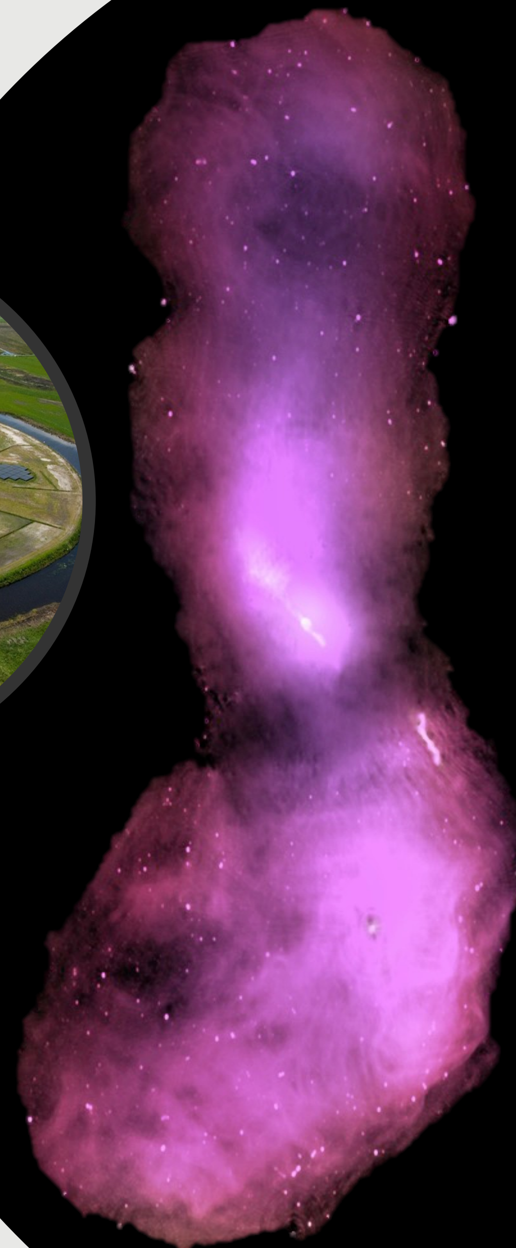
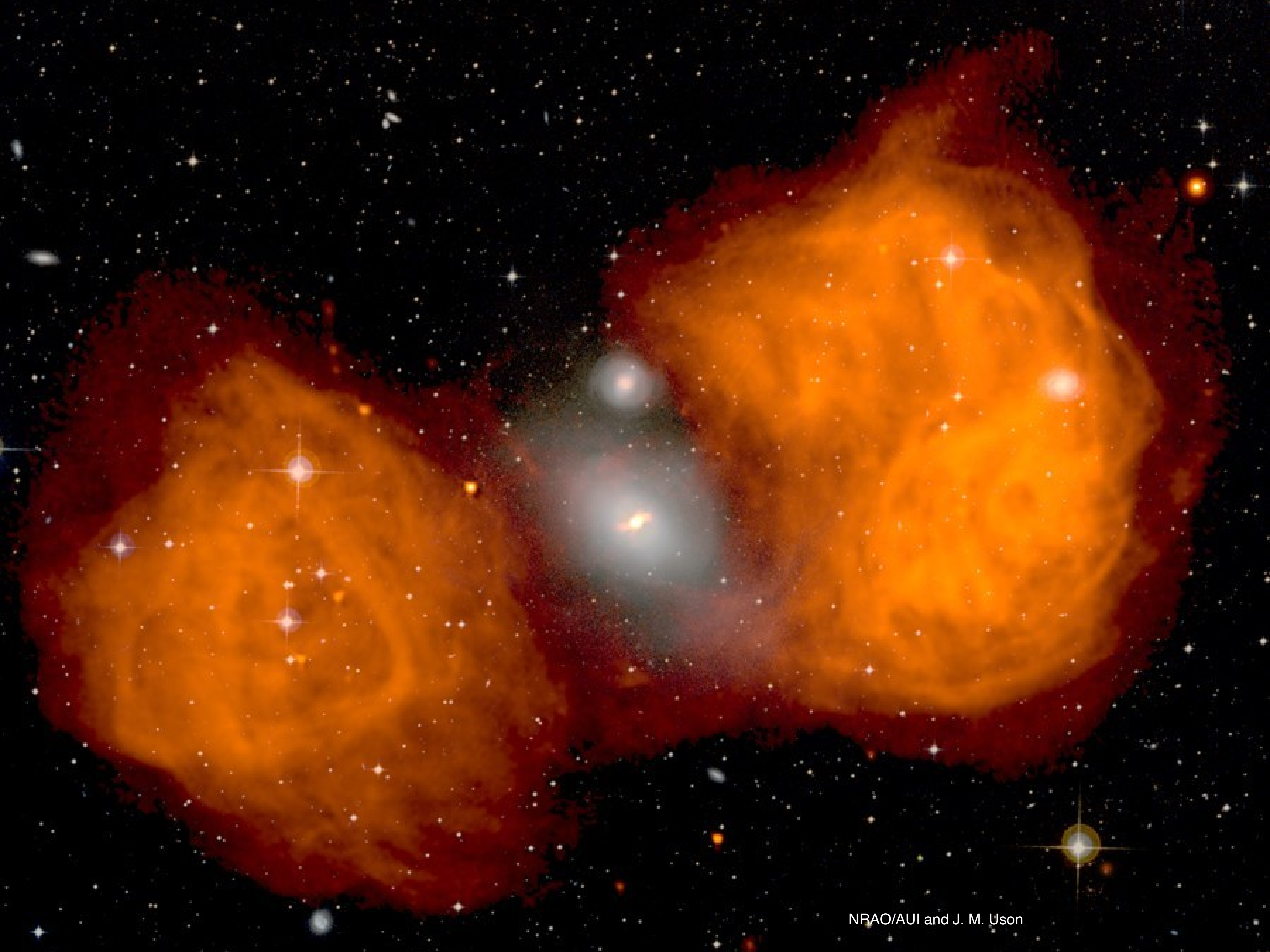
Introduction to (recap of?) Radio Interferometry



Jack Radcliffe and
Joe Callingham

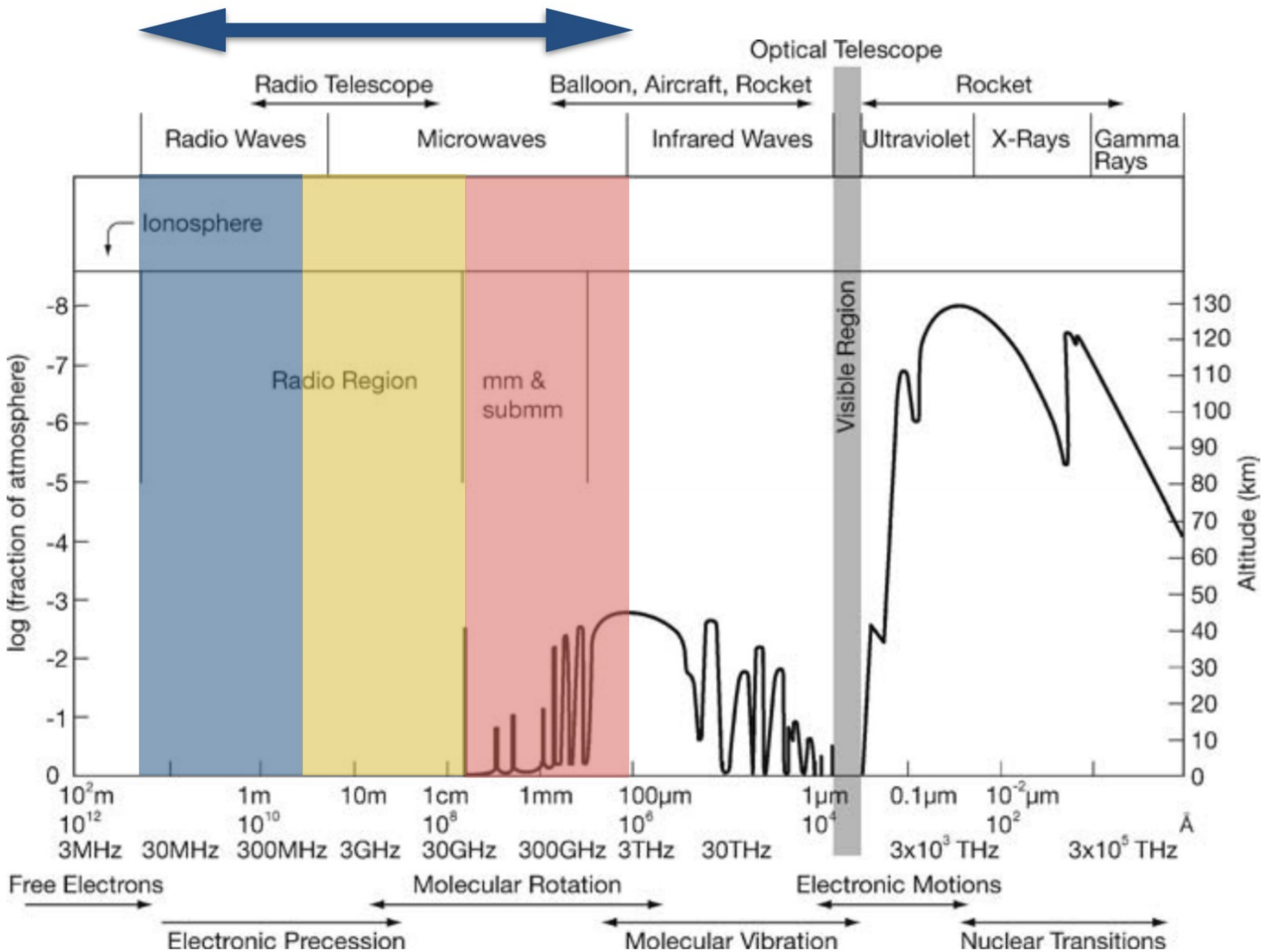
Botswana Radio Astronomy School,
Palapye, Botswana
1st of July 2019

