MANCHESTER 1824

The University of Manchester

DARA Training Unit 1 2018
Radioastronomy
Lecture 4

Noise & receivers

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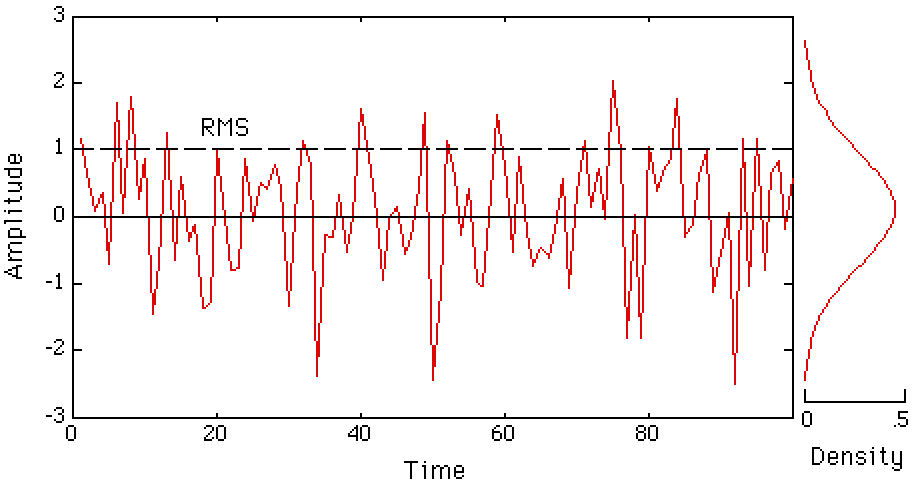
Note 1) this presentation has been put together from more than one presentation given during PW's Manchester lecture course in 2014. Hence narration references to other slides and/or references to material in other lectures are not always correct. I do not think that this greatly affects its usefulness.

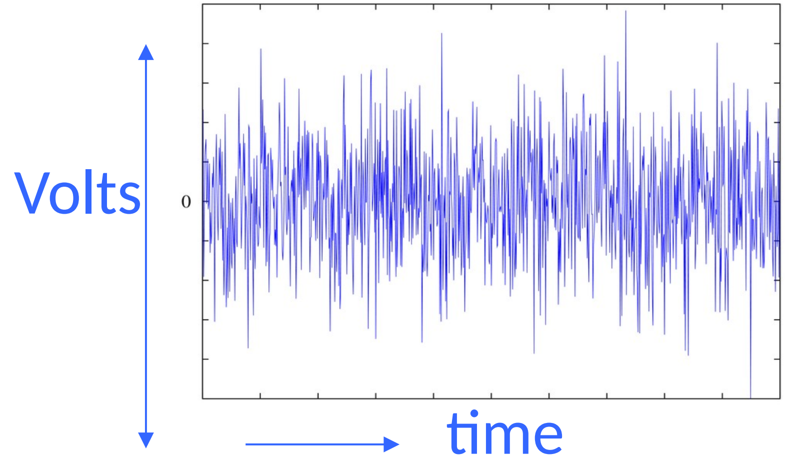
2) Slides removed from original presentations to save time for DARA are retained after "THE END"

3) The handwritten slides (referred to as Notes A, B etc) were written in real-time in Manchester lectures by PW so that students had to pay attention and write this material down in their own notebooks in order to achieve a full set of course material– this scheme also mitigated the deadly "Death by Powerpoint" disease and kept students awake.

Random (“white” or gaussian) noise



http://www.comdis.wisc.edu/vcd202/gaussian.GIF



All the natural signals with which we are dealing have been generated by *random processes* of one form or another & when incident on the receiver have the same basic form - random variations of the incident electric field in amplitude and phase

In the radio receiver the electric field variations are turned into random voltages with a **gaussian probability distribution**

$$P(V)dV = \frac{1}{\sigma(2\pi)^{1/2}} e^{-[V^2/2\sigma^2]}$$

mean $\langle V \rangle = 0$; rms = $\langle V^2 \rangle^{1/2} = \sigma$

Signal voltages are indistinguishable from (and usually much smaller than) the voltages generated by thermal agitations in the resistive components of the receiver. All such voltages are termed “white noise” since the amount of power per unit bandwidth is independent of frequency (as in the Nyquist formula for resistor noise power = kT W Hz^{-1}). The only stable and hence measurable, quantity at a single *point in space is the average power* $\langle V^2 \rangle = \sigma^2$ (i.e. the “variance” of a random variable)

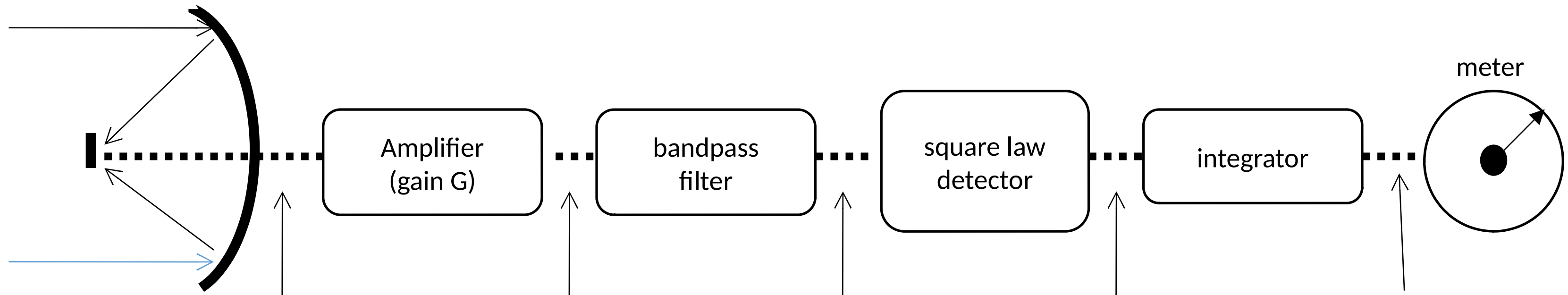
Generic Characteristics of random noise

- At any instant assumes unpredictable values
  → no correlation with what went before
  → no redundancy hence cannot be compressed – every bit is as good as every other bit hence no MPEG or JPEG etc!

Why Gaussian statistics?

