MANCHESTER 1824

The University of Manchester

DARA Training Unit 1 2018
Radioastronomy
Lecture 3

Radio Telescopes

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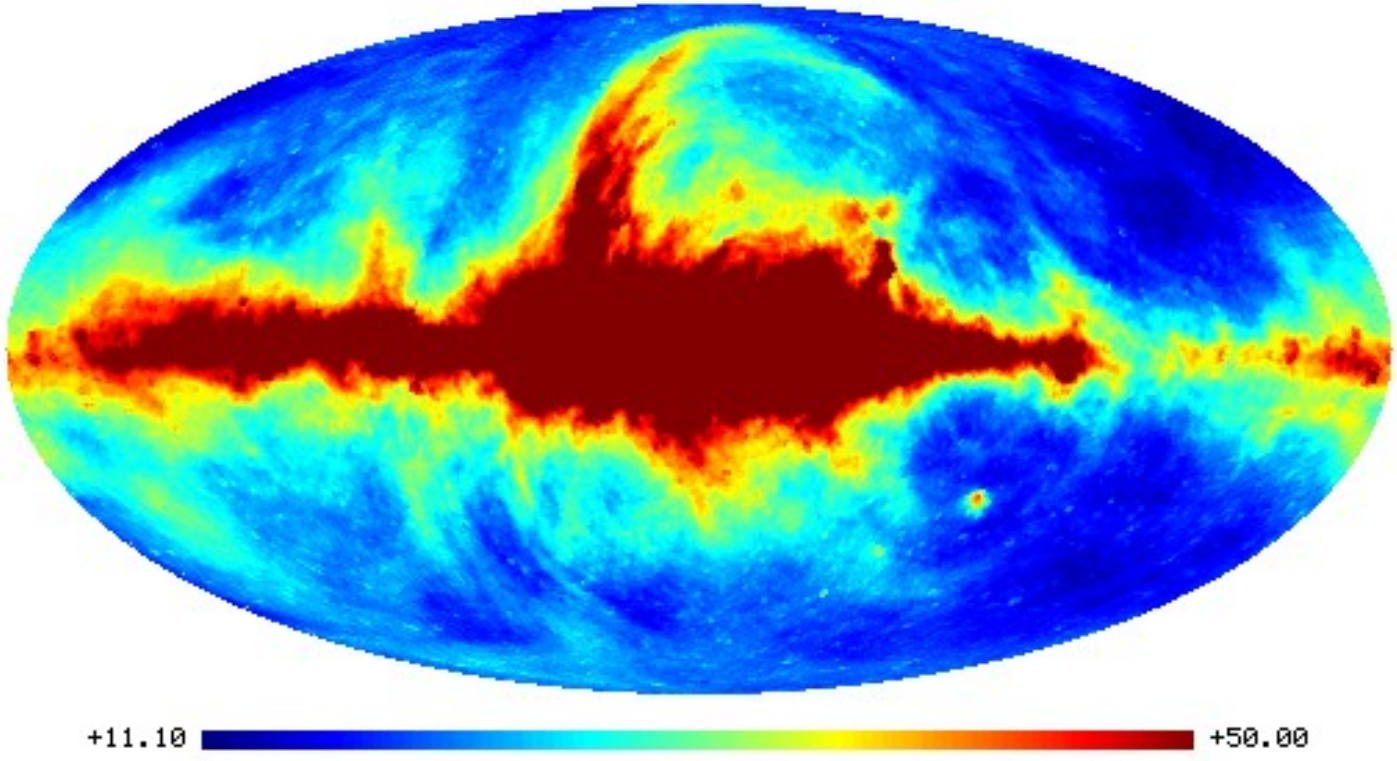
Jodrell Bank Centre for Astrophysics

Note 1) this presentation has been put together from more than one presentation given during PW's Manchester lecture course in 2014. Hence narration references to other slides and/or references to material in other lectures are not always correct. I do not think that this greatly affects its usefulness.
2) Slides removed from original presentations to save time for DARA are retained after "THE END"
3) The handwritten slides (referred to as Notes A, B etc) were written in real-time in Manchester lectures by PW so that students had to pay attention and write this material down in their own notebooks in order to achieve a full set of course material– this scheme also mitigated the deadly "Death by Powerpoint" disease and kept students awake.

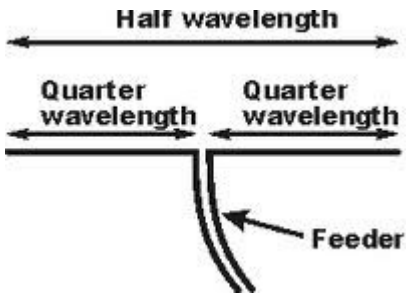
This lecture is aiming towards…

Understanding how the classic map of the entire radio sky at 408 MHz was made



Made >30 years ago with a combination of the Jodrell Bank (76m); Effelsberg (100m Germany); Parkes (64m Australia) radio telescopes by G. Haslam et al.

Radio Antennas

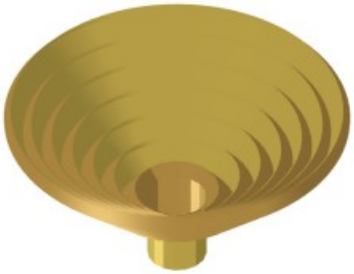


Basic dipole

Yagi antenna

Long Wavelength array (USA)

"Wire" types (many !) operate at low frequencies typically < 500 MHz



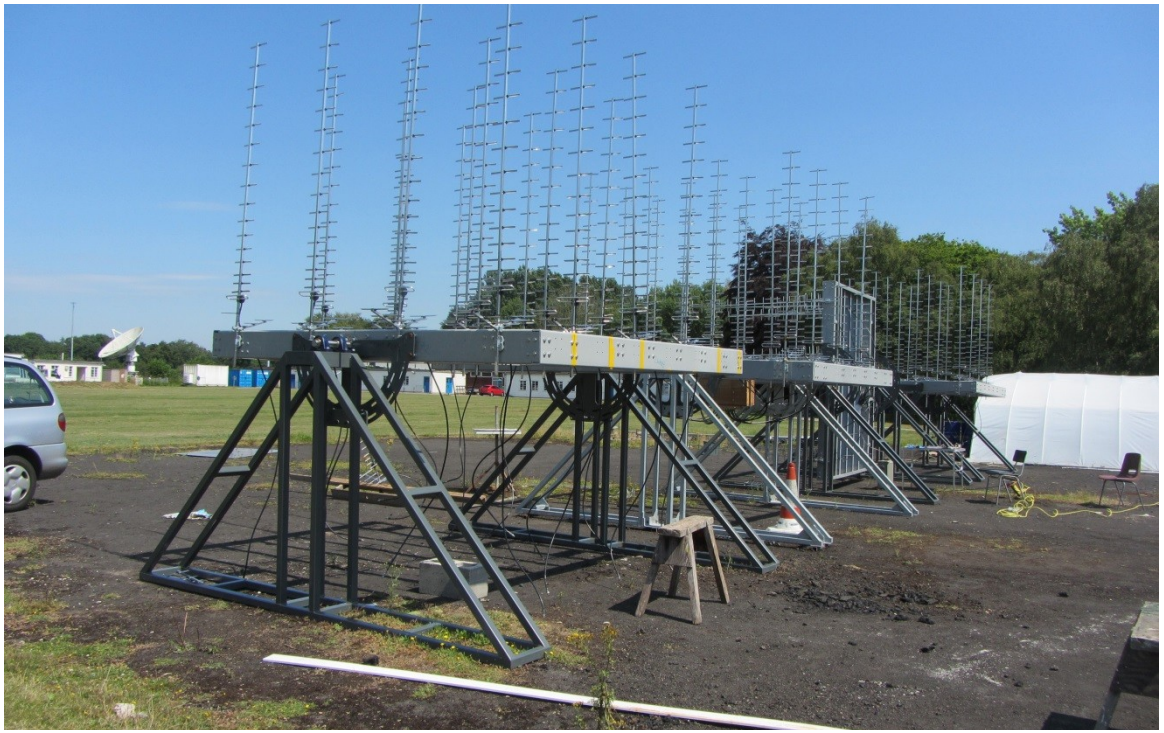
Circular horn

Giant horn

Parabolic reflectors with horns at prime focus

Filled aperture types (many!) operate at frequencies typically >500 MHz

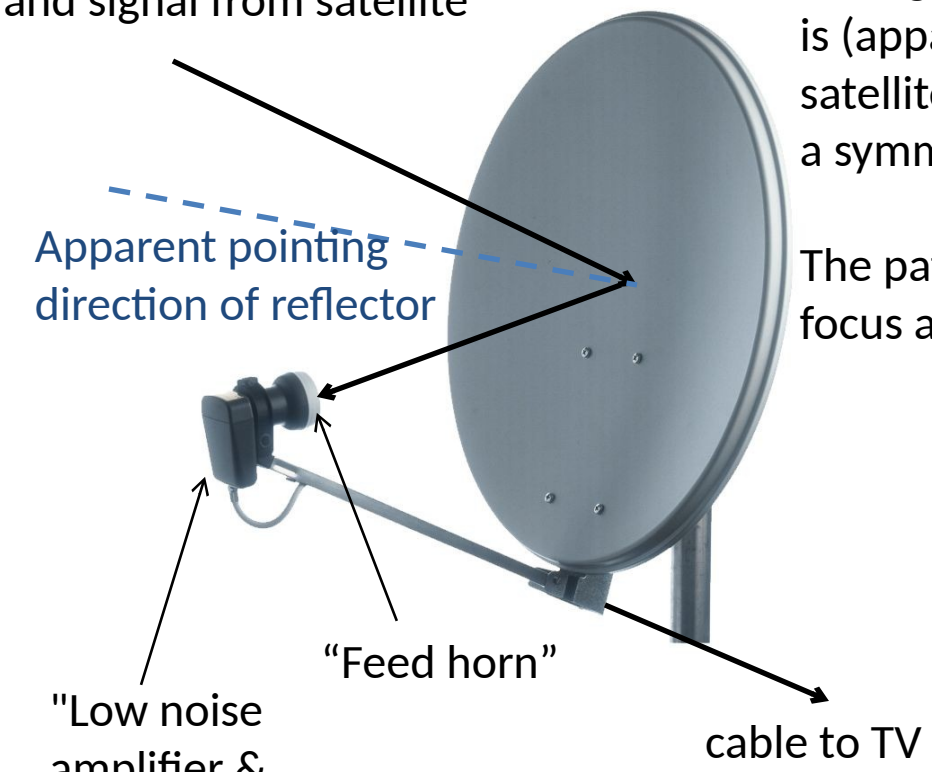
Student antennas at Jodrell Bank



Manchester **U**niversity **S**tudent **T**elescope (**MUST**)
- frameworks of Yagi antennas connected together.

A “prime focus” parabolic antenna diameter 3.1 metres

A satellite TV installation



This is a prime focus system and the antenna is parabolic – but note its asymmetrical "offset" configuration which means that the dish is (apparently) not pointing in the direction to the satellite. Think of the reflector as just one section from a symmetrical "dish" – most of which is not there!

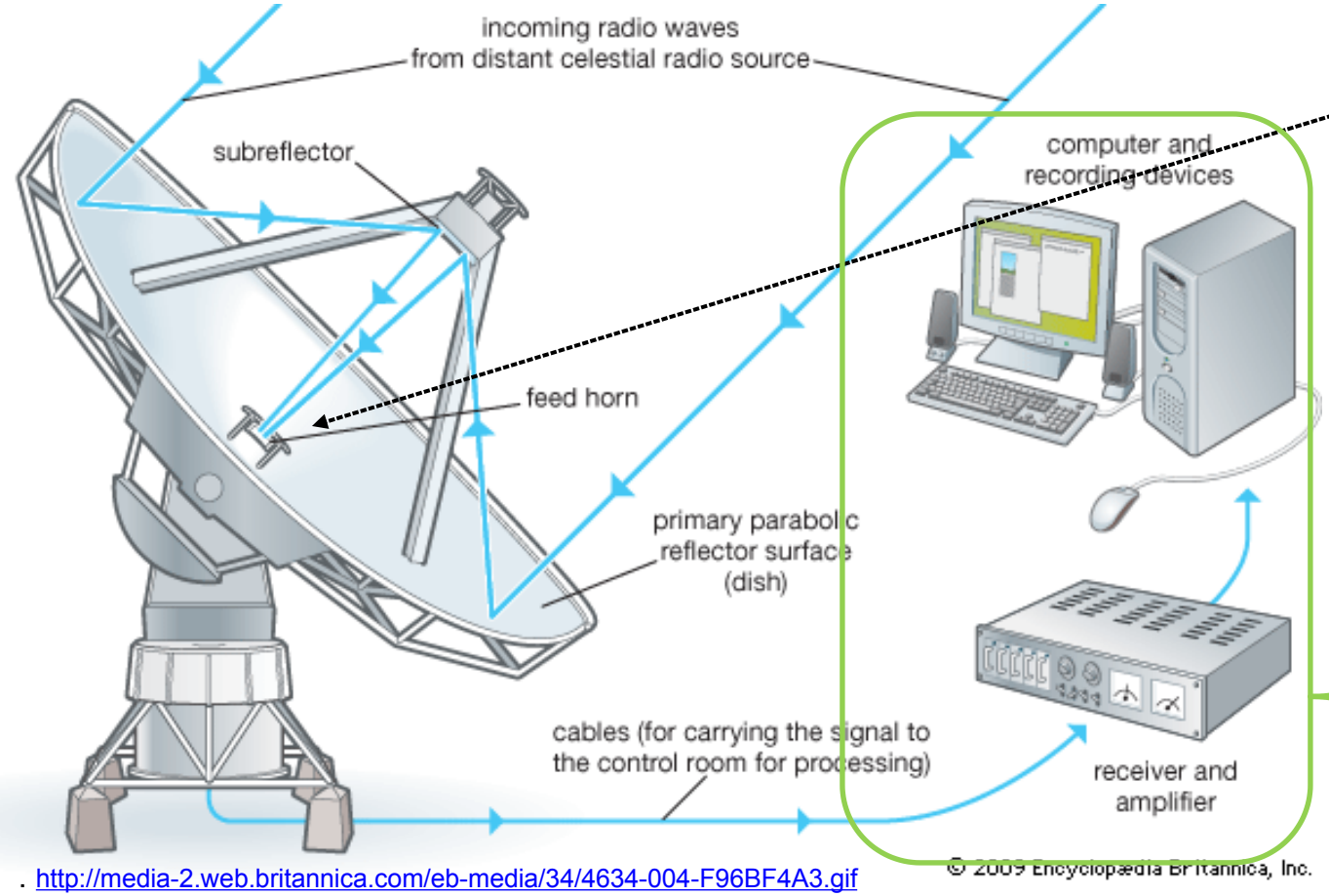
The path of radiation to the dish is not blocked by the focus arrangements → less loss and lower sidelobes

The 100-m diameter Green Bank telescope In West Virginia USA is an "offset" design

http://www.inter-sat.tv/satellite-dish-65-al-gibertiny-e-light-gray_ies6007.jpg

A schematic radio telescope system

We will concentrate on parabolic reflectors – although the concepts are generic. The system collects emission from a limited range of angles (the "beam") and turns it into electrical signal – a fluctuating voltage which is amplified, filtered and then its power measured.

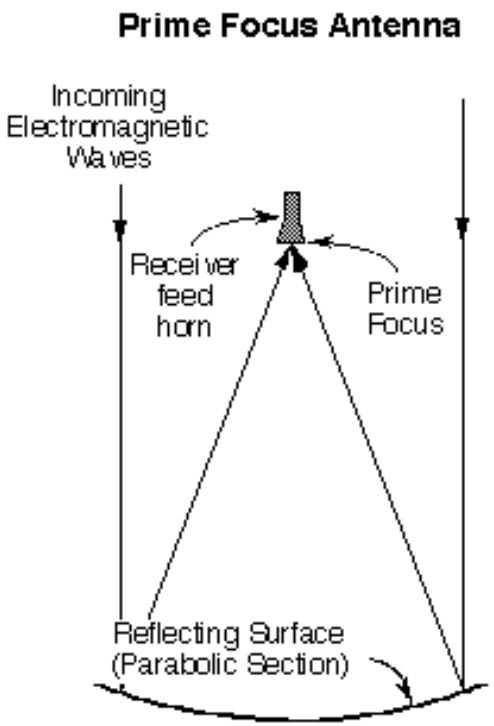


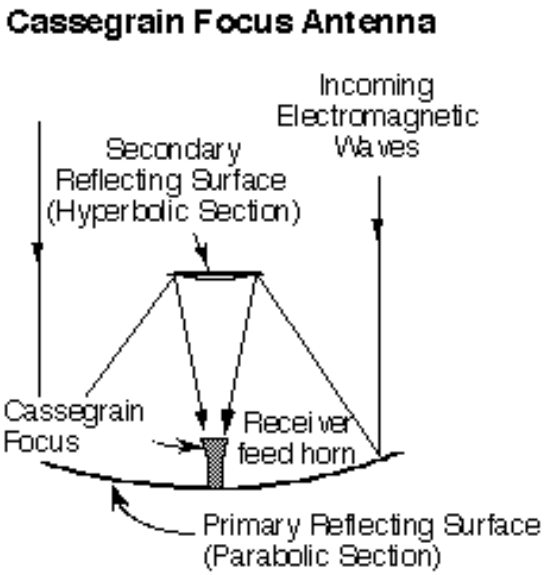
. http://media-2.web.britannica.com/eb-media/34/4634-004-F96BF4A3.gif

Part of the receiver (the "front end") is just behind the focus to amplify the emission collected by the "feed" (here a waveguide "horn"). It is often cooled to temperatures <20K to reduce the noise generated by the amplifiers. N.B. sometimes the feed horn is at the primary focus

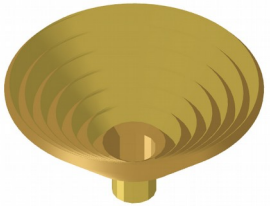
Shown here is the so-called "back-end" of the receiver (e.g. filters and further amplifiers etc) which need not be cooled.