DEVELOPMENT IN AFRICA
WITH RADIO ASTRONOMY


NRAO/AUI and J. M. Uson

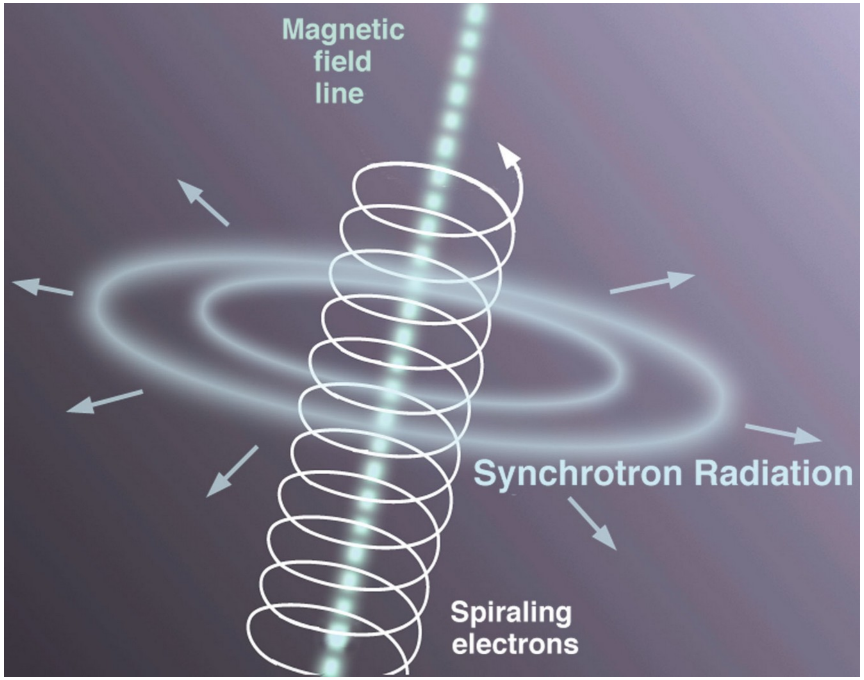
The Radio Sky

› Radio astronomy mostly focuses in the frequency range of ~10 MHz to ~1 THz

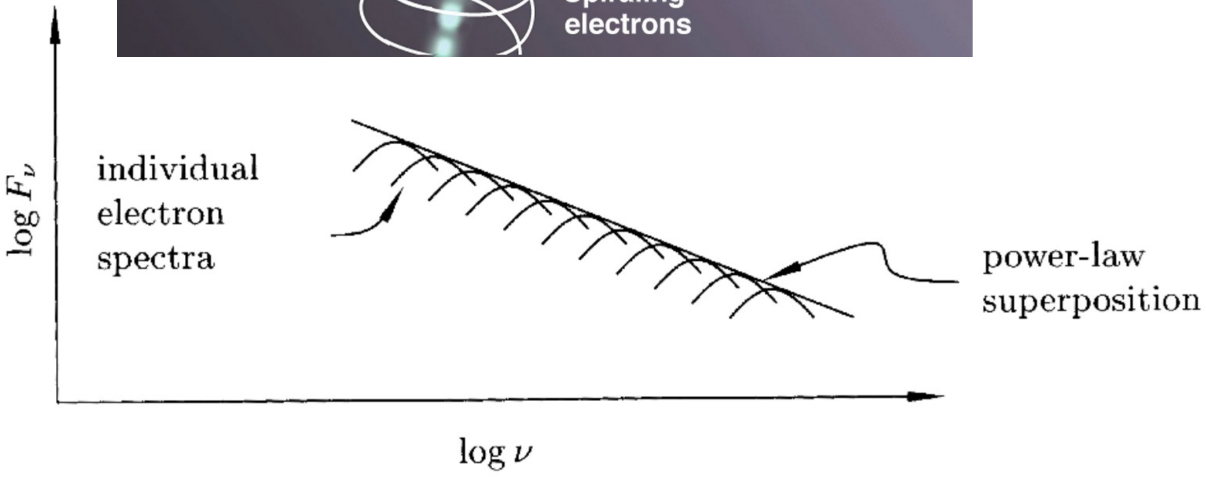


Sync my radiation

- If the energy distribution of electrons is a power-law – the spectrum will be a power-law
- Non-thermal
- Most radio sources are non-thermal

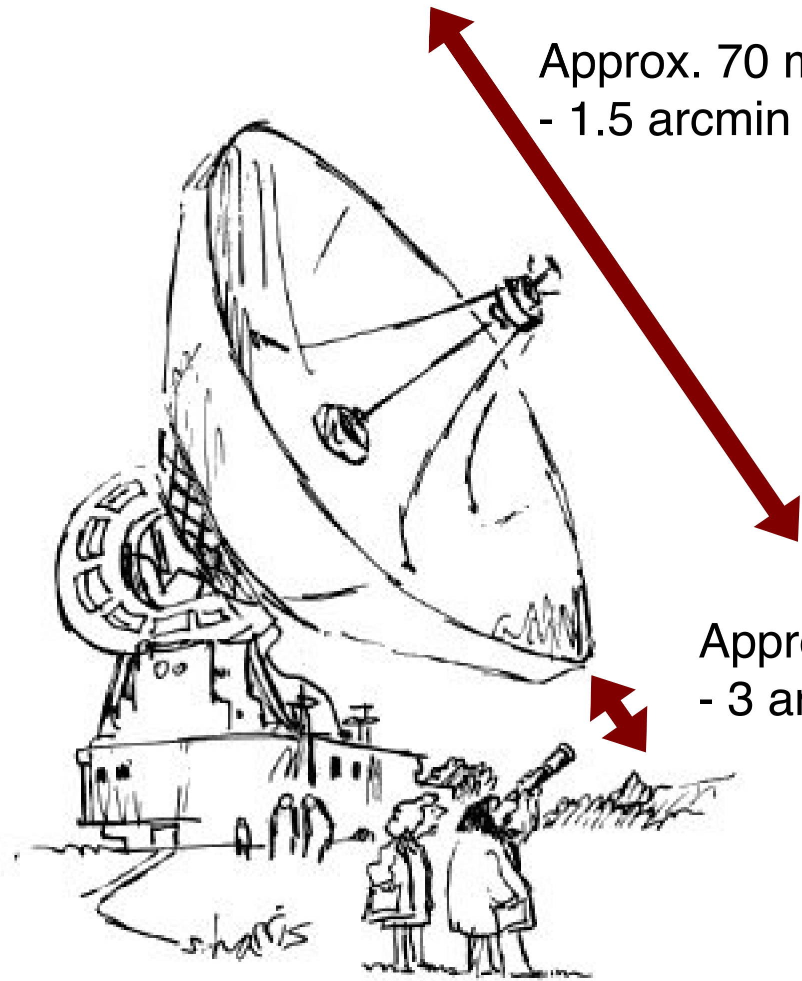


$$S_\nu = a\nu^\alpha$$



Why does size matter?

Resolution and sensitivity



"Just checking."

Approx. 70 metre aperture
- 1.5 arcmin resolution at 3 cm

$$\theta \approx \lambda / D$$

Resolution (radians)

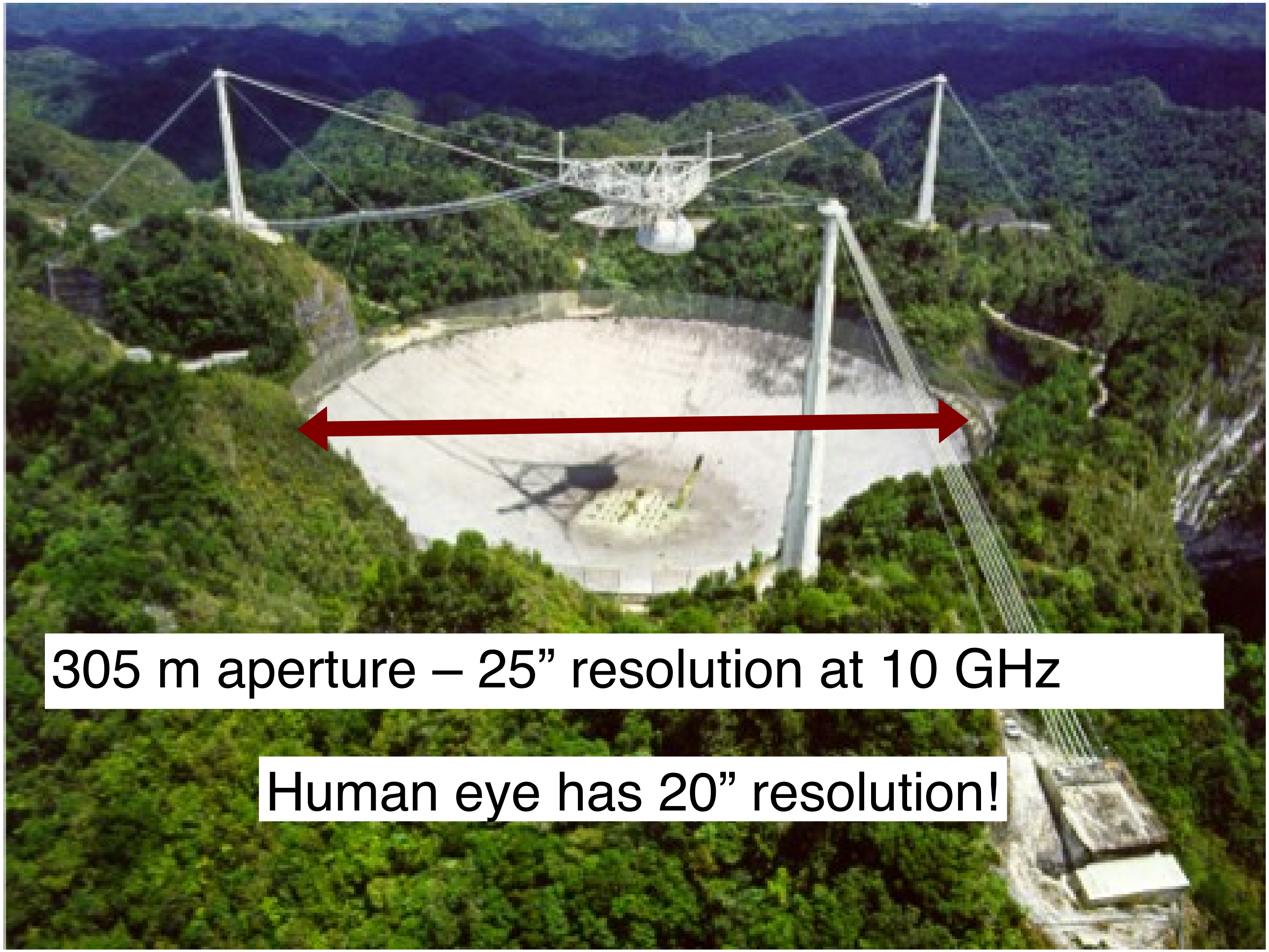
Observing wavelength (m)