- **Central Limit Theorem**: sum or average of samples drawn from ANY distribution with a finite mean and standard deviation will be approximately Gaussian with the approximation improving as the samples get bigger.
- First example was Brownian Motion
- Gaussian processes are of fundamental importance – but do NOT have intrinsic universality e.g. turbulence & other phenomena have power law distributions.
  (see also Lectures 1&2 slide 32)

Basic Receiver – acts as a "radiometer"



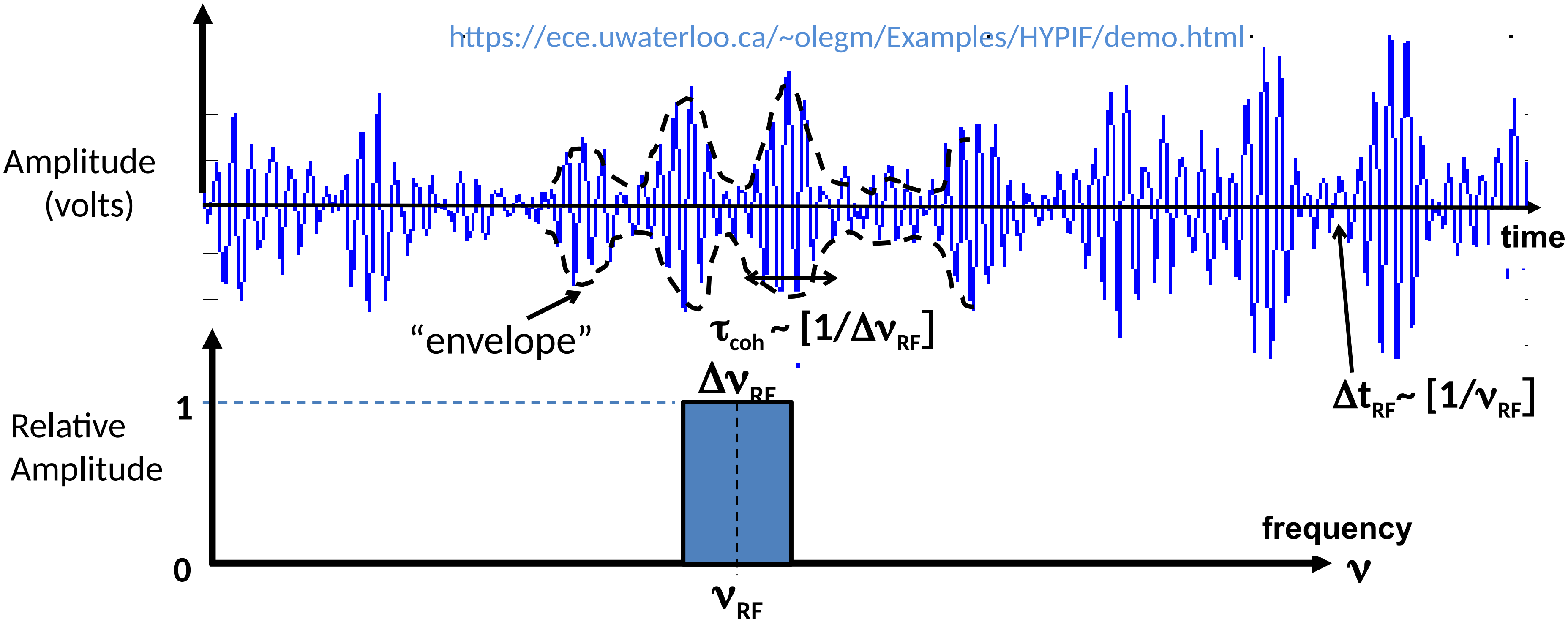
randomly fluctuating electric field: Wide frequency range	**quasi -random voltage** Voltage: $v_i(t)$ Impedance: Z_i Bandwidth: set by antenna & transmission line.	**quasi -random voltage** Voltage = sqrtG $v_i(t)$ Bandwidth: set by amplifier(s) which must be *linear*	**quasi -random voltage** Voltage: $v_o(t)$ Impedance: Z_o Bandwidth: $\Delta\nu$ set by defining filter.	**detected power** Fluctuations on timescale $\tau_{coh} = 1/\ \Delta\nu$	**detected power** Smoothed over selected time period τ_{avg}

A linear radiometer with a parabolic dish with feed horn at the antenna but it could be a simple dipole. Natural sources are broad band so the frequency range of the incident radiation is large. The antenna and transmission line (cable or waveguide) impose their own characteristic bandwidth which can range from ~10% for a dipole to factors up to 10 for modern feed horns. The amplifier system has to provide an overall power gain of >100 dB and must be linear in its voltage response. The amplifier bandwidth usually quite broad so the frequency band is set by a bandpass-defining filter system. The noise power is measured with a diode detector operating in the square-law regime followed by a "video" amplifier (not shown for clarity) which can amplify signals close to d.c. ; its output is smoothed and measured with some form of meter system.

Radio astronomy signals are VERY WEAK so need a huge amount of amplification >100 dB before they can be measured and quantified.

(see slide 17)

Close-up of *band-limited* Gaussian noise



- Noise frequency spectrum restricted by *a band-pass filter covering a range of frequencies* $\boldsymbol{\Delta\nu_{RF.}}$ The result is a *time series* of random noise *but now with some statistical structure imposed*.
- Typical zero crossing times $\boldsymbol{\Delta t_{RF}}$ are separated by $\boldsymbol{1/\nu_{RF}}$ and the amplitude modulation "envelope" exhibits a characteristic "coherence time" $\boldsymbol{\tau_{coh}}$ **~ [1/$\boldsymbol{\Delta\nu}$]** secs. *Within* $\boldsymbol{\tau_{coh}}$ *the voltage is quasi-sinusoidal and hence has a quasi-predictable phase.*
- Extension to sum of random sine waves of the concept of "beats" i.e. modulation resulting when two pure sine waves with frequencies ν_1 and ν_2 are added together. The sum function in time has a modulation period ("beats") =$\boldsymbol{1/[(\nu_1 - \nu_2)]}$ secs.

