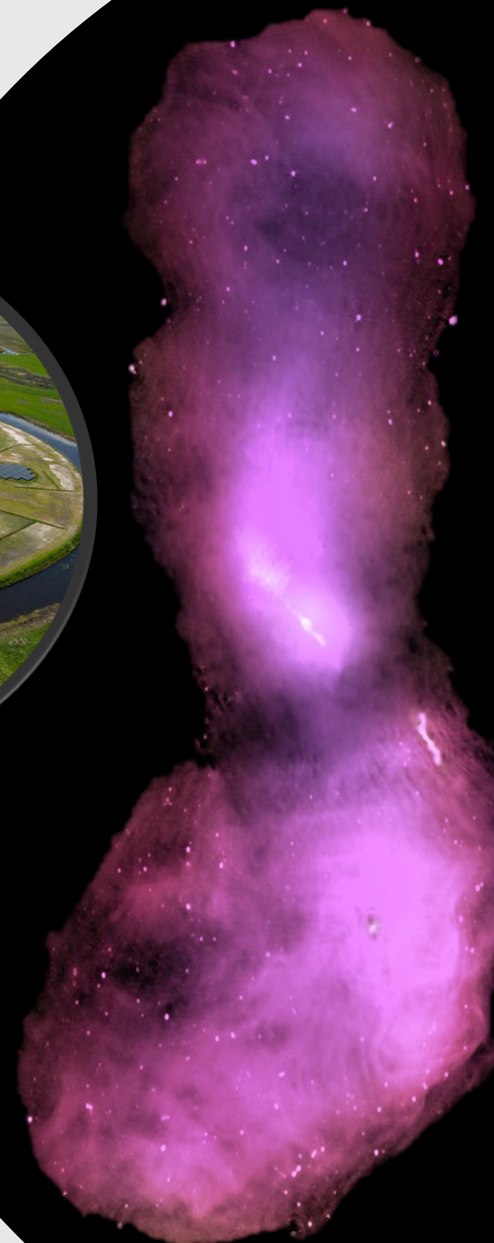
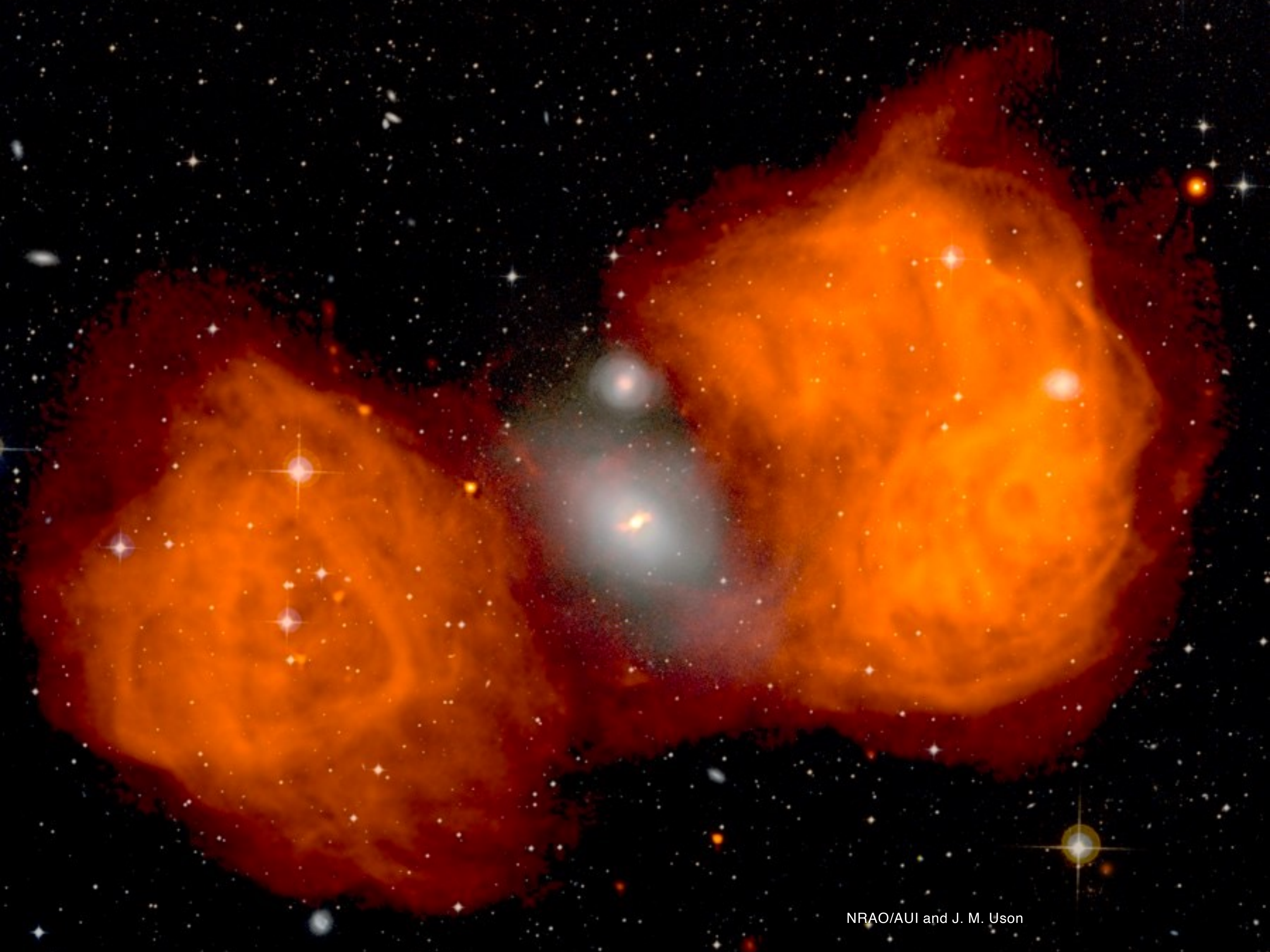
Introduction to (recap of?) Radio Interferometry

Joe Callingham
Veni Fellow
jcal@strw.leidenuniv.nl

DARA Unit 4 Training,
Online
2021

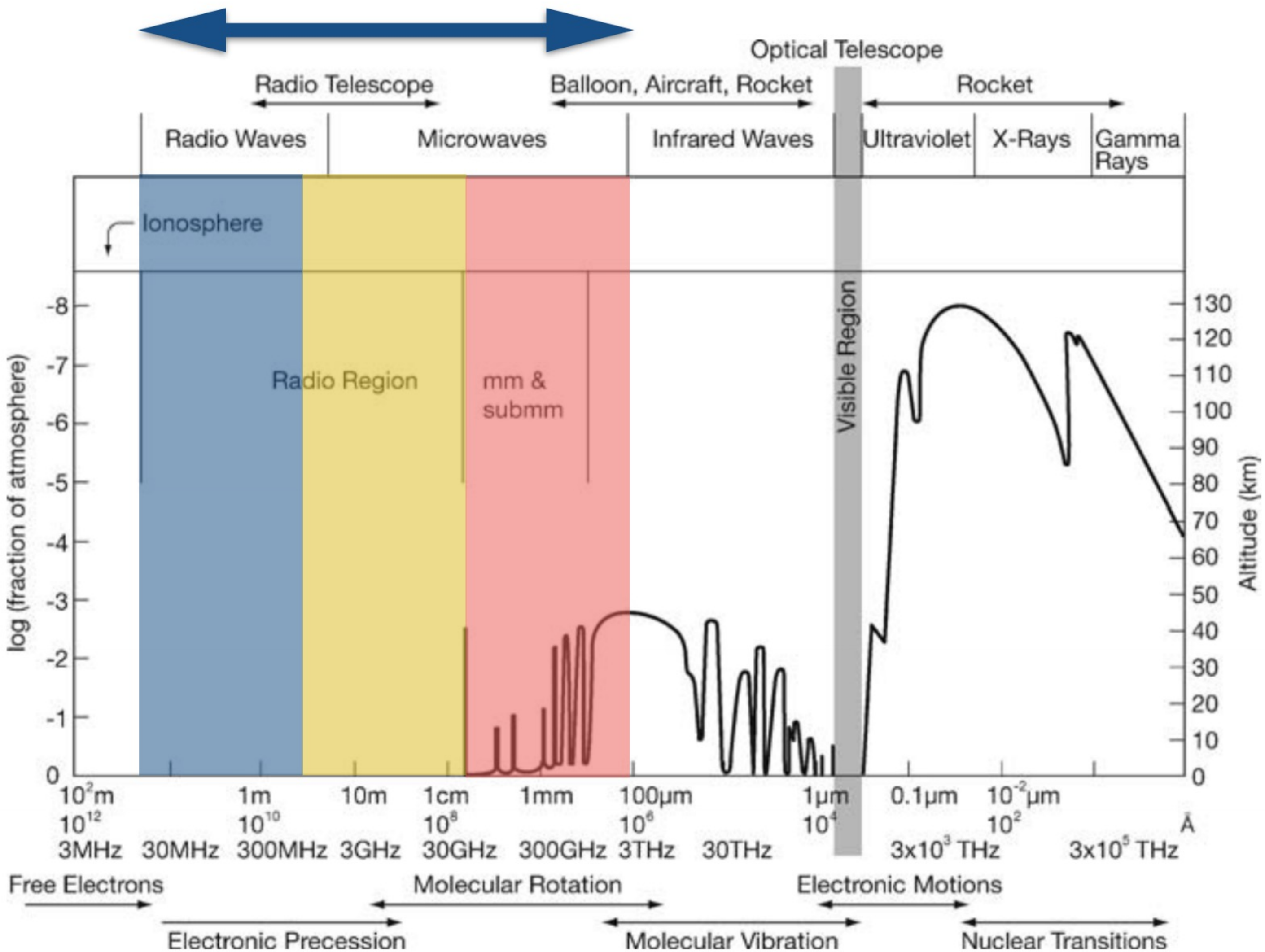




NRAO/AUI and J. M. Uson

The Radio Sky

› Radio astronomy mostly focuses in the frequency range of ~10 MHz to ~1 THz



Why does size matter?
Resolution and sensitivity

"Just checking."

Approx. 70 metre aperture
- 1.5 arcmin resolution at 3 cm

Approx. 5 mm aperture
- 3 arcsec resolution at 600 nm

$$\theta \approx \lambda / D$$

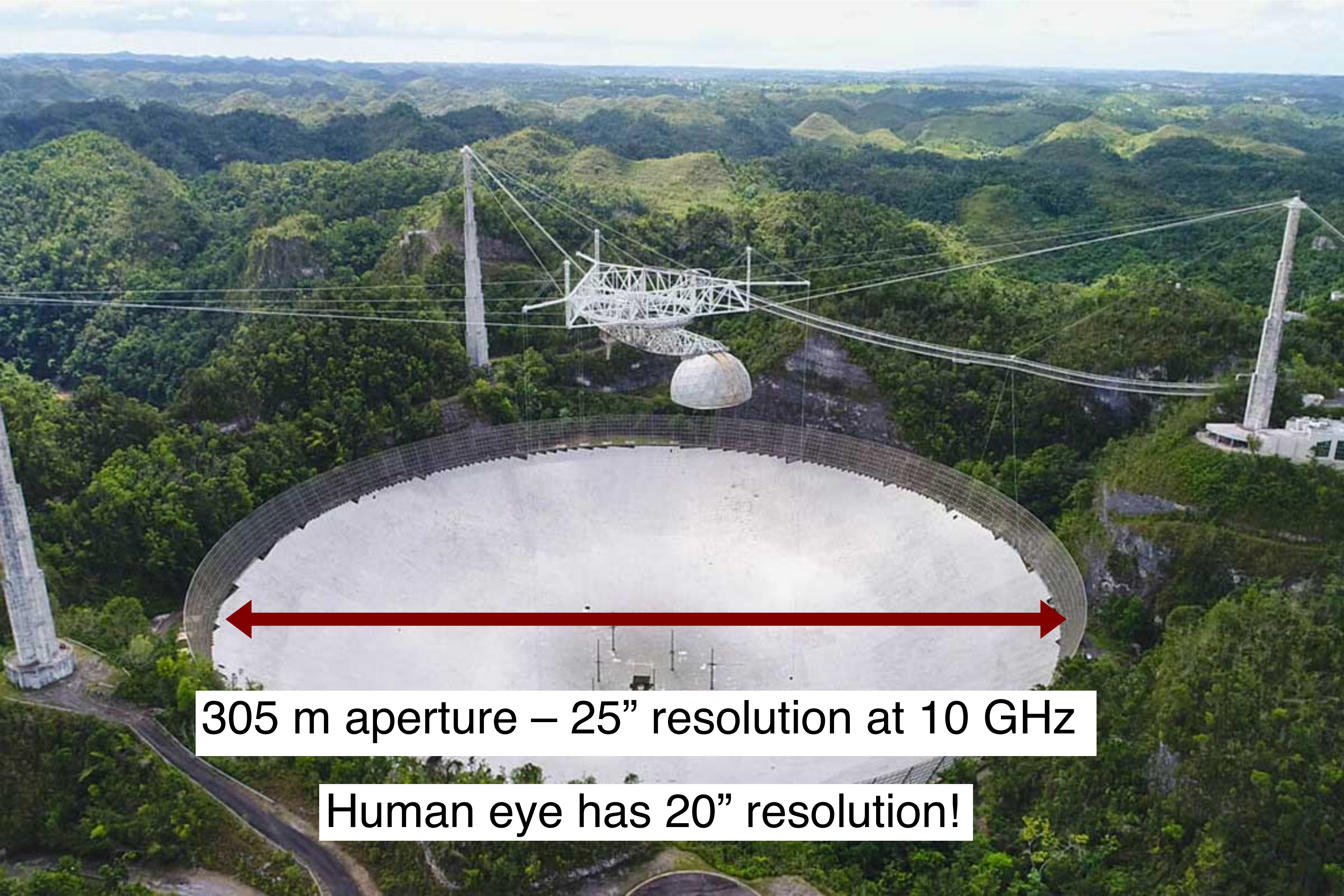
Resolution (radians)