Diameter of telescope (m)

Approx. 5 mm aperture
- 3 arcsec resolution at 600 nm

$$\mathrm{SEFD} = \frac{2kT_{\mathrm{sys}}}{A_e}$$

When big is not big enough



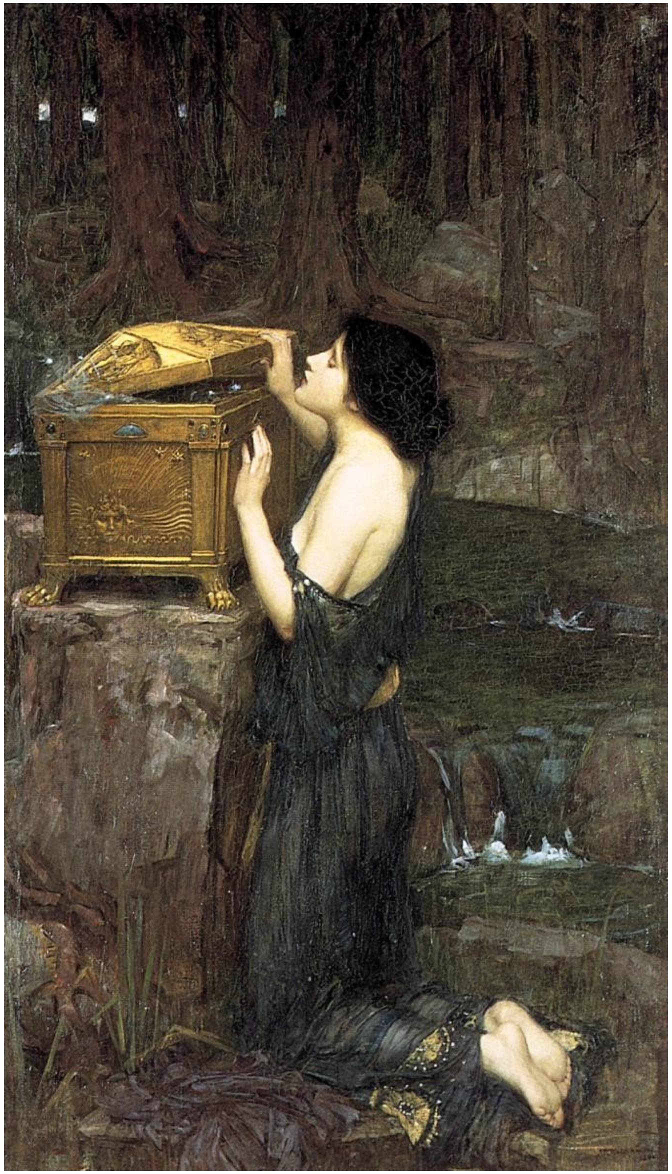
Bigger is better?

Bigger is better?