Radiometers – prologue

- Repeat: Radio astronomy signals need a high degree of amplification (**slide 17**) before they can be measured/quantified
- Noise *voltage* signals have zero mean and fluctuate on rapid timescales $1/\nu_{RF}$
  - ν_{RF} = 1 GHz → voltage varies on 1 nanosecond (10^{-9} s) timescales
  - envelope varies on timescales $\boldsymbol{\tau_{coh}}$~$1/\Delta\nu_{RF}$ ~ 10 nanoseconds for $\Delta\nu_{RF}$ = 100 MHz

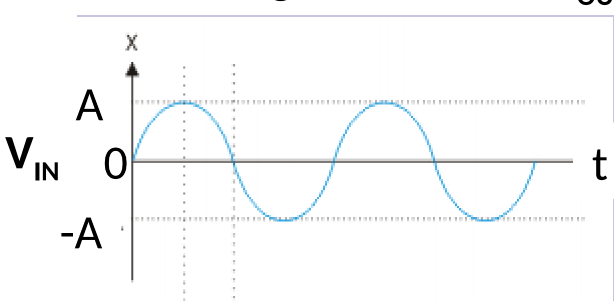
    → envelope typically would “contain” ~10 wavelengths of a quasi-sinusoid

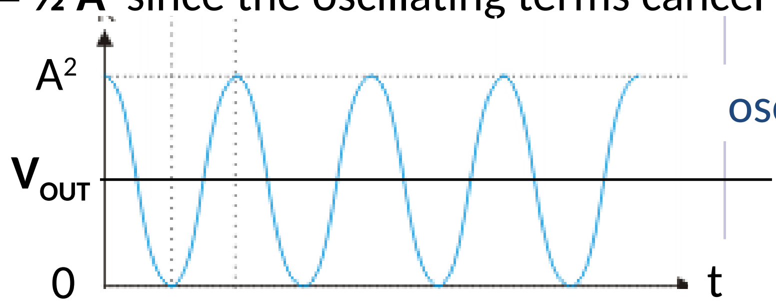
- We need a steady non-zero signal → measure *power* and average for long periods. Simplest way to measure power: put amplified voltage into a semiconductor “square law” detector – for *small input signals* $\mathbf{V_{OUT}\ \alpha\ V_{IN}^2}$ i.e. proportional to the power in the input signal.

$$\mathbf{V_{IN} = A\cos(2\pi\nu_{RF}t) \rightarrow V_{OUT} = A^2\cos^2(2\pi\nu_{RF}t) = \tfrac{1}{2} A^2[1+\cos(2.\{2\pi\nu_{RF}t\})]}$$

trigonometrical identity $\cos^2\theta = \tfrac{1}{2}\,[1+\cos 2\theta]$

Average over time $\mathbf{\langle V_{OUT}\rangle = \tfrac{1}{2} A^2}$ since the oscillating terms cancel out



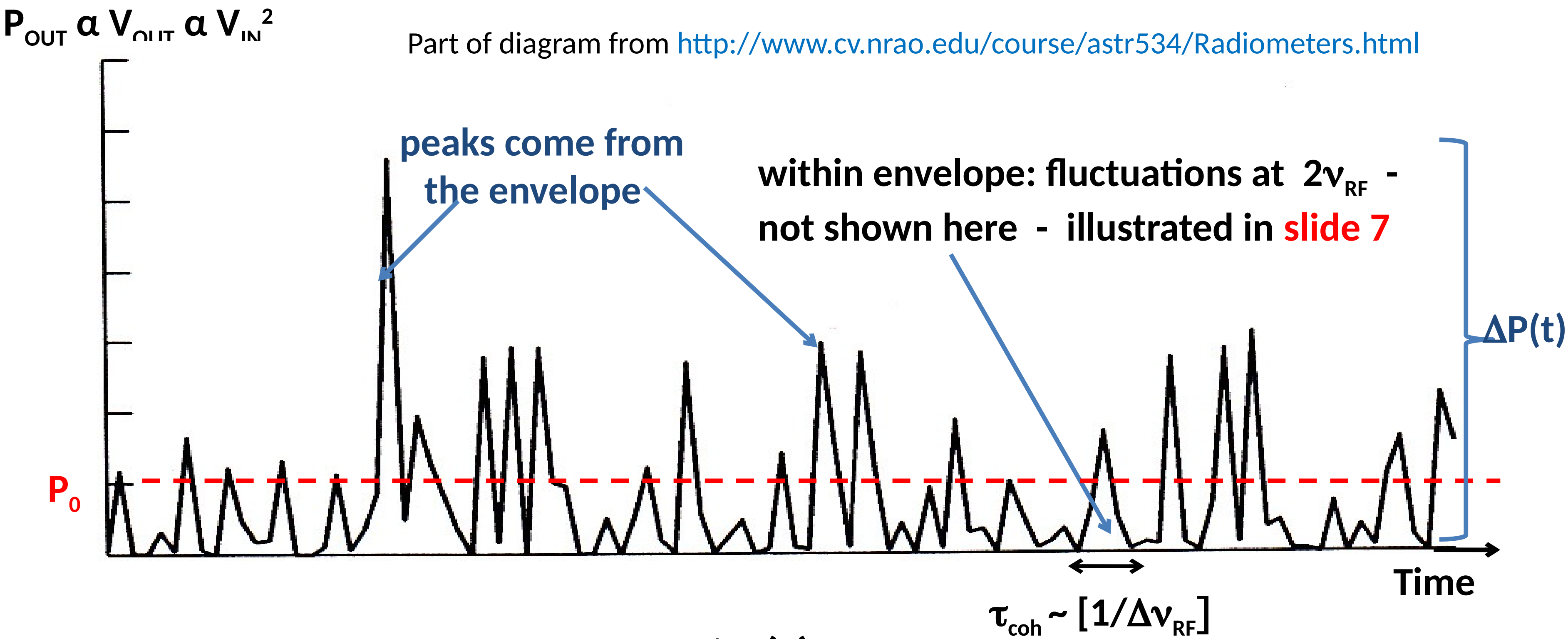


oscillations 2x as fast

average power

Schematic of fluctuating power P(t)

Part of diagram from http://www.cv.nrao.edu/course/astr534/Radiometers.html



$$P_{OUT} = P_0 + \Delta P(t)$$

P_0 = stable component

(what we want)

$\Delta P(t)$ = strongly fluctuating component on timescales $1/\Delta\nu_{RF} = \tau_{COH}$

(what we need to average out)

Tutorial animation illustrating sine waves, noise signals, rms and mean voltages
http://www.fourier-series.com/Noise/flashprograms/noise2.html

Note: it deals directly with fluctuating low frequency signals and so does not capture the concept of a high frequency RF signal within a fluctuating envelope set by the inverse bandwidth (**slide #6**).

Nevertheless it helps you to visualise what's happening & what it looks like when you square a fluctuating voltage.