Observing wavelength (m)

Diameter of telescope (m)

$$\mathrm{SEFD} = \frac{2kT_{\mathrm{sys}}}{A_e}$$

When big is not big enough



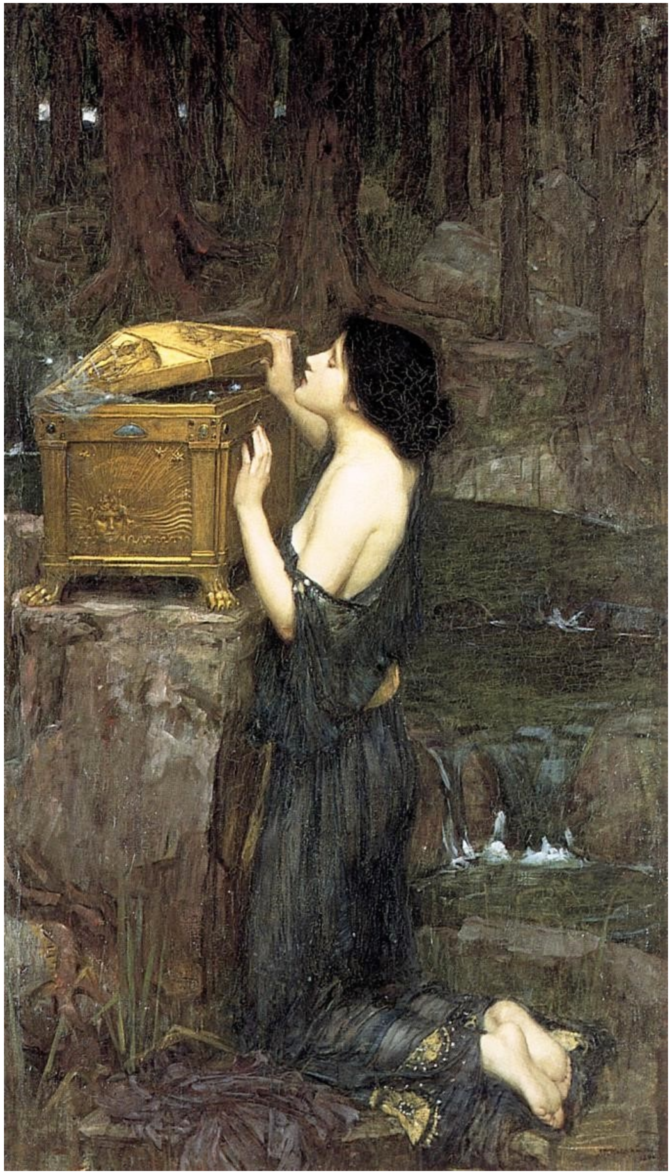
Bigger is better?

Bigger is better?