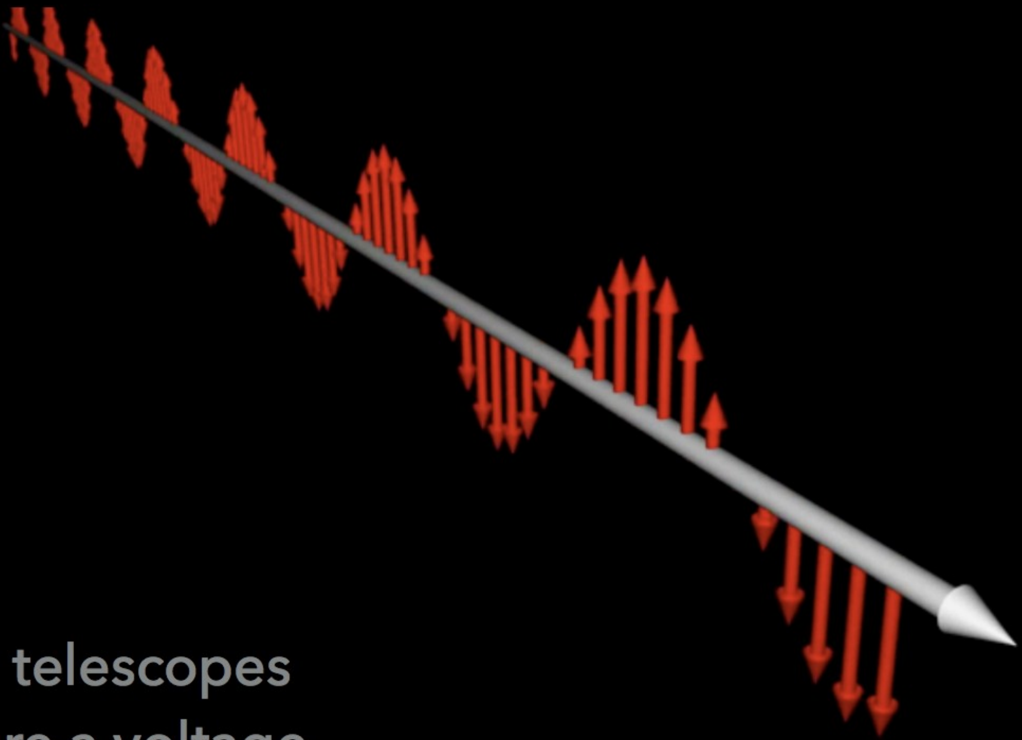
Interferometry to the rescue

$$\theta \approx \lambda / D$$

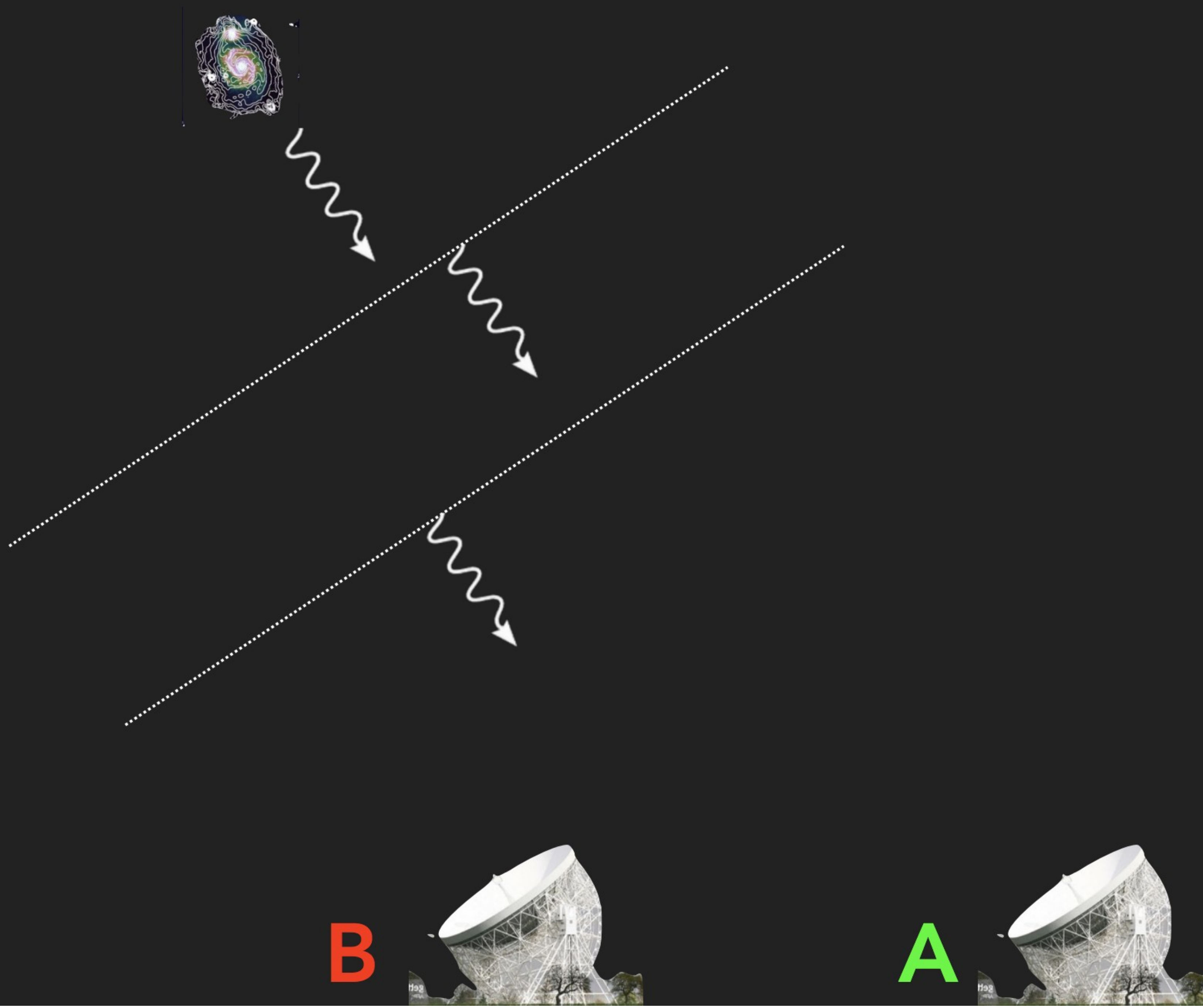
Pandora's Box

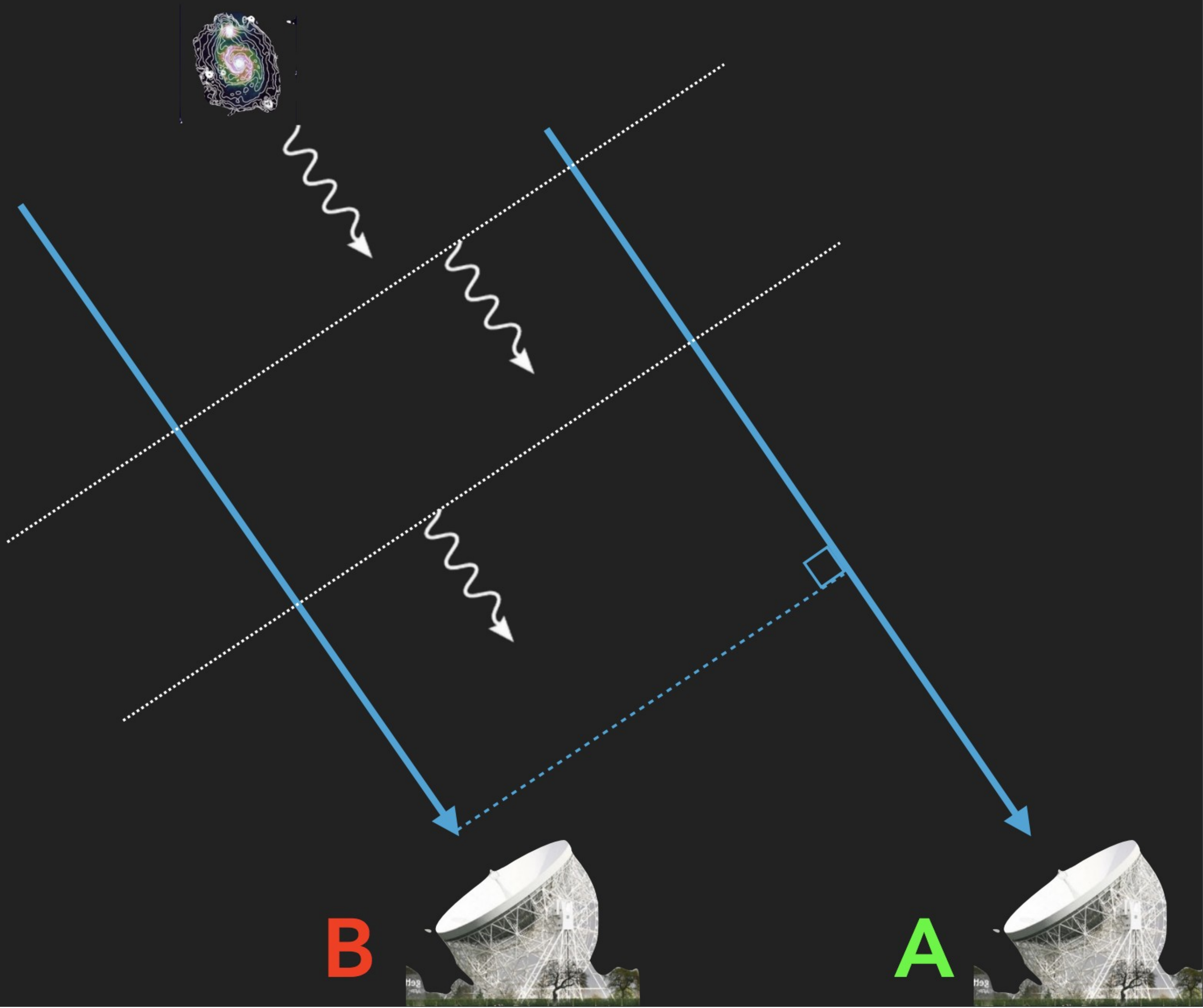


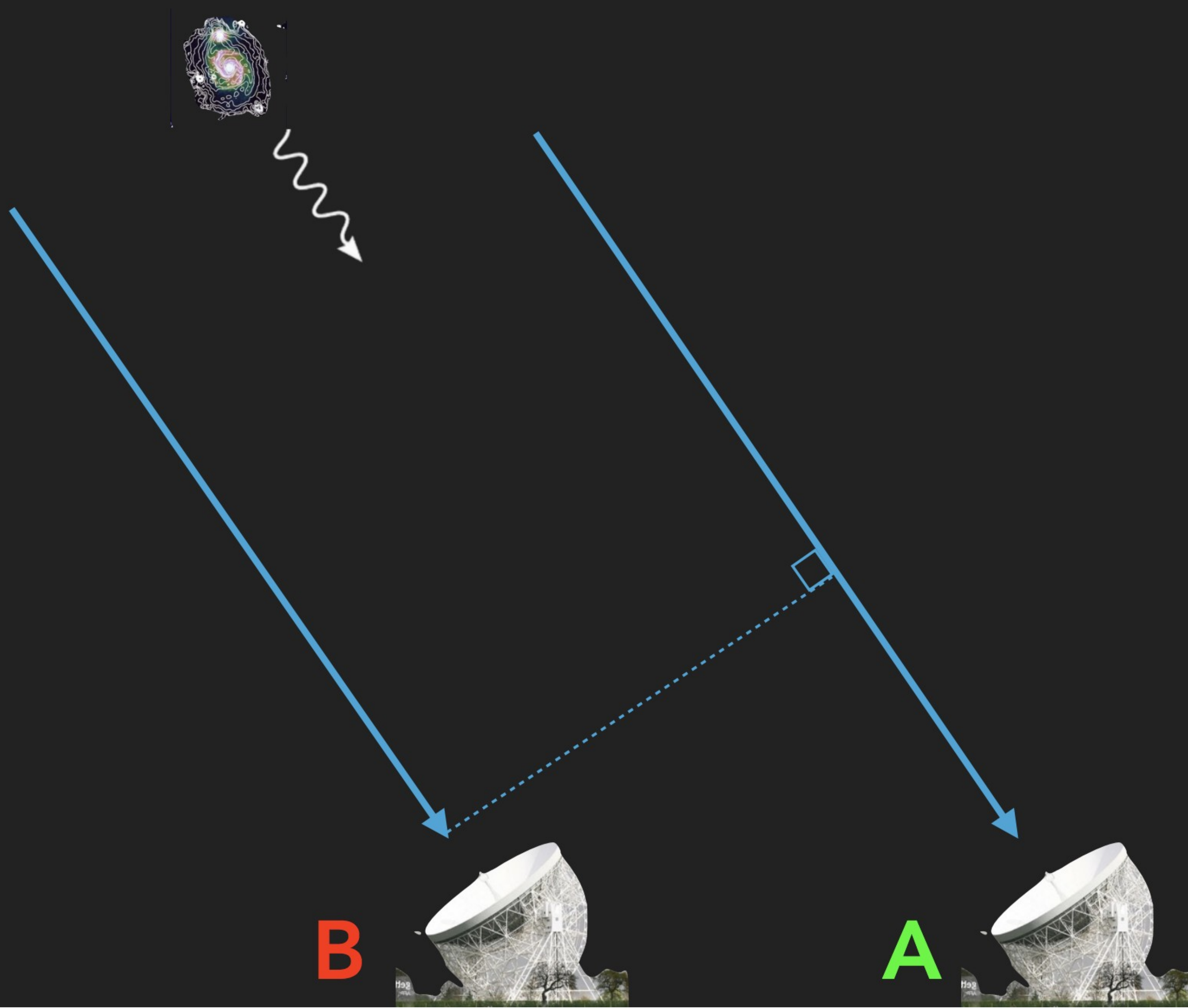
- Calibration is harder
- How do you reconstruct the image?
- What information are you missing?
- Loss of sensitivity