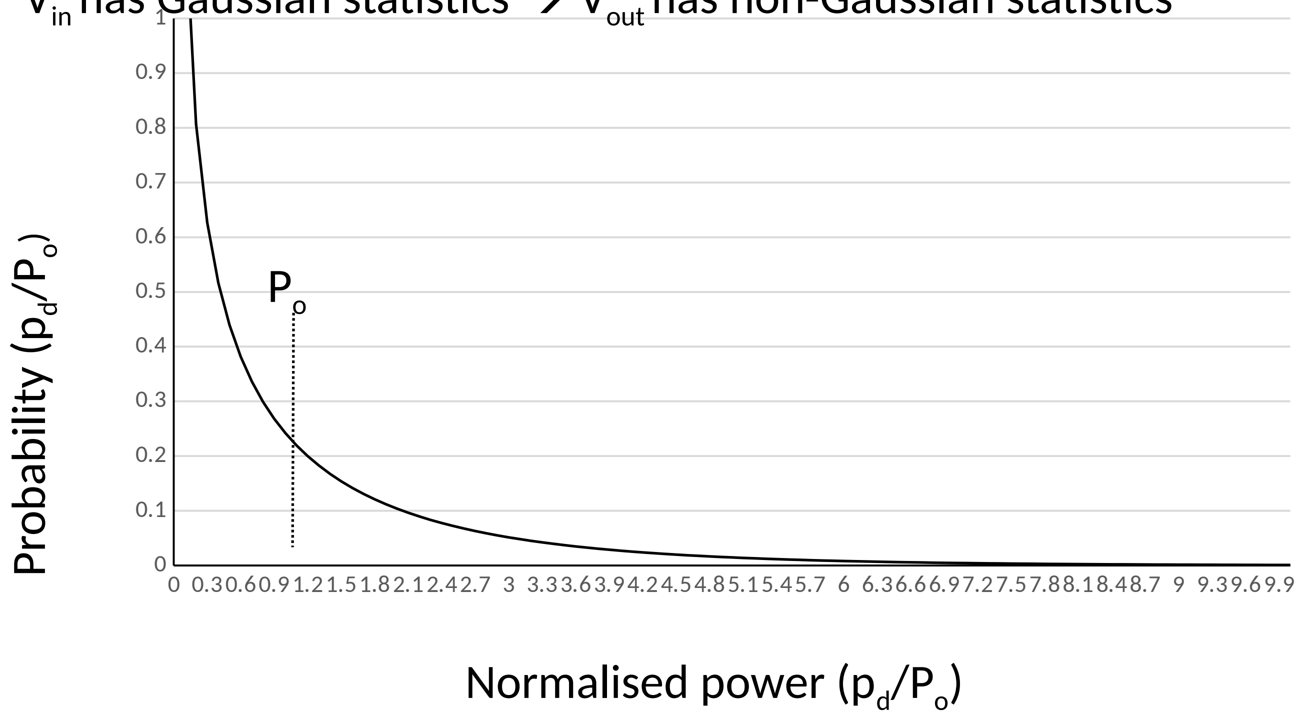
Fluctuating power after square law detection (slides 8&9)

Squaring $V_{in} \rightarrow V_{out}$ (power) produces greater fluctuations about the mean

e.g. V_{in} = 1,2,3,4,5 → mean = 3; rms = 1.6

V_{out}= 1,4,9,16,25 → mean = 11; rms = 9.6

V_{in} has Gaussian statistics → V_{out} has non-Gaussian statistics



The probability distribution of the power p_d of white noise after square law detection is a chi-squared distribution for one degree of freedom. On this normalized plot the steady mean P_o is at 1, the variance of the fluctuations is $2P_o$ and hence the rms $\Delta P(t) = \sqrt{2}P_o$

Averaging P_{OUT} → mean level P_0

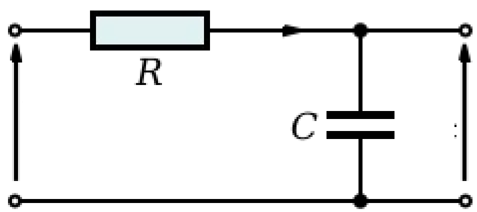
To measure P_0 accurately need to average or "integrate" P_{OUT} (t) over long periods ("integration times") $\tau_{AVG} >> 1/\Delta\nu_{RF}$

- If $\Delta\nu_{RF}$ = 100 MHz → τ_{coh} >> 10 nanoseconds
- Typically τ_{AVG} ~ 1 second for radiometry (shorter for pulsar work – see later)

Classical method: pass P_{OUT} (t) signal into a low pass filter - simplest is an RC circuit

$P_{OUT}=P_{IN}[1-e^{-\tau/RC}]$

$P_{IN} = P_0 \pm \Delta P(t)$



$P_{OUT} = P_0 \pm \delta P(t)$

Modern alternative: digitize $P_0 + \Delta P(t)$ and form series of averages (over τ_{AVG}) in computer

Central limit theorem (repeated!) : *Any* large sample of independent random variables will exhibit a Gaussian distribution → although $P(t) = [P_0 + \Delta P(t)]$ has a very non-Gaussian distribution the averaged power $Po \pm \delta P(t)$, averaged over long $\boldsymbol{\tau}_{AVG}$, is Gaussian.

Fluctuations after averaging $\delta P(t)$

Simple physical argument….

- For an averaging time τ_{AVG} there are $\frac{\tau_{AVG}}{\tau_{COH}}$ coherence times $\equiv \tau_{AVG}\,\Delta\nu_{RF}$
- Nyquist sampling → ≥ 2 samples for each τ_{COH} to capture all information (Lectures 7&8)

  → $(2 . \tau_{AVG} . \Delta\nu_{RF}) = N$ samples of output power P_{OUT}

- Error on the mean $= \sigma/\sqrt{N}$ ← (Central Limit Theorem → gaussian statistics)

  $\sigma = \sqrt{2}\,P_0$ (Notes 'A')

$$\rightarrow \quad \delta P_{rms} = \frac{\sqrt{2}\,P_0}{(2\,\tau_{AVG}\,\Delta\nu_{RF})^{\frac{1}{2}}}$$