Parabolic foci and feed horns





A coffee can is a simple open-ended circular waveguide (a "cantenna"). It provides limited control of the beam pattern – but is very cheap!



A professional waveguide horn – the "corrugations" are used to tailor the beam pattern .

http://www.antennamagus.com/newsletter.php

In the *Cassegrain* focus – close to the reflector surface – complex receivers are easier to get at and hence to maintain. Trade-off against the need for a secondary mirror system → higher initial cost.

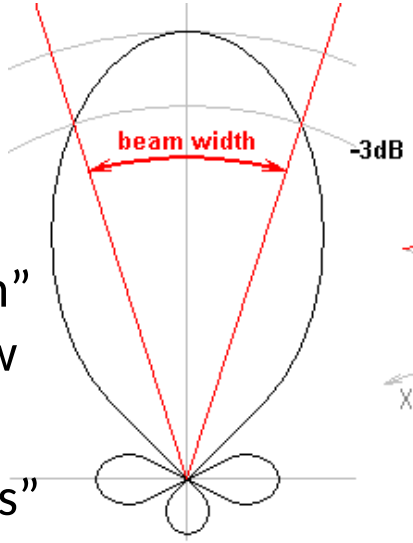
http://www.au.af.mil/au/awc/awcgate/jplbasic/bsf6-5.htm

Introduction to antenna beams

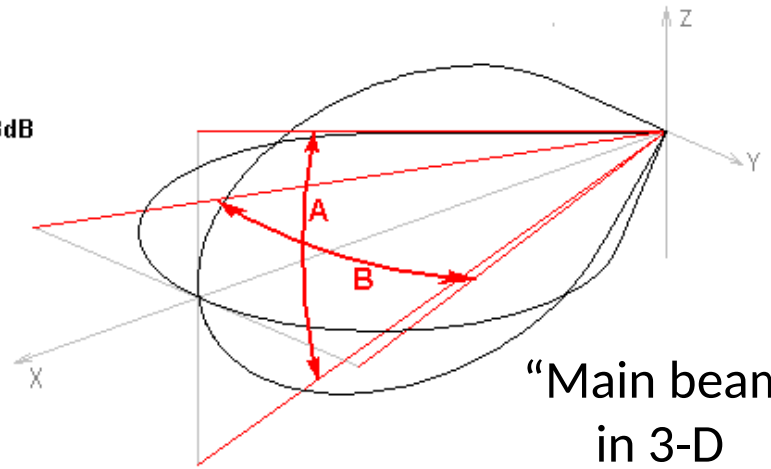
Schematic radio antenna power "beams"

http://www.thescienceforum.com/astronomy-cosmology/14956-radio-telescope-beams.html

"Main beam" in side view



"Sidelobes"



"Main beam" in 3-D

Plotted in polar coordinates

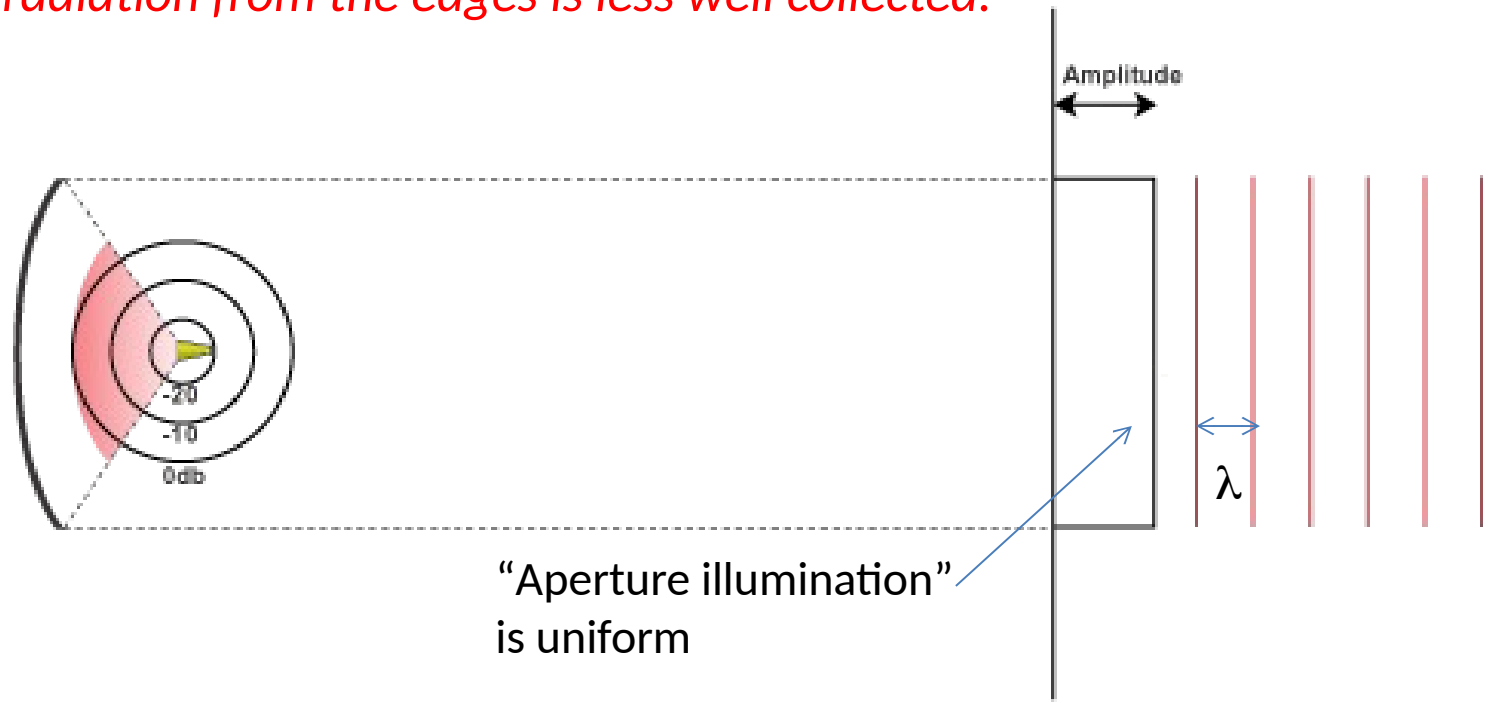
Antennas collect most power over a limited range of angles – the main "beamwidth" is often defined between the half power points (full width half maximum, FWHM or $\Theta_{1/2}$) on either side of the straight ahead direction. These 2-D and 3-D polar plots shows the relative power logarithmically as a function of angle with the FWHM at the -3dB (decibel) points – see later for definitions of dB.

The FWHM is typically ~ λ/D radians– where D is the dimension of the antenna aperture . Power also enters via the "sidelobes" – their structure depends in detail on the antenna design and is always much more complicated than shown here – but it's really true that some radiation often leaks in from behind the antenna!

Adapted from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

Reception with a parabolic antenna

Incoming plane waves from a distant source strike the surface and excite varying currents in it → EM waves radiated back outwards. The parabolic shape ensures that these waves are spherical, converging on the receiving element at the prime focus. In this idealised case the receiving element receives radiation *uniformly* from all points across the aperture – *but usually the radiation from the edges is less well collected.*

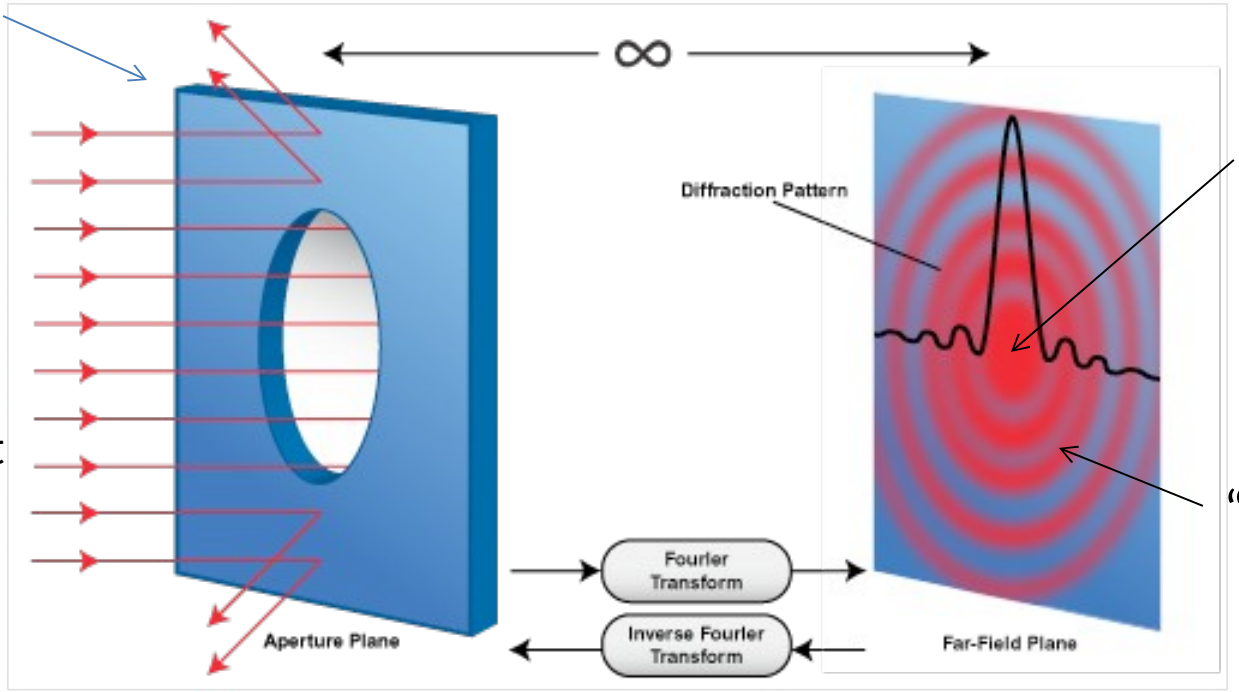


Adapted from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

Parabolic antenna "beams"

Schematic antenna aperture

Parallel rays i.e. plane waves incident from distant source (far-field)



Diffraction pattern from an aperture: the "far-field "beam pattern is the Fourier transform of aperture plane electric field distribution – as in physical optics courses

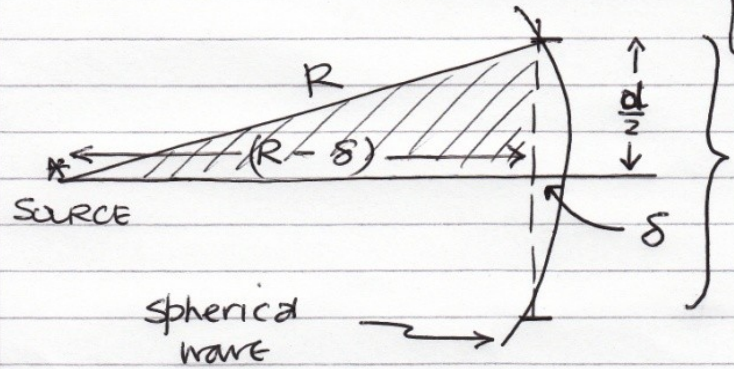
N.B. the diffraction pattern shown on the right actually shows the *square* of the amplitude pattern, since we only detect the *power* of the incident radiation, and not the complex, time varying amplitude pattern itself. This is the same situation as a laser beam shone through an aperture and observing the pattern with your eye which also only detects power (intensity).

RAYLEIGH DISTANCE

BOUNDARY BETWEEN SPHERICAL WAVE / PLANE WAVE

FRESNEL DIFFRACTION "NEAR FIELD"

FRAUNHOFER DIFFRACTION "FAR FIELD"



In $\blacktriangle$ $(R-\delta)^2 + \left(\frac{d}{2}\right)^2 = R^2$ (PYTHAGORAS)

$$R^2 - 2R\delta + \cancel{\delta^2}^{\text{small}} + \frac{d^2}{4} = R^2$$

$$\rightarrow R = d^2/8\delta$$

ARBITARY: "CLOSE ENOUGH" TO PLANE IF $\delta = \lambda/16$

$$\rightarrow \boxed{R_{RAYLEIGH} = 2d^2/\lambda}$$

WHEN $R > R_{RAYLEIGH}$ → CLASSICAL DIFFRACTION (FRAUNHOFER)

E.G. $d = 76\,m$ $\lambda = 1\,m$ → $R_{RAYLEIGH} = 11{\cdot}6\,KM$!

(JODRELL BANK)

→hard to calibrate large telescopes using terrestrial sources

Fourier Transform tutorials with commentary:

http://www.fourier-series.com/f-transform/ OR

https://www.youtube.com/watch?v=ObklYbQaX24 (part 1)

https://www.youtube.com/watch?v=QO3kgwYzpZg (part 2)

https://www.youtube.com/watch?v=6-llh6WJo1U (part 3)

- cosine + sine transforms (provide the phase of the frequency component)
- convenience of the $e^{i\omega t}$ form & concept of +ve and –ve frequencies
- note: these tutorials concentrate on the Fourier pair relationship

time variation ←→ frequency spectrum

(comes in later for signal processing.)

- for antenna beams use d*ifferent* Fourier pair from Fraunhofer diffraction theory:

spatial variation ←→ angular distribution

(complex amplitude across aperture) ←→ (the complex electric field beam pattern)

- Diffraction at a single slit: mathematical analysis

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html#c3

- Diffraction at a single slit: JAVA applet

http://www.walter-fendt.de/ph14e/singleslit.htm

- Diffraction grating: illustrative description

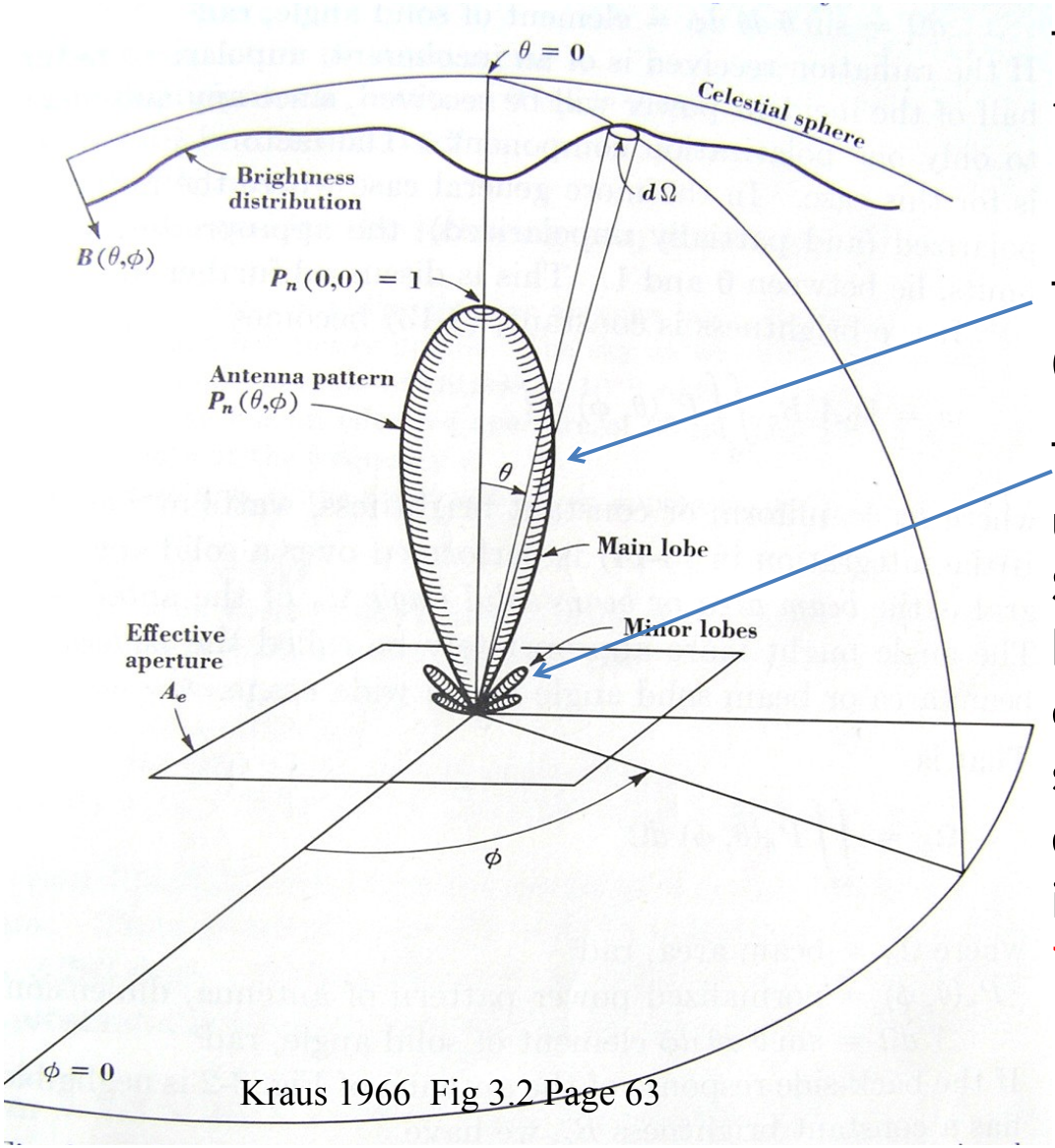
http://www.teralab.co.uk/Experiments/Diffraction/Diffraction_Page1.htm

Shows how one can imagine the grating as being composed of sinusoidal standing waves each of which produces three travelling diffracted waves

Reminder: we can’t measure voltage (it’s changing too fast !) we can only measure the mean power (α voltage^2) in the incoming emission…

Antenna beam pattern and the sky ("celestial sphere")

(in a 3-D polar plot)



Kraus 1966 Fig 3.2 Page 63

The total solid angle *integrated over the entire beam pattern* = Ω_A

(see Additional Notes "C")

The solid angle of the "main lobe" (or main beam) alone = Ω_M

The minor lobes ("sidelobes") pick up radiation in directions away from. Some point backwards (not shown here) indicating that radiation can enter the receiver (via diffraction or scattering) from the opposite direction that at which the antenna is pointed – as already noted in slide 7

Antenna beam pattern : $P_N(\theta, \phi)$ ← normalised to 1 at maximum

- Antenna beam solid angle $\Omega_A = \iint_{4\pi \text{ (all sphere}} P_N(\theta, \phi)\, d\Omega$

≡ Solid angle into which all power would be transmitted if uniform as a function of angle = P_{max} (= 1) across Ω_A & zero outside

Integrate over 4π & compress all power into Ω_A

a cone in polar coordinates

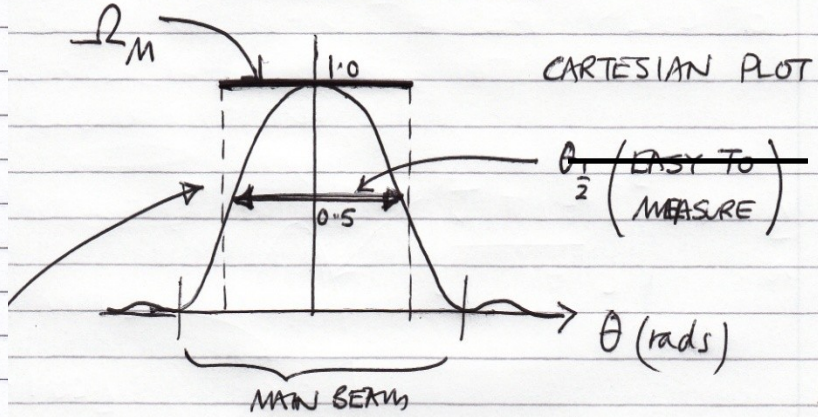
- Main beam solid angle $\Omega_M = \iint_{\text{main lobe only}} P_N(\theta, \phi)\, d\Omega$ } ignore sidelobes

$$\Rightarrow \Omega_A > \Omega_M$$

- Main beam efficiency $\varepsilon = \Omega_M / \Omega_A$
- Want little power in sidelobes for accurate mapping work (SLIDES 33-35 !)
- Can accept more (strong) power in sidelobes for observations of compact sources only seen in the main beam
  — see later slides for how to arrange this.

