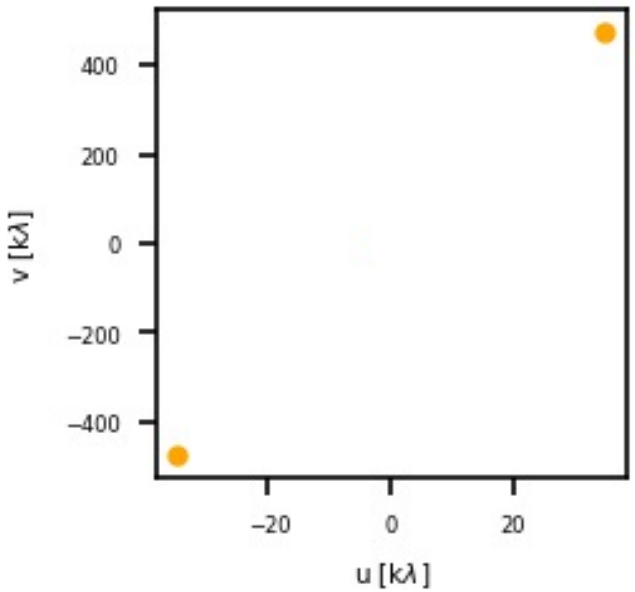
The correlator outputs gives us data with the recovered Amplitudes and Phases of the object we are observing.

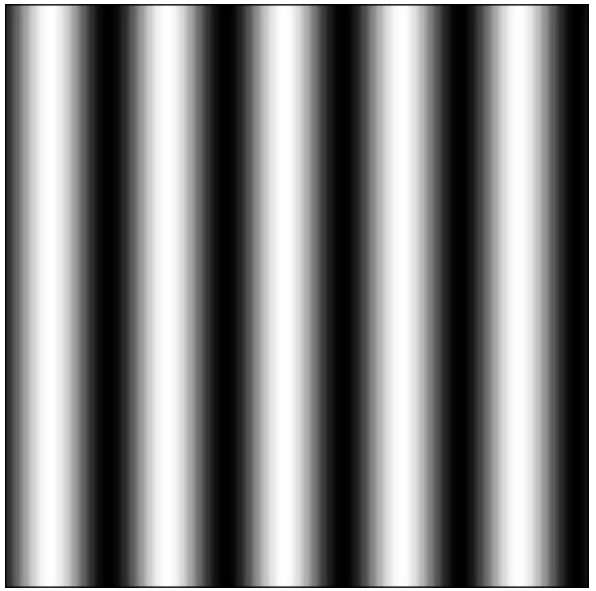
However, as we have a finite number of antennas our recovery (or sampling) of the Fourier *u,v* points is incomplete?



So, what is the effect of incomplete sampling of *u,v* points?

