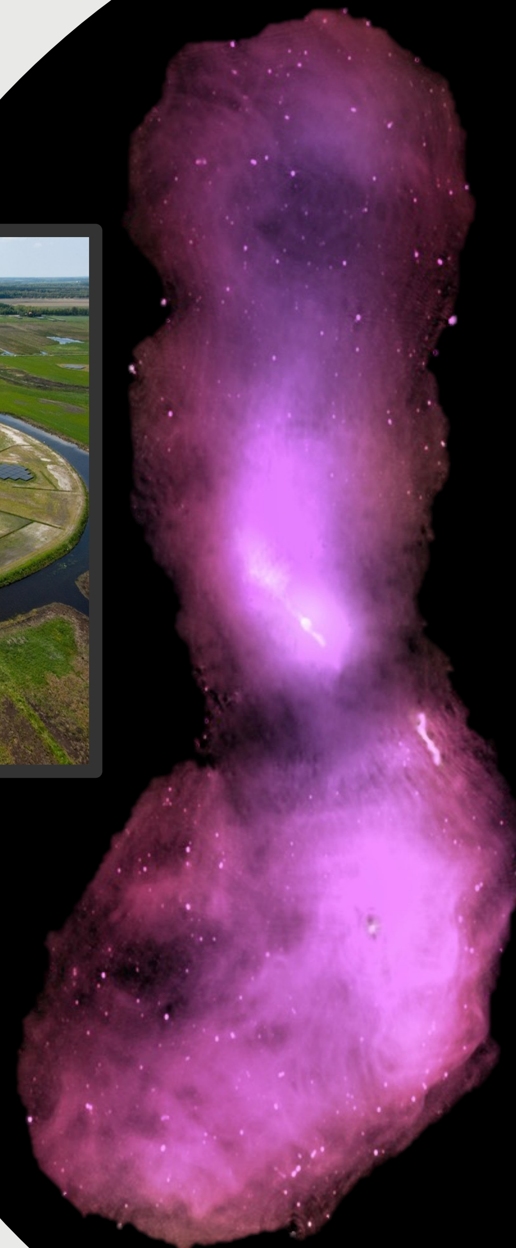
Introduction to polarisation (aka the hard stuff)



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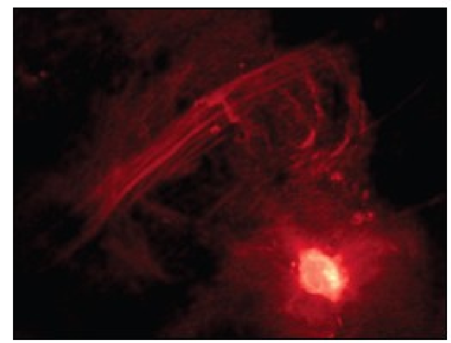
Why is polarisation info interesting?

ASTRON

- Radio is key to measuring polarisation, better than any other wavelength

- High-z seed fields (Widrow 2002; Subramanian 2007) — $B \sim 10^{-30} - 10^{-20}$ G
- Intergalactic Medium — $B \sim$ 1-10 nG ?
- Intracluster Medium — $B \sim$ 0.1-1 μG
- Interstellar medium — $B \sim$ 1 μG – 10 mG
- Galactic Centre (Crocker et al. 2010; Ferrière 2010) — $B \sim$ 50 μG – 1 mG
- Main sequence star: HD 215441 (Babcock 1960) — $B_0 \approx$ 34 kG
- White dwarf: PG 1031+234 (Schmidt et al. 1986) — $B_0 \approx 10^9$ G
- Pulsar: PSR J1847-0130 (McLaughlin et al. 2003) — $B_0 \approx 9 \times 10^{13}$ G
- Magnetar: SGR 1806-20 (Kouveliotou et al. 1998, Israel et al. 2005) — $B_0 \approx 2 \times 10^{15}$ G, $B_i \approx 10^{16}$ G
- Cosmic strings (Ostriker et al. 1986) — $B \sim 10^{30}$ G
- Planck-mass monopoles (Duncan et al. 2000) — $B \sim 10^{55}$ G

Magnetic filaments in Perseus A (Fabian et al. 2008)