EQUIVALENT AREA OF MAIN BEAM

(PRACTICAL ESTIMATES)



CONSTRUCTING A CYLINDER WITH SAME 'VOLUME' AS MAIN BEAM

$$\Omega_M \simeq (1.15 \pm 0.05)\, \theta_{\frac{1}{2}}^2 \quad \text{(depends on taper)}$$

$$\theta_{\frac{1}{2}} \simeq (1.15 \pm 0.05)\, \lambda/d \quad "$$

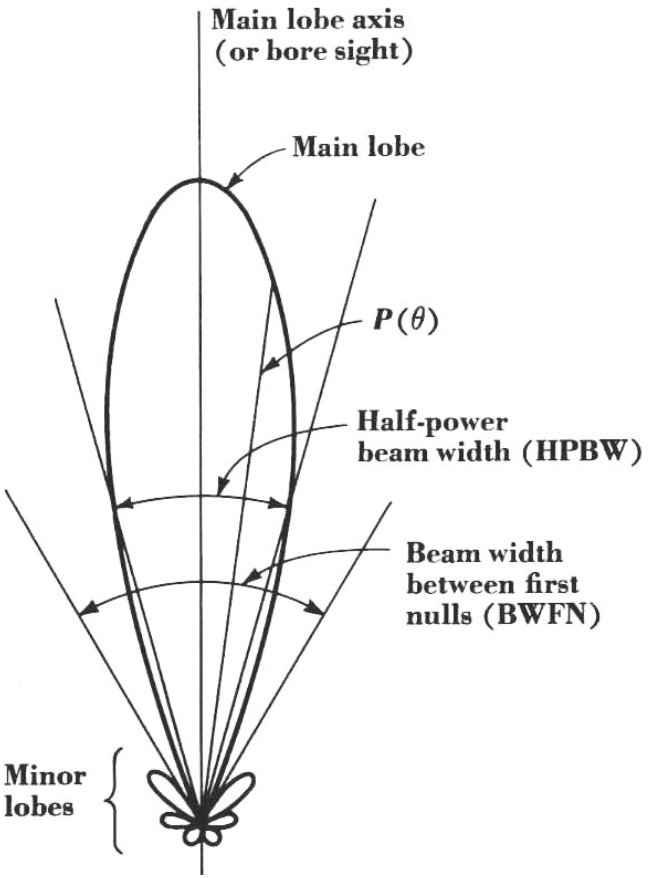
(1.15 factor same by coincidence!)

$$\rightarrow \quad \Omega_M \simeq 1.5 \pm 0.1 \left(\frac{\lambda}{d}\right)^2$$

A "first-order" approximation Ω_M is ≈50% larger than $(\lambda/d)^2$

Antenna Power Beam Pattern

2-D polar plot of normalised power pattern $P_n(\theta,\phi)$



HPBW = $\theta_{1/2}$ = (1.1 to 1.2) λ/D

(depending on aperture taper)

Kraus, 1966. Fig.6-1, p. 153.

Beam solid angle (steradians)

$$\Omega_A = \iint_{4\pi} P_n(\theta,\phi)\,d\Omega$$

Main Beam Solid angle

$$\Omega_M = \iint_{main\ lobe} P_n(\theta,\phi)\,d\Omega$$

The "main beam efficiency" $\varepsilon = \Omega_M/\Omega_A$ hence the fraction of the "beam solid angle" Ω_A arising from sidelobes is $(1-\varepsilon)$

What factors control the beam pattern?

Aperture Illumination & shaping the beam

Uniform illumination across an aperture (2-D cut):

→collecting all the radiation across the aperture

→ maximum gain : analytically this gives….

a) [sin(x)/x] (type) complex *voltage* beam pattern

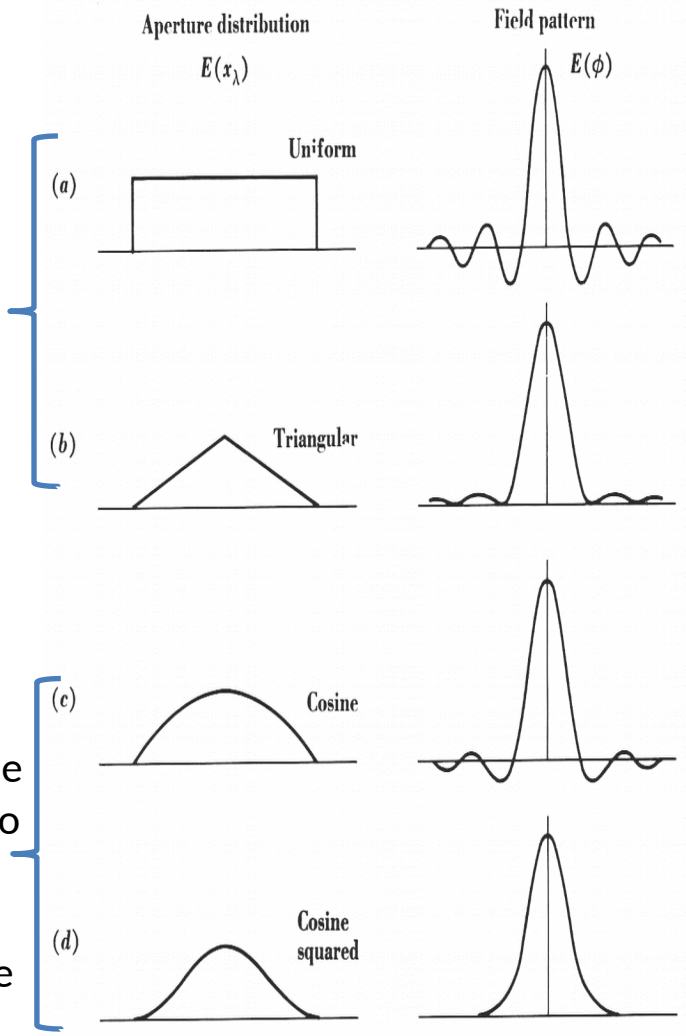
b) $[\sin(x)/x]^2$ (type) *power* beam pattern

Uniform illumination is never achieved - impossible to arrange the sharp transition from full illumination to zero at the edge of the aperture without some "spillover" of radiation from the ground (slide 32) which is unwanted since it adds to the "system noise" T_{sys} (see later).

In practice various compromise, "tapered", illuminations are adopted , usually seeking to maintain the power collected (the signal) while minimising the spillover noise. One example is: cosine illumination across the aperture : c) and d) being the equivalents to a) and b) (gaussian beams are similar to d).

The resultant main beams are little broader – but the sidelobes are lower.

→ *General point: abrupt edges inevitably result in "sidelobes" away from the main beam.*



Kraus, 1966. Fig.6-9, p. 168.

(Answer to question from Manchester student)

"Taper" just means that the amount of power collected from unit area near the edge of the dish is less than from the middle – hence the "illumination pattern" as we refer to it (even though we are talking about reception!) falls off towards the edge of the dish
→ lower sidelobes but a broader main beam.

See some quantitative calculations in :
"The Perfect Single Dish Antenna" (IRAM)
http://www.iram.fr/IRAMFR/IS/IS2002/html_1/node5.html

Adapted from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

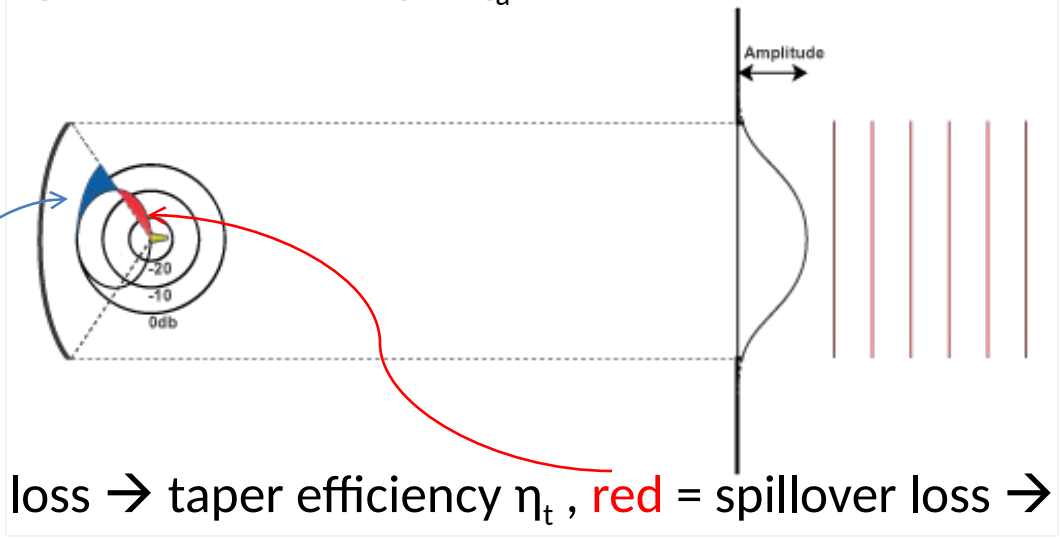
Aperture Efficiency and Illumination

Area of geometric aperture, A_{geom}

Area of effective aperture, A_{eff}

→ Aperture efficiency, η_a

$$\eta_a = A_{eff} / A_{geom}$$



blue = taper loss → taper efficiency η_t , red = spillover loss → spillover efficiency η_s

Gaussian-illuminated circular antenna:

$\eta_a = [\eta_t \eta_s]$ = 0.82 for 11dB edge taper (power down by 1/12.6)

$\eta_a = [\eta_t \eta_s]$ = 0.7 for 15dB edge taper (power down by 1/31.6)

But also need to take two more sources of efficiency losses into account !

Slide from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

Blockage losses

Effelsberg 100 m

NRAO GBT 100m

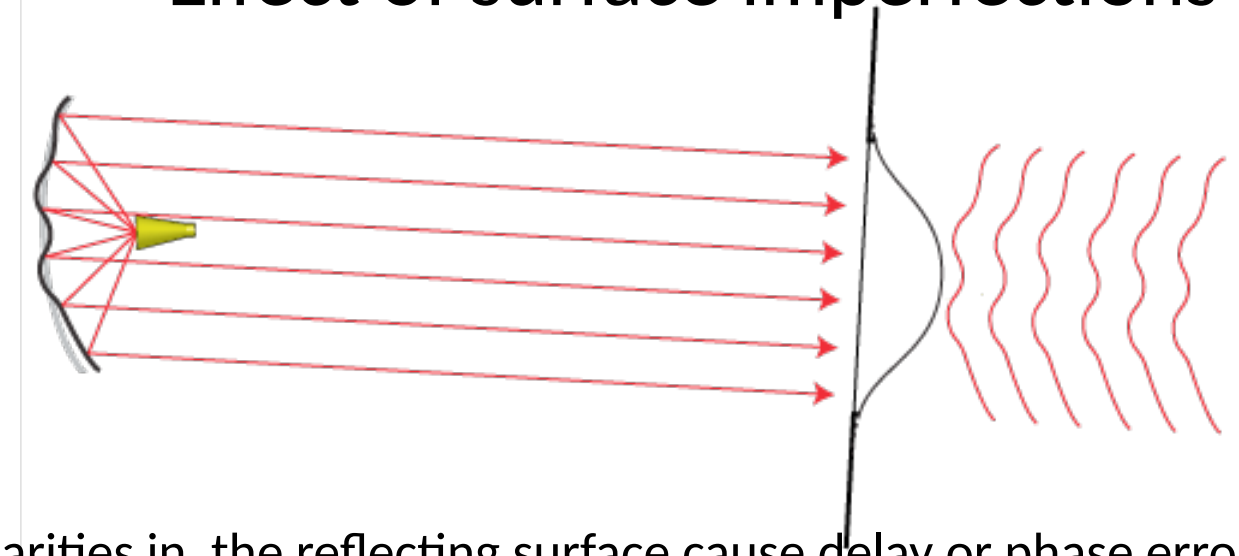
Physical blockage due to focal legs, receiver cabin etc – reduces the signal captured and also scatters power into the "sidelobes " .

The reduction in efficiency due to blockage is typically in the range η_b = 0.9-0.95.

The GBT was specifically designed to have zero blockage

Slide adapted from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

Effect of surface imperfections - I

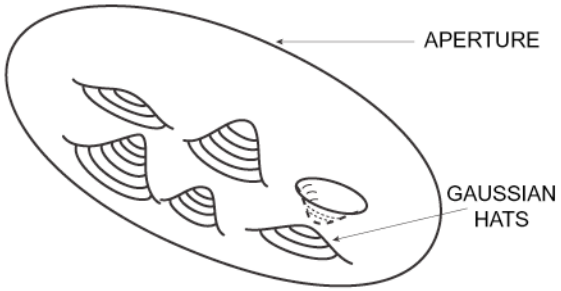


Irregularities in the reflecting surface cause delay or phase errors in the wavefronts. Here we imagine the telescope radiating to produce a non-planar wavefront at a distance. In the receiving mode, with plane wavefronts arriving the surface, the signals received at the focus from different parts of the surface do not all arrive at the same time→ loss of coherence → resultant signal not as big as it should be (see next slide).

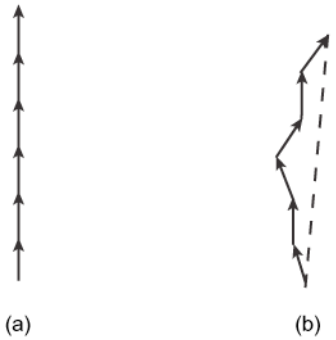
The schematic above also shows the effect of an overall misalignment of the reflecting surface, resulting in the beam not pointing exactly in the forward direction. This can arise as a result of varying gravitational deformation at different pointing angles.

Slide from presentation "Adjusting the GBT Surface: Towards 100 GHz operation" by R.Prestage, B. Nikolic, D.Balser , US National Radio Astronomy Observatory 18th August 2006

Effect of surface imperfections - II



Dish surface irregularities were modelled by Ruze as a distribution of gaussian "hats" (b) with respect to perfectly smooth reflector surface (a).



Resultant phase differences on arrival at focus → imperfect vector addition → loss of gain

Ruze formula:

$\eta_p = \exp[-(4\pi\delta/\lambda)^2]$

δ= rms surface error from perfect shape

$\eta_p = 0.96$ for $\delta = \lambda/60$

$\eta_p = 0.91$ for $\delta = \lambda/40$

$\eta_p = 0.84$ for $\delta = \lambda/30$

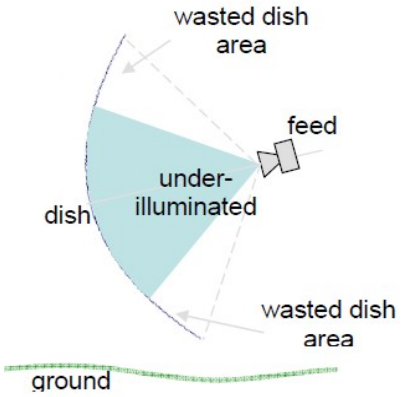
$\eta_p = 0.50$ for $\delta = \lambda/16$

$\eta_p = 0.37$ for $\delta = \lambda/4\pi$

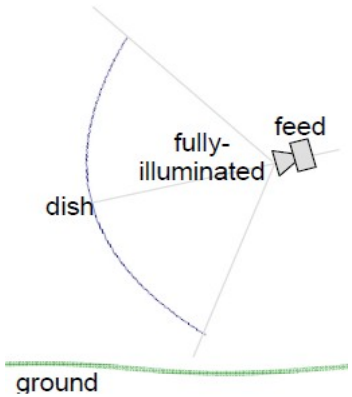
η_p is an effective surface reflectivity

Aperture & beam efficiencies and illumination

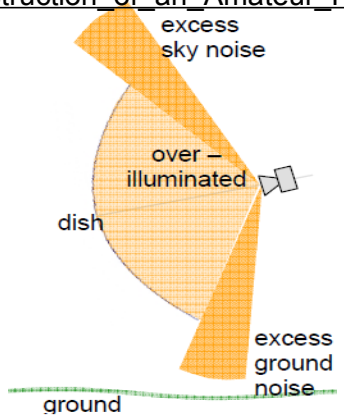
http://www.dmradas.co.uk/new%20files%20dec%202011/RAG%20site%20pdfs/Construction_of_an_Amateur_Radio_Astronomy_Antenna.pdf



under illumination,



full illumination



over-illumination

The ***aperture efficiency*** rises as the illumination gets closer to uniform but this produces a sharp transition at the edge:

→ higher sidelobes

→ the *relative* amount of power in the main beam falls

→ ***main beam efficiency*** ε falls as the ***aperture efficiency*** η_a rises

More taper → smoother transition at the edge

→ the edge of the aperture collects less power

→ the ***aperture efficiency*** η_a falls

→ lower sidelobes → ***main beam efficiency*** ε rises

Both efficiencies fall as δ, the rms deviation from a perfect reflector,rises the loss is given by the Ruze formula $\exp[(-4\pi\delta/\lambda)^2]$.