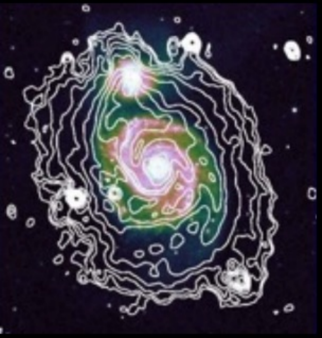
Interferometry to the rescue

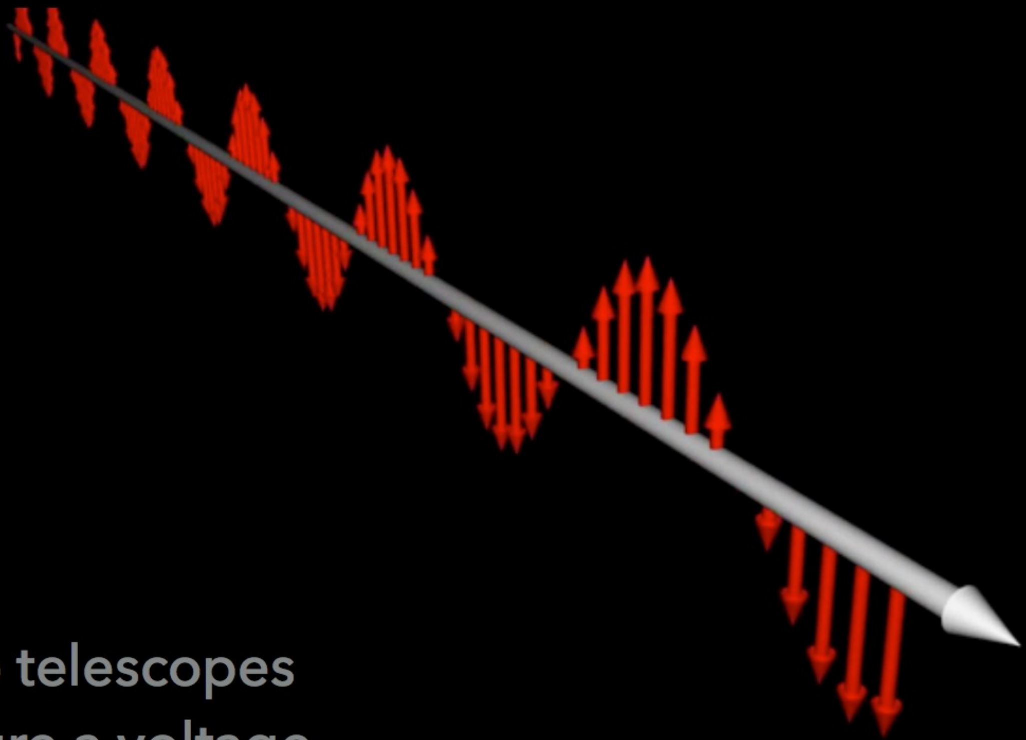
$$\theta \approx \lambda / D$$

Pandora's Box

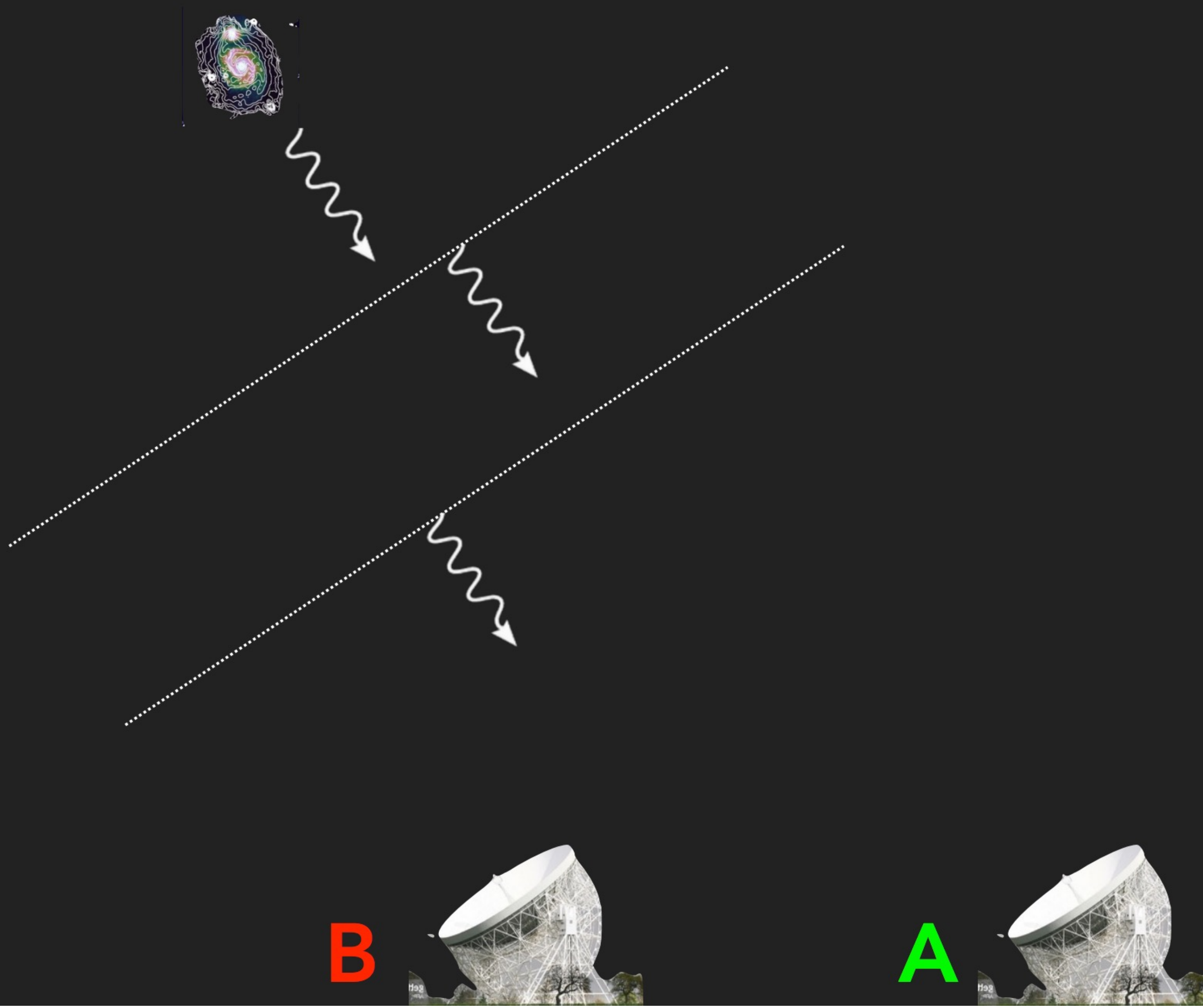


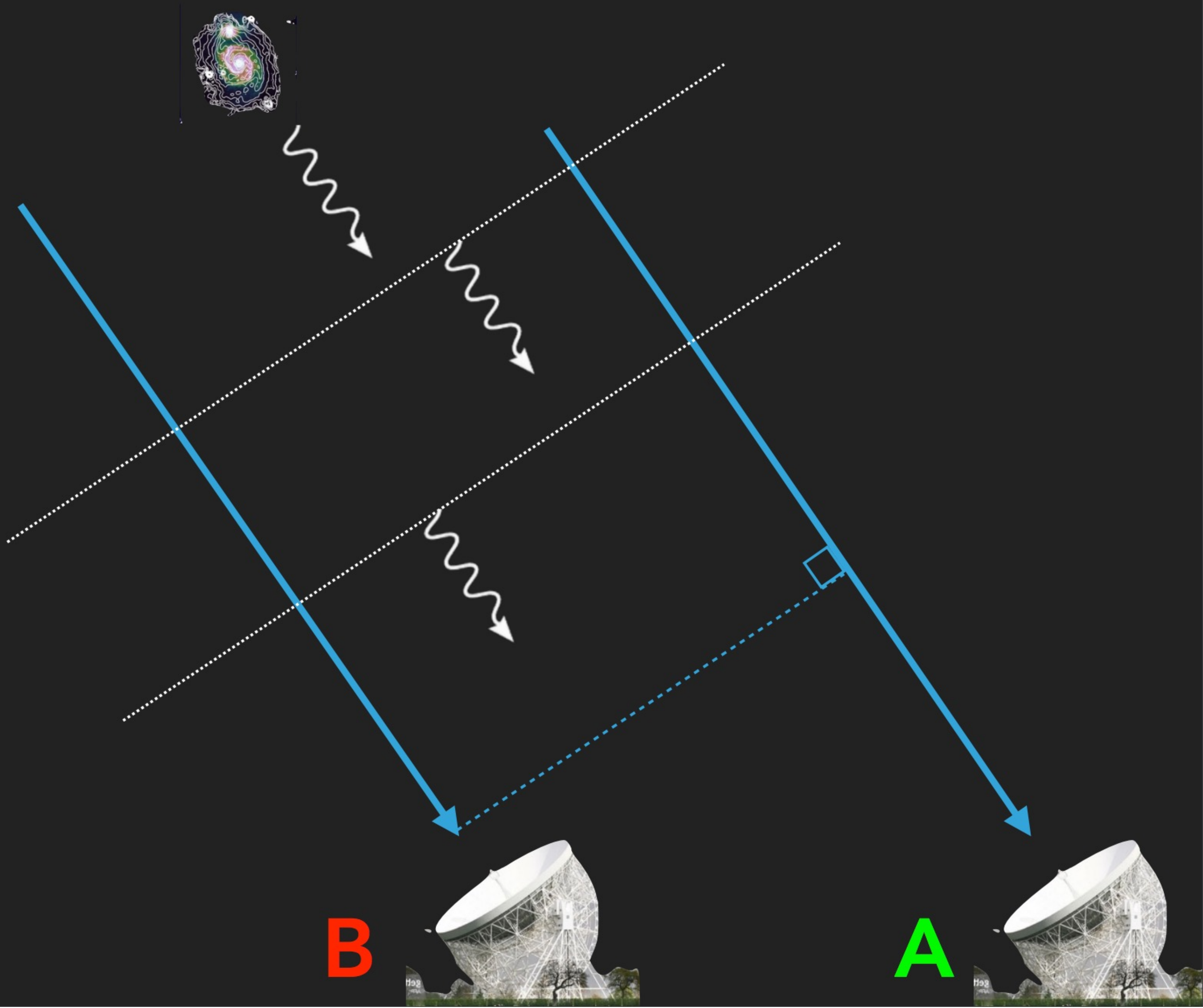
- Calibration is harder
- How do you reconstruct the image?
- What information are you missing?
- Loss of sensitivity





Radio telescopes measure a voltage due to the incident EM radiation


B
A


B
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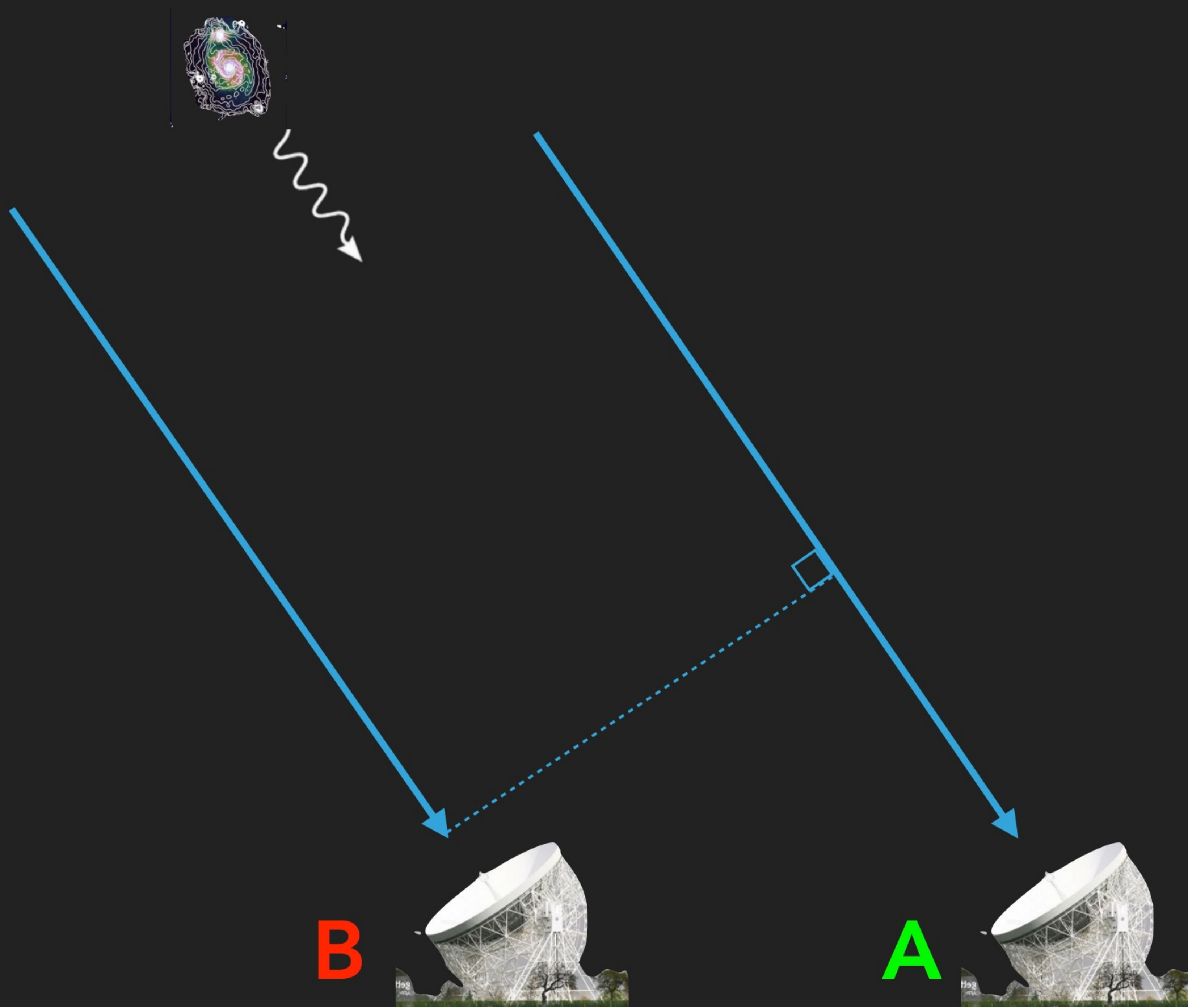
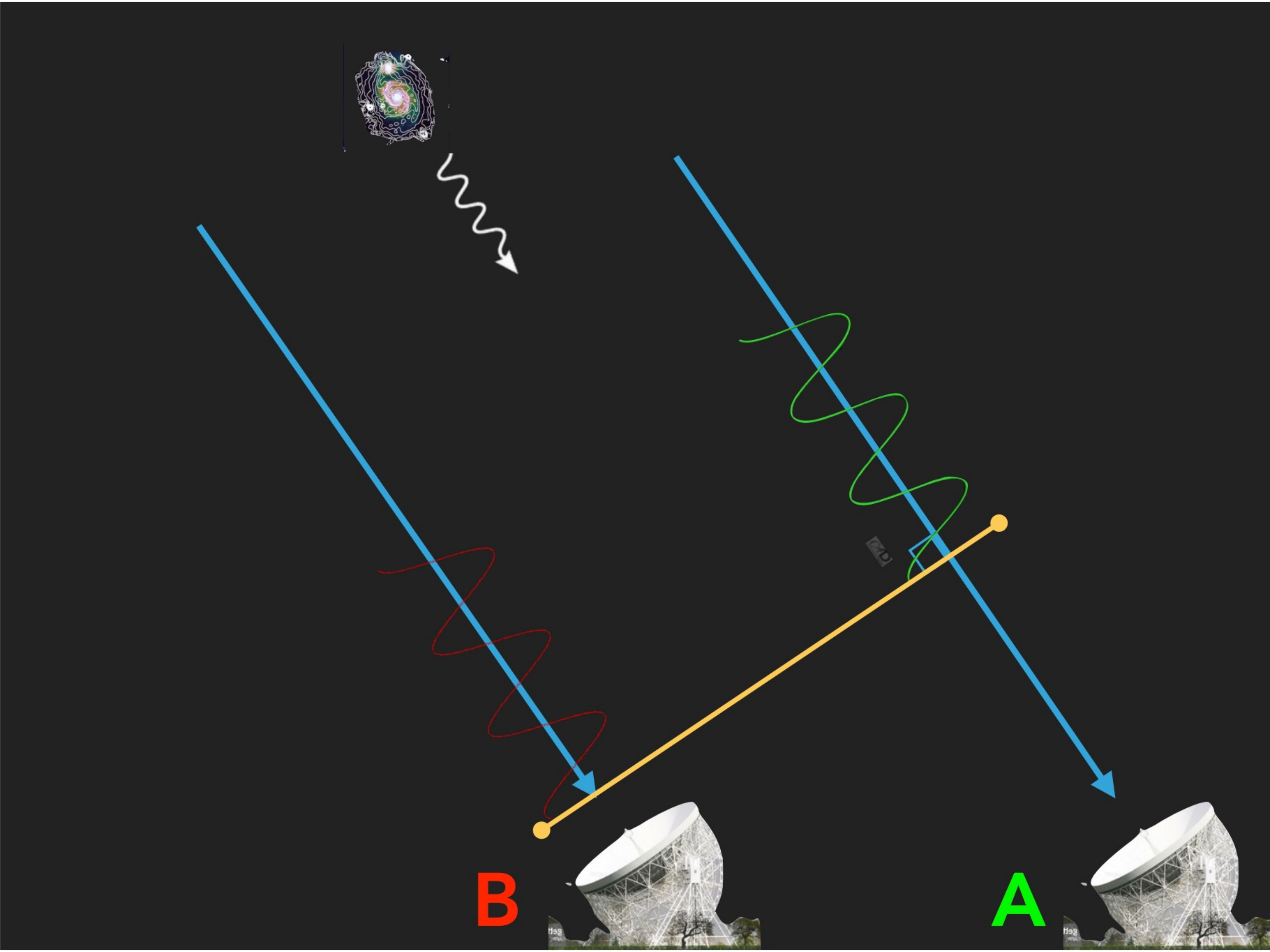
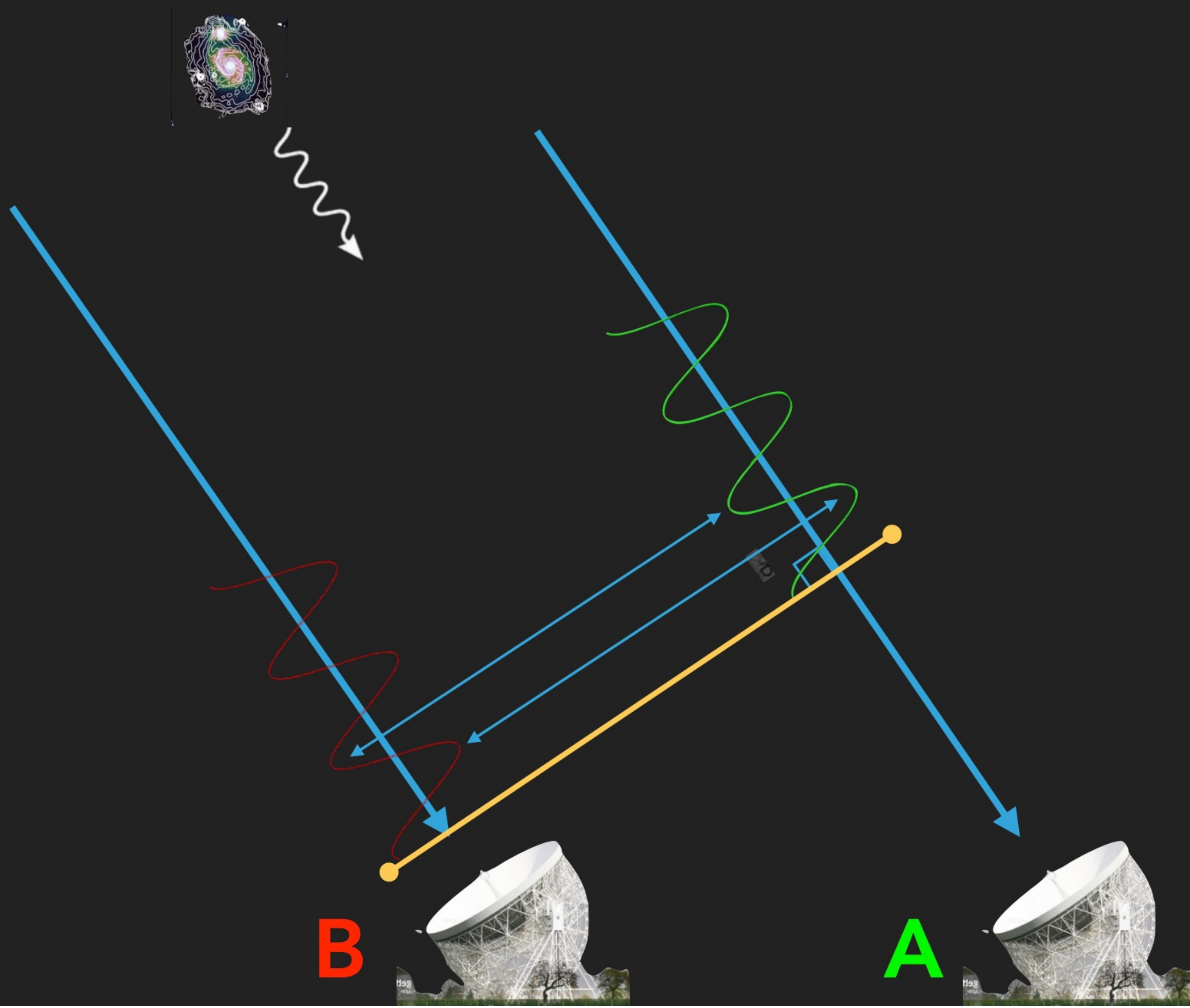

