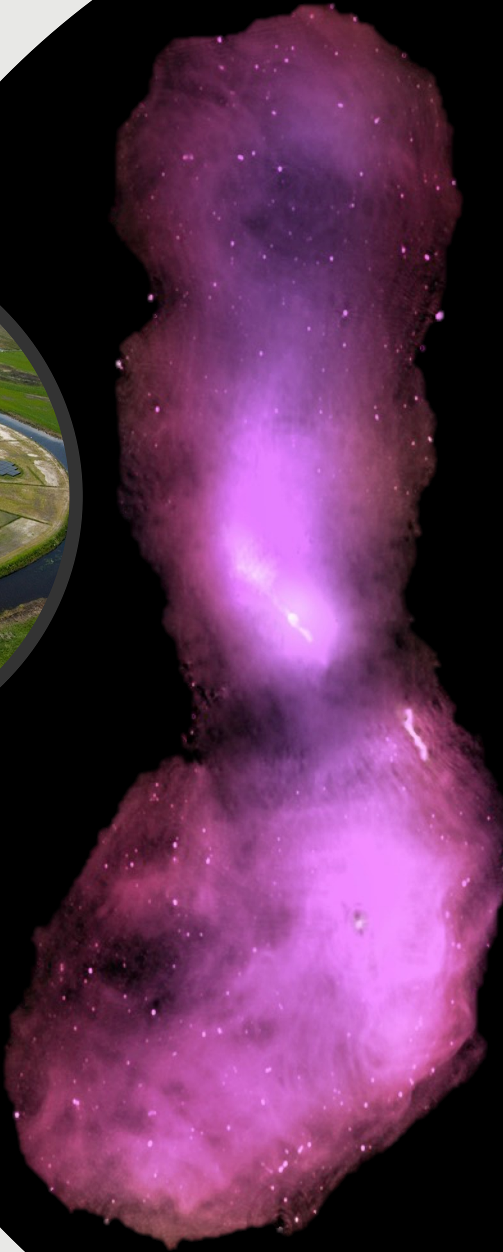
Introduction to polarisation (aka the hard stuff)



Joe Callingham (ASTRON)

Botswana Radio Astronomy School,
Palapye, Botswana
11th of July 2019

Thanks to Dave McConnell and Bob Sault

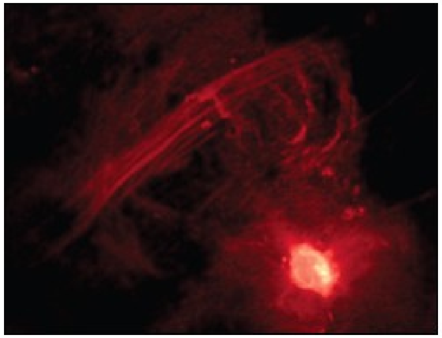
Why is polarisation info interesting?

ASTRON

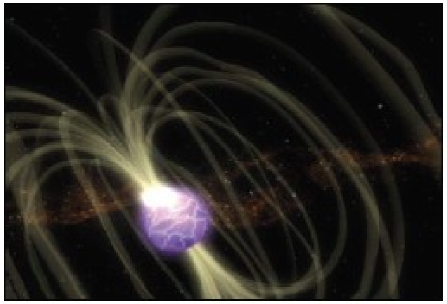
- Radio is key to measuring polarisation, better than any other wavelength

