Radiation Transfer

- How does radiation interact with matter
  - Sources of opacity
    - Absorption processes
    - Scattering
- Radiative transfer equation
  - Solution
  - Radio regime solution

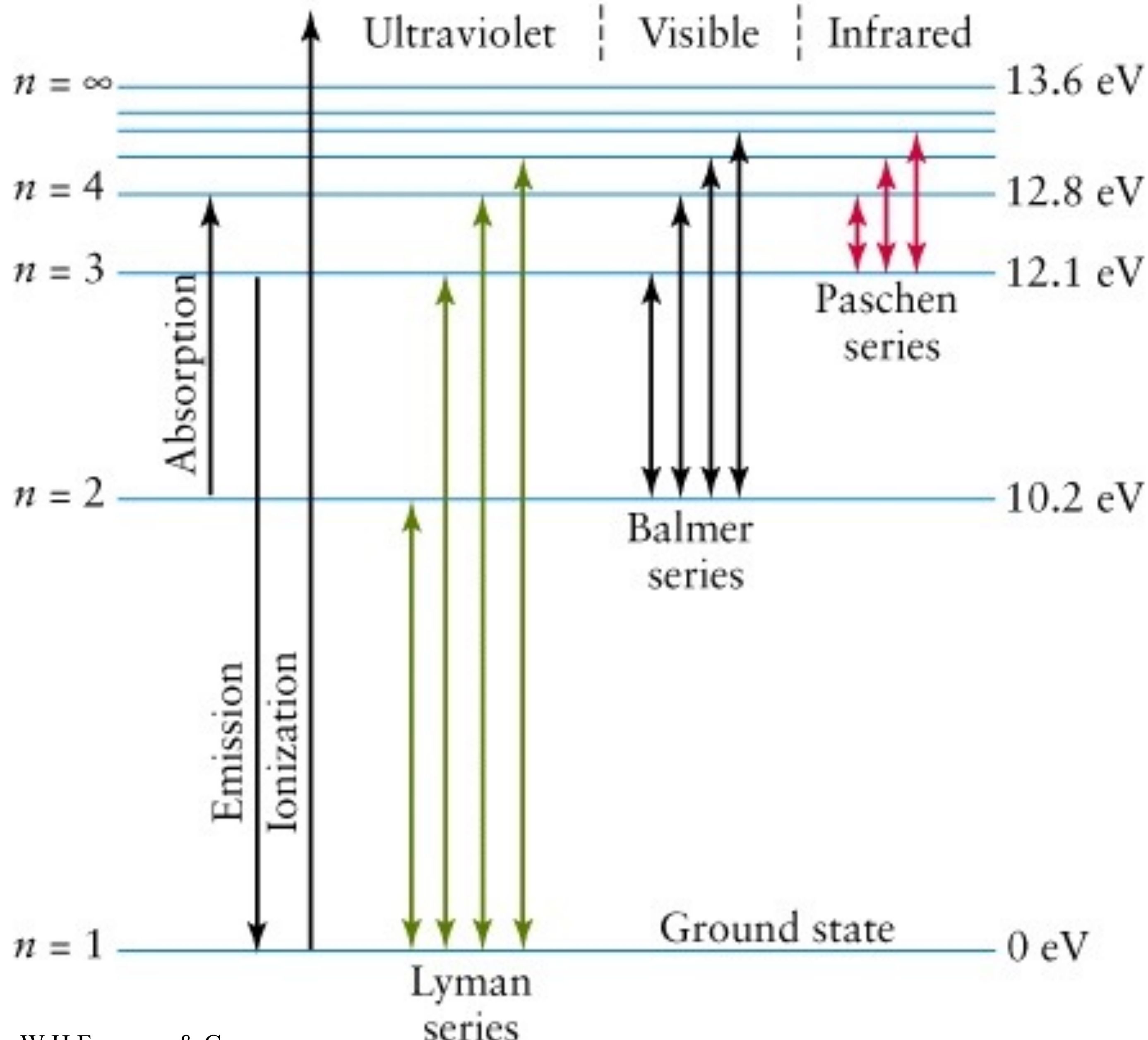
Radiation and Matter

- Radiation (photons) interact with matter (ions, atoms or molecules) either via absorption or scattering
- The quantity describing this process is called the opacity of the material