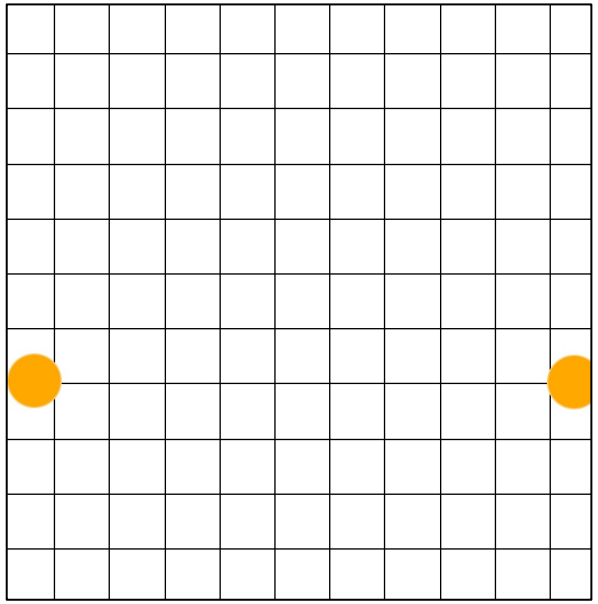


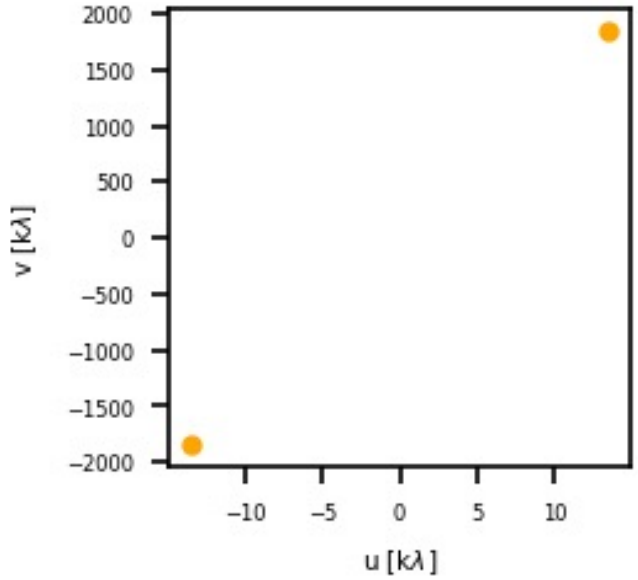


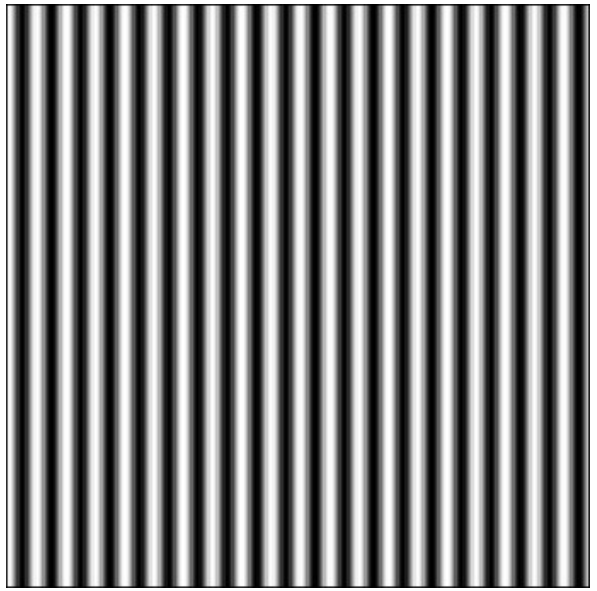


Our array has 2 dishes, about 4km apart.

The resulting image is a sinusoidal pattern. For each pair of antennas we will get one sinusoid.



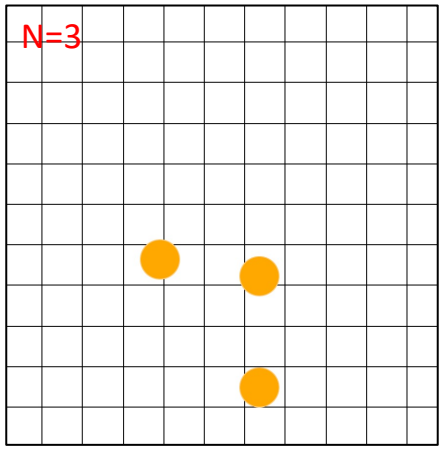


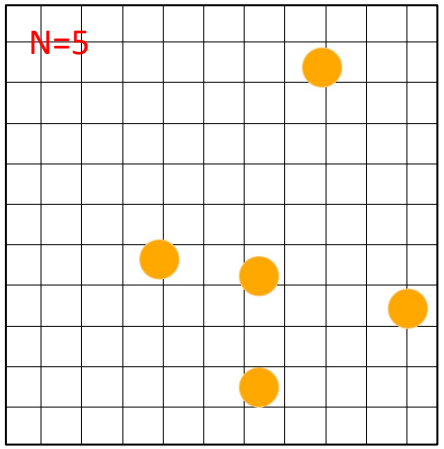


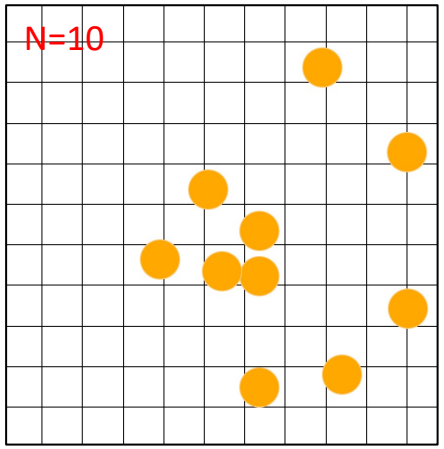
Now our array has 2 dishes, ~18km apart.

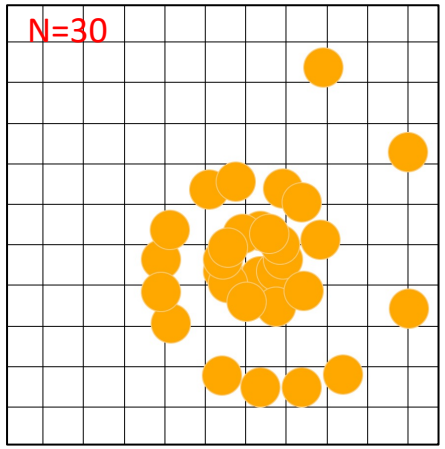
The sinusoid has become much higher frequency.
Remember, for each pair of antennas we will get one sinusoid.