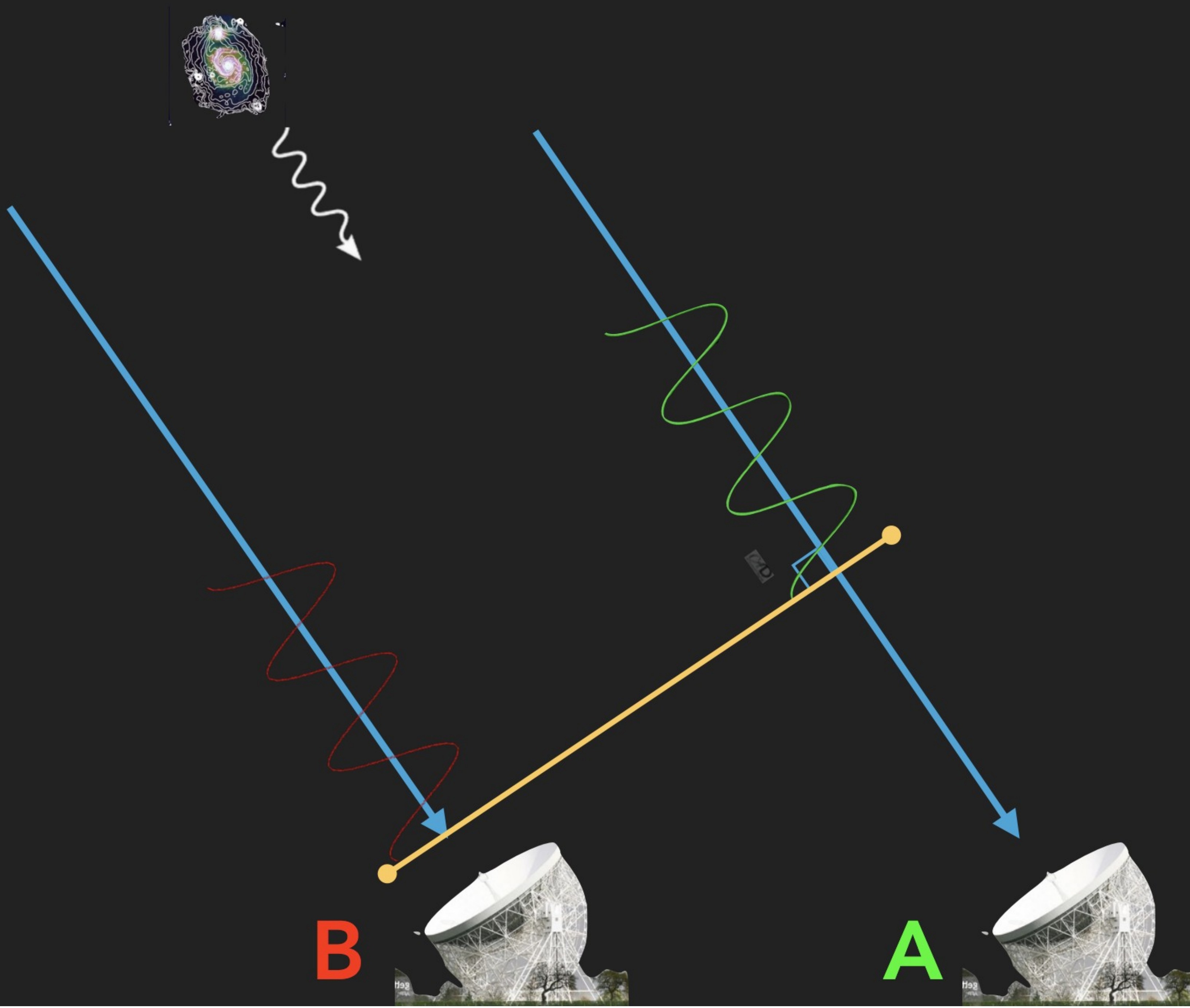


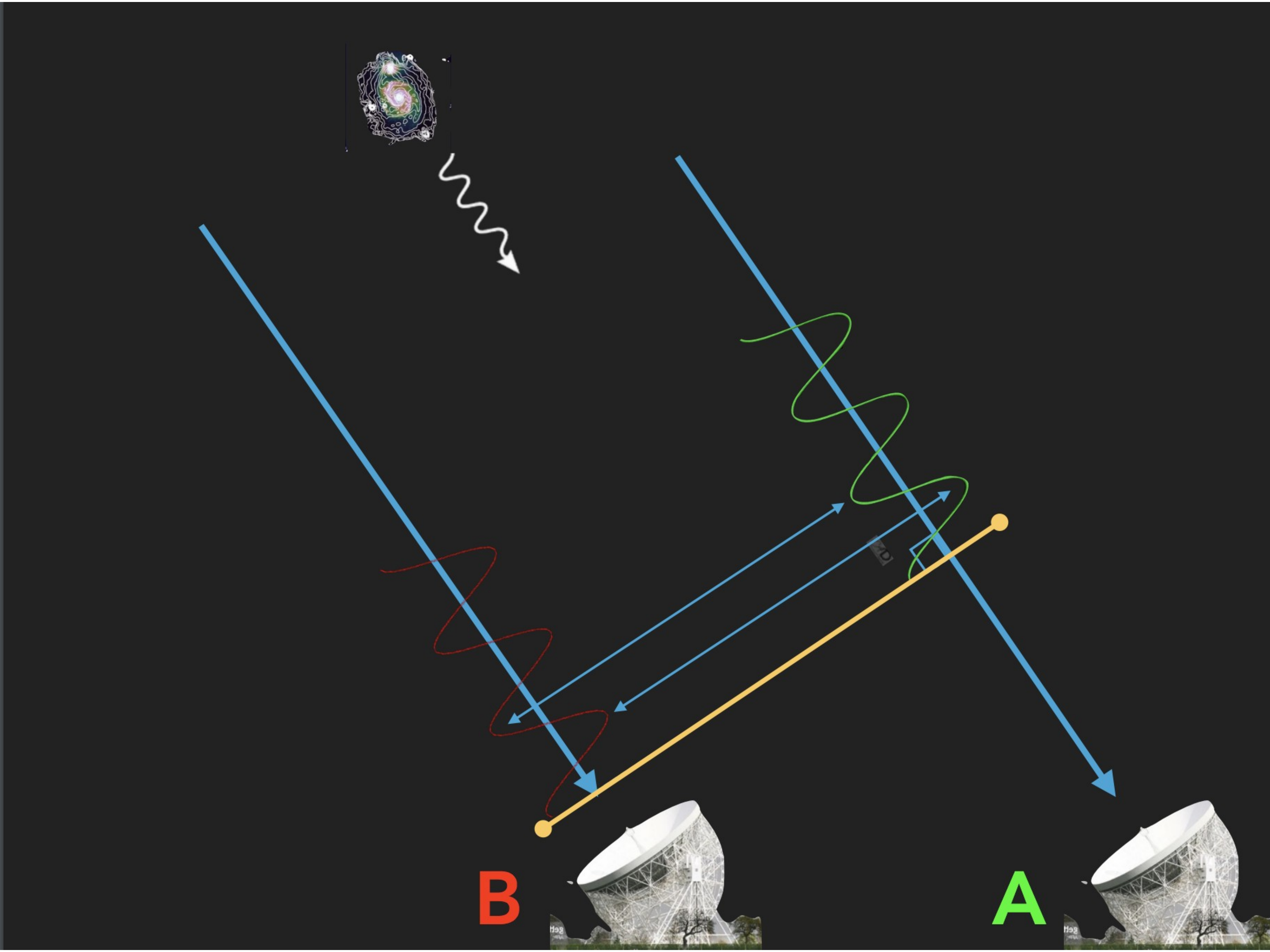
Radio telescopes measure a voltage due to the incident EM radiation