.

Summary: effective aperture & efficiency losses

Geometric aperture, A_{geom}
Effective aperture, A_{eff}
Aperture efficiency, η_a

$$\eta_a = A_{eff} / A_{geom}$$

Slide ##26&28: Illumination $\eta_a = \eta_t \eta_s =$ 0.82 for 11dB edge taper

Slide #29: Blockage $\eta_a = \eta_t \eta_s \eta_b$ = 0.82 x 0.93 (typical blockage 7% of geom. area)

Slides #30&31: Irregularities $\eta_a = \eta_t \eta_s \eta_b \eta_p$ = 0.82 x 0.93 x 0.83 (for λ/30 rms errors)

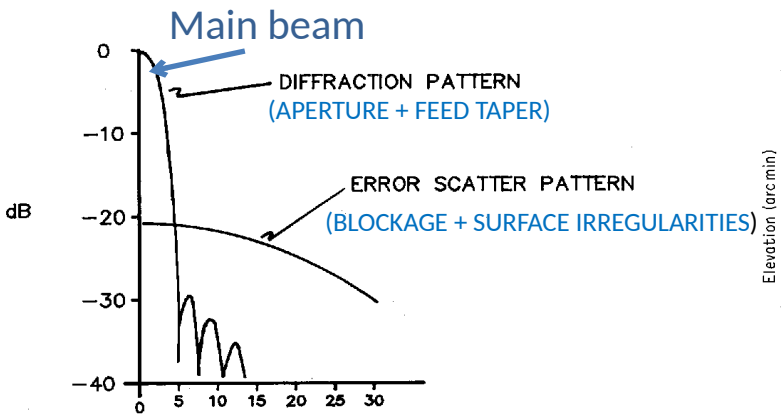
→ Total aperture efficiency η_a = 0.63 - quite good for prime focus

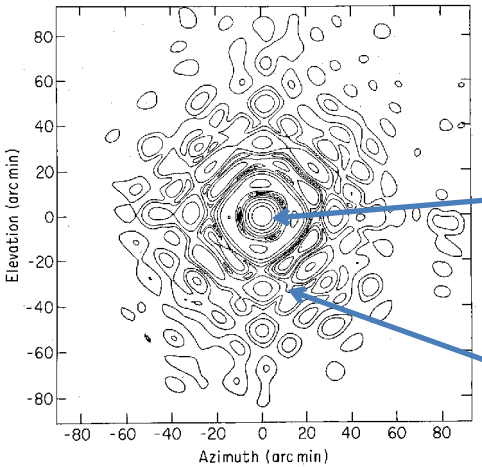
- Maximum aperture efficiency achieved ~0.8
- Typical for prime focus ~0.6

(Reducing further if the surface errors become larger in terms of the observing λ)

Main beam and sidelobes – another look

P. Napier, Ninth Synthesis Imaging Summer School, June 15-22 2004





Main beam

near-in sidelobes only – the far sidelobes cover all angles!

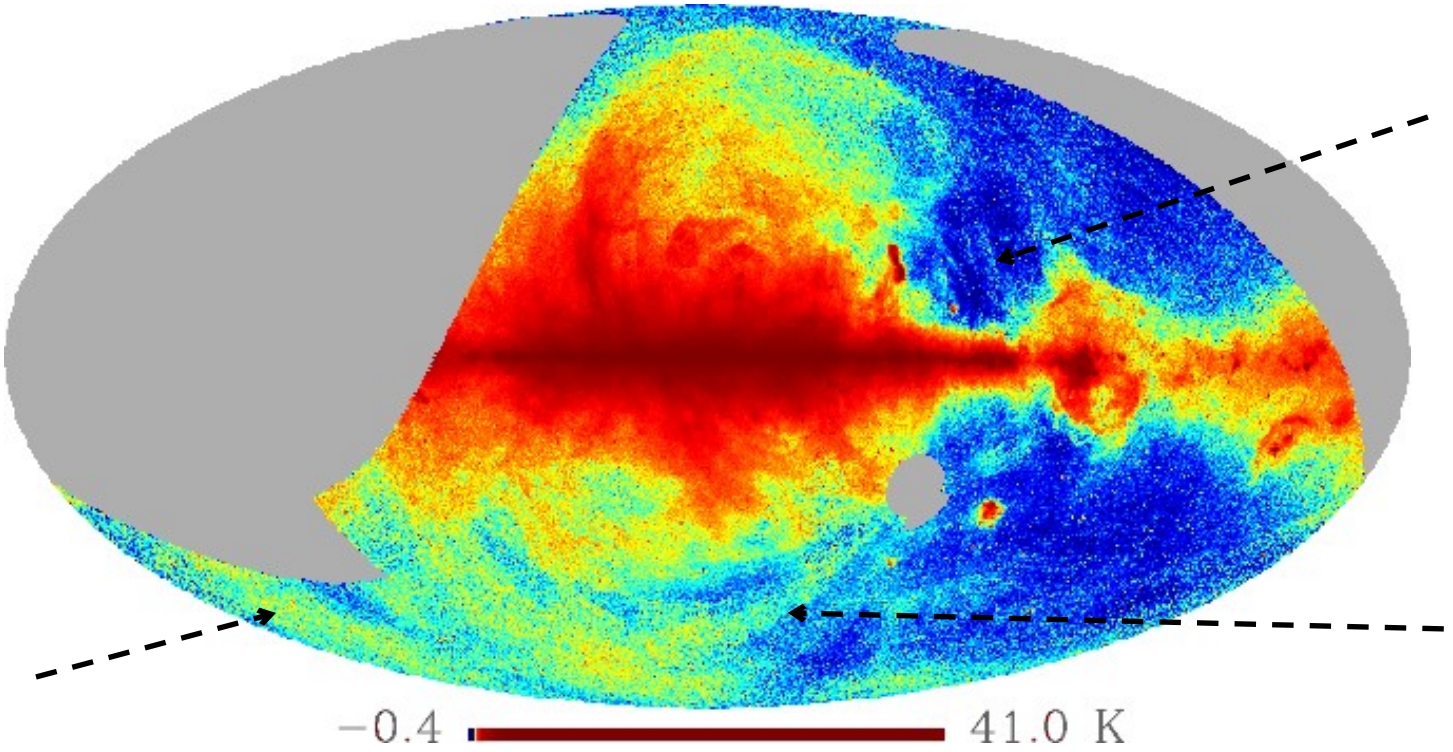
By scanning the telescope beam across a strong source e.g. Cass A one can map out the power pattern of the main beam and the sidelobes

For parabolic telescopes ε is typically between 0.6 & 0.8 i.e. 40% -20% of the power comes in through the sidelobes rather than the main lobe ($\Omega_{sidelobes} = \Omega_A - \Omega_M$). The sidelobe *levels* are much lower but

a) they cover a much larger area; b) some intercept the ~300K ground and this "spill-over" power changes as the elevation of the telescope changes.

For accurately -calibrated radiometry need high ε → taper to reduce sidelobes at expense of A_e

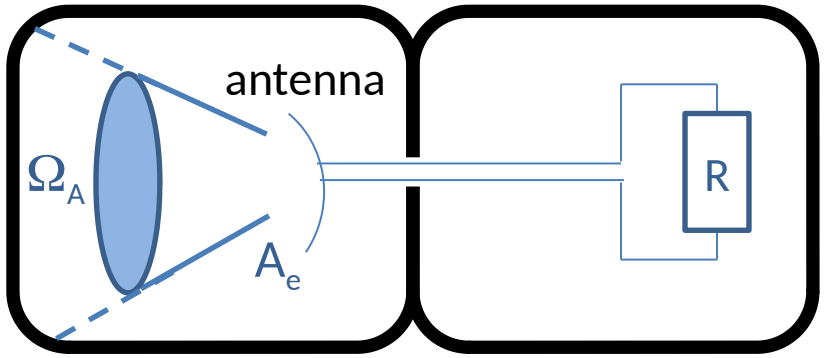
Large area map showing calibration artefacts



http://www.aanda.org/index.php?option=com_article&access=standard&Itemid=129&url=/articles/aa/full/2001/20/aah2559/aah2559.right.html

A map of part of the radio sky – it is dominated by bright emission (rendered as red) from the plane of the Milky Way across the centre. The "stripey" features - - - - in the image are artefacts arising from the difficulty of making absolute calibration measurements on different days under different observing conditions – in significant part due to the variations in the power coming in via the sidelobes which is hard to control/measure. The grey area to the top left is the part of the sky which is unobservable from the South Africa – where the data were taken.

The Antenna Theorem



Antenna & receiver (represented by a matched load resistance R) enclosed within black body cavity at temperature T. *Everything in thermodynamic equilibrium i.e. at temperature T.*

- That part of the cavity which subtends the antenna beam solid angle Ω_A (*input via sidelobes as well as main beam*) radiates towards antenna but only *part* will be absorbed – antenna is not fully efficient and its effective aperture is A_e (*reasons summarised in slide #33*)

Black Body has brightness $B_\nu = 2kT/\lambda^2$ W m^{-2} Hz^{-1} sr^{-1} (Rayleigh-Jeans regime)

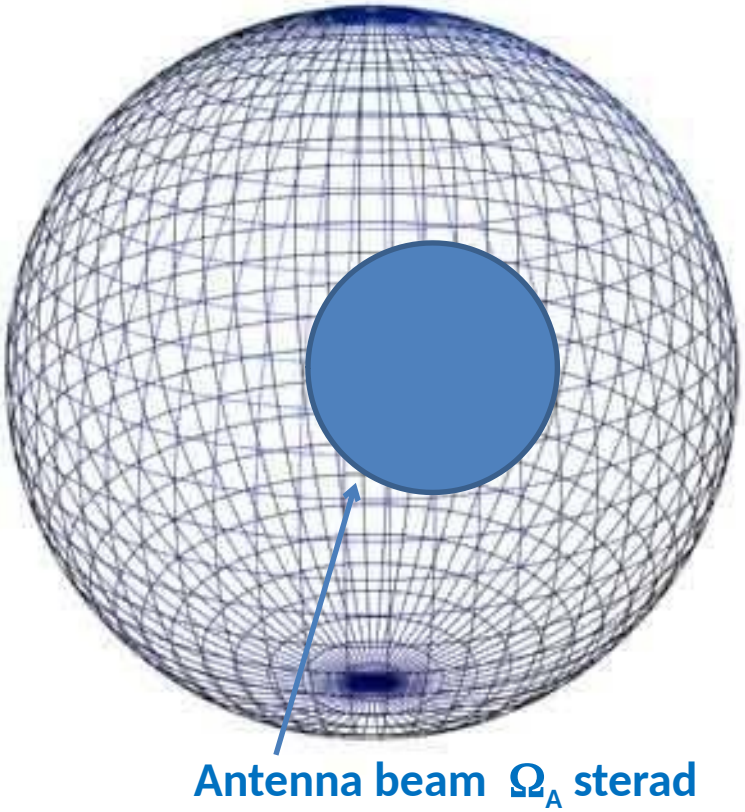
Antenna collects power $W = (kT/\lambda^2)\,\Omega_A\,A_e$ W Hz^{-1} (lost a factor 2 since *one receiver can only collect power from one component of polarisation)*

- The receiver radiates power back into the cavity via the antenna - cavity absorbs it. Receiver is effectively a matched load resistance, also at temperature T, which produces transferable power = kT per unit bandwidth or $kT\Delta\nu$ over a given bandwidth.

In thermodynamic equilibrium $(kT/\lambda^2)\,A_e\,\Omega_A = kT \rightarrow$ $A_e\,\Omega_A = \lambda^2$

Directivity & Gain of Antennas

Solid angle of Sphere = 4π sterad



"Directivity": $D = 4\pi/\Omega_A$

(Concentration enabled by antenna beam compared with radiating/receiving isotropically)

"Gain" G = D x losses in antenna + transmission line
(Usually G ~ D - and we assume this)

e.g. G=1000 (30dB) → $\Omega_A = 4\pi/1000$
= 0.01257 sterad (41 deg²)

Antenn

$G = 4\pi A_e / \lambda^2$

http://www.sciforums.com/showthread.php?100501-Area-of-a-sphere

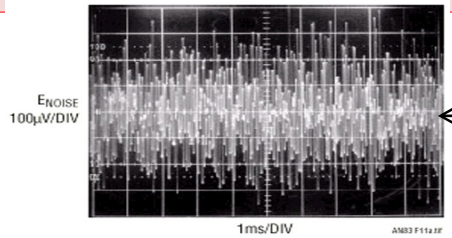
Power transfer in radio telescopes and “antenna temperature”

Antenna Temperature - 1

Resistors as 1-D Black Bodies

A resistance R at a physical temperature T contains B-Body radiation
→ causes electrons to fluctuate about their equilibrium positions
→ produces a *single* fluctuating voltage across resistor terminals

A resistor behaves like an electronic B-body at temperature T



Johnson-Nyquist noise
voltage has zero mean

http://www.eetimes.com/electronics-news/4163475/-Silence-of-the-Amps--Noise-performance-of-LDOs-depends-on-filters--Part-1

- In 1-D the Planck formula: $h\nu/[e^{h\nu/kT} - 1]$ W Hz^{-1} (per polarisation per unit bandwidth)
- **In Rayleigh Jeans regime = kT** W Hz^{-1} (per polarisation per unit bandwidth)

the mean power depends only on T since it's associated with B-Body radiation

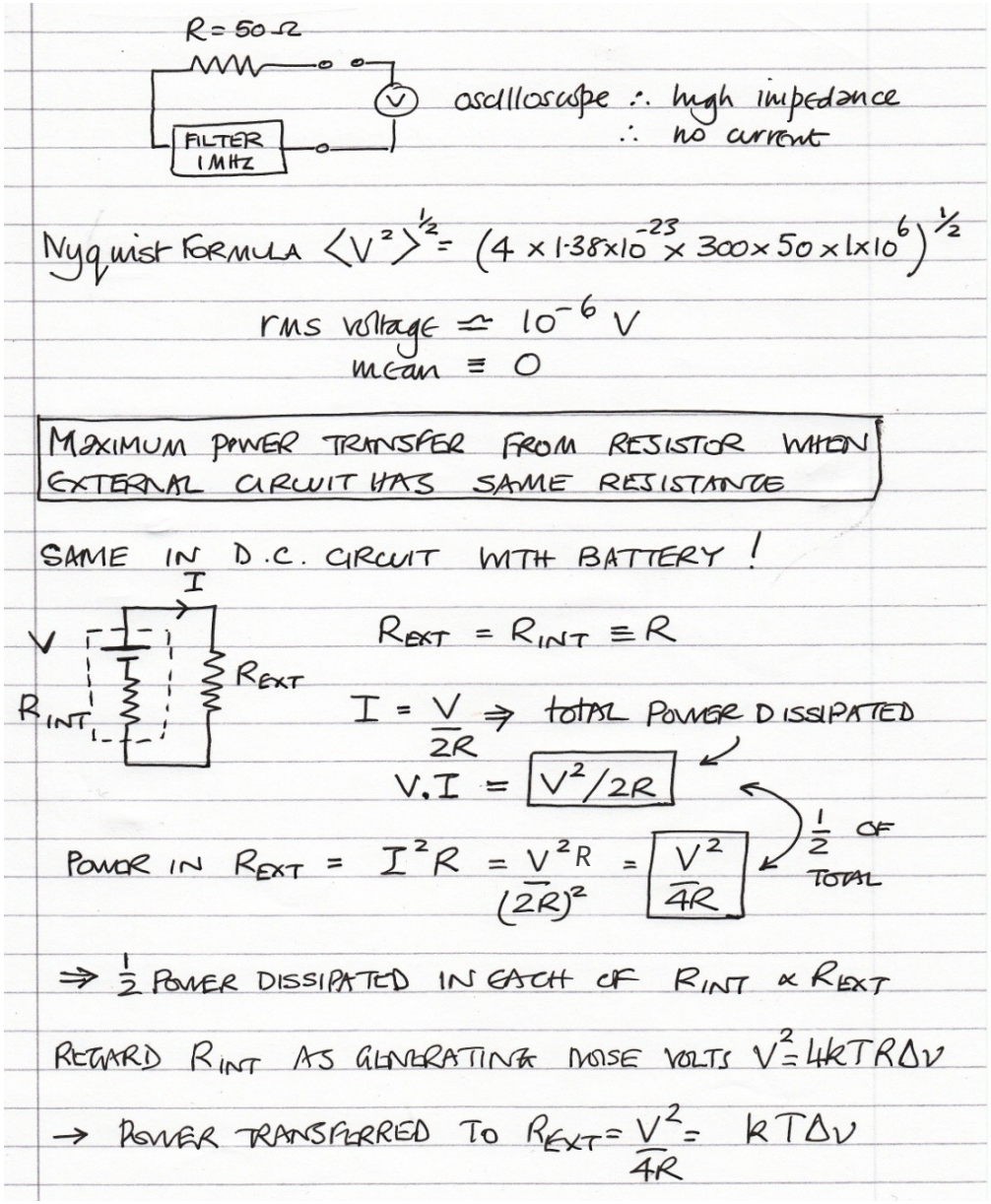
- **Mean power available = $kT\Delta\nu$ watts** ($\Delta\nu$ is the measurement bandwidth)

the voltage depends on the value of R as well

- RMS voltage $\langle V^2 \rangle^{1/2} = (4kTR\Delta\nu)^{1/2}$ volts (Nyquist formula - not proved here)

***All* electronic devices & components exhibiting resistance at radio frequencies produce "noise" - characterised by a "noise temperature" T of a resistor producing the same *power*. (Note: we rarely have to be concerned with the voltage).**

Antenna Temperature - 2



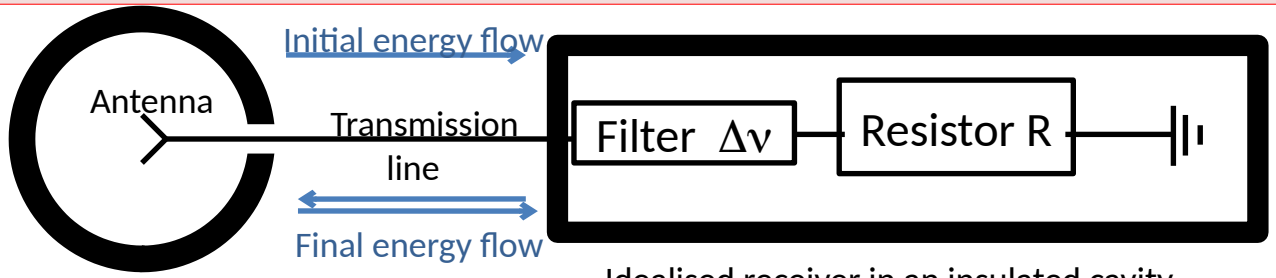
For visual illustrations of resistor thermal noise

http://www.fourier-series.com/Noise/flashprograms/noise1.html

Antenna Temperature - 3

Power Transfer in a "matched" system

The action of an antenna is to couple the incident radiation field to the receiver system and thus transfer power into a receiver – where it can then be measured.