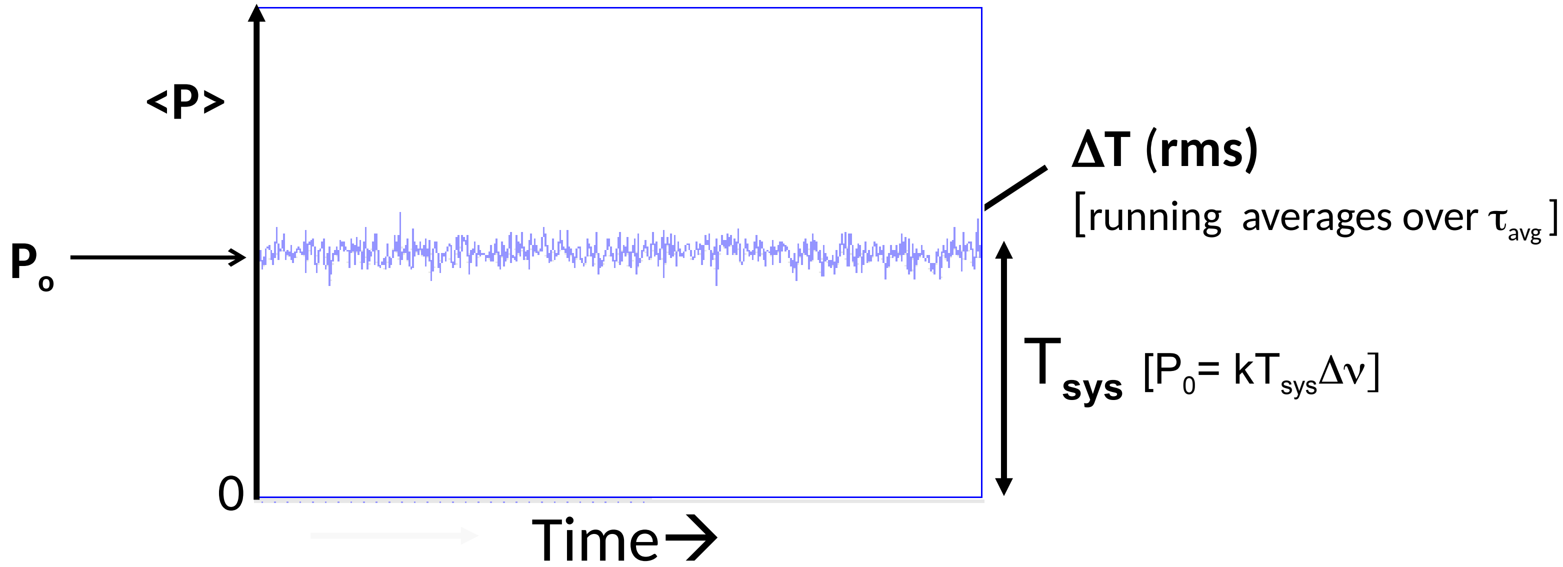
$$= \frac{P_0}{(\tau_{AVG}\,\Delta\nu_{RF})^{\frac{1}{2}}}$$

But $P_0 = kT$ $\mathrm{W\,Hz^{-1}}$ (resistor Rayleigh-Jeans)

$$\rightarrow \quad \boxed{\Delta T_{rms} = \frac{T_{SYSTEM}}{(\tau_{AVG}\,\Delta\nu_{RF})^{\frac{1}{2}}} \quad \text{RADIOMETER EQUATION}}$$

coming to this….

White noise power after square law detection and integration



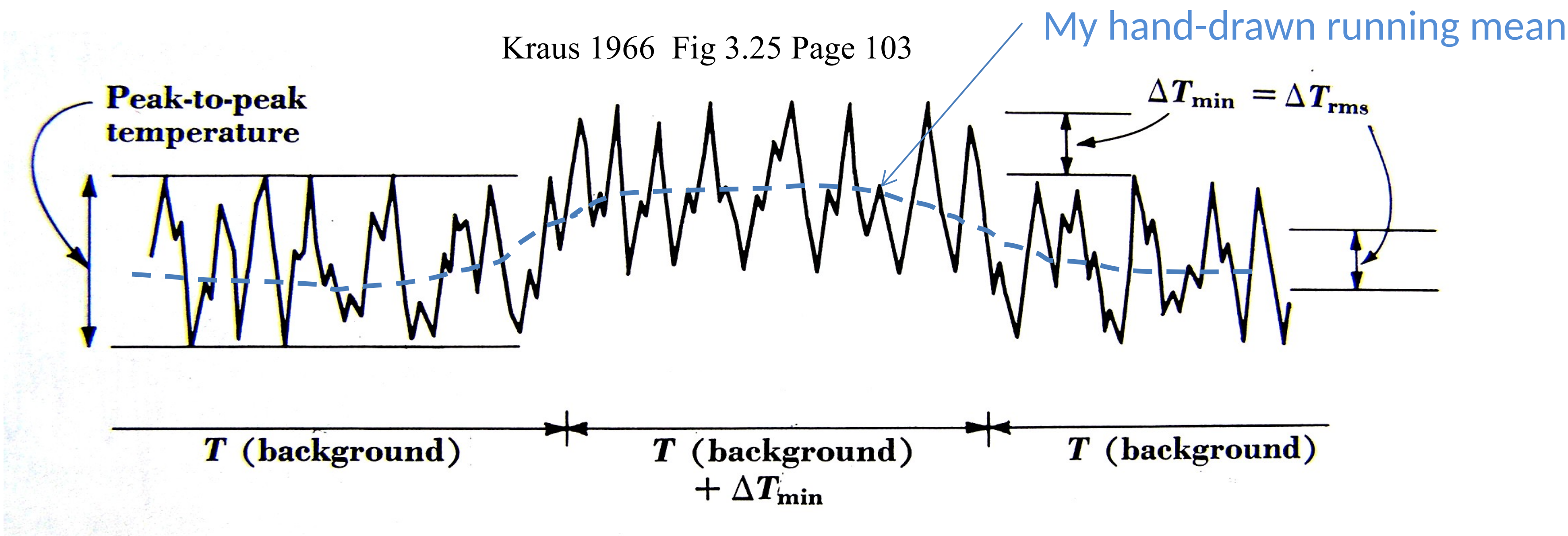
ΔT (rms) = Tsys / $\sqrt{(\tau_{avg}\,\Delta\nu)}$

THE RADIOMETER EQUATION

With random noise signals ***all we can do*** is observe the fluctuating power output of the receiver over a long enough time $\boldsymbol{\tau}_{avg}$ to obtain a result of the desired accuracy. Increasing the receiver bandwidth $\boldsymbol{\Delta\nu}$ also helps – but the reduction in the level of fluctuations only goes as the square root of their product.

Natural signals are very weak and hence long integation times are often required for their detection.

Detecting a weak source



Kraus 1966 Fig 3.25 Page 103

A schematic radio telescope record of the output power of a receiver after final averaging over a timescale seconds as the beam moves over a weak source. *The (slowly-varying) component of T_{sys} (due mainly to T_{LNA}) has been suppressed to emphasise the fluctuations.* On the left and right the telescope "sees" only the background brightness temperature. In the central region the beam is pointed at a weak source comparable to ΔT_{rms} (~1 sigma for the chosen $\boldsymbol{\tau}_{avg}$). Because the telescope is pointed at the source for a time longer than $\boldsymbol{\tau}_{avg}$ the rise in mean temperature due to the source can be detected.