B
A


B
A


B
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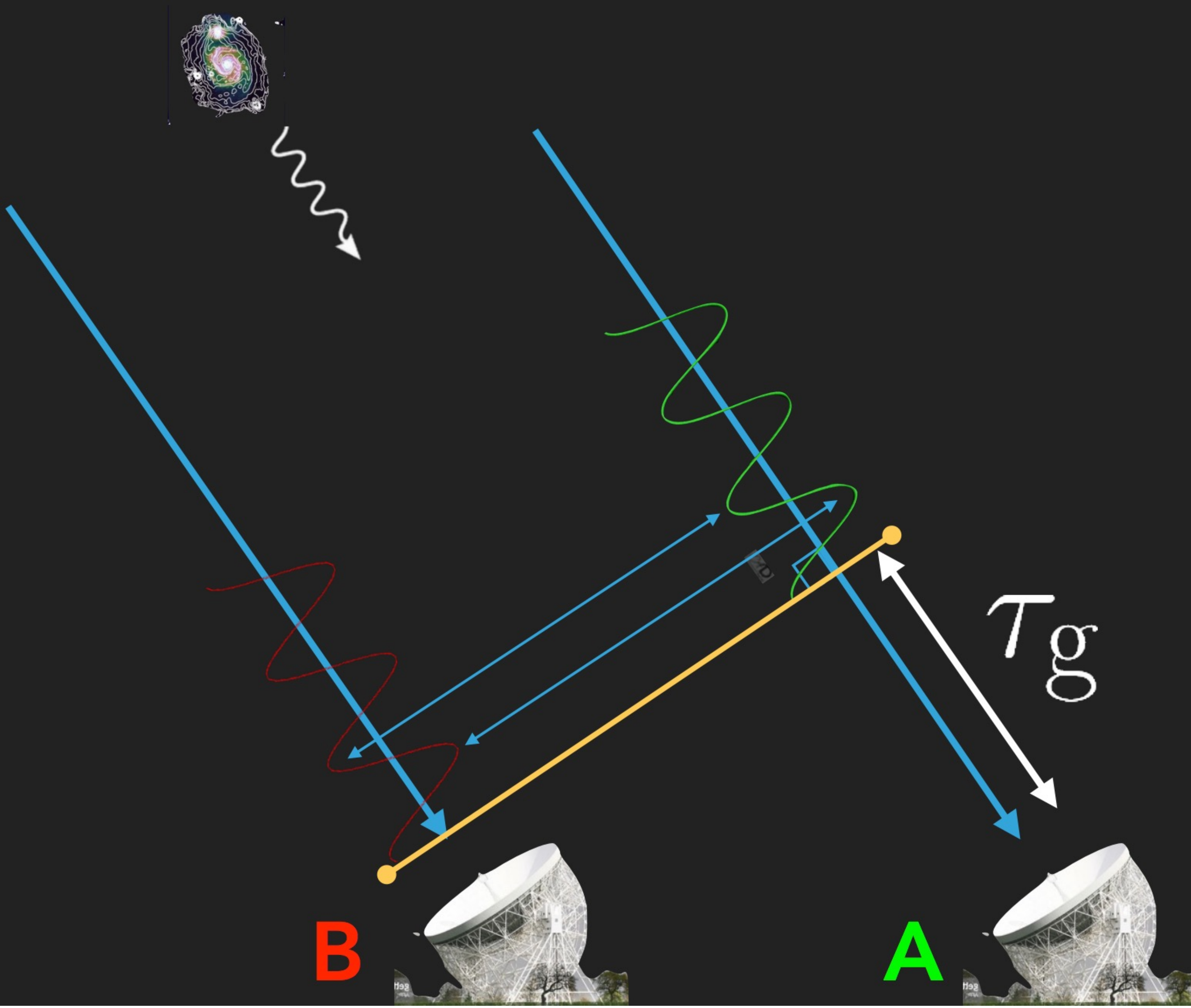
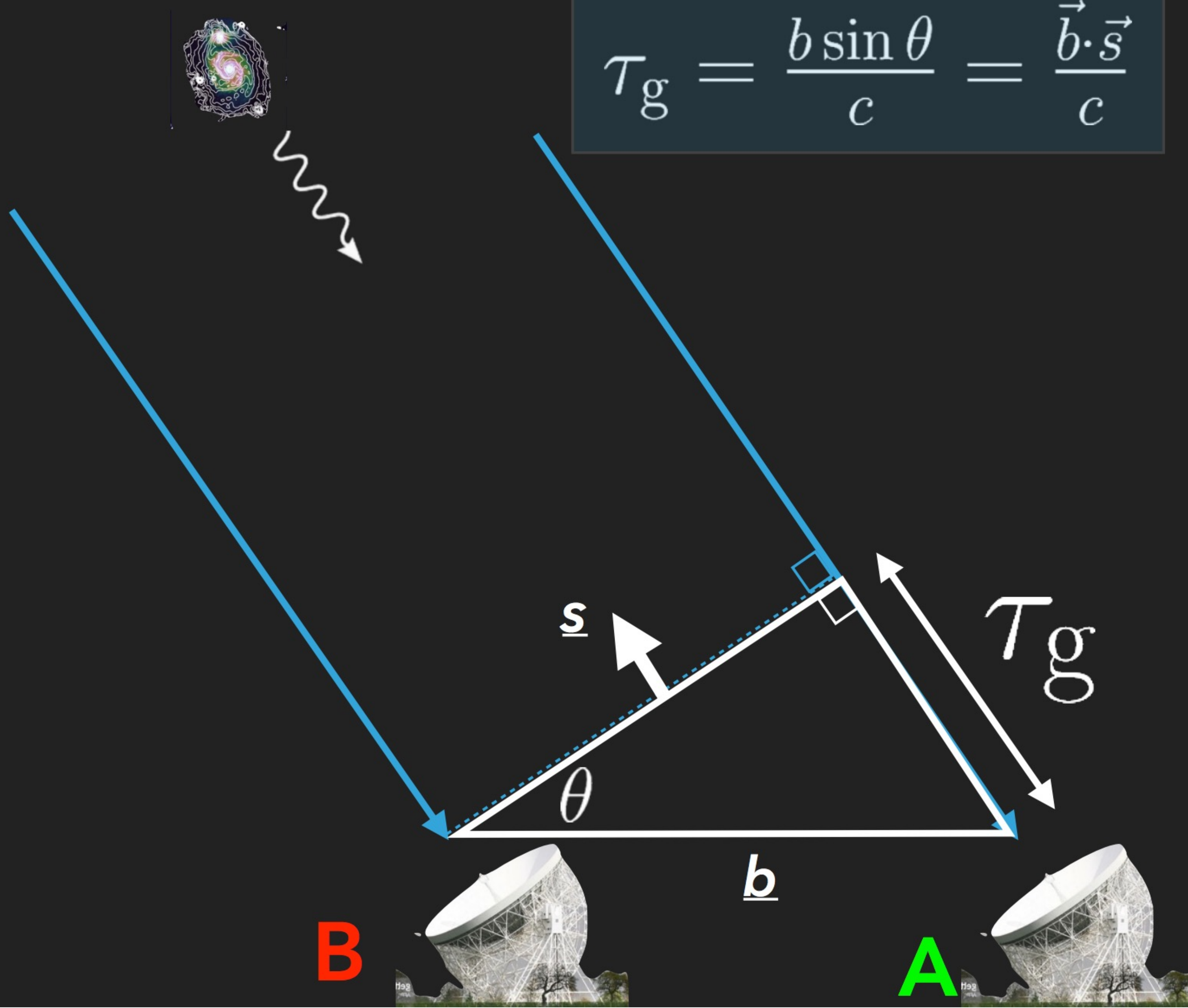
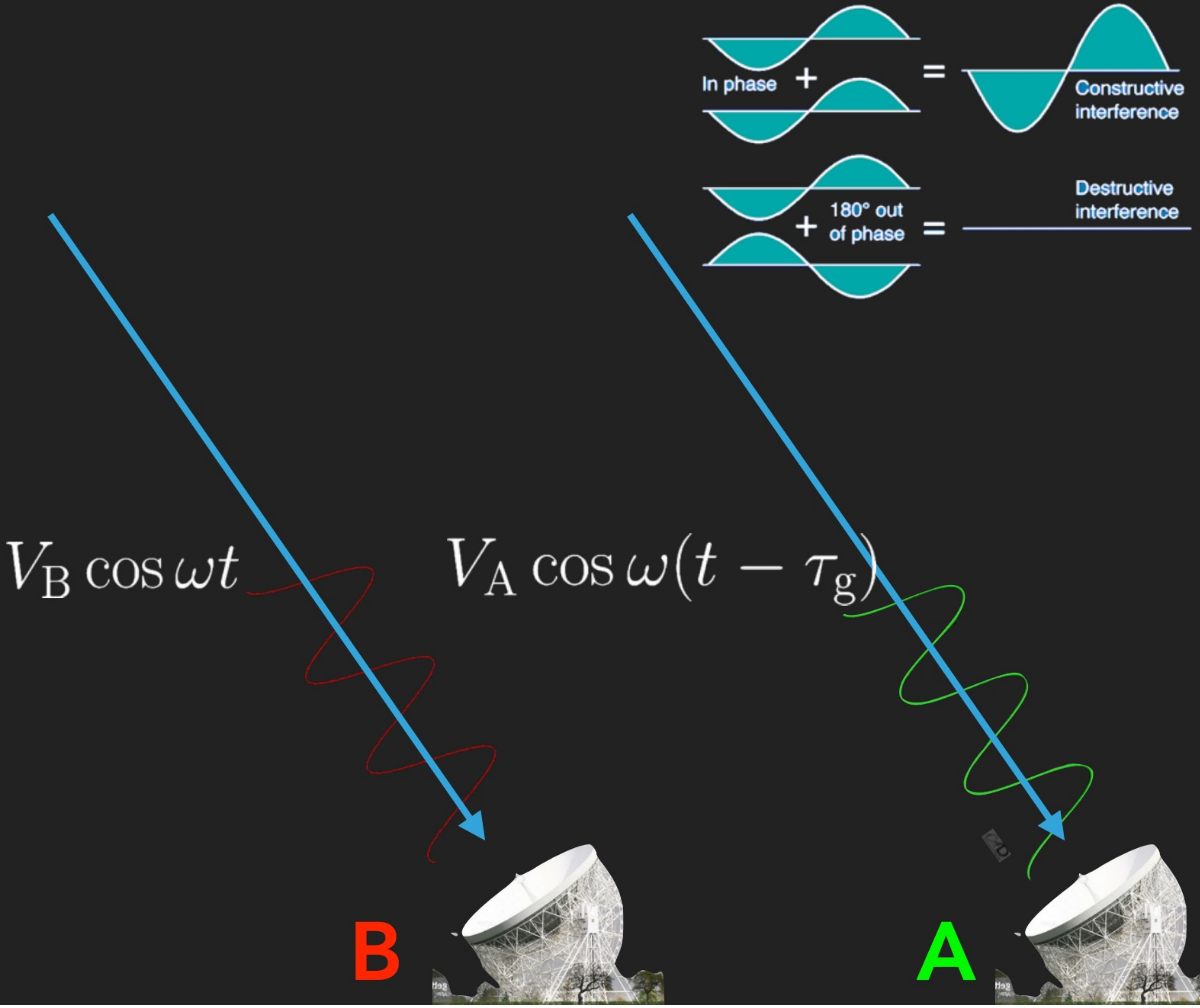
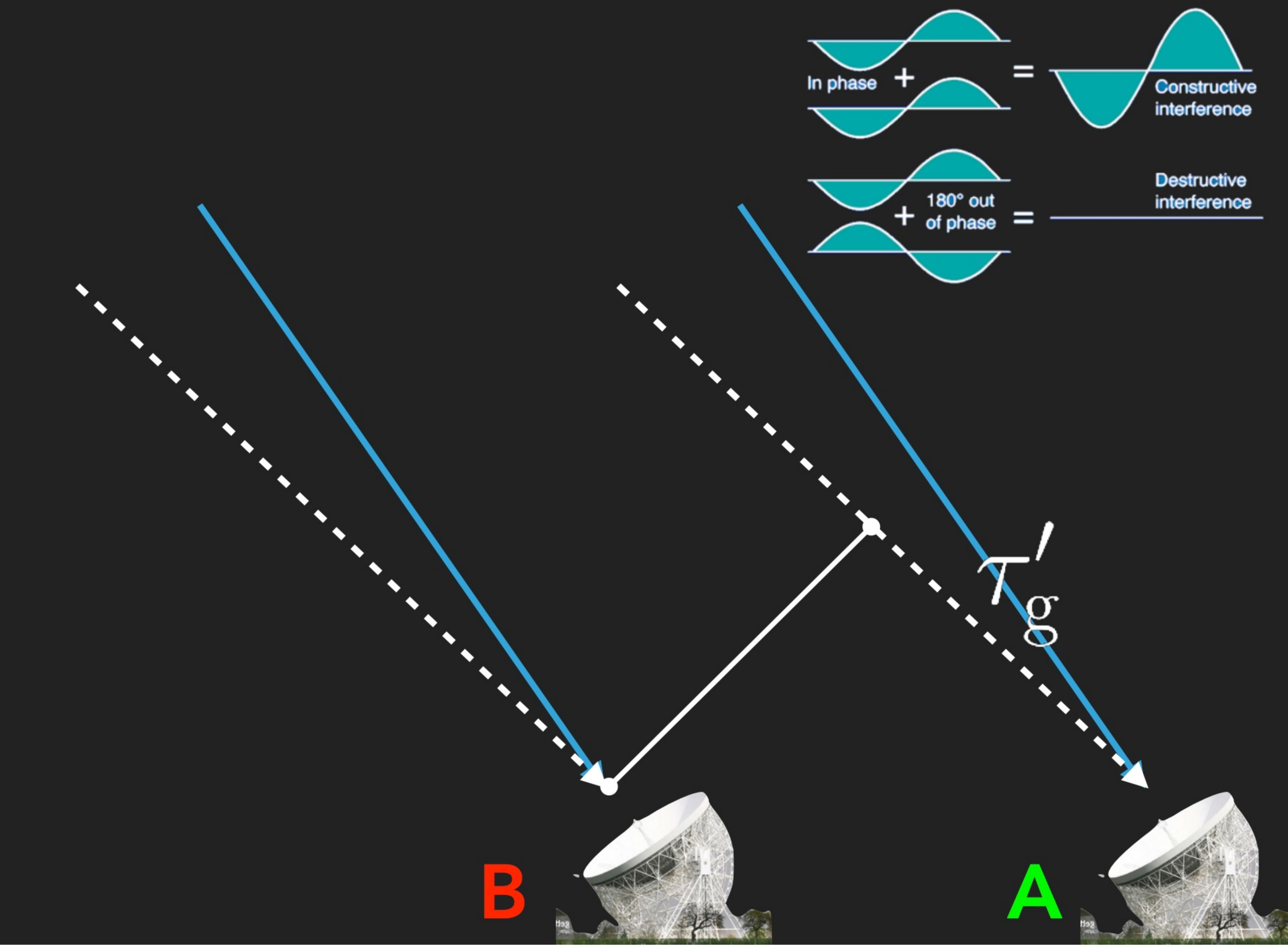