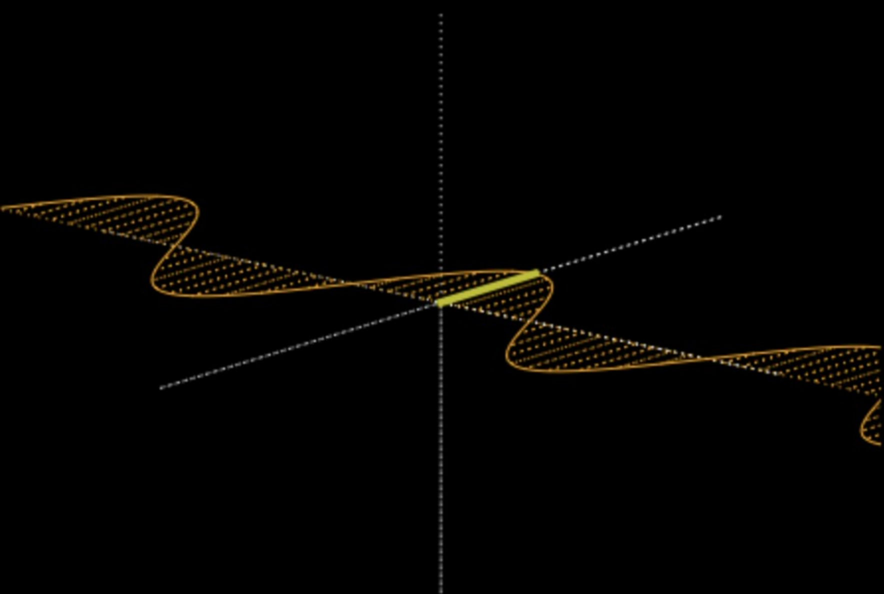


Galactic Centre (Yusef-Zadeh et al. 1984)

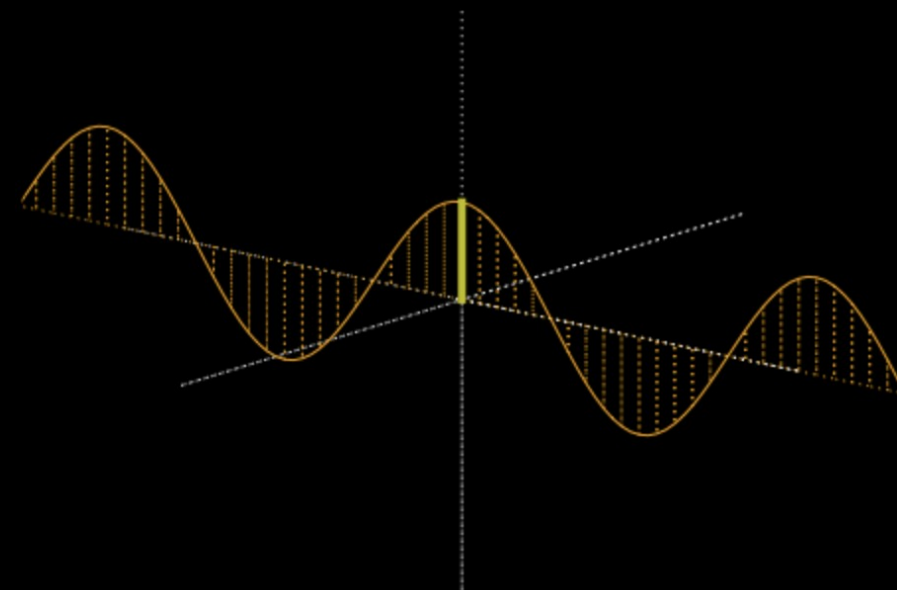
SGR 1806-20 giant flare (NASA)