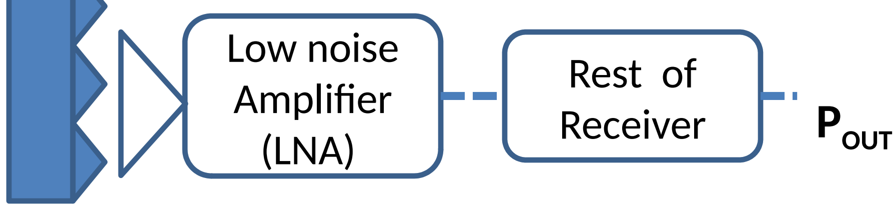
Example: for a 1 sigma "detection" of an antenna temperature T_A= 1 milliK = ΔT_{MIN} with a total system temperature T_{sys} = 100K and $\Delta\nu_{RF}$ = 100 MHz requires $\boldsymbol{\tau}_{avg}$ ~ 100 s ; to increase this to 5 sigma requires an integration time of 2500 sec → *for the detection of weak sources* the *system performance must stay stab*

Measuring the receiver temperature T_{rec}
"Hot and cold load" calibration

http://www.hartrao.ac.za/xdm/xdm_07_jun.html

Subject the system to known sources of input power which are comparable or larger than associated with T_{rec} → little or no integration needed to overcome the output fluctuations.

"ECCOSORB" is carbon loaded polystyrene foam with large "spikes" → excellent absorber over a range of frequencies → acts as b-body at $T_{PHYSICAL}$



Measure P_{OUT} with absorber at two different physical temperatures:

$$P_{OUT,HOT} \propto \text{Receiver power gain x } (T_{rec} + T_{HOT})$$

$$P_{OUT,COLD} \propto \text{Receiver power gain x } (T_{rec} + T_{COLD})$$

Define "Y factor" : $Y = P_{OUT,HOT} / P_{OUT,COLD} = (T_{rec} + T_{HOT})/(T_{rec} + T_{COLD})$

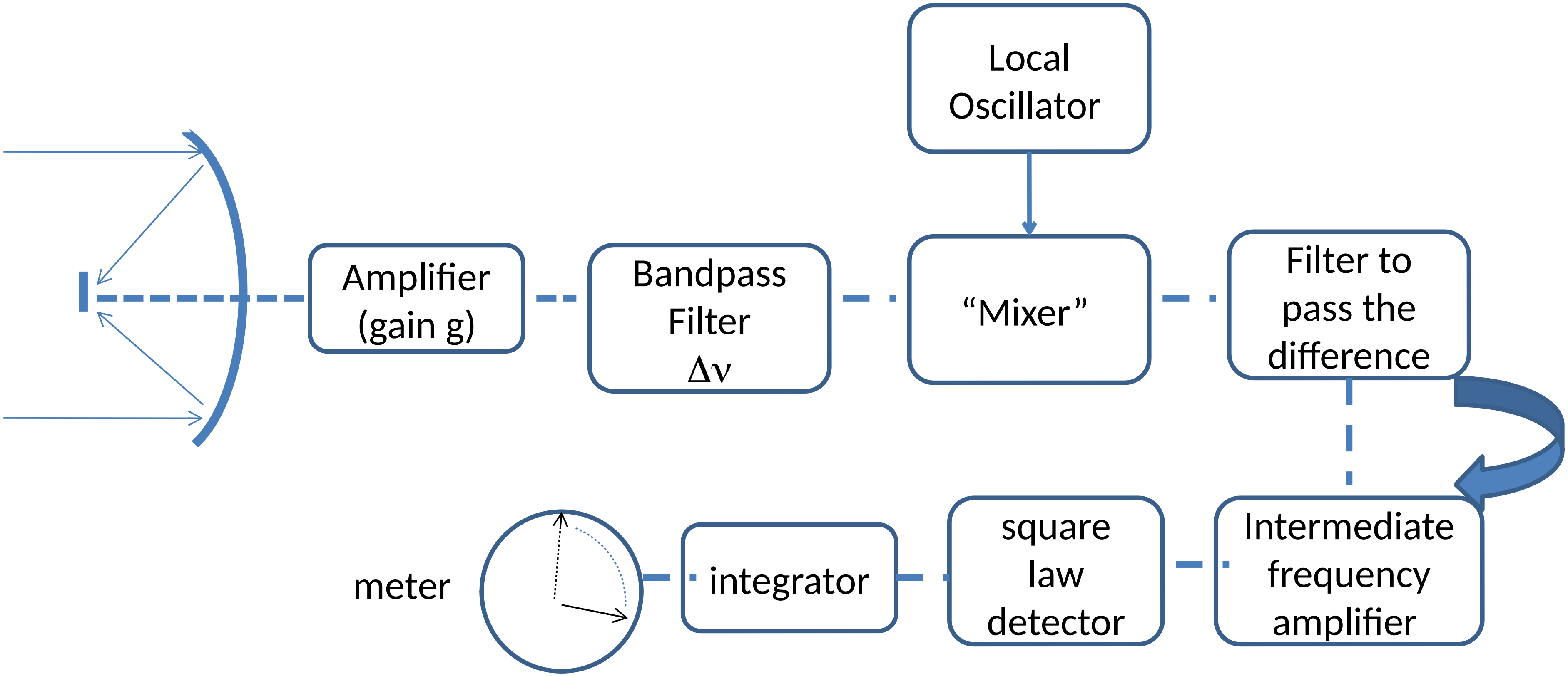
Calculate $T_{rec} = [T_{HOT} - Y.\, T_{COLD}]/(Y-1)$

Usually for convenience use: T_{HOT} ~ 293K (room temperature)

T_{COLD} = 77K (liquid nitrogen – soak the ECCOSORB in it, as in picture)

N.B. implicitly assumes receiver responds linearly to wide range of power input!

Basic "Heterodyne" Receiver



- Easier and cheaper to amplify, filter and generally manipulate signals at lower radio frequencies →the "heterodyne" architecture is common.
- A non-linear mixer circuit (see slides #19-20) is driven by a local oscillator (a sinewave or square wave at frequency ν_{LO}) to shift the band of frequencies $\Delta\nu$ down to a lower centre frequency.
- Band-pass filters always have loss – and hence generate unwanted noise (see later) ; the narrower the band the higher is the loss – hence they are best placed after a stage of amplification.
- Often there is more than one filter in the system e.g. after the IF amplifier as well.