τ_g
B
A

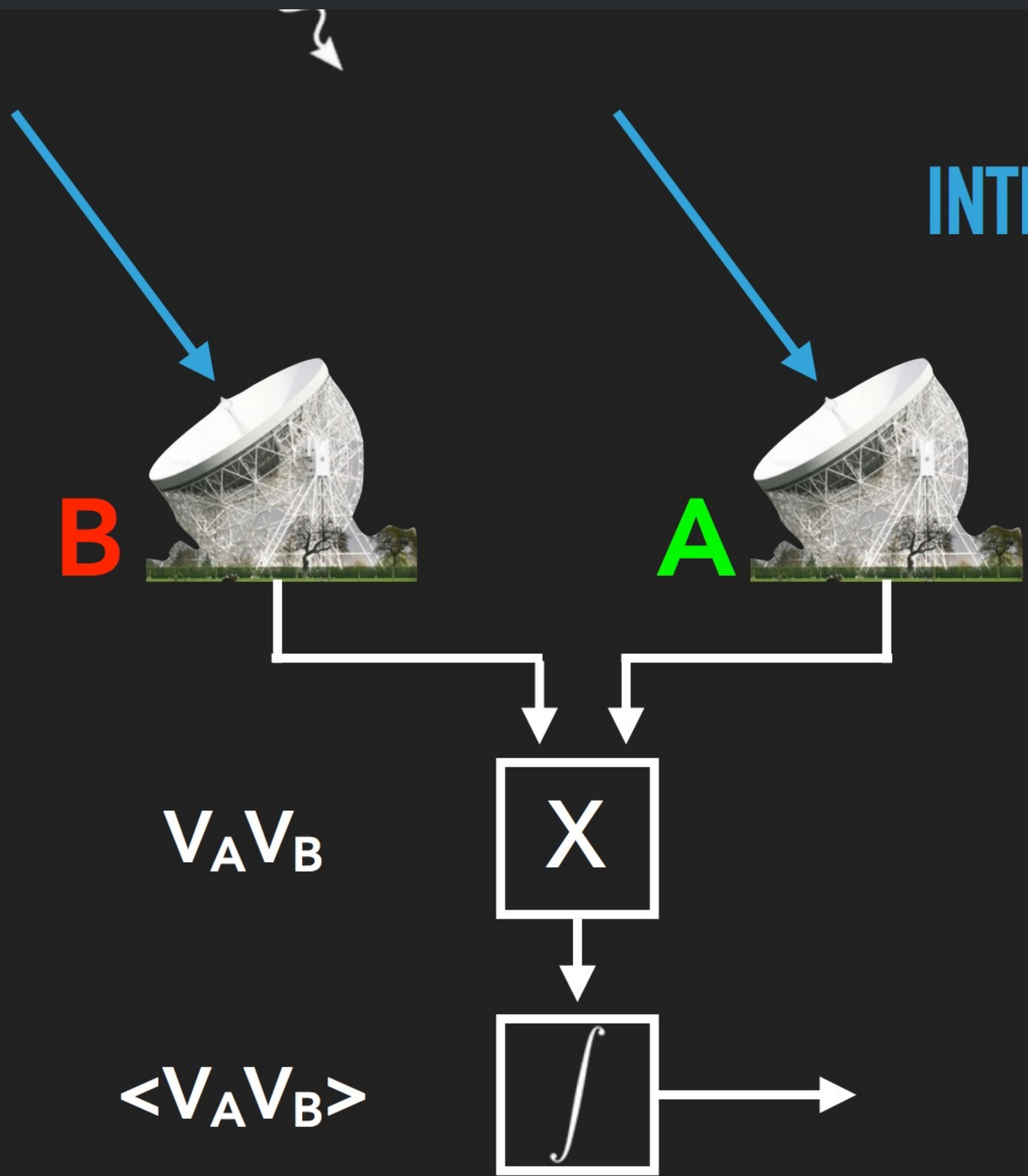
$$\tau_g = \frac{b \sin\theta}{c} = \frac{\vec{b}\cdot\vec{s}}{c}$$



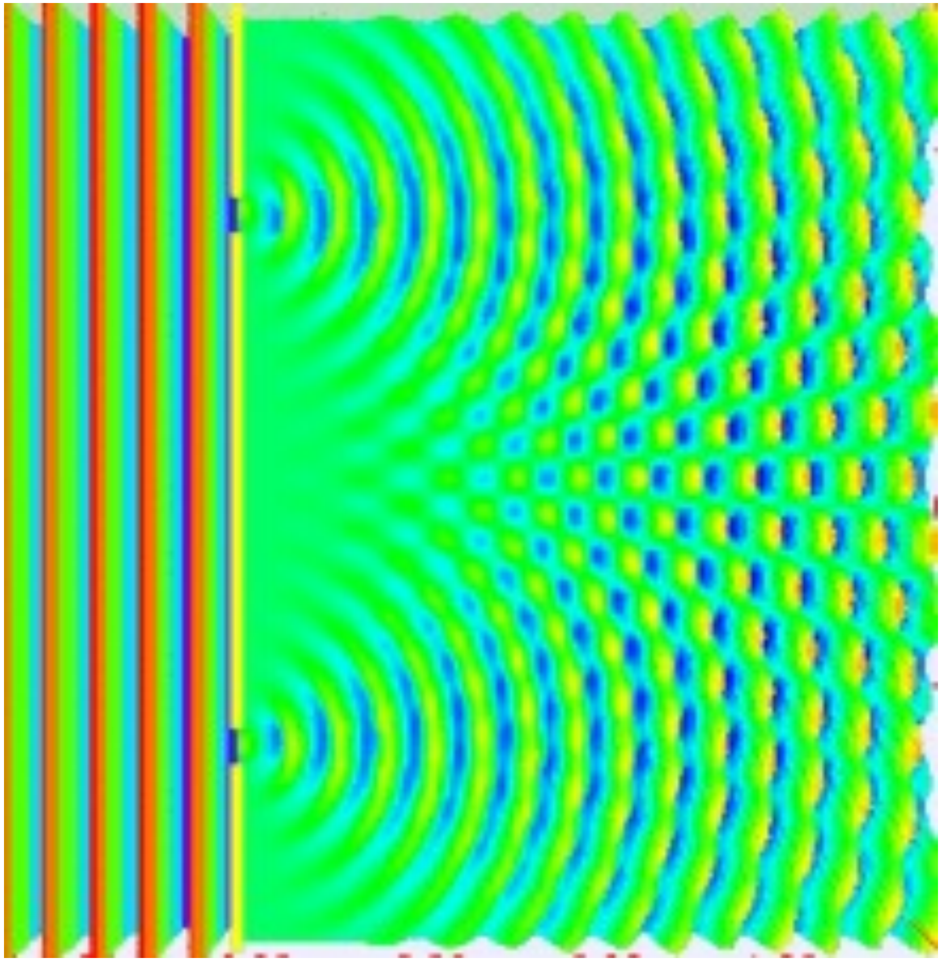

In phase
Constructive interference
180° out of phase
Destructive interference
$V_{\mathrm{B}} \cos \omega t$
$V_{\mathrm{A}} \cos \omega (t - \tau_{\mathrm{g}})$
B
A


In phase
Constructive interference
180° out of phase
Destructive interference
τ_g'
B
A