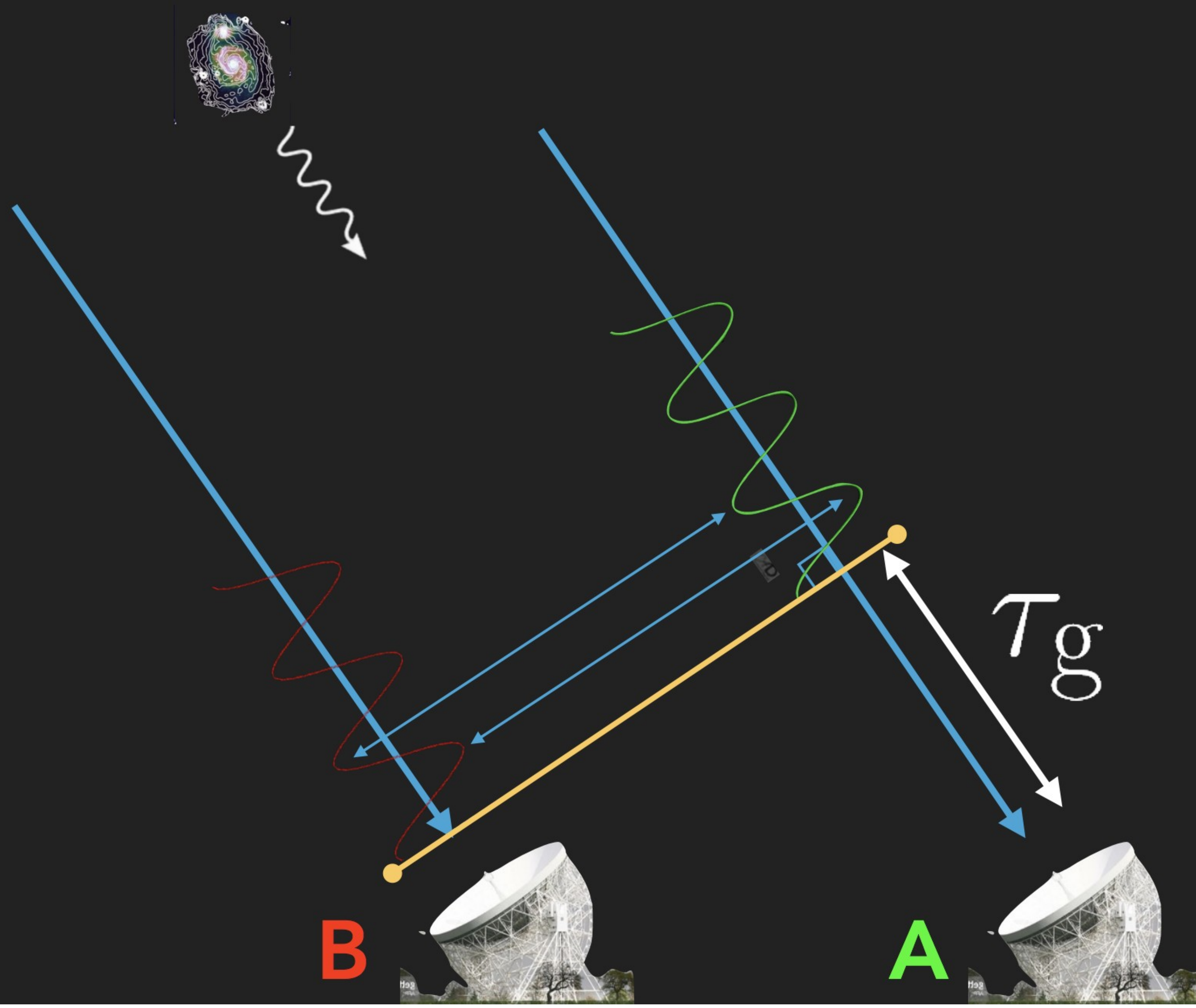

B
A


B
A

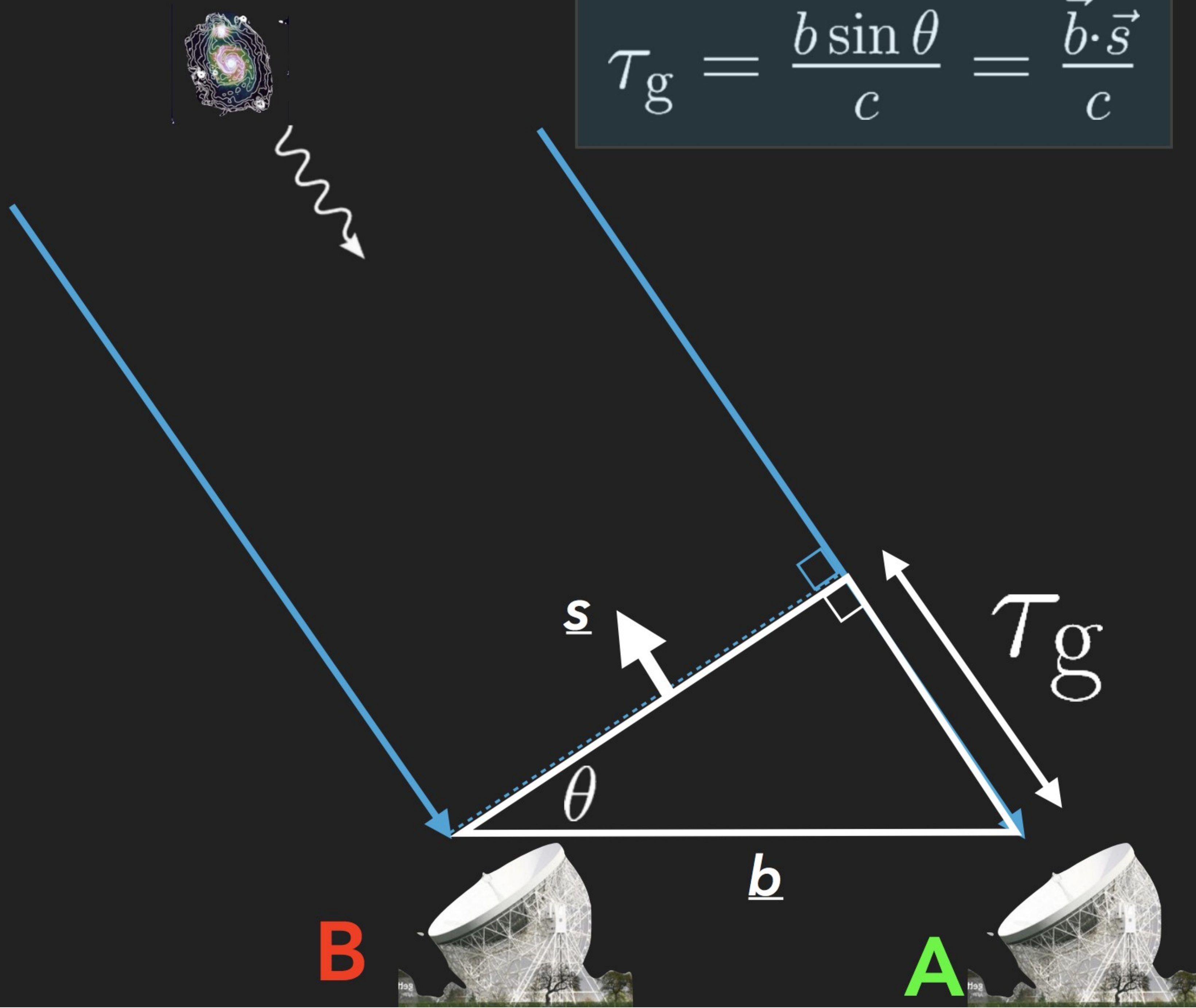

B
A


B
A

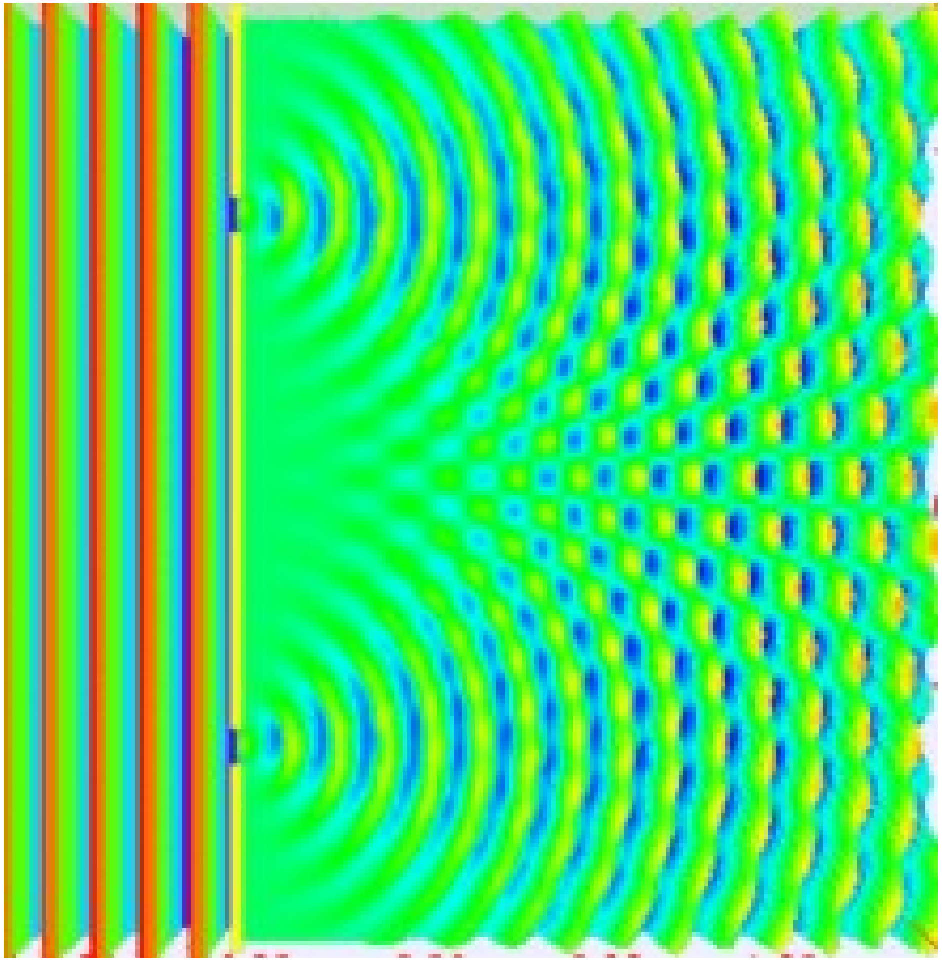

B
A

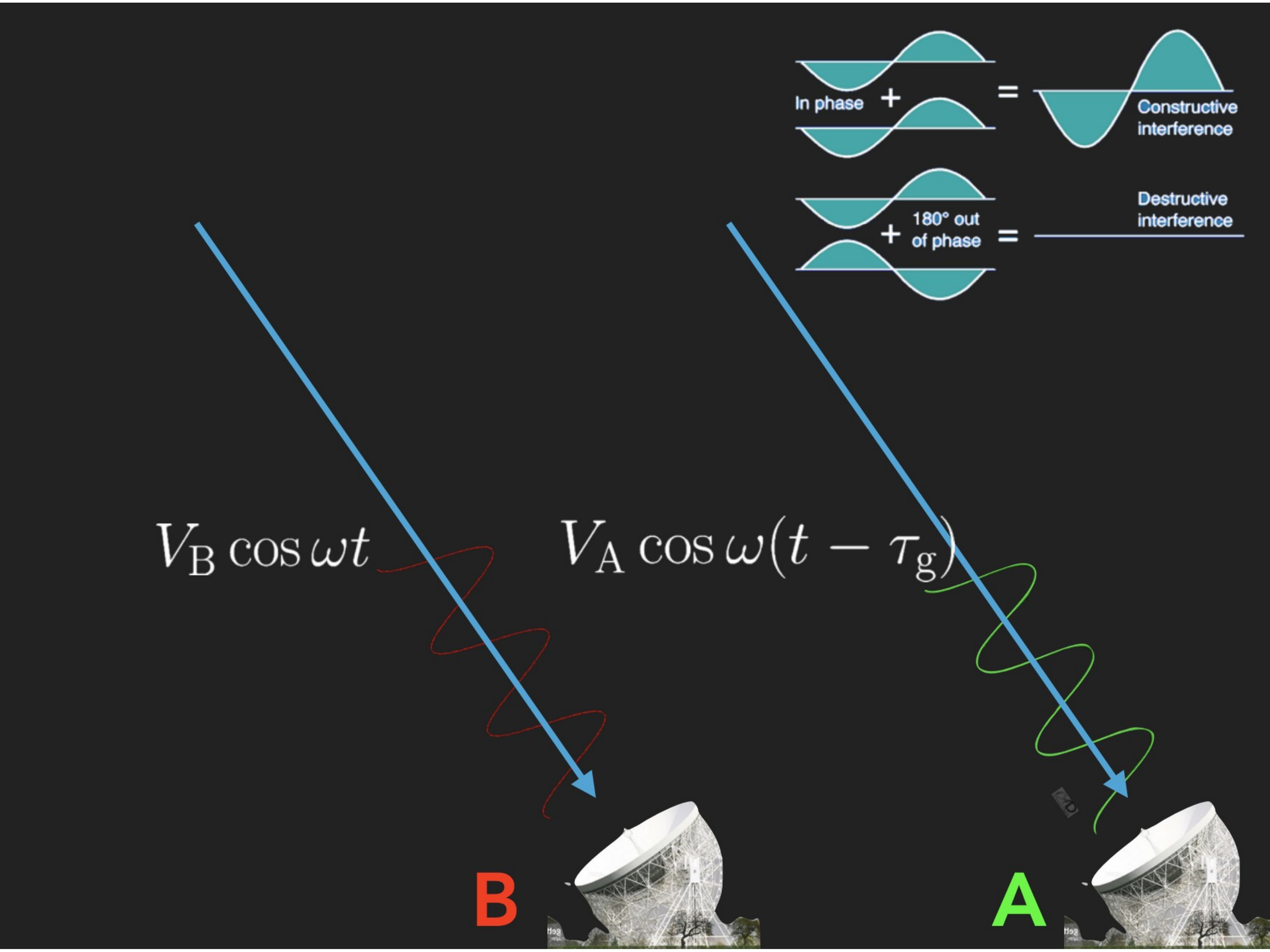

τg
B
A

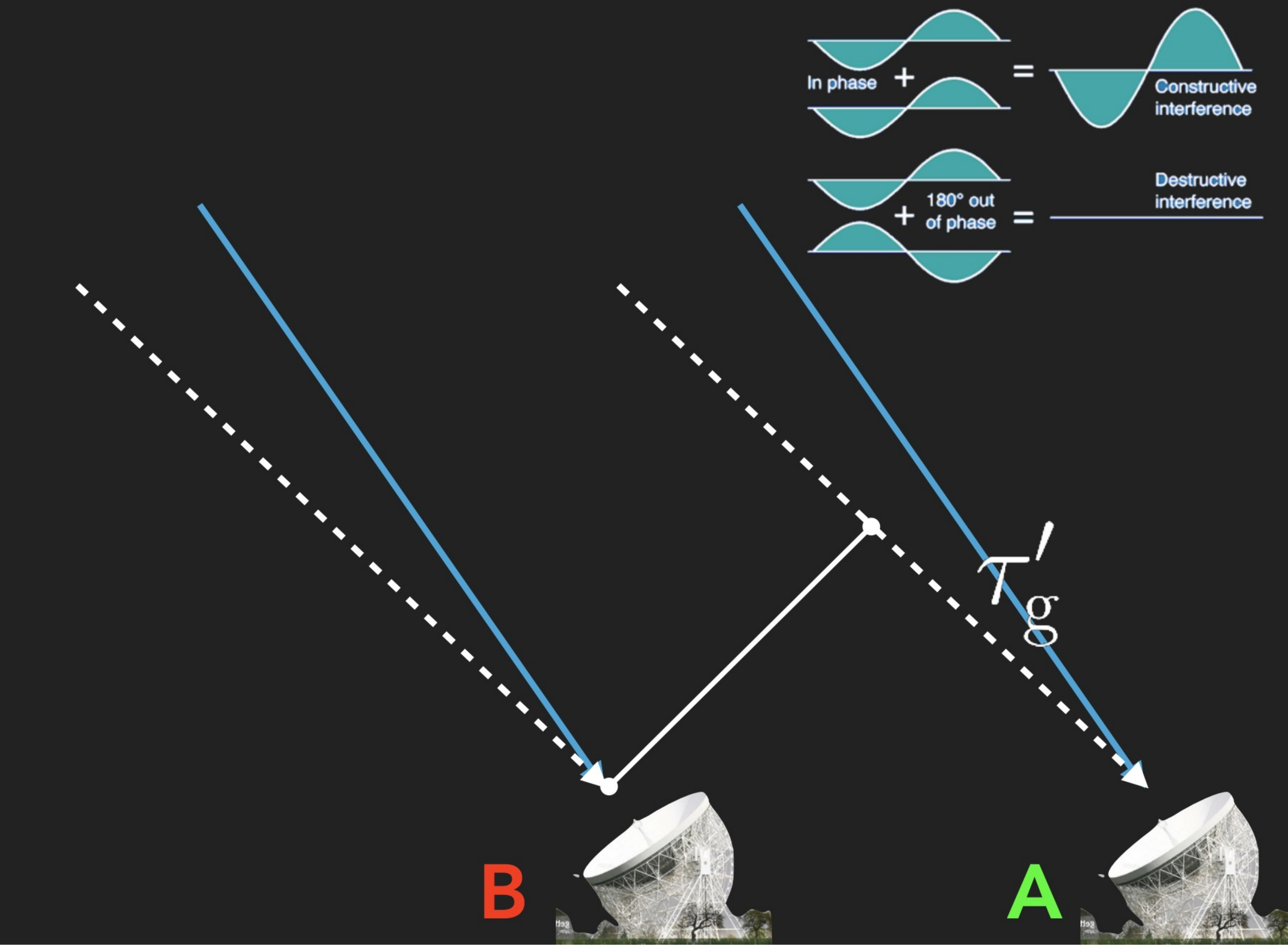
$$\tau_g = \frac{b \sin\theta}{c} = \frac{\vec{b} \cdot \vec{s}}{c}$$