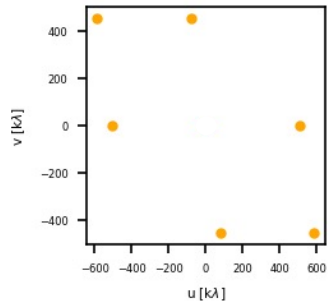
Increasing the number of dishes

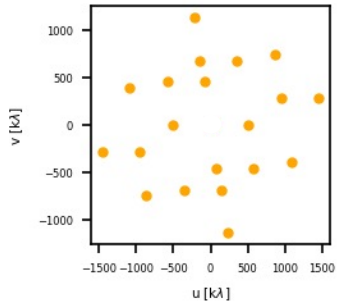


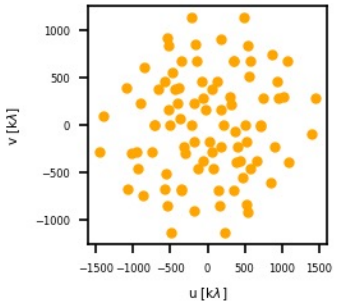


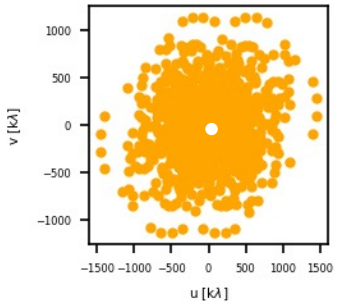


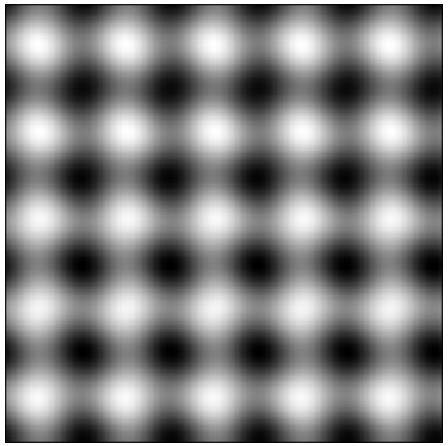


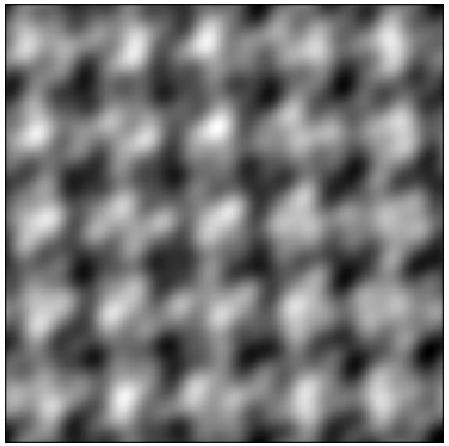


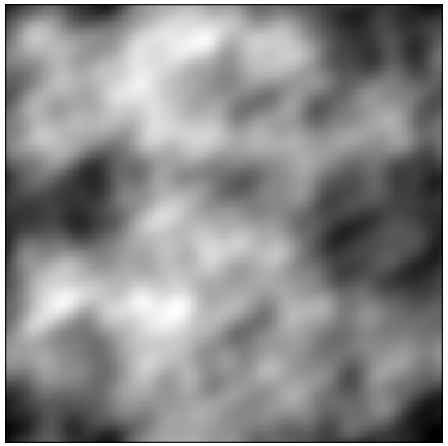


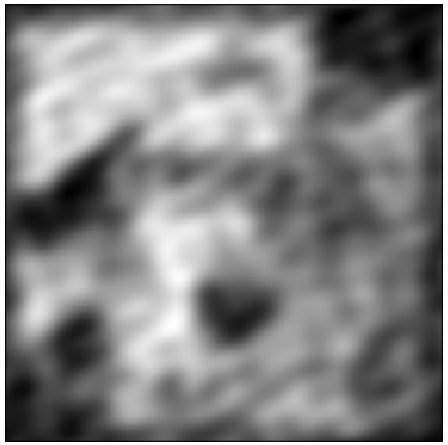












The Earth’s Rotation is your Friend!

Increasing the number of dishes *and* observing over 12 hours