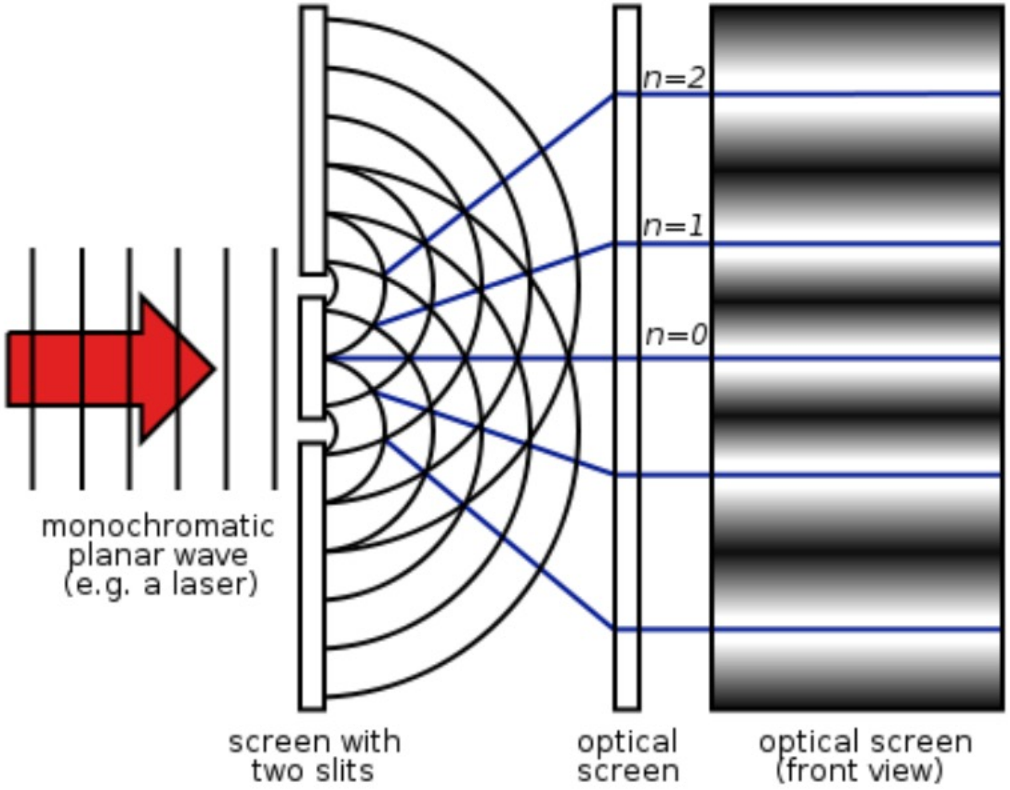
MULTIPLYING INTERFEROMETER



Like Young's Double Slit Experiment



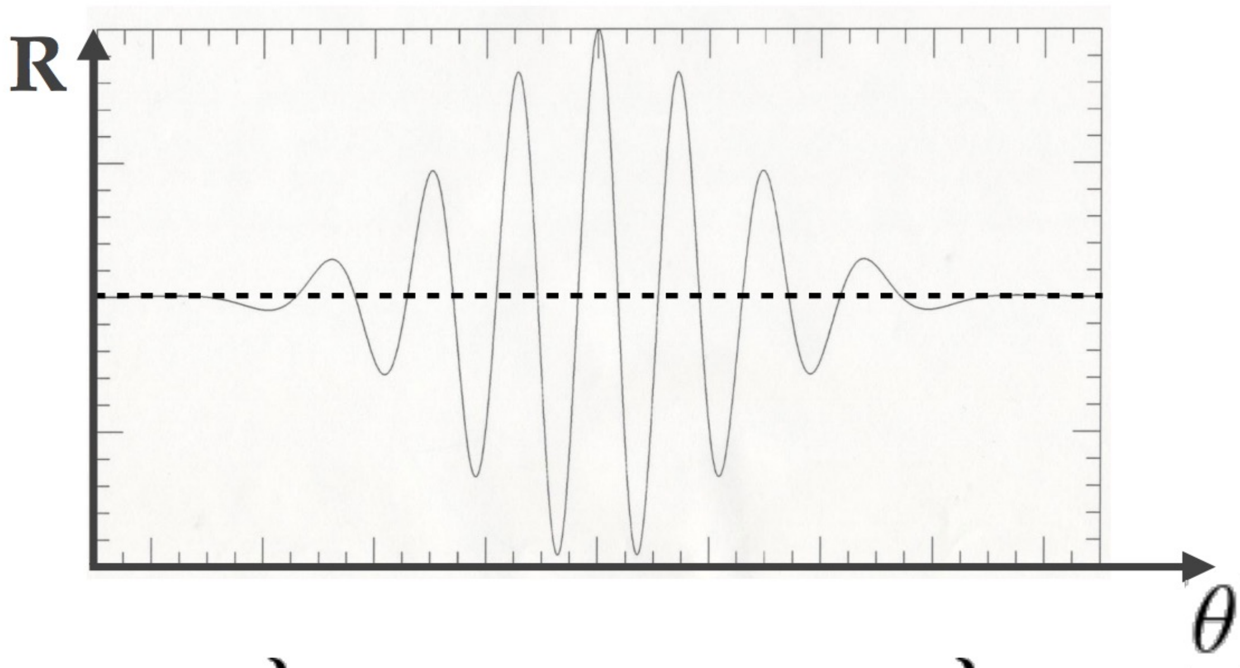
Like Young's Double Slit Experiment



Correlation

MULTIPLYING INTERFEROMETER

$$R \propto \langle V_{\rm A} \cos \omega (t - \tau_{\rm g}) \cdot V_{\rm B} \cos \omega t \rangle = \tfrac{1}{2} V_{\rm A} V_{\rm B} \cos \tau_{\rm g}$$

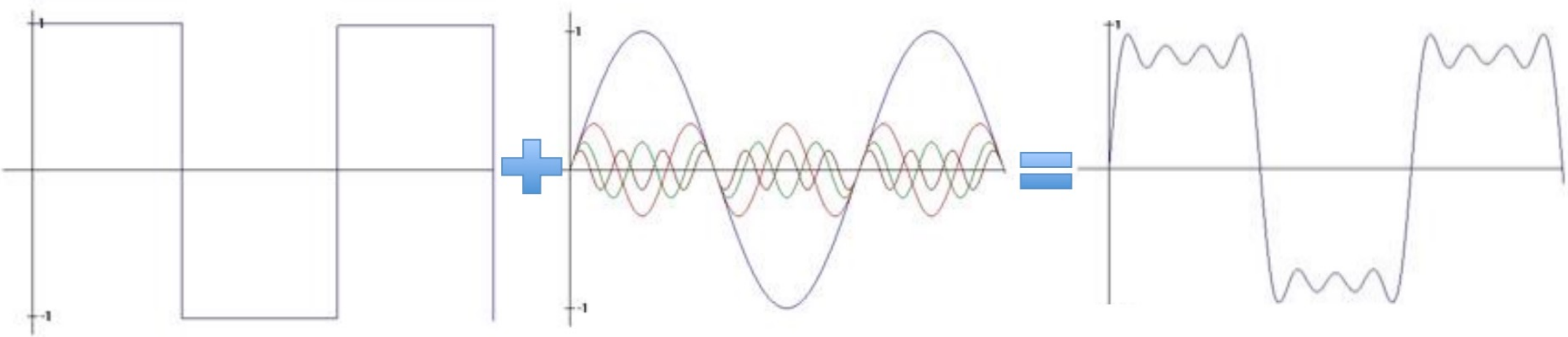


primary beam envelope: $\dfrac{\lambda}{D}$

fringe spacing: $\dfrac{\lambda}{b}$

The Fourier Transform

Fourier theory states that any well behaved signal (including images) can be expressed as the sum of sinusoids



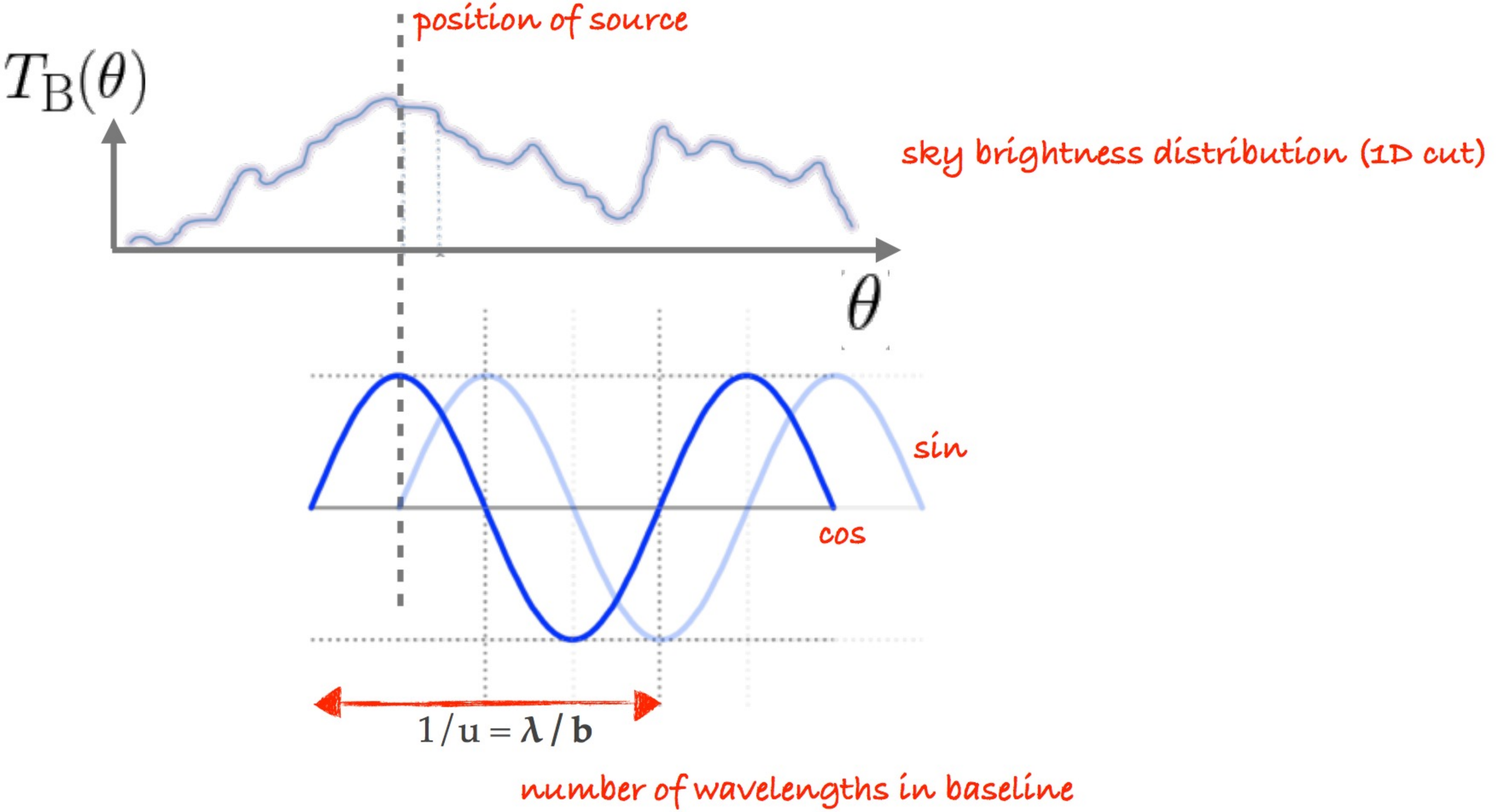
Reference signal

4 sinusoids

Sum of sinusoids & signal

- The Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- The Fourier transform contains *all* of the information of the original signal

Measuring the sky



Visibilities

In reality the response will be 2D, but in 1D for simplicity:

power out as a function of baseline

$$R_{\cos}(u) = \int_{\mathrm{src}} B(\theta)\cos(2\pi\, u\, \theta)\mathrm{d}\theta$$

$$R_{\sin}(u) = \int_{\mathrm{src}} B(\theta)\sin(2\pi\, u\, \theta)\mathrm{d}\theta$$

The sky brightness distribution is **not an even function**. If we want to reconstruct it from its Fourier components then we need **both the cos and sin terms**.

Van Cittert Zernike function

The (2-D) lateral coherence function of the radiation field in space is the Fourier Transform of the (2-D) brightness (or intensity) distribution of the source.

$$\langle V(x_1,t)V(x_2,t)\rangle = \int\int B(\theta,\phi)\mathrm{e}^{-2\pi i(u\theta+v\phi)}\mathrm{d}\theta\mathrm{d}\phi$$

$$u = \frac{(x_1 - x_2)}{\lambda} \qquad v = \frac{(y_1 - y_2)}{\lambda}$$

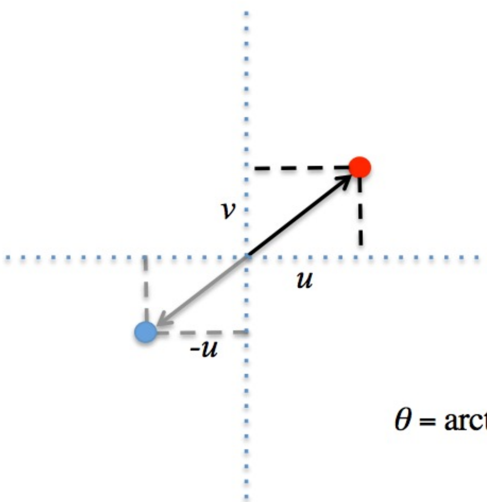
The Visibility Function is therefore another name for the spatial correlation function.