Light can have a preferential direction

› Linear polaristion

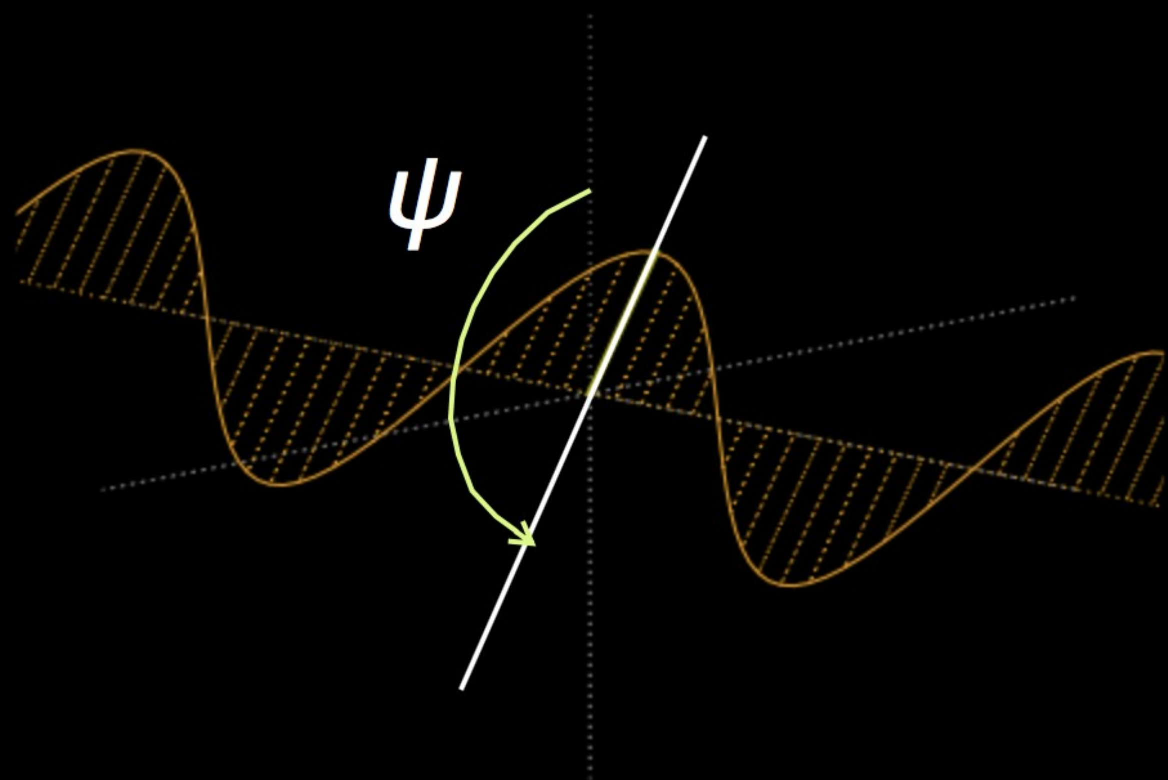


$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}}$$



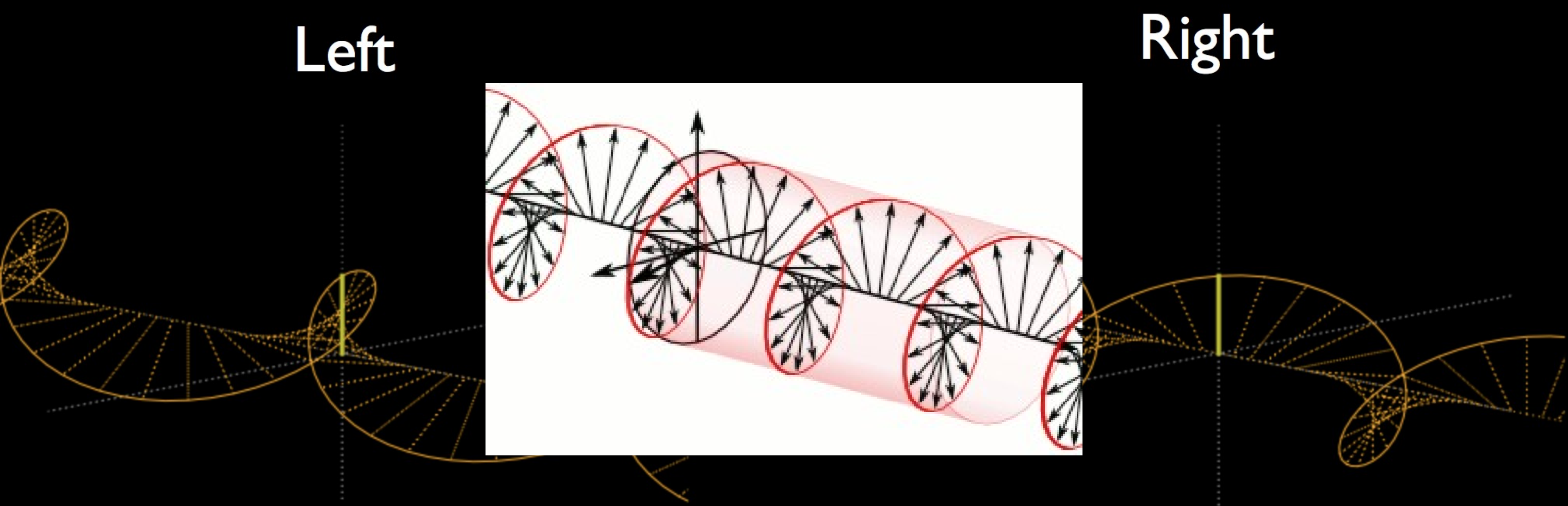
$$\mathbf{E} = E_y \cos(\omega t - kz)\underline{\hat{y}}$$

And any angle inbetween



$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}} + E_y \cos(\omega t - kz)\underline{\hat{y}}$$

Light can also be circularly polarised



$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz + \frac{\pi}{2})\underline{\hat{y}}$$

$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz - \frac{\pi}{2})\underline{\hat{y}}$$

How to describe polarisation?