- High-z seed fields — $B \sim 10^{-30} - 10^{-20}$ G
  (Widrow 2002; Subramanian 2007)
- Intergalactic Medium — $B \sim$ 1-10 nG ?
- Intracluster Medium — $B \sim$ 0.1-1 μG
- Interstellar medium — $B \sim$ 1 μG – 10 mG
- Galactic Centre — $B \sim$ 50 μG – 1 mG
  (Crocker et al. 2010; Ferrière 2010)
- Main sequence star: HD 215441 — $B_0 \approx$ 34 kG
  (Babcock 1960)
- White dwarf: PG 1031+234 — $B_0 \approx 10^9$ G
  (Schmidt et al. 1986)
- Pulsar: PSR J1847-0130 — $B_0 \approx 9 \times 10^{13}$ G
  (McLaughlin et al. 2003)
- Magnetar: SGR 1806-20 — $B_0 \approx 2 \times 10^{15}$ G, $B_i \approx 10^{16}$ G
  (Kouveliotou et al. 1998, Israel et al. 2005)
- Cosmic strings (Ostriker et al. 1986) — $B \sim 10^{30}$ G
- Planck-mass monopoles — $B \sim 10^{55}$ G
  (Duncan et al. 2000)

Magnetic filaments in Perseus A (Fabian et al. 2008)



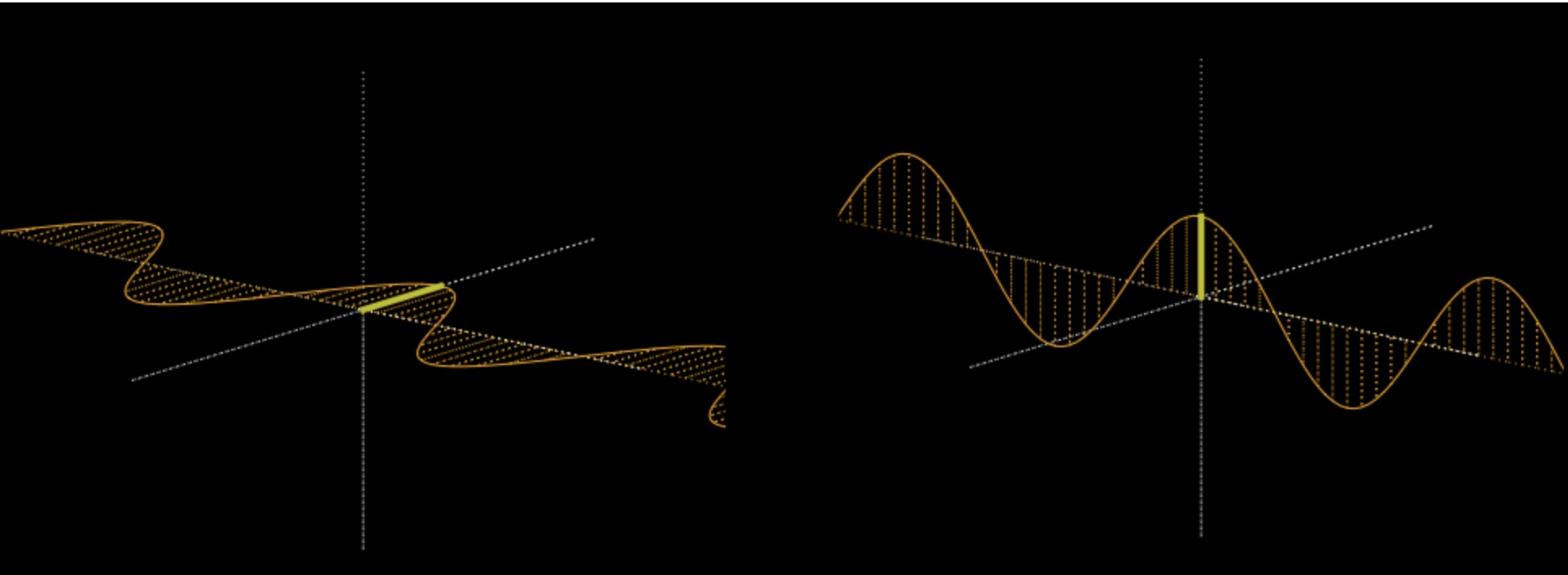
Galactic Centre (Yusef-Zadeh et al. 1984)



SGR 1806-20 giant flare (NASA)