- If the system is "matched " i.e. if (effective resistance of antenna + transmission line) = effective resistance R of receiver → no reflections at the interface → get the maximum available power transmitted (or transferred).

- When equilibrium is reached, R and the interior of its surrounding cavity, has been heated to a temperature T - *at which point there is an equal two-way flow of energy.*

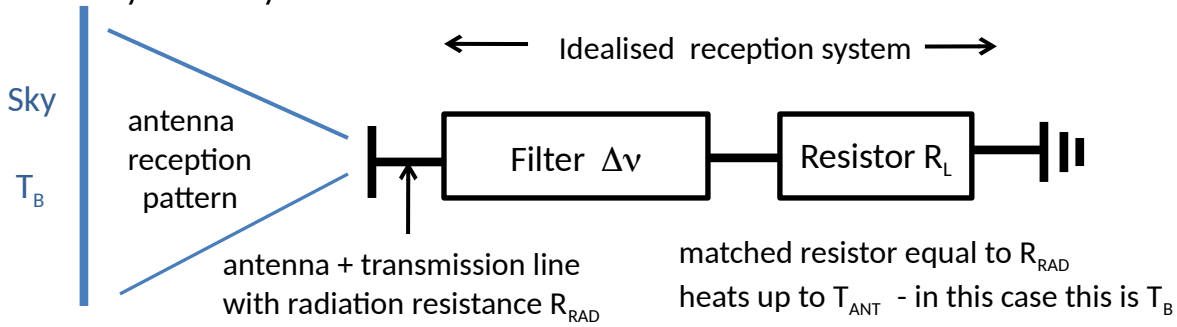
The power produced by the antenna in the B-B cavity = power in R = $kT\Delta\nu$ Watts

(In practice power flow is measured electronically not thermally – except in "bolometers")

Antenna Temperature - 4

Antenna Temperature and Radiation Resistance

Practical situation, when radiation fills the beam, is effectively the same as if the antenna surrounded by a B-Body enclosure.



Antenna Temperature T_{ANT} = temperature of a resistor (a B-Body!) which would generate the same power over bandwidth $\Delta\nu$ as that coming from the (antenna +transmission line).

When the antenna reception pattern is filled with radiation at temperature T_B , $T_{ANT} = T_B$ and the (antenna+receiver) acts like a radio thermometer → a "radiometer" .

Antenna Radiation Resistance R_{RAD}: EM power is absorbed by the antenna and sets up currents *i* in the transmission line; this power = i^2R_{RAD} (NOT ohmic losses – usually much lower). If the load resistance R_L of the receiver is matched (equal) to the radiation resistance R_{RAD} maximum power is transferred *but as much power is reradiated from the antenna as is delivered to the load* (see the previous slide and slide 23). Impedance matching is critical in radio receivers.

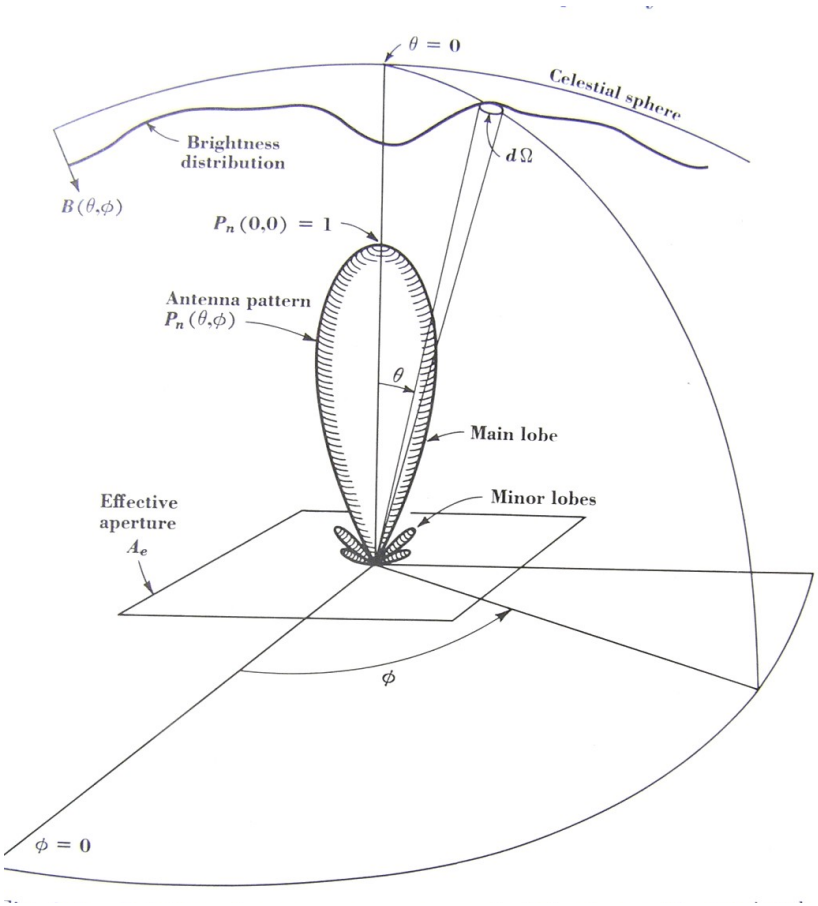
http://www.cv.nrao.edu/course/astr534/AntennaTheory.html

IMPORTANT POINTS TO REMEMBER

- Radio astronomy (and indeed all passive imaging and detection) is all about measuring the tiny amounts of power in random noise signals in the face of (much larger) unwanted sources of noise – typically from the receiver itself – but also from other "stray" radiation entering the system.

- In the Rayleigh-Jeans regime the power per unit bandwidth is directly proportional to temperature – hence we have the brightness temperature, antenna temperature, noise temperature concepts. This is a convenient way to characterise amounts of power – we don't actually measure temperatures with a thermometer !

Antenna temperature – in practice



Kraus 1966 Fig 3.2 Page 63

The power W (WHz^{-1}) collected by the antenna, per unit bandwidth, is its effective area A_e x the sky brightness temperature $B(\theta,\phi)$ ($Wm^{-2}\,Hz^{-1}$ $sterad^{-1}$) weighted by the antenna beam pattern $P_n(\theta,\phi)$ (sterad) integrated over the whole sphere x ½ (only one polarisation per receiver).

Next slide → antenna temperature

$$T_A = 1/\Omega_A \iint_{4\pi \text{ (all angles)}} T(\theta,\phi)\, P_n(\theta,\phi)\, d\Omega$$

or, equivalently

$$T_A = A_e/\lambda^2 \iint_{4\pi} T(\theta,\phi)\, P_n(\theta,\phi)\, d\Omega$$

ANTENNA TEMPERATURE

point antenna to position (θ, ϕ) on sky ($r = \infty$!)
where brightness = $B_\nu(\theta, \phi)$

power received (per Hz)

$$W = \frac{1}{2} A_e \iint_{4\pi} B_\nu(\theta,\phi) P_N(\theta,\phi)\, d\Omega \qquad (W\,Hz^{-1}) \qquad (A)$$

only 1 polaris" per receiver (see later lectures)

$$B_\nu = 2k T_B(\theta,\phi)/\lambda^2 \qquad \text{(Rayleigh-Jeans)}$$

$$\rightarrow \quad W = \frac{A_e k}{\lambda^2} \iint_{4\pi} T_B(\theta,\phi) P_N(\theta,\phi)\, d\Omega = k T_A$$

$(W\,Hz^{-1})$ $\qquad$ $(W\,Hz^{-1})$

$$A_e = \lambda^2/\Omega_A \qquad \text{Antenna theorem}$$

$$\rightarrow \quad T_A = \frac{1}{\Omega_A} \iint T_B(\theta,\phi)\, P_N(\theta,\phi)\, d\Omega \qquad (B)$$

N.B. this is at a single position on the sky

$T_A \equiv T_B(\theta,\phi)$ averaged over the antenna beam
ie. a smoothed version ....

Examples: (i) point source → "∞ brightness"

$$S = \iint_{\text{source}} B(\theta,\phi)\, d\Omega$$

flux density

: [don't know Ω_{source} but can get S from power, $A_e \propto \Delta\nu$]

Lecture 2 slides 6&7

Using: (A) power collected / Hz $= \frac{A_e}{2} \iint \underbrace{B(\theta,\phi)\, P_N(\theta,\phi)}_{=S}\, d\Omega$ ($P_N \equiv 1$ at max of beam)

(Slide 2) $kT_A = \frac{A_e}{2} \cdot S$

$\rightarrow$ $\boxed{S = \frac{2kT_A}{A_e}} \equiv \boxed{T_A = \frac{SA_e}{2k}}$ useful formulae

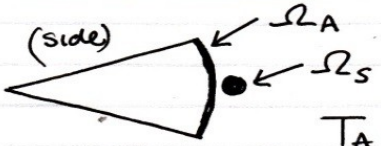
$A_e/2k$ can be expressed in units K/Jy (with $1\ \text{Jy} = 10^{-26}\ \text{W m}^{-2}\ \text{Hz}^{-1}$) & is good indicator of sensitivity — check out!

eg (i) $D = 32\text{m}$ $\eta = 0.6 \Rightarrow A_e = 0.6 \times \pi \times 16^2 = 483\ \text{m}^2$
$A_e/2k = 1.75 \times 10^{25} = \underline{0.175}\ \text{K/Jy}$

(ii) $D = 100\text{m}$ $\eta = 0.7 \Rightarrow A_e/2k = 2 \times 10^{26} = \underline{2\text{K}}\ \text{/Jy}$

(see also Examples 7 & 8 Q2

(ii) Extended source smaller than Ω_A



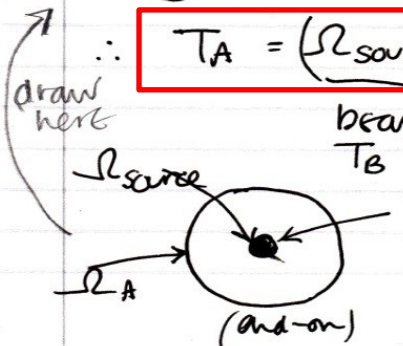
Assume $P_N(\theta,\phi) \approx 1$ across source

(using (B))

$$T_A = \frac{1}{\Omega_A} \iint_{\text{source}} T_B(\theta,\phi)\, d\Omega \equiv T_{AVG}\ (\text{over source}) \times \Omega_{source}$$

$\therefore$ $\boxed{T_A = (\Omega_{source}/\Omega_A) \cdot T_{AVG}}$

beam dilution factor: reduction in true T_B since source does not fill Ω_A



See also lecture 2 slides 6&7

Early (1958) Microwave Measurements of Mars Surface Temperature

Measured data from telescope:

- D =15.2m ; $\eta_a = 0.56 \rightarrow A_{eff} = 102\ m^2$
- Wavelength $\lambda = 3.15$ cm (0.0315 m)
- Beamwidth $\theta_{1/2} = 522$ arcsec (~1.2 λ/D)
- T_A (measured) = 0.24K on Mars

Mars solid angle

- Disk diameter = 24.8 arcsec (*optical telescope!*)

  Diameter = 24.8/206265 = 1.20 x 10^{-4} radians

  (180/π).60.60 = 206265 arcseconds per radian

  Disk area = $\pi D^2/4$ = p [1.20 x 10^{-4}]^2/4 = 1.135 x 10^{-8} rad^2

$\rightarrow \Omega_{MARS} = 1.14 \times 10^{-8}$ rad^2 (or sterad)

1) Via the "beam dilution" approach

$T_{MARS} = T_A\ (\Omega_A/\Omega_{MARS}) = 0.24\ (\Omega_A/1.14 \times 10^{-8})$

$\Omega_A A_{eff} = \lambda^2$ (Antenna Theorem)

$T_{MARS} = 0.24 \times (0.0315)^2 / 102 \times 1.14 \times 10^{-8} =$ **205K**

2) Cross-check via the flux density S (since $\Omega_{MARS} << \Omega_A$)

$S = 2kT_A/A_e = 2.76 \times 10^{-23} \times 0.24 / 102 = 6.5 \times 10^{-26}\ Wm^{-2}\ Hz^{-1}$ (i.e. 6.5 Jy)

$S = [2kT_{MARS}/\lambda^2].\ \Omega_{MARS}$ (Flux density = average brightness x solid angle)

$T_{MARS} = S\lambda^2/2k.\ \Omega_{MARS} = 6.5 \times 10^{-26} \times (0.0315)^2 / 2.76 \times 10^{-23} \times 1.14 \times 10^{-8}$

= **205K**

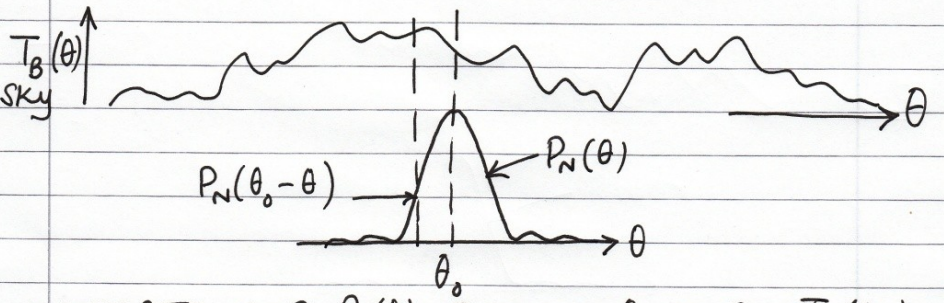
Similar measurements on Venus showed its surface temperature to be >600K

This was the first indication of a runaway greenhouse effect on Venus.