- Polarized state of light can be described by 4 parameters e.g.
  - Total power;
  - Fractional powers in horizontal and vertical linear components;
  - Phase relationship between the horizontal and vertical components.

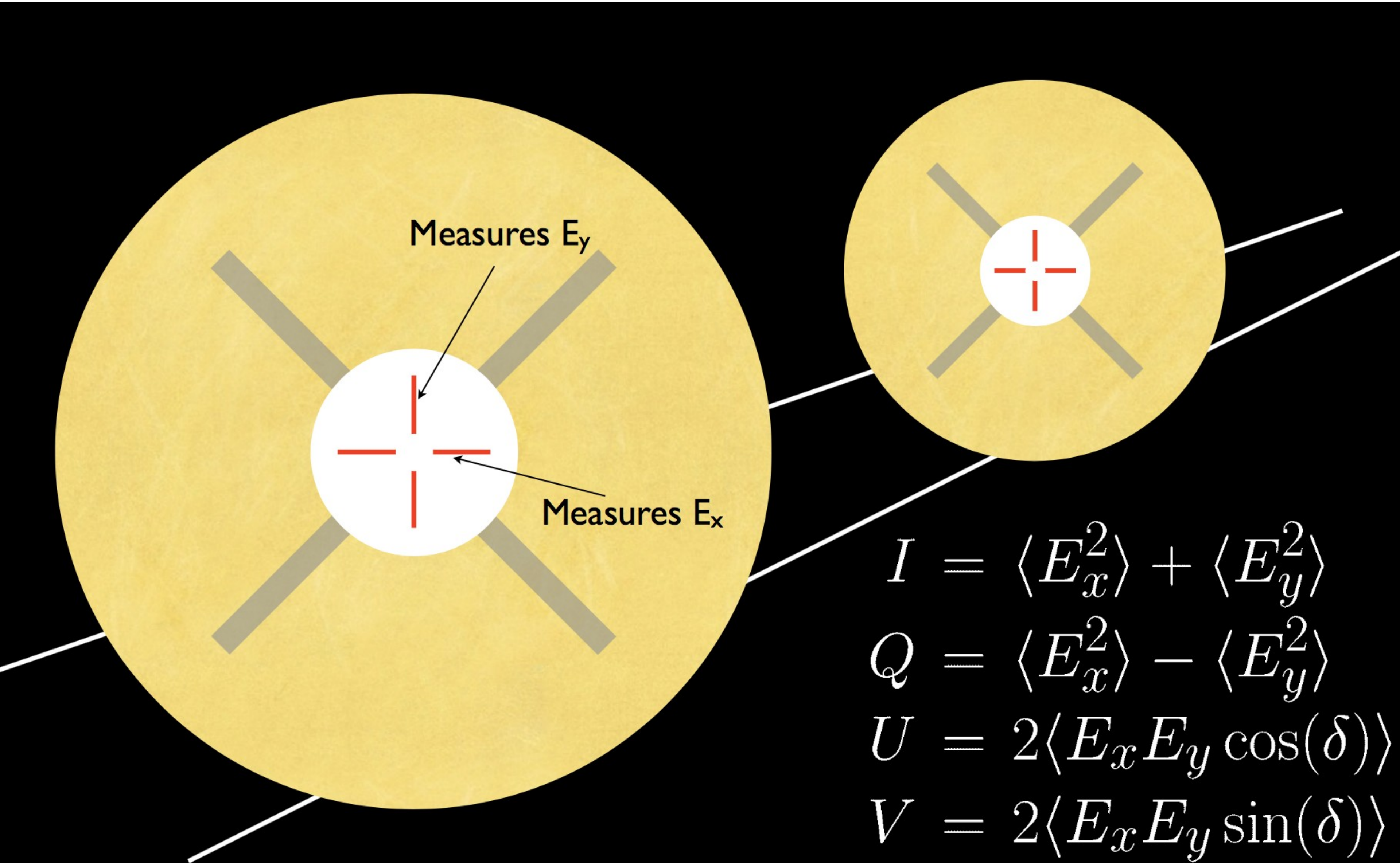
Stokes Parameters

- Ease of use over direct
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)
- Exact definition of Stokes parameters dependent on feeds of telescope.