Light can have a preferential direction

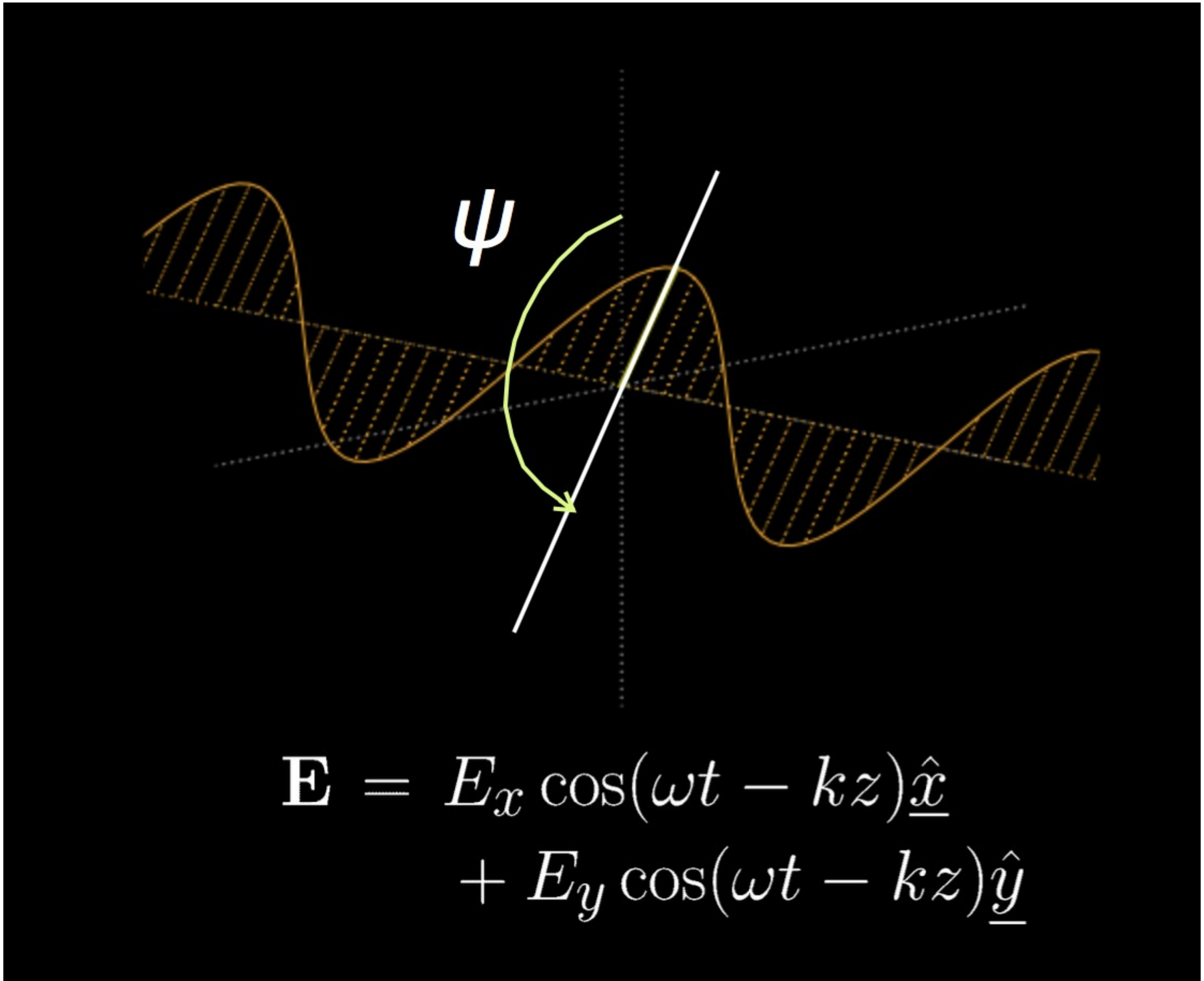
› Linear polaristion



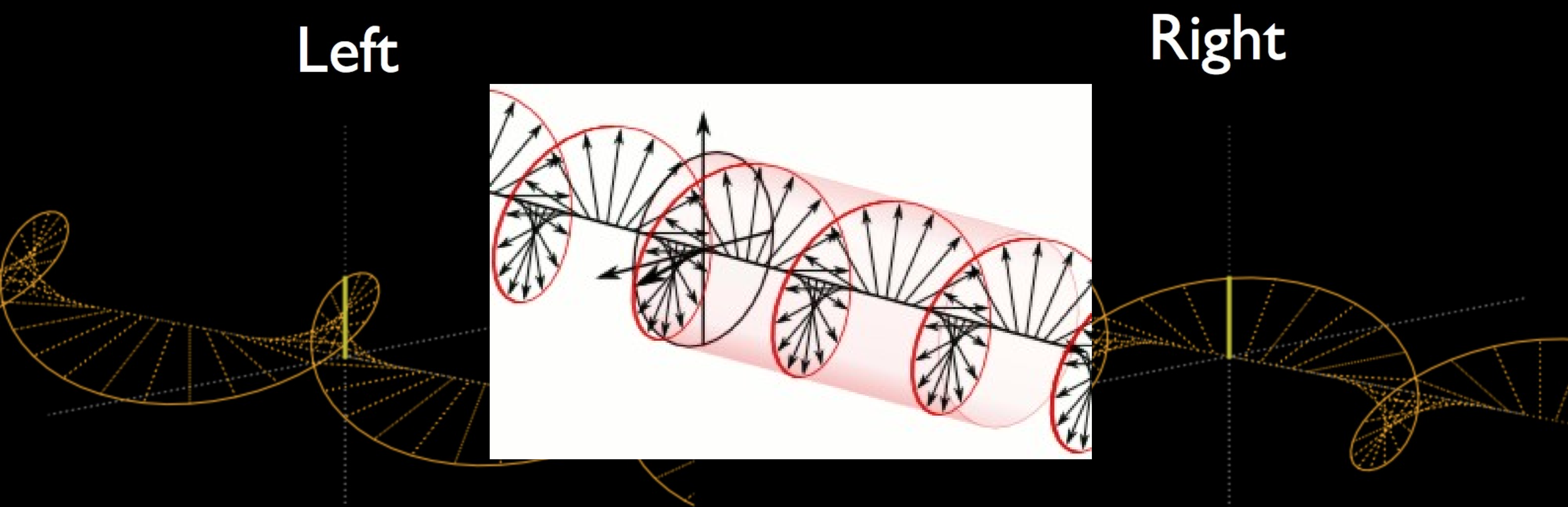
$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}}$$

$$\mathbf{E} = E_y \cos(\omega t - kz)\underline{\hat{y}}$$

And any angle inbetween



Light can also be circularly polarised



Left:

$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz + \frac{\pi}{2})\underline{\hat{y}}$$

Right:

$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz - \frac{\pi}{2})\underline{\hat{y}}$$

How to describe polarisation?

- Polarized state of light can be described by 4 parameters e.g.
  - Total power;
  - Fractional powers in horizontal and vertical linear components;
  - Phase relationship between the horizontal and vertical components.