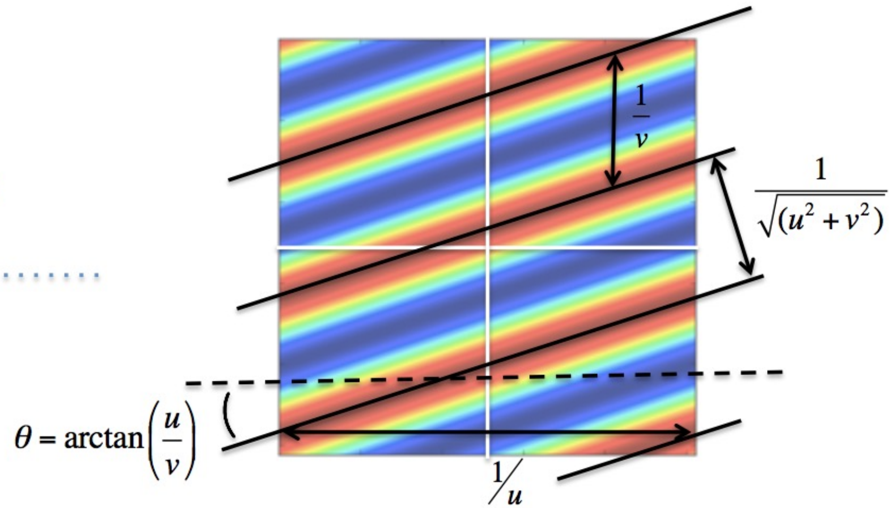
If we change our notation slightly, so that $V=Ae^{i\phi}$, we can write:

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \phi_i]$$

Visibilities

FOURIER COMPONENTS



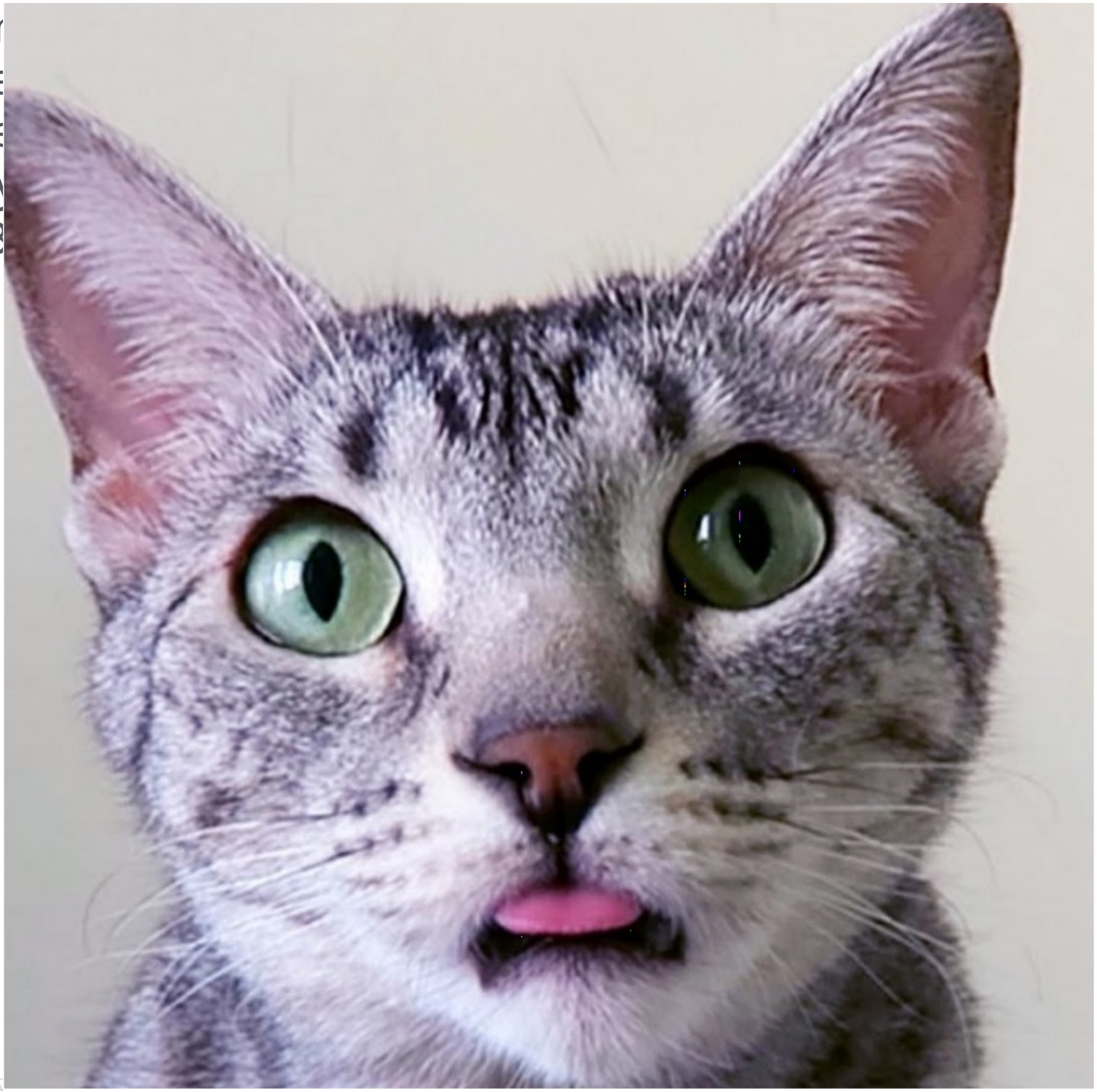


Writing the equation in this way allows us to visualise how our image is composed.

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \phi_i]$$

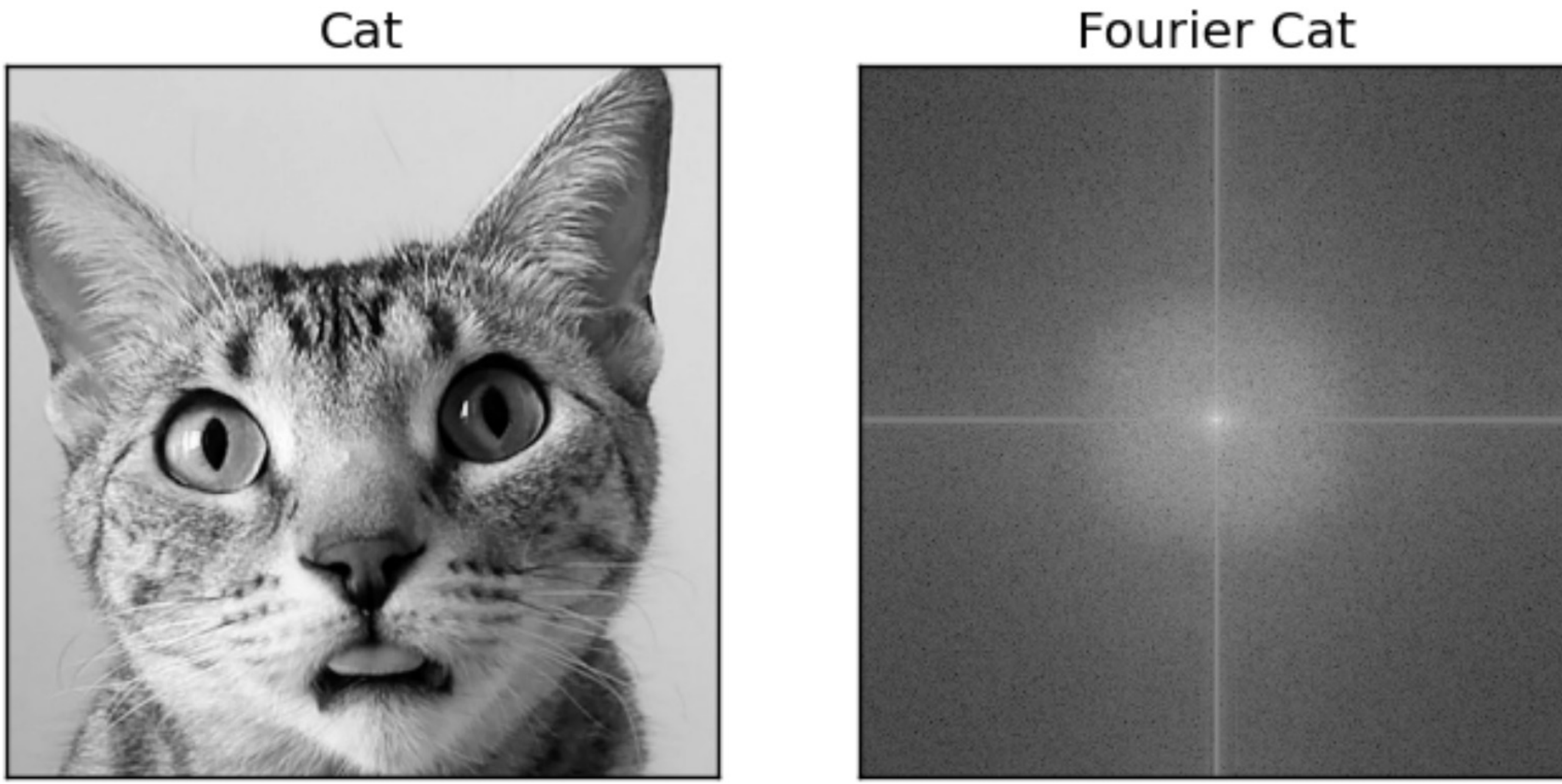
Title Text

- Body Level On
  - Body Leve
    - Body Le
      - Body
        - B



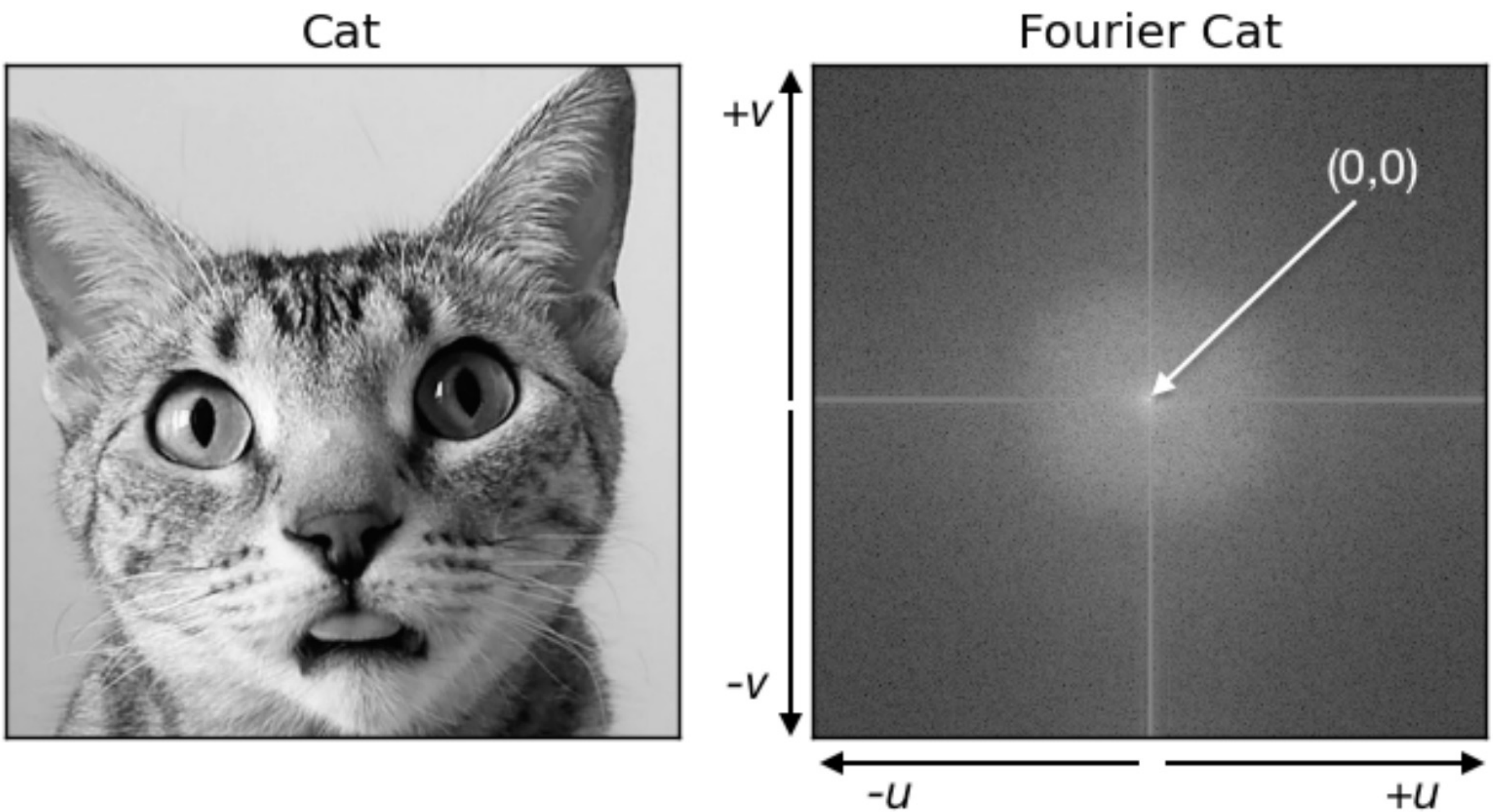
Disclaimer: This is not my cat.