Like Young's Double Slit Expertiment

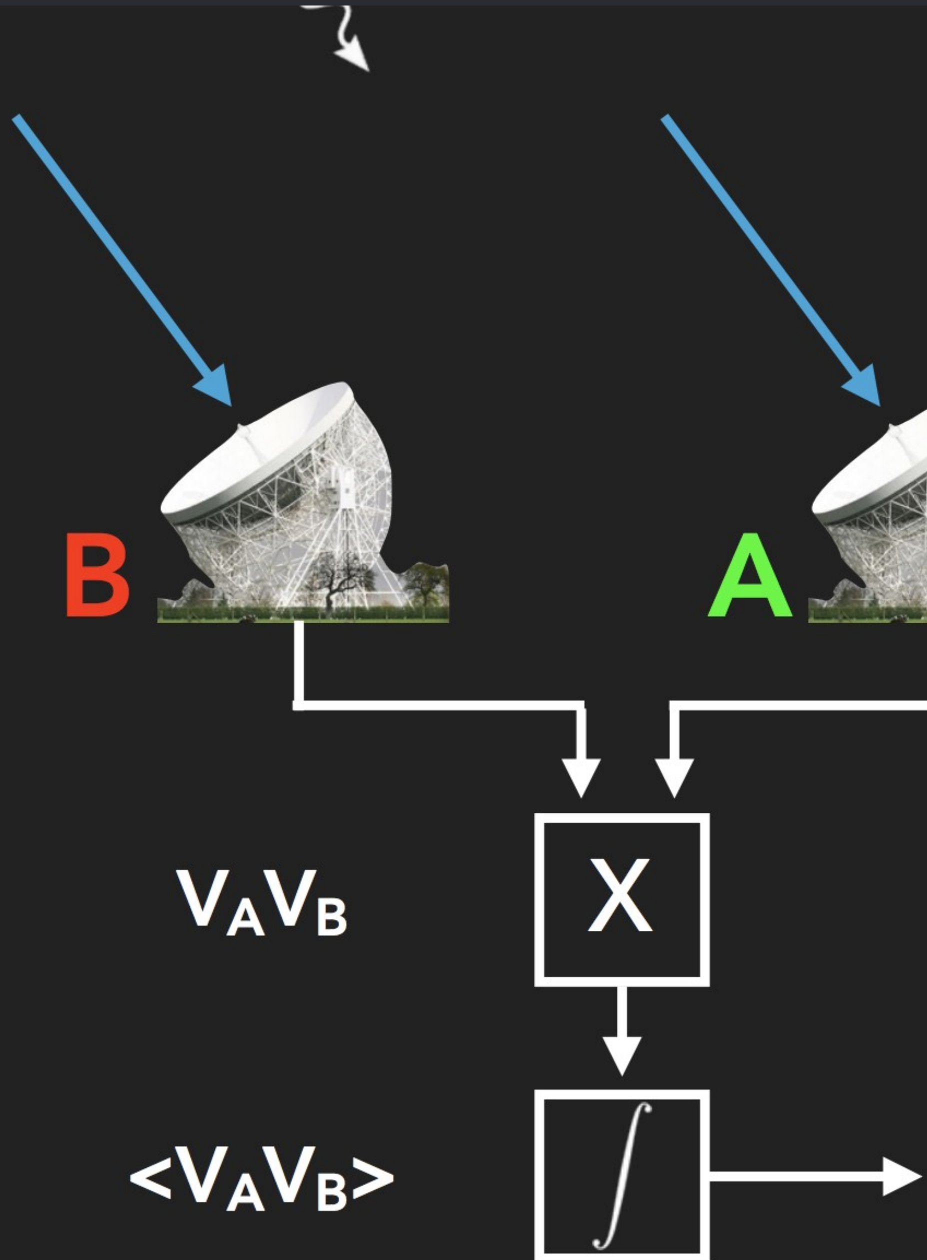



In phase
+
=
Constructive interference
180° out of phase
+
=
Destructive interference
$V_{\mathrm{B}} \cos \omega t$
$V_{\mathrm{A}} \cos \omega (t - \tau_{\mathrm{g}})$
B
A



In phase
Constructive interference
180° out of phase
Destructive interference
τ_g'
B
A