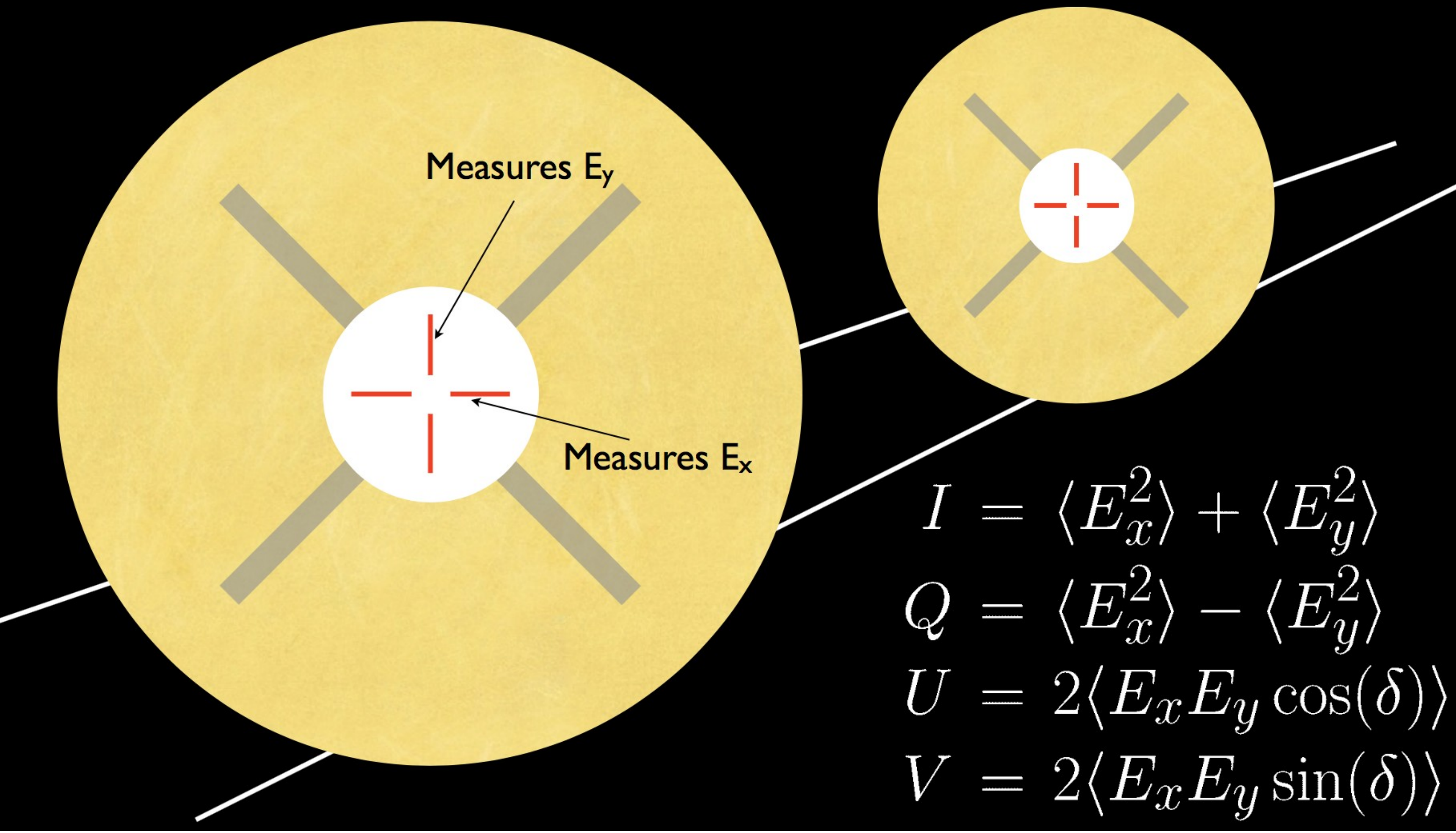
Stokes Parameters

- Ease of use over direct
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)
- Exact definition of Stokes parameters dependent on feeds of telescope.

$$I = E_x^2 + E_y^2$$
$$Q = E_x^2 - E_y^2$$
$$U = 2E_xE_y\cos(\delta)$$
$$V = 2E_xE_y\sin(\delta)$$