Absorption

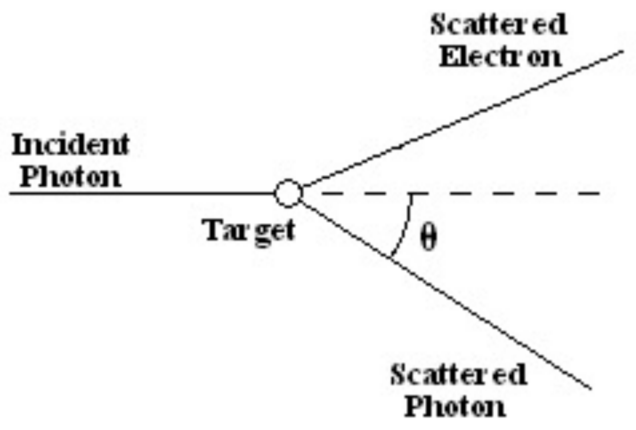
- Absorption processes
  - photon is absorbed (destroyed) (although usually re-emited a short time later, but in a random direction)
  - can be classifed by the type of electron transition taking place in the particle doing the absorbing

- bound-bound
  - photon absorbed when e^- undergoes transition between two bound energy levels in an ion, atom or molecule - spectral lines
- bound-free
  - transition from bound level to free the e^- - photoionization – continuum with thresholds
- free-free
  - e^- absorbs photon energy and is accelerated in vicinity of an ion – bremsstrahlung - continuum


Ultraviolet
Visible
Infrared
$n = \infty$
13.6 eV
$n = 4$
12.8 eV
$n = 3$
12.1 eV
Paschen series
Absorption
$n = 2$
10.2 eV
Balmer series
Emission
Ionization
Ground state
$n = 1$
0 eV
Lyman series

Scattering

- E.g. Thomson scattering by free electrons where photon is re-directed, but frequency unchanged - elastic collision – continuum



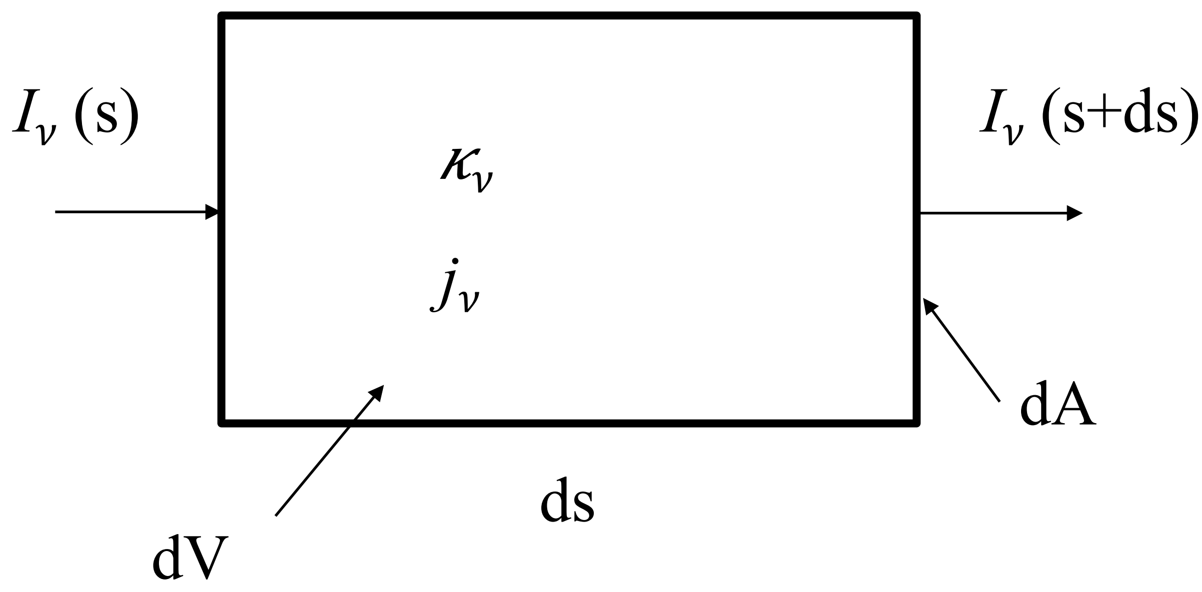
Discussion

- Can you think of examples of absorption and scattering processes for optical light travelling through the Earth's atmosphere?