3) EXTENDED SOURCES $\gtrsim \Omega_A$

EQUATION (B) REFERS TO T_A FOR SINGLE POINTING DIRECTION (θ_0, ϕ_0). FOR EXTENDED SOURCES WANT TO MAP OUT T_A OVER MANY (θ_0, ϕ_0) DIRECTIONS Slide #39
Slide #55

SCHEMATICALLY (IN CARTESIAN 1-D ANGLE θ)



- INTEGRATE OVER $P_N(\theta)$ FOR ALL θ TO GET $T_A(\theta_0)$ $\equiv$ WEIGHTED SUM OF $T_B(\theta)$ OVER THE BEAM
- MOVE TO A NEW (θ_0, ϕ_0) AND REPEAT INTEGRATION $\rightarrow$ MAP OUT $T_A(\theta)$
- FORMALLY IN 2-D

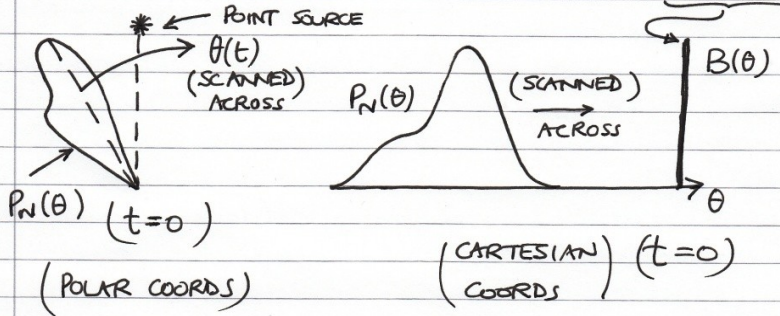
$$T_A(\theta_0, \phi_0) = \frac{1}{\Omega_A} \iint_{4\pi} \overbrace{T_B(\theta, \phi)}^{\text{SKY}} P_N(\theta_0 - \theta, \phi_0 - \phi)\, d\Omega$$

MAP OUT $T_A(\theta, \phi)$ BY MOVING TO SERIES OF θ_0, ϕ_0 I.E. SCAN ANTENNA ACROSS SKY POINT BY POINT NOTING $T_A(\theta_0, \phi_0)$ AT EACH

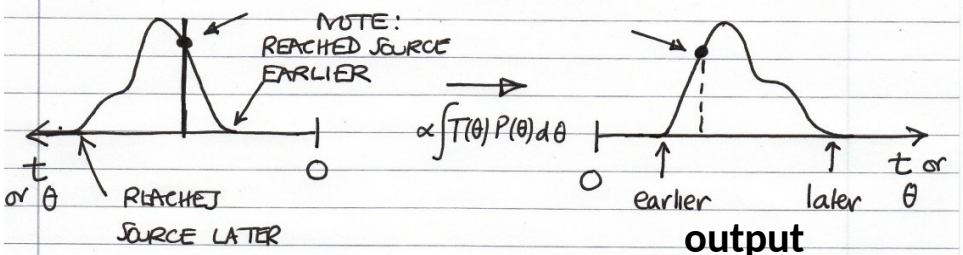
(RECALL AUTOCORRELATION Slides 18-20

PICTORIALLY : SIMPLE CASE TO ILLUSTRATE

ASYMMETRIC BEAM SCANNED ACROSS POINT SOURCE



AS t ↑ ANTENNA MOVES IN θ: THUS TRACE OUT BEAM



output

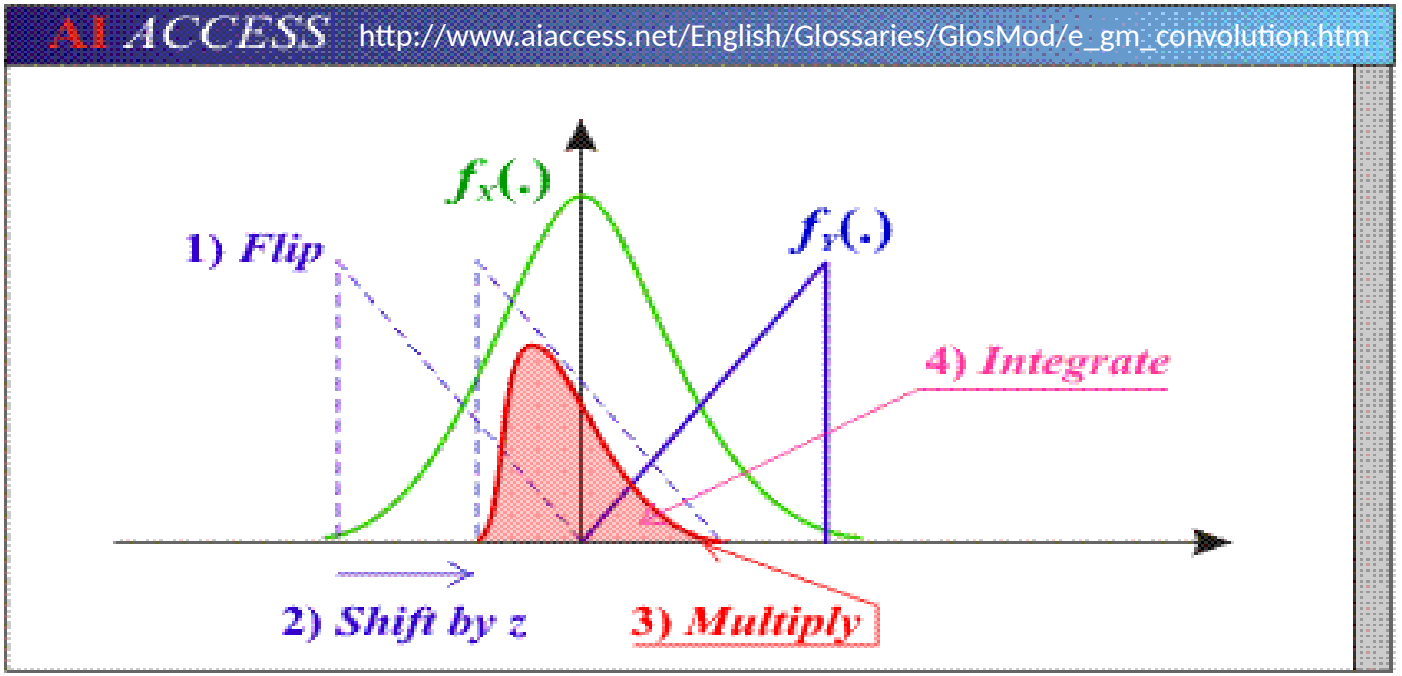
NOTE: BEAM PATTERN $P_N(\theta)$ HAS AUTOMATICALLY BEEN "FLIPPED" AROUND" IN ANGLE OR TIME

WE SAY SKY HAS BEEN "CONVOLVED" WITH BEAM PATTERN : HAS SIMILARITY TO CORRELATION BUT VITAL DIFFERENCE IS "FLIPPING" OF ONE FUNCTION

SEE SLIDE (next) ON CONVOLUTION

NOTE: BEAM $P_N(\theta, \theta)$ MAY BE CALLED "POINT SPREAD FUNCTION" OR "IMPULSE RESPONSE"

Convolution: pictorially



Convolution of the triangular (blue) function with the gaussian (green) function . The first step is to flip the blue function around (two dotted cases) – and then multiply the two functions (point by point) and integrate (i.e. add up the results of the multiplications) to produce a single number. Here two position shifts are shown with the overlap integral in red shown for the second one. *For each output point this entire process must be repeated and then at many different shifts to produce the convolved version of the input signal . It does not matter which function is flipped around – you can check this out using the applet on the next slide.*

Tutorial animation illustrating the convolution process
http://www.fourier-series.com/Convolution/index.html

It deals with the smoothing effect of a “system” or process on a time series input (1-D) – but it’s the same concept and maths (in 2-D) for an angular distribution smoothed by a telescope beam.

This one http://www.jhu.edu/signals/convolve/ lets you swap around the input signal and system response and to show for yourself that the result is the same – convolution is commutative.

The Wikipedia entry on convolution also has some simple animations
http://en.wikipedia.org/wiki/Convolution

Convolution: applicability

A smoothing i.e. integrating operation over (θ,ϕ) or (x,y) or t

- radioastronomy –sky brightness (θ,ϕ) smoothed by telescope beam
- image processing : 2-D photograph (x,y) "blurred" due to motion or out-of-focus
- electrical circuits – time varying voltages passing through filters, amplifiers etc
- there are many more (e.g. http://en.wikipedia.org/wiki/Convolution).... why?

Applies to 1) linear, 2) translational-invariant processes (very common)

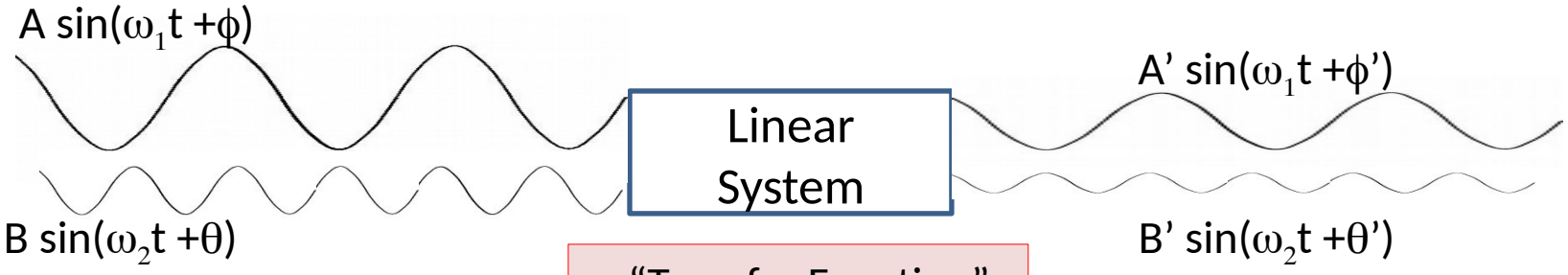
1) The output amplitudes **O** are simple *linear* multiples of the input amplitudes **I**
   - **O= α I** rather than some non-linear relationship e.g. **O= I** $^{\alpha}$

2) The output variation does not depend on the *absolute* arrival time (or equivalently the absolute angle or location) – the same relative input changes produce the same output variations regardless of where they start.

   → preserves the *relative positions* of temporal, spatial, or angular features of the input hence the *relative phases* of the frequency components of that signal

Properties 1) and 2) →Fourier Theory can be used to analyse the system

Convolution and linear systems – Fourier approach



i.e. Fourier components of the input signal spectrum

"Transfer Function" i.e. how each input Fourier component is weighted and shifted

i.e. Fourier components of the output signal spectrum

Sinusoidal signals input to a linear time invariant system are passed through:

- *unchanged in angular frequency* ω_1 → ω_1 *etc*
- *with different amplitudes A* → *A' etc* (often smaller but can be larger in case of amplifier)
- *with different phases* ϕ → ϕ' *etc* (nearly always a delay through the system)
- Each sinusoid passes through in general with different weightings and different delays i.e. phase shifts.
- Input signal is superposition of Fourier components (sine waves) : output is a superposition of different Fourier components: the "Transfer Function" is the recipe for changing i/p → o/p

[Spectrum of input] x [Transfer Function] = [Spectrum of output signal]

Linear systems do not generate any extra Fourier components i.e. harmonics of input ω's

The convolution theorem: illustrated

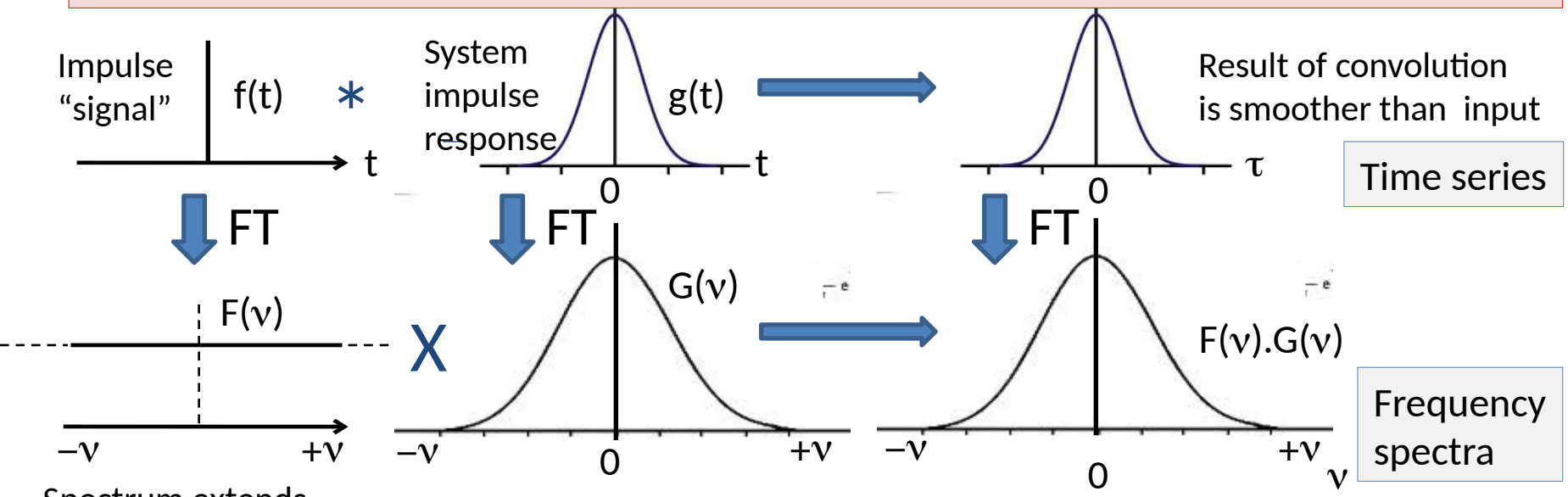
In terms of time t and frequency ν and *time-symmetric* (for ease of drawing)

$$C(\tau) = [f(t) * g(t)] = \int f(t).g(\tau - t)dt$$

"flipped " but no effect if function is symmetric in time

$$FT\ [\ f(t) * g(t)] = F(\nu).G(\nu)$$

Spectrum of convolved output = product of the individual spectra



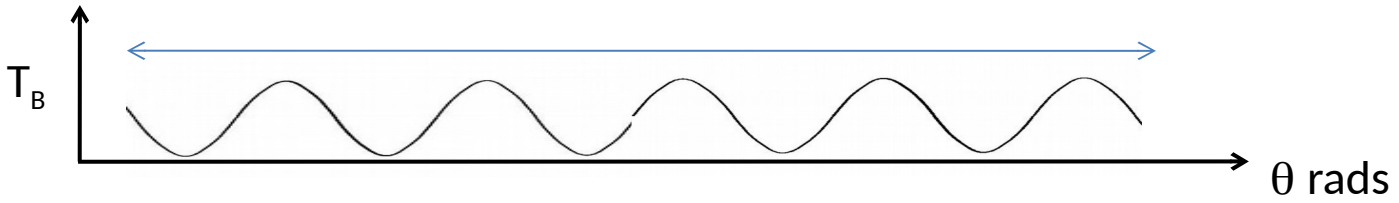
Convolution Theorem and the angular frequency domain

Convolution theorem → the "true" sky brightness distribution (intensity or power) can *either* be viewed as:

- a fine grid of point sources to be convolved with the antenna beam

or

- a spectrum of angular frequency components with units of cycles radian^{-1} (compare with Hz ≡ cycles s^{-1}) to be weighted by the angular frequency "transfer function" of the antenna for intensity.



e.g. 5 cycles across an angle of 0.01 radian = 500 cycles rad^{-1}

Angular frequency transfer function of the antenna for intensity (power)

=

ACF of the aperture distribution (from Wiener-Khinchine theorem)

Antenna smoothing and "cut-off" - 1

$T_B(\theta,\phi)$ * $P_N(\theta,\phi)$ ≡ $T_A(\theta,\phi)$

true sky temp distribution **convolved** **antenna beam** ≡ **smoothed sky distribution i.e. antenna temperature**

"point spread function"
OR "impulse response"

Fourier Transform **Fourier Transform** **Fourier Transform**

sky spectrum **X** **Transfer Function (antenna ACF)** **=** **weighted sky spectrum**

(All spectra expressed in angular frequencies of cycles per radian)

Convolution with the antenna beam smooths out fine details in the sky temperature distribution on angular scales smaller than ~λ/d i.e. smaller than $1/u_{max}$

EQUIVALENTLY

The antenna ACF acts as a low pass filter cutting out high angular frequencies $>u_{max}$

→ the smoothed sky distribution is "band-limited" i.e. "cut-off" beyond u_{max} cycles/radian

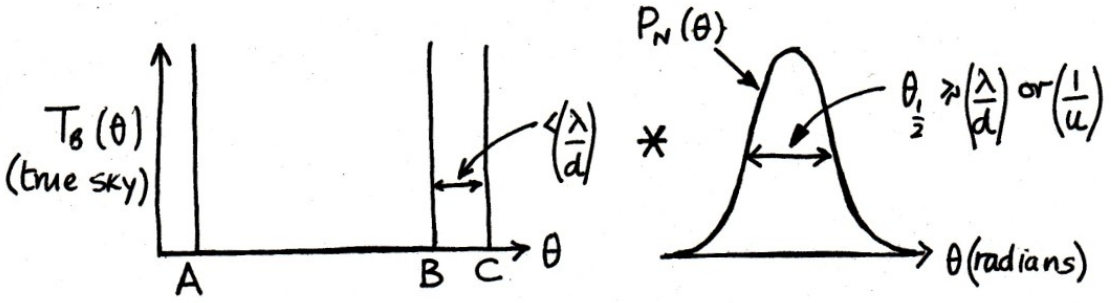
Antenna smoothing and "cut-off" - 2

Simple illustration of convolution smoothing in 1-D

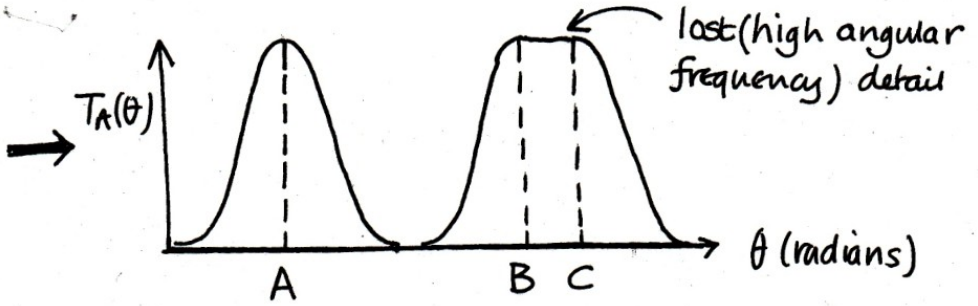
Treat sky distribution as a series of impulses (point sources) and convolve with simple symmetric beam $P_N(\theta)$ → lose detail on scales smaller than ~$\theta_{1/2}$ ~ λ/d

e.g. if antenna dimension d = 500 λ can't see details in map <1/500 radians (0.1146 degrees)

Sky as series of "impulses" point sources



After convolution with beam lose fine detail



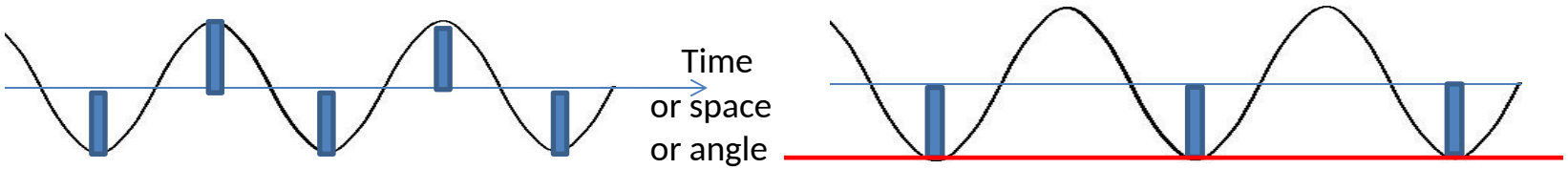
(Similar argument to Rayleigh resolution criterion in physical optics)

EQUIVALENTLY: the angular spectrum of the map is cut-off beyond ~500 cycles per radian

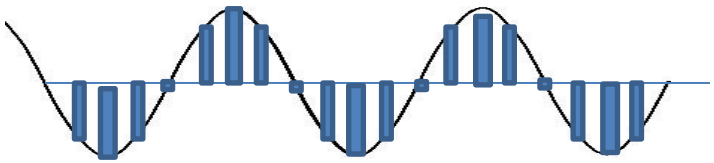
Nyquist-Shannon Sampling Theorem

A fundamental result applicable widely in information theory and data analysis.