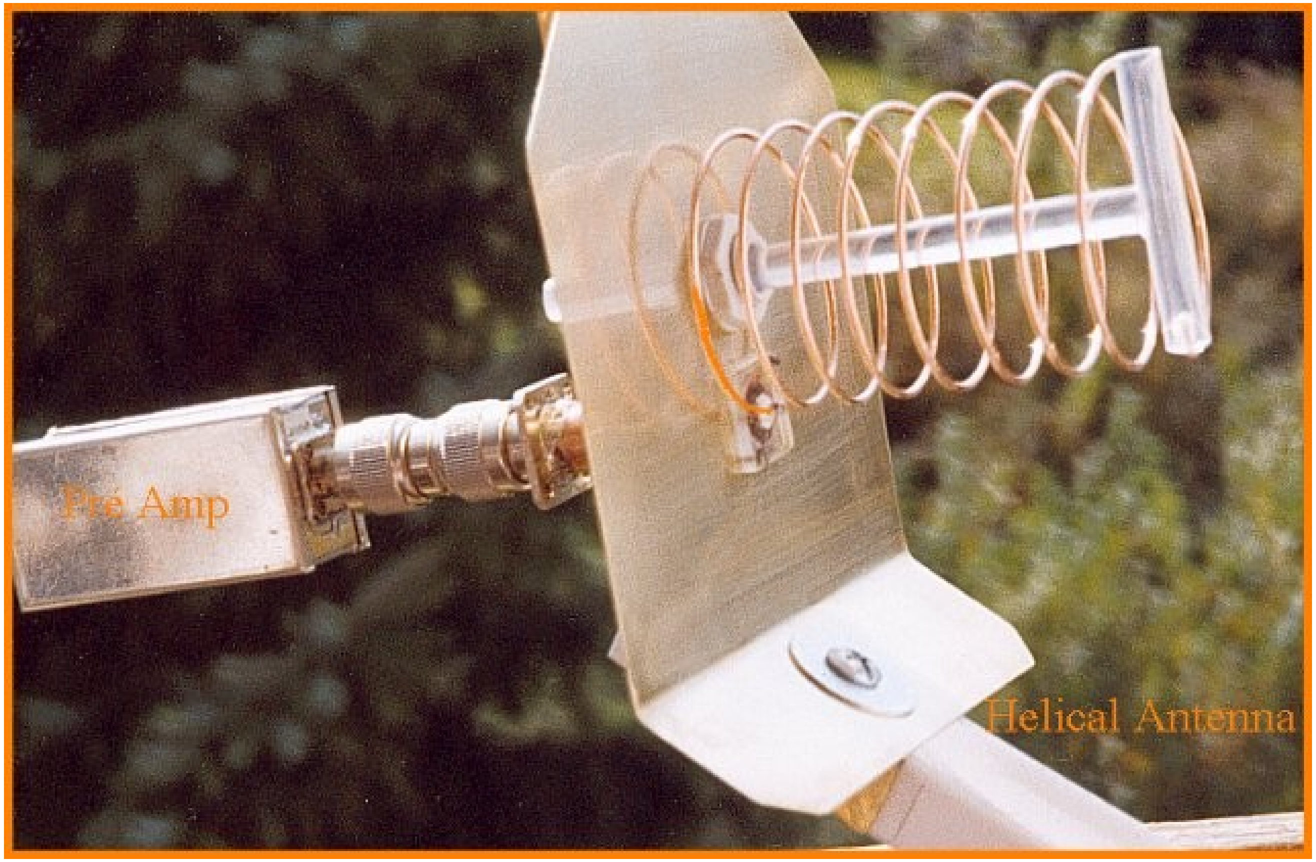
- For monochromatic waves
- I : total intensity
- Q : linear
- U : linear
- V : circular
- $I^2 = Q^2 + U^2 + V^2$

How to measure polarisation – linear feeds



$$I = \langle E_x^2 \rangle + \langle E_y^2 \rangle$$
$$Q = \langle E_x^2 \rangle - \langle E_y^2 \rangle$$
$$U = 2\langle E_x E_y \cos(\delta) \rangle$$
$$V = 2\langle E_x E_y \sin(\delta) \rangle$$

How to measure polarisation – circular feeds



Measurement of Stokes

› Conceptually use many feeds to measure the different orthogonal polarizations

› At radio frequencies voltages can be measured and correlated!

$$I = XX + YY,$$
$$Q = XX - YY,$$
$$U = XY + YX,$$
$$iV = XY - YX.$$

where XX is $< E_0 \overline{E_0} >$ and YY is $< E_{90} \overline{E_{90}} >$ ("parallel hand" correlations)
and XY is $< E_0 \overline{E_{90}} >$ and YX is $< E_{90} E_0 >$ ("cross hand" correlations).