Here we have missed out detailed description of mixing in heterodyne receivers

Main advantage of heterodyne approach: Translating the received signals down to lower frequency makes it easier and cheaper to manipulate them to produce the desired output.

(Some other advantages in the next slide)

Further advantages of heterodyne systems

1) Helps to avoid danger of oscillation in the system arising from positive feedback given the enormous RF amplifier gain required from the input to the output of the receiver

e.g. T_{sys} = 100K ; $\Delta\nu$ = 100 MHz → P_{out} = $k\,T_{sys}\,\Delta\nu$ = 1.4 x 10^{-13} W

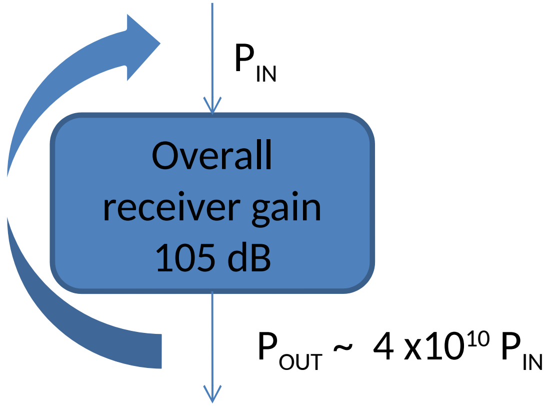
Power for i/p to square law detector typically ~0.01 mW = 10^{-5} W

→ required gain ~ 10^8 i.e. 80dB *(followed by further >20dB of low freq. amplification)*

Power for i/p to a digitizer typically ~5 mW = 5 x 10^{-3} W

→ required gain = 3.6 x 10^{10} ~ 105dB (amplification by >10^{10})

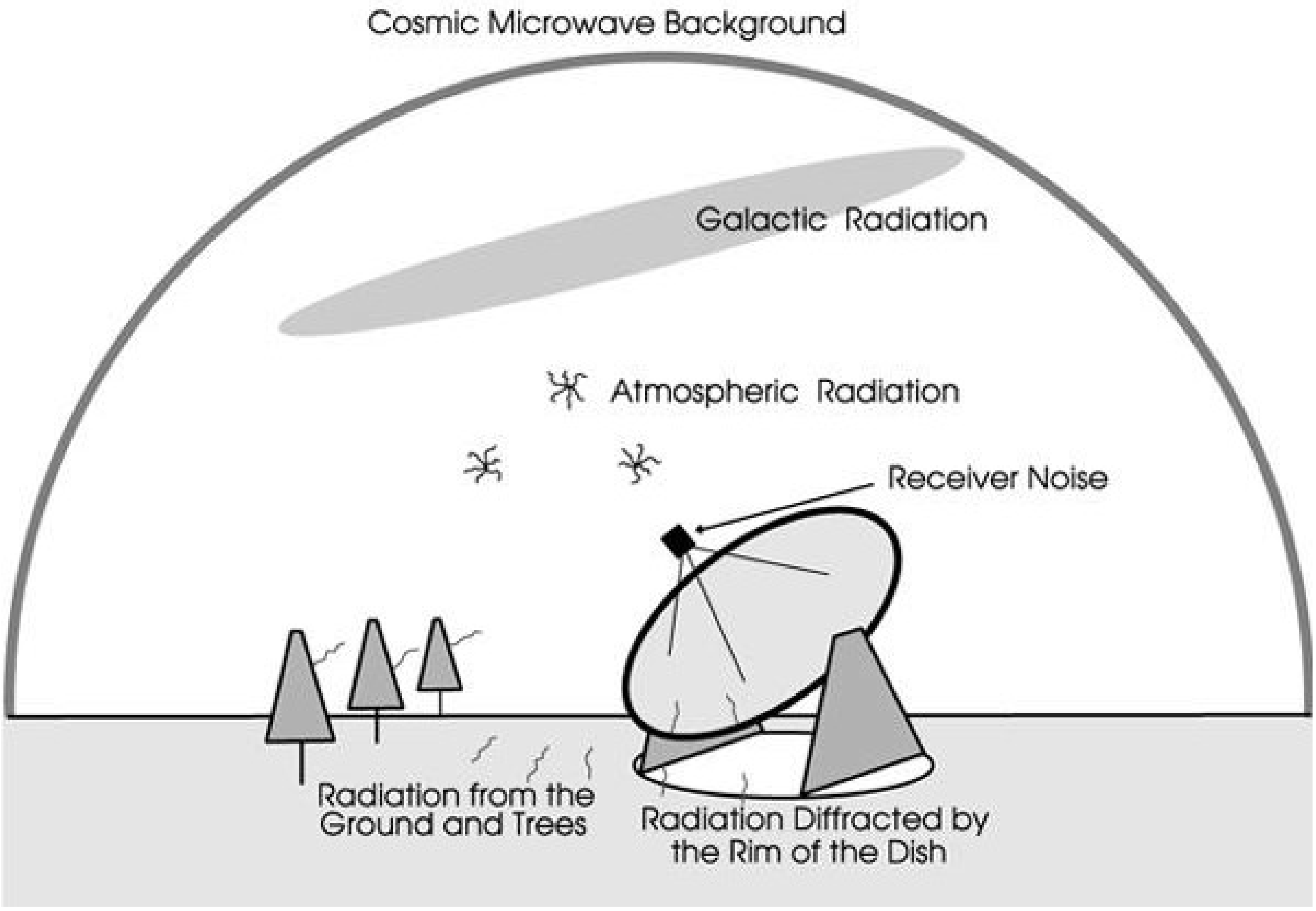
Small leakage of power δP_{OUT} back up the receiver chain can give rise to oscillation if all the gain is at the same frequency.