An arbitrary function – containing no frequencies higher than ν_{max} - can be reconstructed exactly (by interpolation) from a series of samples of that function taken at a frequency rate of at least $2\nu_{max}$.



sampled at the Nyquist rate i.e. twice a period of the highest frequency



oversampled

Undersampled – and with sampling at exactly once per period - looks like a constant signal - the variation is missed completely – an example of "aliassing"

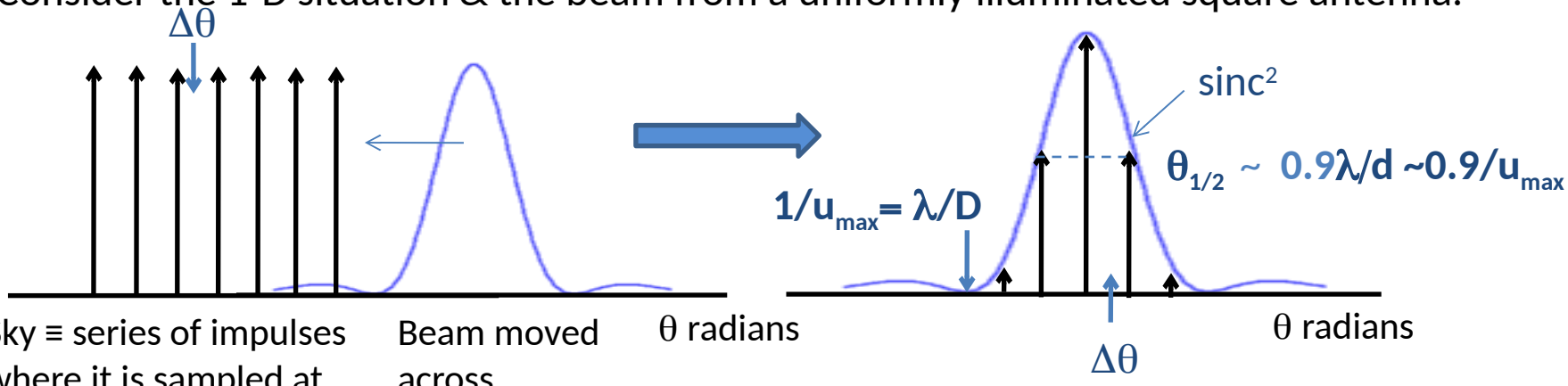
In applying the theorem to antennas we are concerned with *angular* frequencies.

Antenna cut-off and sampling in angle

To make a 2-D map of the radio sky brightness temperature distribution we shift the antenna beam in angle and collect samples of the antenna temperature $T_A(\theta,\phi)$.

How frequently should we sample T_A in angle?

Consider the 1-D situation & the beam from a uniformly illuminated square antenna.

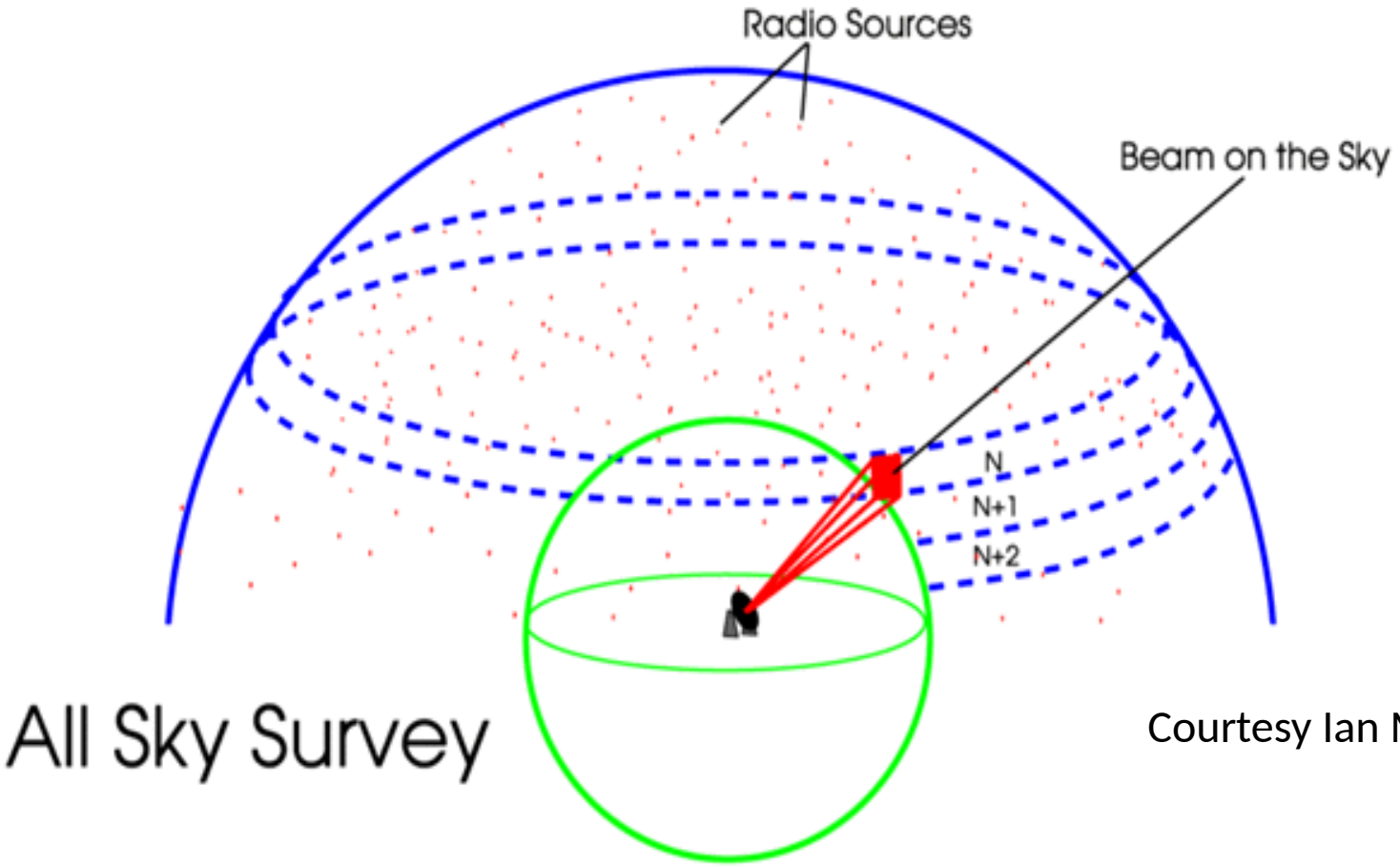


Sky ≡ series of impulses where it is sampled at intervals of $\Delta\theta$ radians

Beam moved across

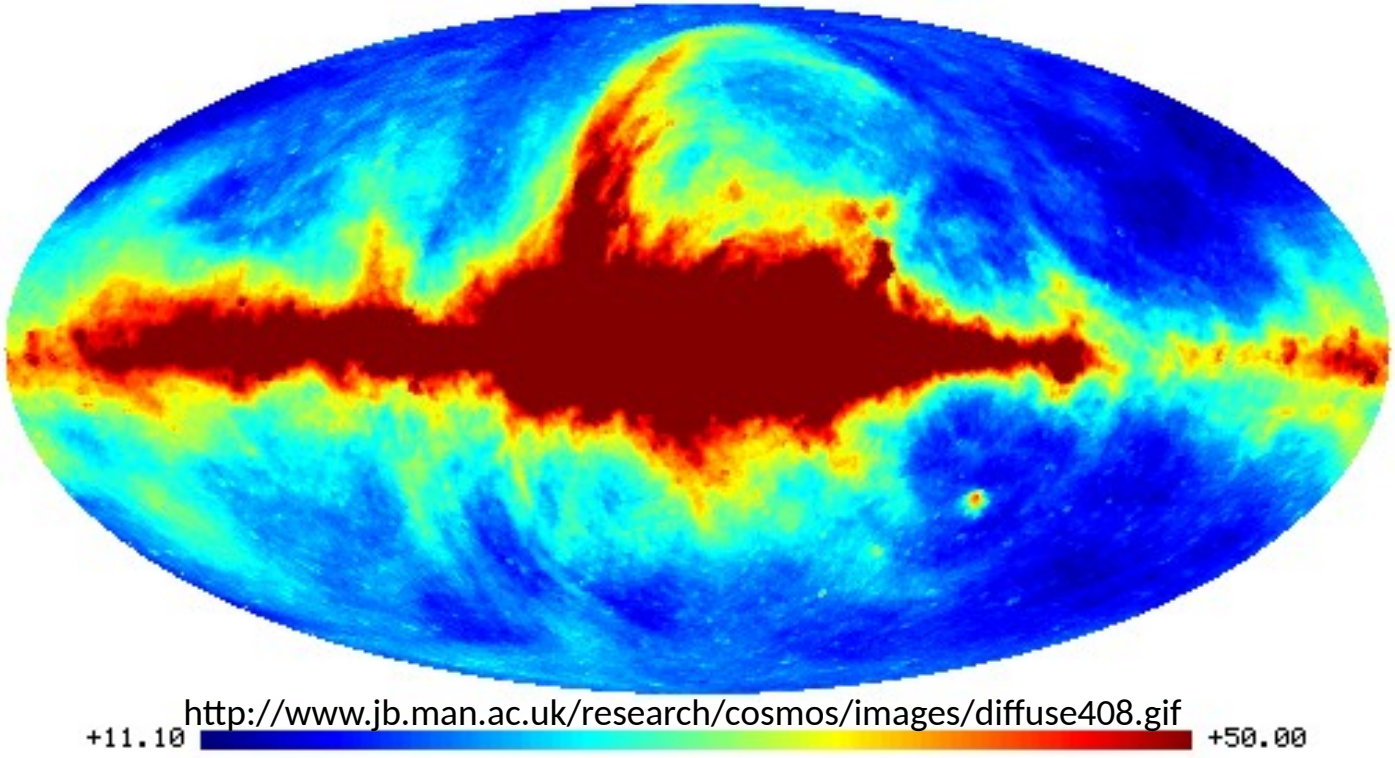
- **Slides 48-49:** → an antenna diameter $u_{max} = d/\lambda$ wavelengths is sensitive to Fourier components of the sky brightness distribution up to u_{max} cycles per radian.
- For Nyquist sampling only have to sample at $>2u_{max}$ points per radian – the antenna cut-off means no other angular frequencies are passed.
- In angle therefore sample at $\Delta\theta < 1/2u_{max}$ or for a sinc² beam >2 sample points between the peak and the first null or across the half power beamwidth.
- Radio astronomers are conservative and usually sample at a rate of ~3 points across the main beam $\theta_{1/2}$.

Cartoon of All-Sky Survey



Courtesy Ian Morison

Classic map of the entire radio sky at 408 MHz



http://www.jb.man.ac.uk/research/cosmos/images/diffuse408.gif

Made >30 years ago with a combination of the Jodrell Bank (76m); Effelsberg (100m Germany); Parkes (64m Australia) radio telescopes by G. Haslam et al.

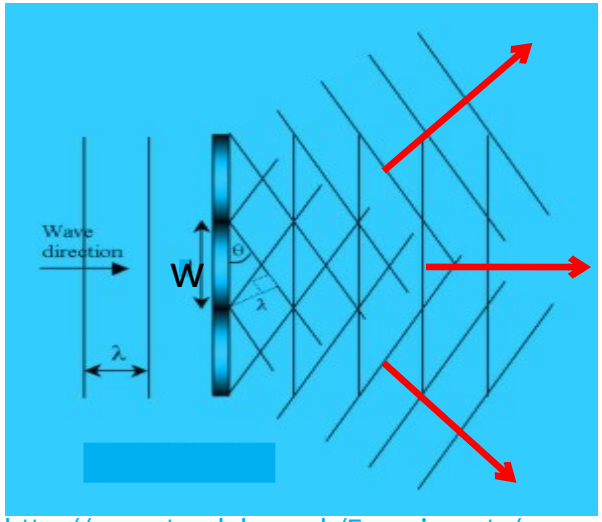
λ=0.74m; d(smallest telescope) = 64m → λ/d = 0.012 radians = 0.66 degrees
map resolution = 0.85 degrees i.e. 1.28 λ/d → highly tapered aperture distribution for low sidelobes

Map was made by scanning the telescope across the sky and collecting data point-by-point at angles separated by ~0.25 degrees → (41,259 deg^2)x (16 points/square degree) ~ 660,000 separate points.
Main problems: i) pick-up ground + sky radiation via residual sidelobes which varies with pointing position and is different for each of the three telescopes
ii) residual variations in gain of receivers despite careful calibration (see next lectures)

THE END

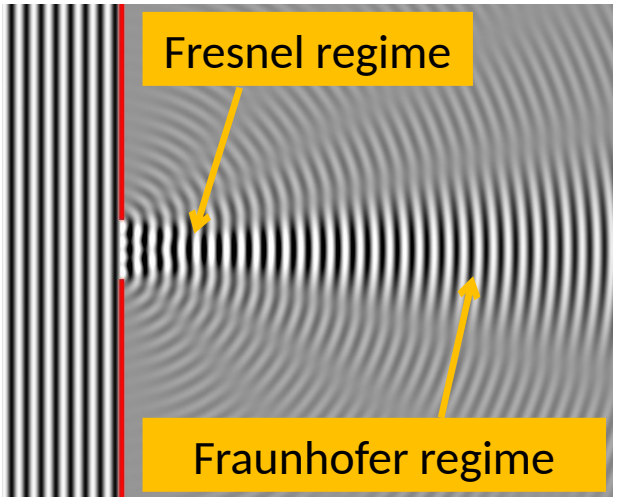
SLIDES REMOVED FROM ORIGINAL VERSIONS

Fraunhofer diffraction - reminders



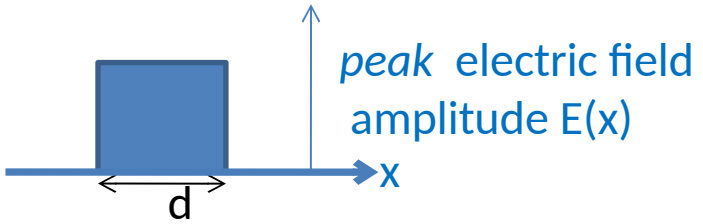
http://www.teralab.co.uk/Experiments/Diffraction/Diffraction_Page1.htm

A grating with a *sinusoidal* variation in transmission, illuminated by plane monochromatic waves, produces only three diffracted waves – one straight through and two at angles $\pm\theta = \lambda/w$ (small angle approximation); w is the *spatial* wavelength of the variation in transmission across the grating.



http://www.ualberta.ca/~pogosyan/teaching/PHYS_130/FALL_2010/lectures/lect35/lecture35.html

A single slit uniformly illuminated across its aperture i.e.

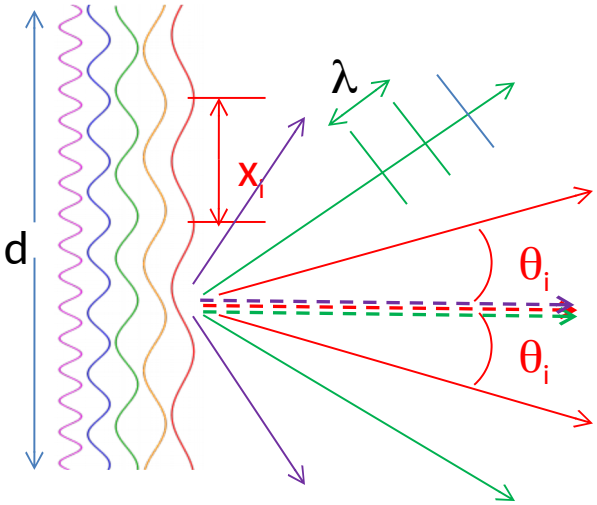


produces many diffracted beams. It is equivalent to a combination many sinusoidal gratings each of a different spatial wavelength, limited by the extent of the slit width d.

A physical picture of beam formation

The aperture distribution E(x) = Σ standing sinusoidal waves

= Σ sinusoidal "diffraction gratings"



• In the "far-field" each "grating" of *spatial wavelength* x_i metres gives a "straight ahead" wave and two waves at angles $\pm\theta_i = \lambda/x_i$ radians (small angle approximation). *The spatial and angular patterns constitute a "Fourier pair" - which have an inverse relationship to each other.*

• Equivalently the extent of the aperture can be described in units of λ ($u = x_i/\lambda$) where *u is a spatial frequency* i.e. the number of RF cycles of λ across the aperture.

• The combined effect of the imagined grating is the complex angular pattern of electric field variations $E(\theta,t)$. As the aperture size increases, lower spatial frequencies *u* can be fitted in → more power is radiated at small angles (1/*u* radians) → a narrower beam!

•The complex electric field response $E(\theta,t)$ *varies at the radio frequency* ν – is not measurable – instead the average power response is detected : $[E(\theta)]^2 = E(\theta)\,E^*(\theta)$ [recall $(x+iy)(x-iy)= x^2+y^2=r^2$] and thus the relative phase information – i.e. the relationships between the spatial and the temporal variations in the angular pattern $E(\theta,t)$ (e.g. sidelobes can be π out of phase) is lost .