$$
\begin{aligned}
I &= E_x^2 + E_y^2 \\
Q &= E_x^2 - E_y^2 \\
U &= 2E_xE_y\cos(\delta) \\
V &= 2E_xE_y\sin(\delta)
\end{aligned}
$$

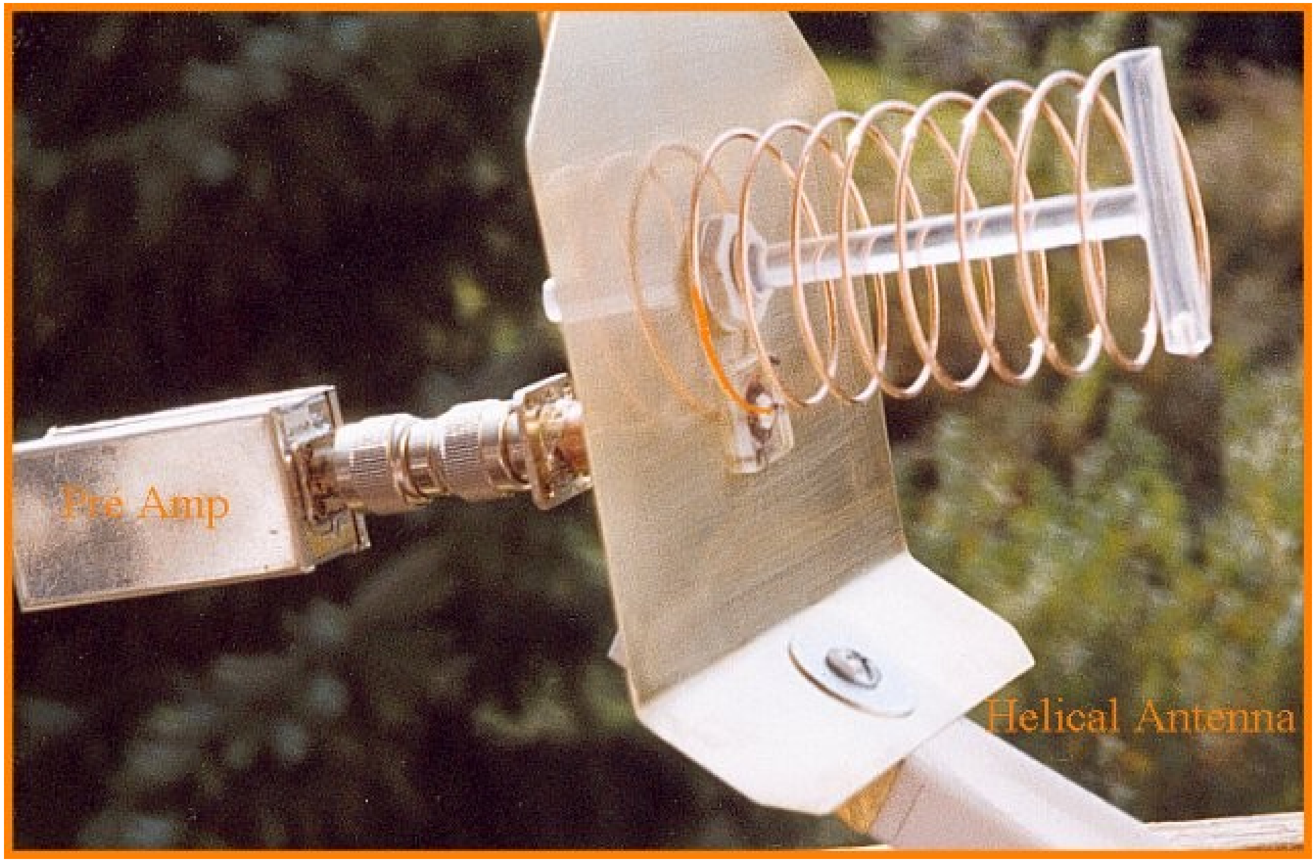
- For monochromatic waves
- I : total intensity
- Q : linear
- U : linear
- V : circular
- $I^2 = Q^2 + U^2 + V^2$