Radiation Transport

- Two quantities define radiation interation
- Opacity κ_ν = fractional reduction in intensity per unit path length (m^{-1})
- Emissivity j_ν = amount of energy emitted per unit volume per unit time per unit frequency interval in to unit solid angle ($Jm^{-3}s^{-1}Hz^{-1}\ st^{-1}$)

- How does the intensity of radiation, I_ν, change when passing through a medium



Radiative Transfer Equation

$$I_\nu(s + ds) - I_\nu(s) = -\kappa_\nu \, I_\nu ds + j_\nu \frac{dV}{dA}$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu \, I_\nu + j_\nu$$

- Now define optical depth, τ_ν, as

$$d\tau_\nu = \kappa_\nu ds$$

- The radiative transfer equation can be written as

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where

$$S_\nu = \frac{j_\nu}{\kappa_\nu}$$

is known as the source function.

Formal Solution

- The formal solution to this equation is

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu e^{-(\tau_\nu - \tau_\nu^{'})} d\tau_\nu^{'}$$

- If the source function is constant along the path and there is no incident radiation ($I_\nu(0)=0$) then it becomes

$$I_\nu(\tau_\nu) = S_\nu(1 - e^{-\tau_\nu})$$

Optional Exercise

- What is the solution of the radiative transfer equation when there is only absorption and no emission, i.e.

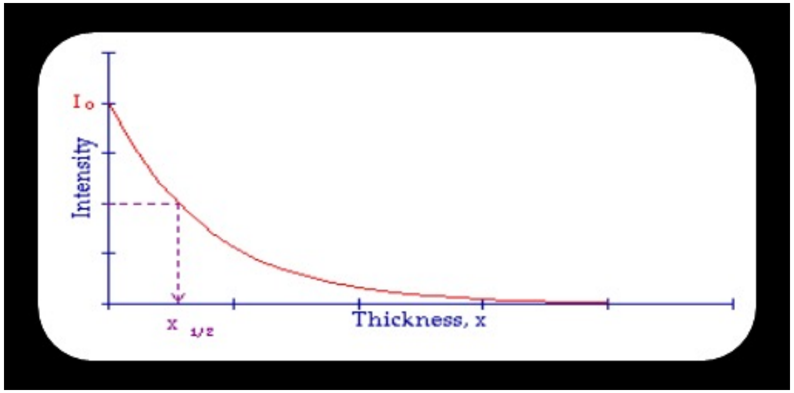
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu$$

$$\frac{dI_\nu}{I_\nu} = -d\tau_\nu$$

$$\int \frac{dI_\nu}{I_\nu} = -\int d\tau_\nu$$

$$lnI_\nu = -\tau_\nu + C$$

$$I_\nu = I_0 e^{-\tau_\nu}$$



Optical Depth

- This means that if the medium is optically thick (τ_ν>>1)

$$I_\nu(\tau_\nu) = S_\nu$$

- And if it is optically thin (τ_ν<<1)

$$I_\nu(\tau_\nu) = S_\nu \tau_\nu$$

Source Function

- In thermal equilibrium the source function is just the Planck function

$$S_\nu = B_\nu$$

- and in the radio regime we can use the Rayleigh-Jeans approximation so

$$S_\nu = B_\nu = \frac{2kT\nu^2}{c^2}$$

Brightness Temperature

- The Rayleigh-Jeans approximation also means that we can use temperature as a measure of intensity
- Brightness temperature T_B is the temperature a blackbody would have to give the same intensity

$$I_\nu = \frac{2kT_B\nu^2}{c^2}$$

RT Equation in Radio Regime

- Rewrite the transfer equation such that

$$T_B = T(1 - e^{-\tau_\nu})$$

where T is the temperature of the emitting region

- Now when optically thick (τ_ν>>1)

$$T_B = T$$

- And if it is optically thin (τ_ν<<1)

$$T_B = T\tau_\nu$$

Summary

- Opacities determine how photons interact with matter
- The radiative transfer equation can be used to predict the intensity of radiation we observe given the physical conditions
- Optical depth and brightness temperature are useful concepts