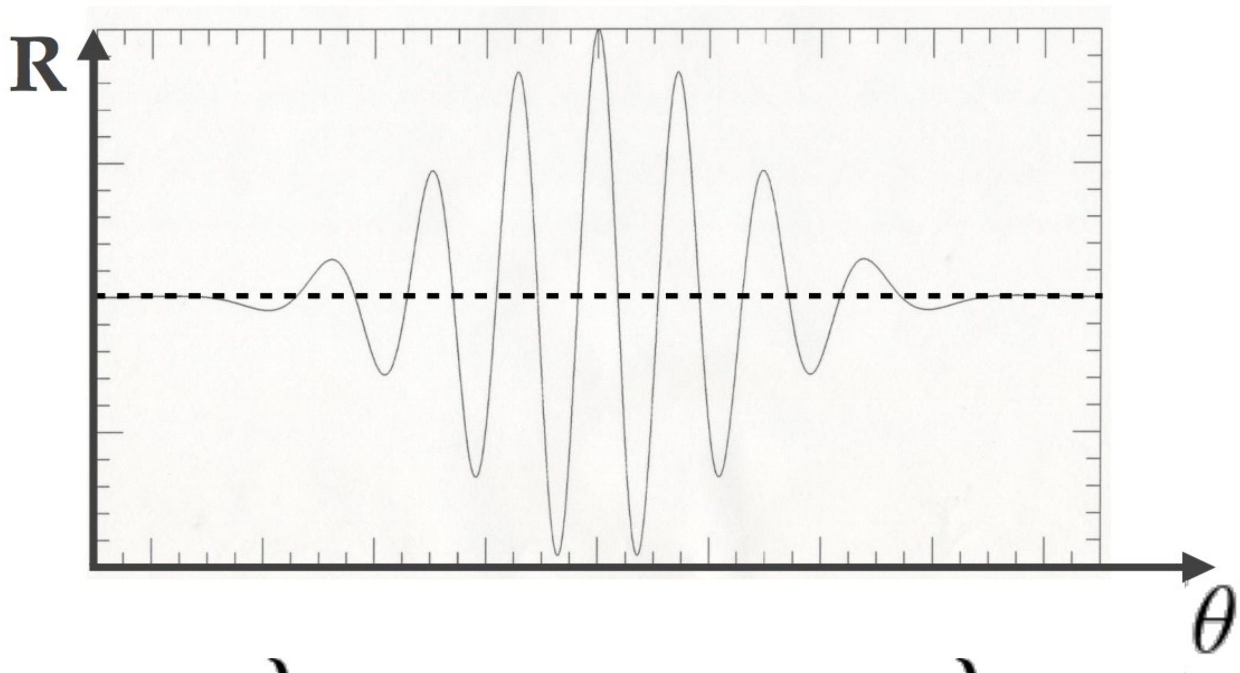
MULTIPLYING INTERFEROMETER



Correlation

MULTIPLYING INTERFEROMETER

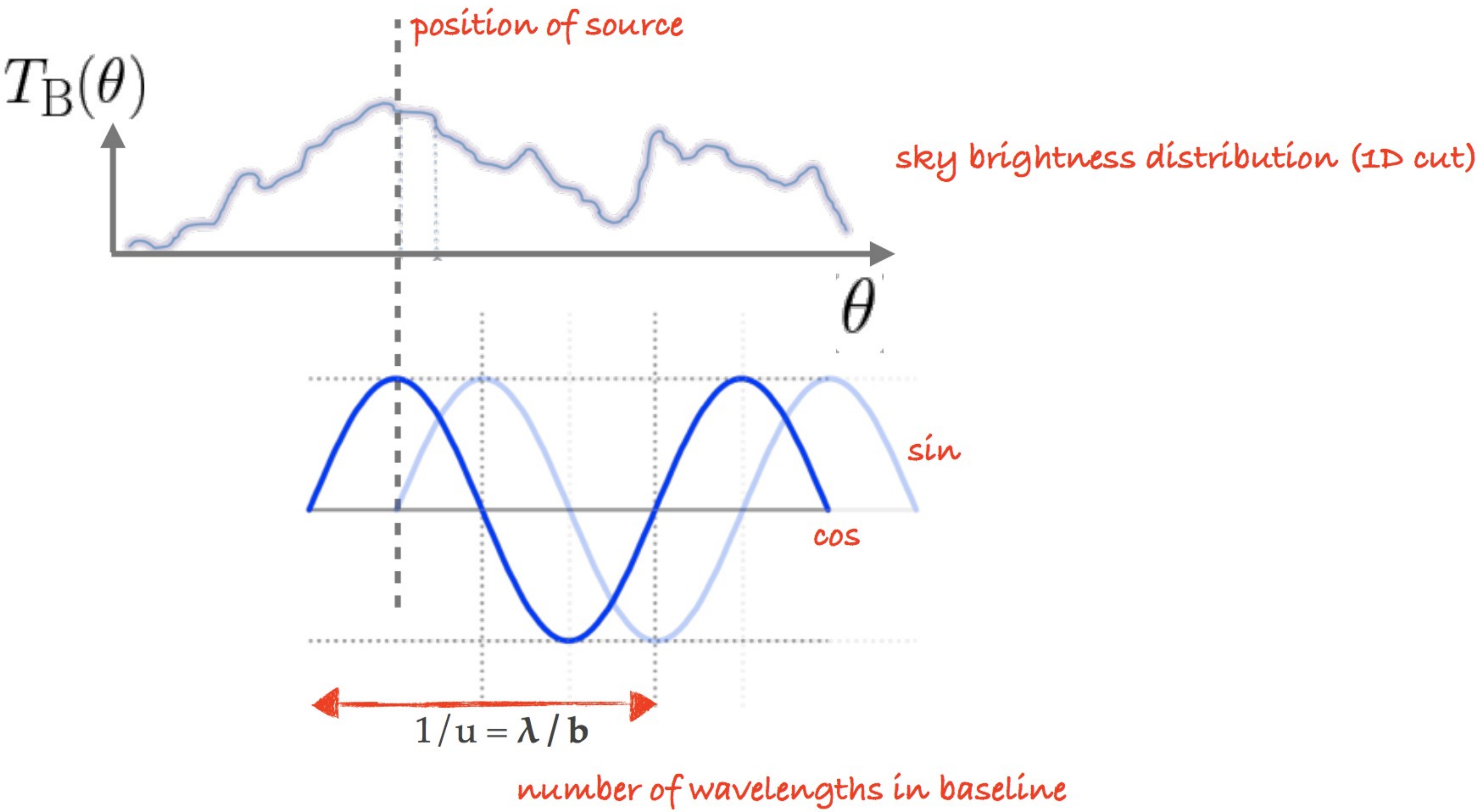
$$R \propto \langle V_{\rm A} \cos\omega(t-\tau_{\rm g}) \cdot V_{\rm B} \cos\omega t \rangle = \tfrac{1}{2} V_{\rm A} V_{\rm B} \cos\tau_{\rm g}$$



primary beam envelope: $\dfrac{\lambda}{D}$

fringe spacing: $\dfrac{\lambda}{b}$

Measuring the sky



Visibilities

In reality the response will be 2D, but in 1D for simplicity:

$$R_{\cos}(u) = \int_{\mathrm{src}} B(\theta)\cos(2\pi\, u\, \theta)\mathrm{d}\theta$$

power out as a function of baseline

$$R_{\sin}(u) = \int_{\mathrm{src}} B(\theta)\sin(2\pi\, u\, \theta)\mathrm{d}\theta$$

The sky brightness distribution is **not an even function**. If we want to reconstruct it from its Fourier components then we need **both the cos and sin terms**.

Van Cittert Zernike function

The (2-D) lateral coherence function of the radiation field in space is the Fourier Transform of the (2-D) brightness (or intensity) distribution of the source.

$$\langle V(x_1, t) V(x_2, t) \rangle = \int\int B(\theta, \phi) \mathrm{e}^{-2\pi i (u\theta + v\phi)} \mathrm{d}\theta \mathrm{d}\phi$$

$$u = \frac{(x_1 - x_2)}{\lambda} \qquad v = \frac{(y_1 - y_2)}{\lambda}$$

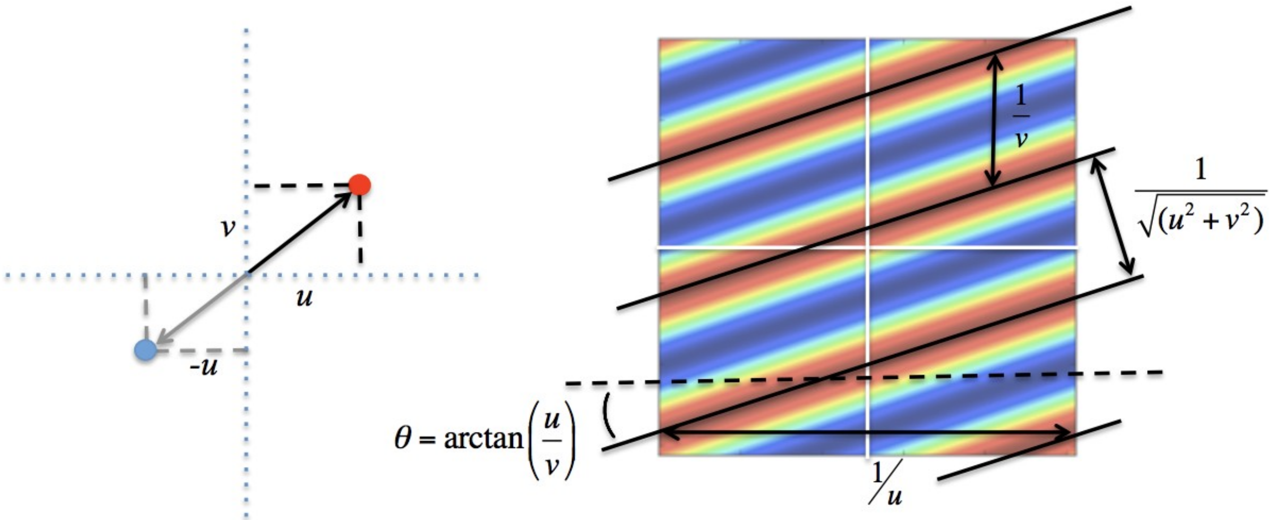
The Visibility Function is therefore another name for the spatial correlation function.

If we change our notation slightly, so that $V = Ae^{i\phi}$, we can write:

$$I_{meas}(l,m) = \frac{1}{M} \sum_{i=1}^{M} A(u_i, v_i) \cos[2\pi(u_i l + v_i m) + \phi_i]$$

Visibilities

FOURIER COMPONENTS



Writing the equation in this way allows us to visualise how our image is composed.

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \phi_i]$$

Summary

› The key to interferometry is the geometric delay

› The sky is not symmetric - we need both cosine & sine waves to make a picture of it

› Interferometers measure complex visibilities, which are the Fourier components of the sky brightness.

› Thanks to Anna Scaife, Ron Ekers, and John McKean