How to measure polarisation – linear feeds



$$
\begin{aligned}
I &= \langle E_x^2 \rangle + \langle E_y^2 \rangle \\
Q &= \langle E_x^2 \rangle - \langle E_y^2 \rangle \\
U &= 2\langle E_x E_y \cos(\delta) \rangle \\
V &= 2\langle E_x E_y \sin(\delta) \rangle
\end{aligned}
$$

How to measure polarisation – circular feeds



Measurement of Stokes

› Conceptually use many feeds to measure the different orthogonal polarizations

› At radio frequencies voltages can be measured and correlated!

$$I = XX + YY,$$
$$Q = XX - YY,$$
$$U = XY + YX,$$
$$iV = XY - YX.$$

where XX is $< E_0 \overline{E_0} >$ and YY is $< E_{90} \overline{E_{90}} >$ ("parallel hand" correlations)
and XY is $< E_0 \overline{E_{90}} >$ and YX is $< E_{90} \overline{E_0} >$ ("cross hand" correlations).

Making stokes images

- Each antenna measures two orthogonal polarizations - X and Y.
- For every baseline, form all four possible correlations - XX, YY, XY, YX.
- Calibration and other tricks
- Appropriately combine the four correlations to get four Stokes “visibilities”.
- Perform standard imaging with these Stokes visibilities to make Stokes images.

Talking about the Jones

- Jones matrices are "antenna based"
- Antenna gain:

$$\begin{pmatrix} g_x & 0 \\ 0 & g_y \end{pmatrix}$$

- Polarization leakage:

$$\begin{pmatrix} 1 & d_x \\ d_y & 1 \end{pmatrix}$$

- Rotation:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Leakage

- For real-world feeds, the feed will respond both to the desired polarization, and to a small degree the orthogonal polarization:

$$E'_X = E_X + d_x E_Y$$
$$E'_Y = E_Y + d_y E_X$$

- The leakage ("d term") is typically ~10^{-2}. It is caused by alignment error, feed ellipticity, etc. Generic linear model (will suit practically anything).

Rotation and parallactic angle

Rotation and parallactic angle

- Alt-az mount
- For a conventional alt-az mount, the sky rotates relative to the antenna feed - ``parallactic angle rotation''
- Instrumental polarization (leakage) will be in the frame of the antenna
- Astronomical polarization will be in the frame of the sky.
- Instrumental and astronomical characteristics can be decoupled if the your observation spans a good range of parallactic angles.

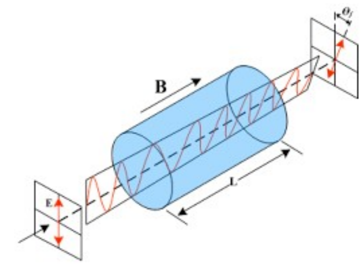
Depolarisation

- Depolarization = loss of polarimetric signal.
- Caused by the system polarimetric response not being constant (i.e. varying spatially, with time, across bandwidth etc etc) smearing out the polarimetric signal.
- Calibrated interferometers generally have very low depolarization (in polarimetric jargon, a system with no depolarization is called “pure”).

Some of the quanities we measure with polarisation

- Rotation measures – provide integrated magnetic fields through Faraday rotation of linear polarisation – integral along the line of sight of the electron density and line-of-sight magnetic field

$$\mathrm{RM} = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e(l)\, B_{\parallel}\, dl \ \ \mathrm{rad\,m^{-2}}.$$



- Polarisation vectors – provide field strength and direction in plane of sky through computation of measured Stokes Q and U to obtain linear polarisation and polarisation angle

$$P = \sqrt{U^2 + Q^2} \qquad \Theta = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

- Gradient of Stokes Q and U – provide direct imaging of interstellar turbulence – changing of magnetic field orientation with gas motions

$$|\nabla \mathrm{P}| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

- Circular polarisation from synchrotron emission (it has a small, <0.1% Stokes I, component in Stokes V) – provide direct measurement of field strength and direction

$$m_{\mathrm{c}} = \epsilon_{\alpha}^{\nu} \left(\frac{\nu_{B\perp}}{\nu}\right)^{0.5} \frac{B_{\mathrm{u,los}}}{B_{\perp}^{\mathrm{rms}}}$$

Conclusions

- Huge amount of information in polarised light – an something radio astronomy (since we store phase) is uniquely suited to exploit
- It is hard… instrumental calibration, weak signals, depolarisation, many structures between us and the source
- Worth doing, but make sure your know your instrument and calibrators!