Title Text



Title Text

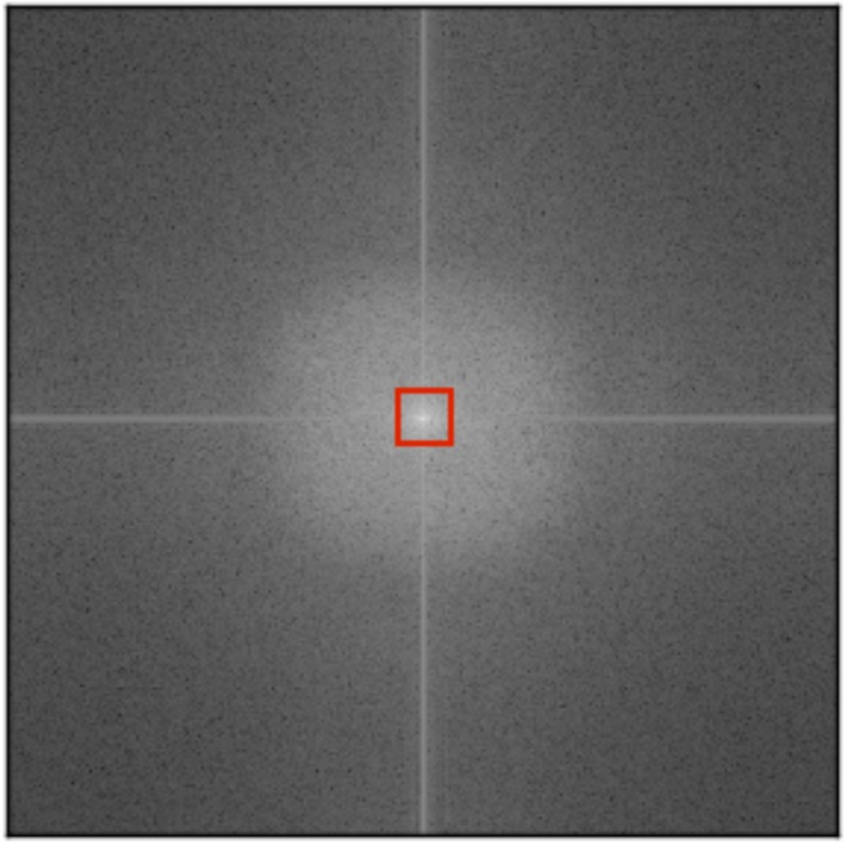
- Body Level One



Title Text

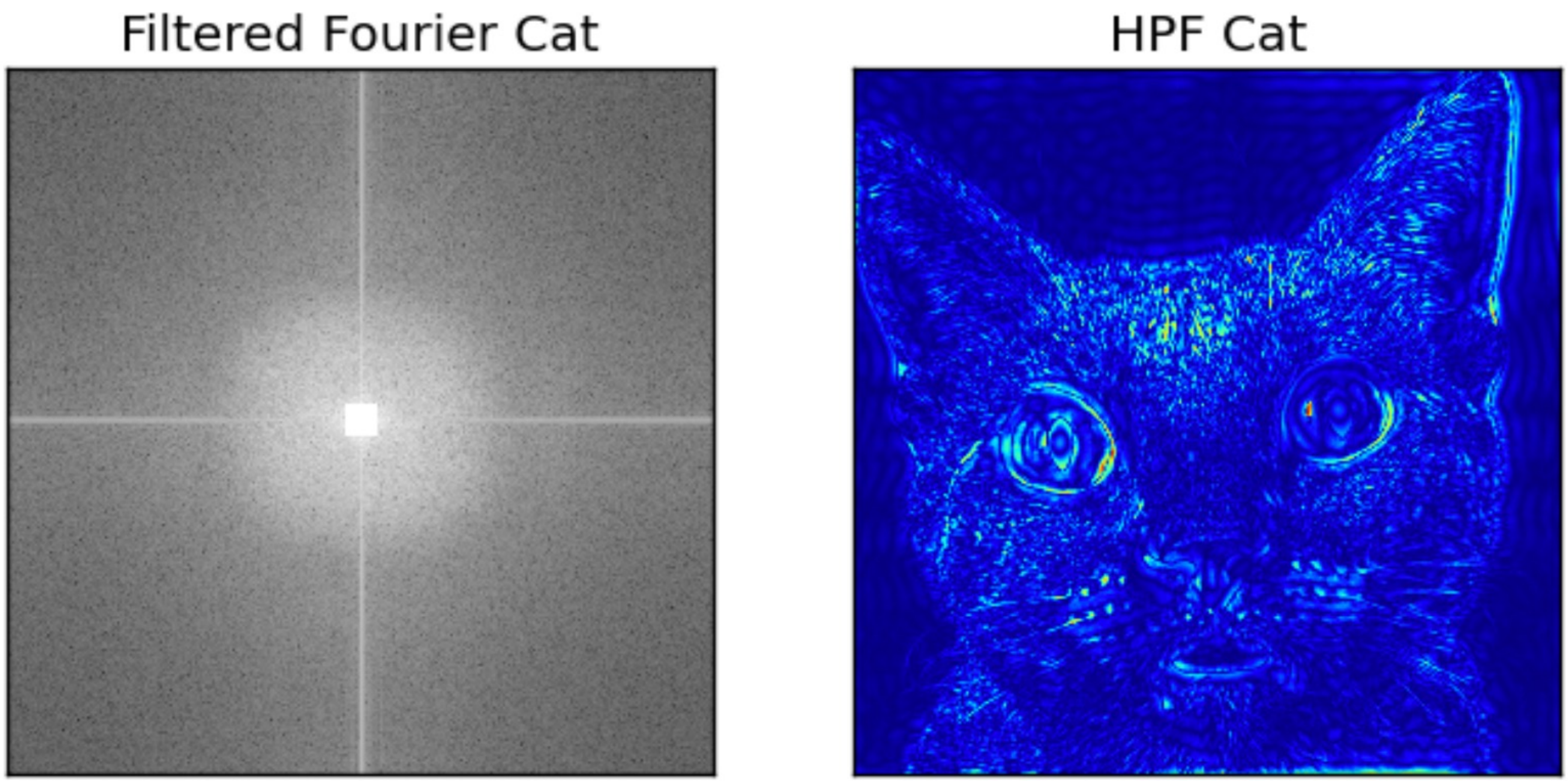
Cat

Fourier Cat



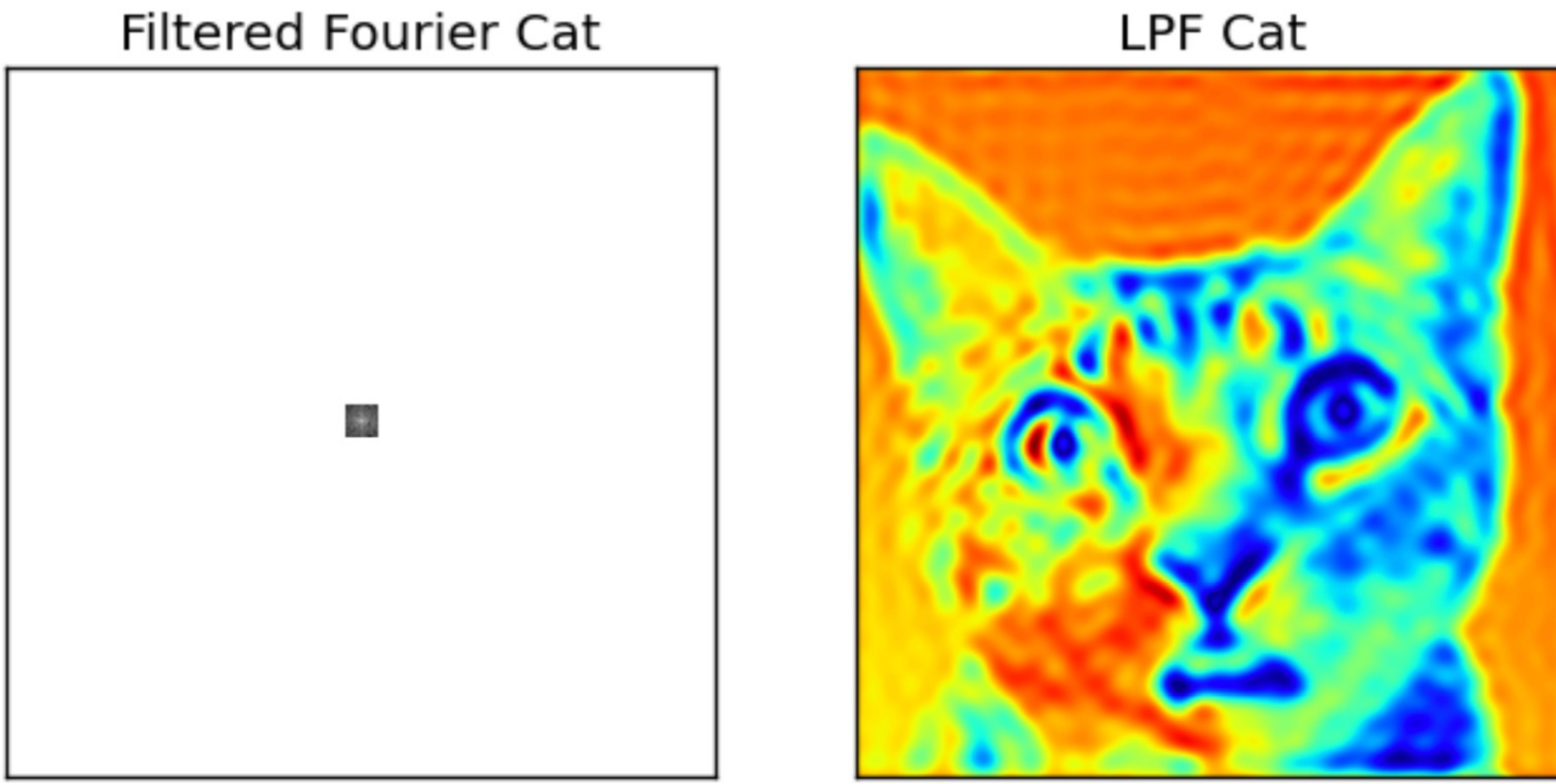
Title Text

Body Level One



Title Text

- Body Level One
  - Body Level Two



Title Text

- Body Level One
  - Body Level Two
    - Body Level Three

Small Fourier Frequencies = Large Scale Image Structure
Large Fourier Frequencies = Small Scale Image Structure

Big is Small & Small is Big

Summary

- The key to interferometry is the geometric delay
- The sky is not symmetric - we need both cosine & sine waves to make a picture of it
- Interferometers measure complex visibilities, which are the Fourier components of the sky brightness.

- Thanks to Anna Scaife, Ron Ekers, and John McKean