Making stokes images

› Each antenna measures two orthogonal polarizations - X and Y.

› For every baseline, form all four possible correlations - XX, YY, XY, YX.

› Calibration and other tricks

› Appropriately combine the four correlations to get four Stokes “visibilities”.

› Perform standard imaging with these Stokes visibilities to make Stokes images.

Talking about the Jones

- Jones matrices are “antenna based”
- Antenna gain:

$$\begin{pmatrix} g_x & 0 \\ 0 & g_y \end{pmatrix}$$

- Polarization leakage:

$$\begin{pmatrix} 1 & d_x \\ d_y & 1 \end{pmatrix}$$

- Rotation:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Leakage

› For real-world feeds, the feed will respond both to the desired polarization, and to a small degree the orthogonal polarization:

$$E'_X = E_X + d_x E_Y$$

$$E'_Y = E_Y + d_y E_X$$

› The leakage ("d term") is typically ~10^{-2}. It is caused by alignment error, feed ellipticity, etc. Generic linear model (will suit practically anything).

Rotation and parallactic angle

Rotation and parallactic angle

- Alt-az mount
- For a conventional alt-az mount, the sky rotates relative to the antenna feed - ``parallactic angle rotation''

- Instrumental polarization (leakage) will be in the frame of the antenna

- Astronomical polarization will be in the frame of the sky.

- Instrumental and astronomical characteristics can be decoupled if the your observation spans a good range of parallactic angles.

Depolarisation

- Depolarization = loss of polarimetric signal.

- Caused by the system polarimetric response not being constant (i.e. varying spatially, with time, across bandwidth etc etc) smearing out the polarimetric signal.

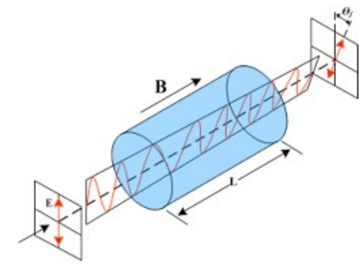
- Calibrated interferometers generally have very low depolarization (in polarimetric jargon, a system with no depolarization is called “pure”).

Some of the quanities we measure with polarisation

ASTRON

- Rotation measures – provide integrated magnetic fields through Faraday rotation of linear polarisation – integral along the line of sight of the electron density and line-of-sight magnetic field

$$\mathrm{RM} = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e(l)\, B_{\parallel}\, dl \;\; \mathrm{rad\,m^{-2}}.$$



- Polarisation vectors – provide field strength and direction in plane of sky through computation of measured Stokes Q and U to obtain linear polarisation and polarisation angle

$$P = \sqrt{U^2 + Q^2} \qquad \Theta = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

- Gradient of Stokes Q and U – provide direct imaging of interstellar turbulence – changing of magnetic field orientation with gas motions

$$|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

- Circular polarisation from synchrotron emission (it has a small, <0.1% Stokes I, component in Stokes V) – provide direct measurement of field strength and direction

$$m_{\mathrm{c}} = \epsilon_{\alpha}^{\nu} \left(\frac{\nu_{B\perp}}{\nu}\right)^{0.5} \frac{B_{\mathrm{u,los}}}{B_{\perp}^{\mathrm{rms}}}$$

Conclusions

- Huge amount of information in polarised light – an something radio astronomy (since we store phase) is uniquely suited to exploit
- It is hard… instrumental calibration, weak signals, depolarisation, many structures between us and the source
- Worth doing, but make sure your know your instrument and calibrators!