→ *power beam is everywhere +ve (see also slide **10**) (squaring a number loses evidence of its sign !)*

- Diffraction at a single slit: mathematical analysis

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html#c3

- Diffraction at a single slit: JAVA applet

http://www.walter-fendt.de/ph14e/singleslit.htm

- Diffraction grating: illustrative description

http://www.teralab.co.uk/Experiments/Diffraction/Diffraction_Page1.htm

Shows how one can imagine the grating as being composed of sinusoidal standing waves each of which produces three travelling diffracted waves

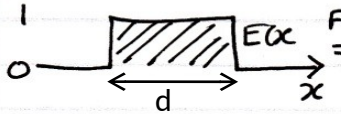
Fraunhofer diffraction at an aperture

ANGULAR PATTERN ≡	FOURIER TRANSFORM OF
OF ELECTRIC FIELD	ILLUMINATING FIELD
(θ)	(x)

$$(1\text{-}D)\quad E(\theta) = \int_{\text{aperture}} E(x)\, e^{i(2\pi\theta x/\lambda)}\, dx$$

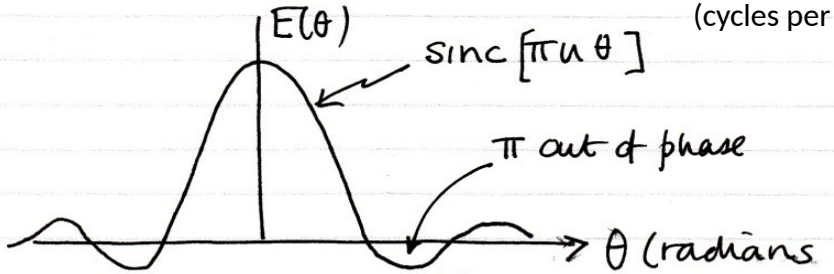
phase term varies across aperture with θ (refer back to slide 17)

For single slit:



$$E(\theta) = \frac{\sin\left[\pi\theta\, d/\lambda\right]}{\left[\pi\theta\, d/\lambda\right]} = \frac{\sin[\pi\theta u]}{[\pi\theta u]} = \operatorname{sinc}[\pi u\theta]$$

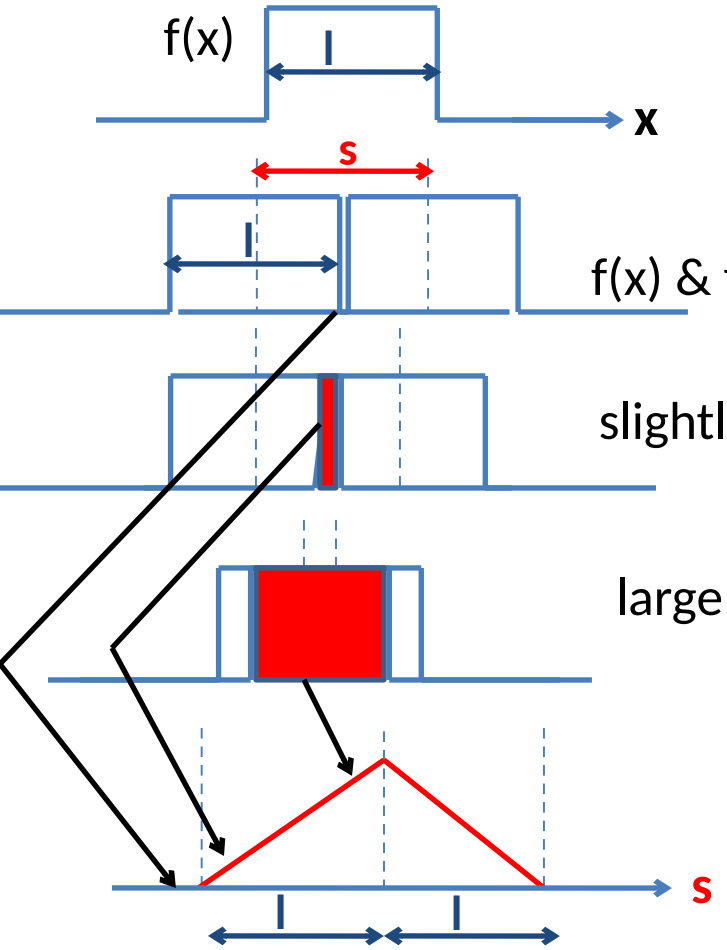
$u = \frac{d}{\lambda}$ = # RF wavelengths across the aperture ≡ a spatial frequency (cycles per metre)



N.B. Oscillating at RF frequency ν

Autocorrelation

Multiply a function f(x) (here a rectangle of length l) with a copy f(x-s) shifted by an amount s and ∫ (i.e. form an "overlap" integral) then repeat for all shifts s and plot the results of each integration as a function of s → ACF(s).



f(x) & f(x-s) not quite overlapping → ACF(s) = 0

slightly overlapping → ACF(s) = small

largely overlapped → ACF(s) = nearly maximum

etc etc repeating the process to map out the Autocorrelation Function ACF(s)

Wiener-Khinchine Theorem

The autocorrelation function(ACF) $F(s) = \int_{-\infty}^{+\infty} f(x) f(x-s) dx = f(x) \bigstar f(x)$

The Autocorrelation Theorem or Wiener-Khinchine Theorem

The Fourier transform of the auto-correlation of a function is equal to the Fourier transform of the function squared: (stated not proved here – justification in lecture 5)

$$\text{FT}\{ f(x) \bigstar f(x) \} = [\text{FT}\{ f(x) \}]^2$$

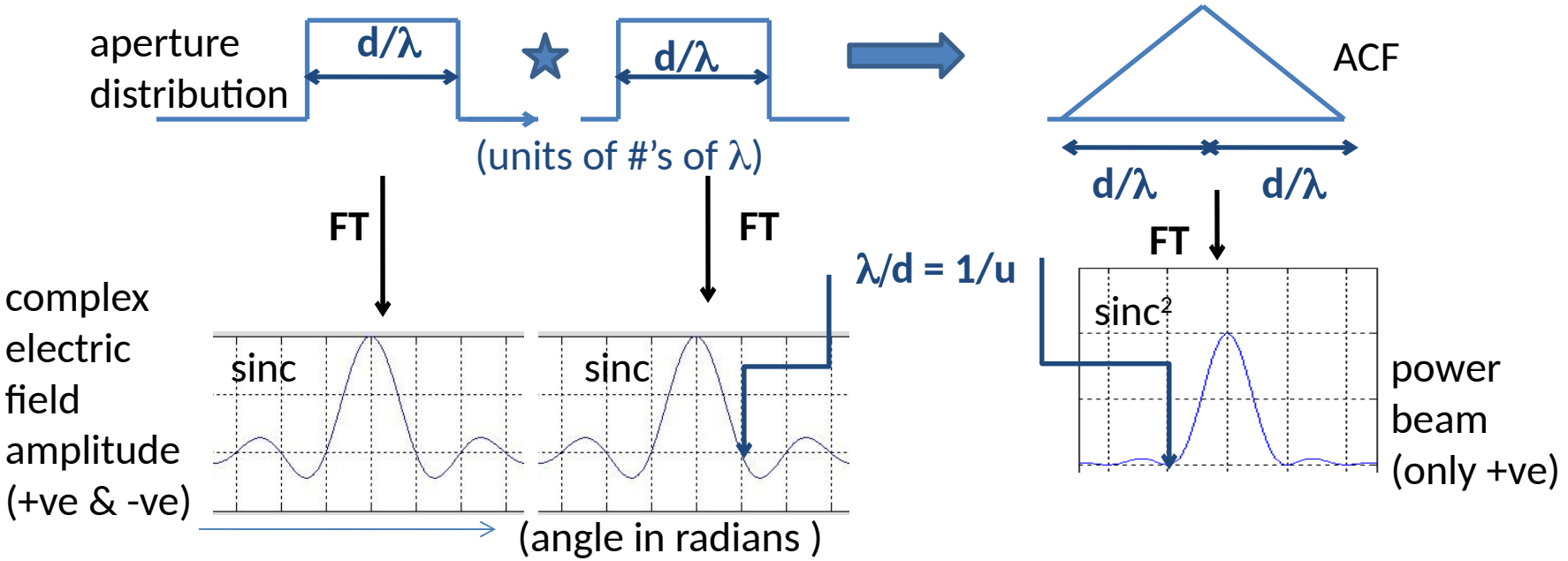
= a *power* spectrum

Wiener-Khinchine Theorem applied to antennas

Fourier Transform [ACF of aperture distribution (in spatial coordinates)]
= Power spectrum (in angular coordinates)
= Antenna power beam pattern

Autocorrelation function and power beams

1. There is a "Fourier pair" relationship between the *spatial* pattern of the illumination across the aperture (the aperture distribution) and the *angular* pattern of the complex electric field in the far-field (the "voltage beam") – this is a basic result from diffraction theory.
2. The Fourier Transform of the ACF is the Fourier Transform of the original function squared – this is the *Autocorrelation or W-K Theorem*
3. Putting these ideas together we see the FT of the ACF is the "power beam"

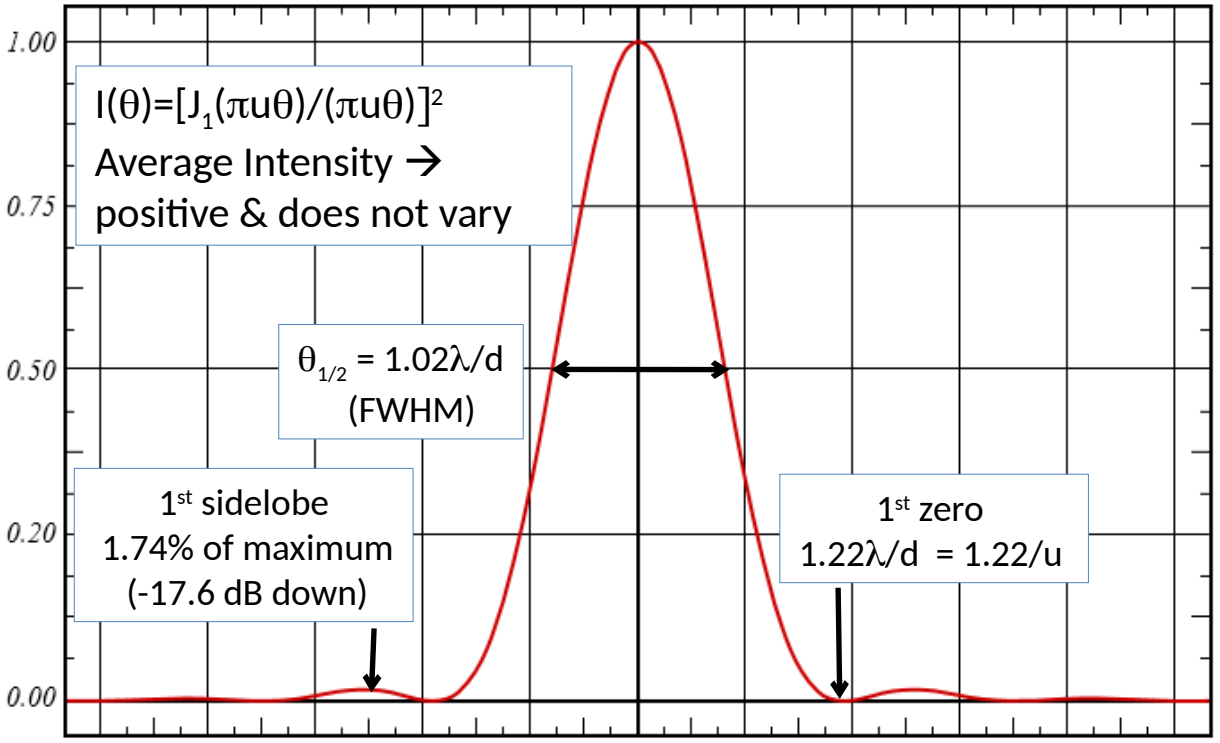


http://www.thefouriertransform.com/pairs/box.php

The Airy Disk

The Wiener-Khinchine Theorem applied to a circular aperture with uniform Illumination results in the intensity pattern $I(\theta)$ known as the Airy Disk. Shown here is a 1-D cut.

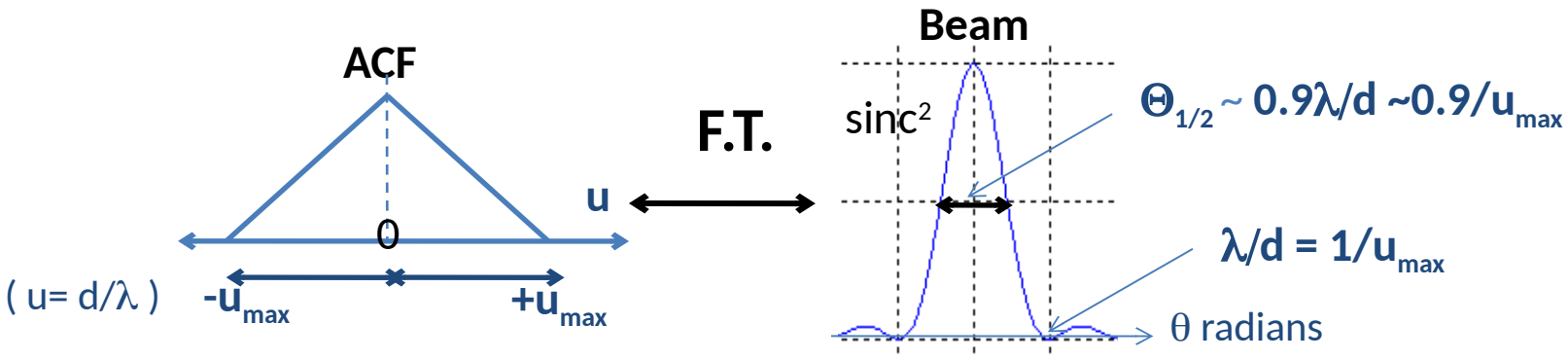
http://upload.wikimedia.org/wikipedia/commons/9/93/Airy_Pattern.svg



u = d/λ : a "spatial frequency"
i.e. # of cycles of the RF wavelength across the aperture

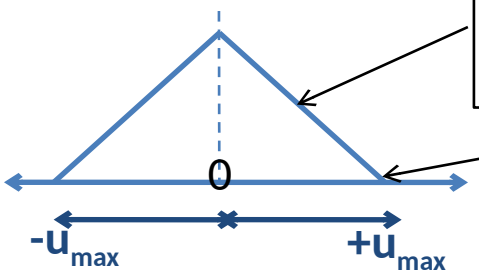
The angular frequency transfer function for intensity

Example: square antenna (as in slide #20)



Diffraction theory (in the small angle approximation) means that
u_{max} = d/λ *spatial* frequency (# cycles of λ across aperture) corresponds directly to an *angular* frequency (# cycles rad^{-1} across sky):
e.g. d= 500 λ → u_{max}=500 cycles aperture^{-1} → 500 cycles rad^{-1}

The *angular* frequency transfer function is the same as the weighted *spatial* frequencies which make up the virtual "intensity aperture".

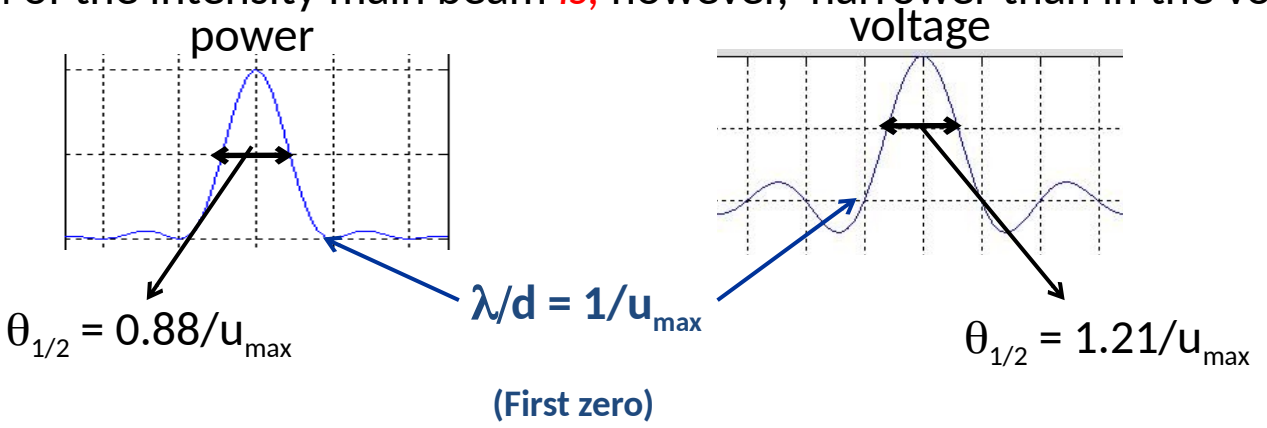


Angular frequency components passed by antenna but with reduced magnitude

Angular frequency components higher than u_{max} are not passed by antenna

Voltage and power beams – another look

- A triangular ACF is appropriate for a uniformly illuminated antenna measuring intensity (power α voltage^2). The antenna can be regarded as having a virtual "intensity aperture" of this shape. This extends from $-u_{max}$ to $+u_{max}$ and its triangular shape implies an increasingly low weighting to the higher spatial frequency Fourier components making up the virtual aperture and thus to the angular frequency components of the sky – see previous slide).

- The weighting means that intensity beam obtained from the Fourier transform of the ACF therefore does *not* have double the formal resolution of the voltage beam obtained from the Fourier transform of a uniform distribution with only half the extent; the 1st zero in sinc^2 and sinc are both at $1/u_{max}$ (slide 20 and below) .

- The FWHM of the intensity main beam *is,* however, narrower than in the voltage main beam.