2) Attenuation in signal transport cables is less at low frequencies – see "cable to observatory" **in lecture 3 Slide 5**

3) Varying the Local Oscillator Frequency ν_{LO} enables the centre frequency of the band under study to be varied – useful for spectroscopic observations – see later

Schematic of receiver noise contributions



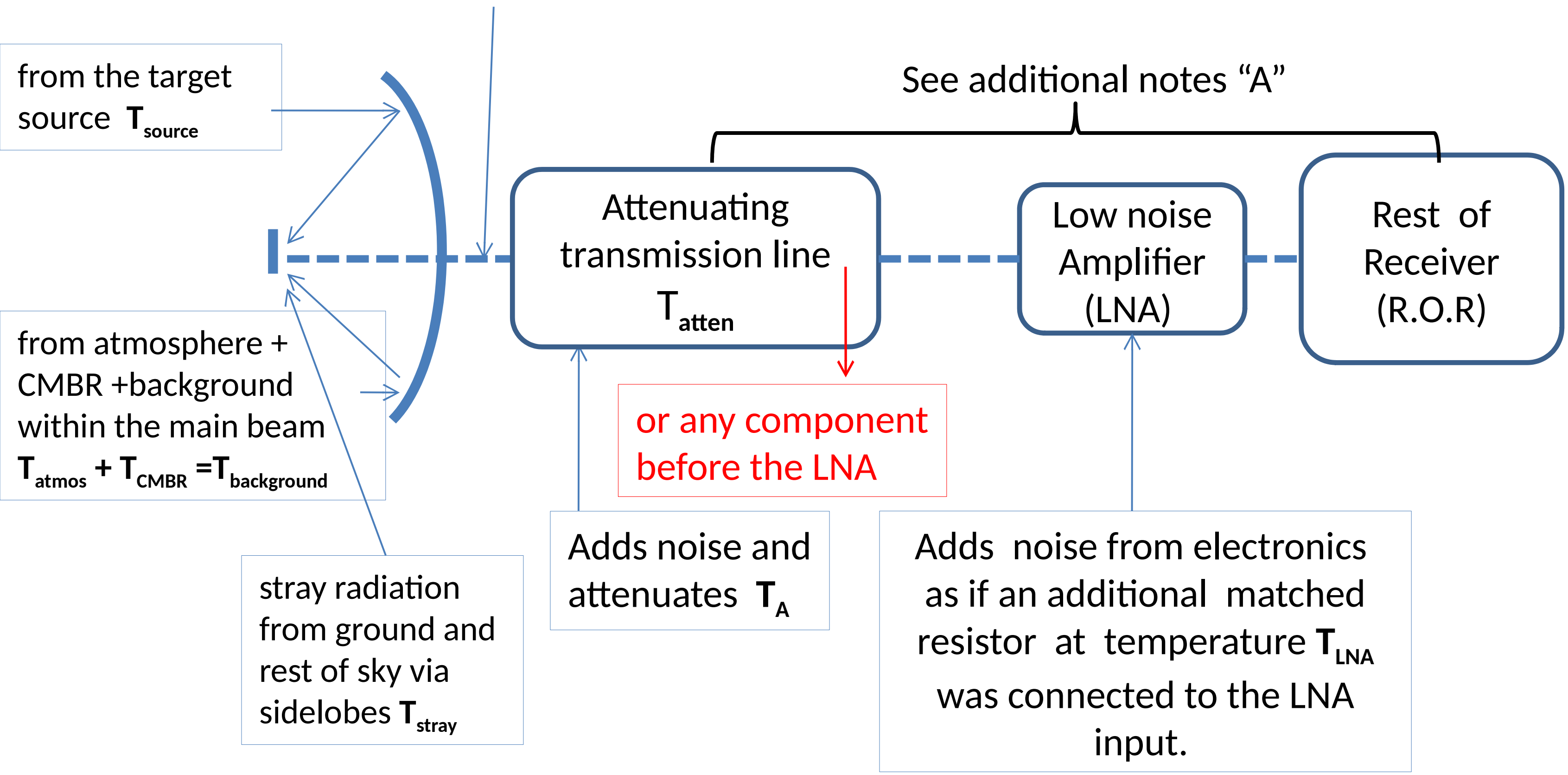
Noise into the receiver

Radiation from target and other directions each produce independent noise *voltages* in the receiver. These voltages have no phase relationships between them , they are "uncorrelated" so the products after the square law detector average to zero *(N.B. averaging over very many coherence times)*

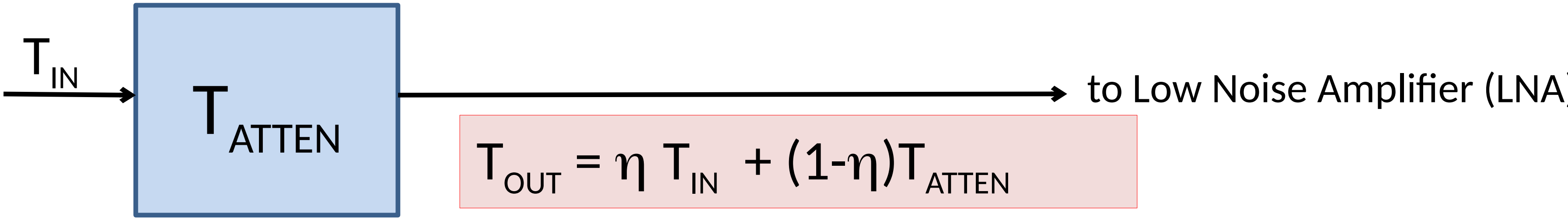
i.e. for a pair of noise voltages: $<(V_1+V_2)^2> = <(V_1)^2> + <(V_1)^2> + \cancel{<2V_1V_2>} \equiv T_1 + T_2$

→ the noise *powers* add, not the voltages. The antenna temperature T_A can therefore be written

$$\mathbf{T_A = T_{source} + T_{CMBR} + T_{atmos} + T_{background} + T_{stray}}$$



Effects of attenuation



$$T_{OUT} = \eta\, T_{IN} + (1-\eta)T_{ATTEN}$$

- Attenuator at a physical temperature T_{ATTEN} with a *transmission* factor $\boldsymbol{\eta}$ (in range 0 → 1) reduces the input signal and also emits its own random noise (see Lecture 2 slide 50)
- As $\boldsymbol{\eta}$ falls the transmitted signal gets smaller *and* the noise generated increases
  → a "lose - lose" situation !
- More usual to quote attenuator's loss in dB i.e. $\eta = 10^{-(\text{dB loss}/10)}$

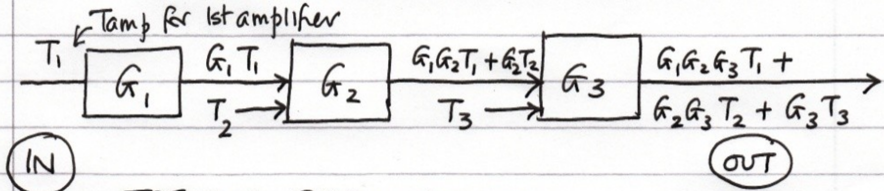
- Example: loss = 0.1 dB and T_{ATTEN} = 290K between antenna and the LNA
  → $\eta = 10^{-0.01} = 0.977$ → transmitted signal = $0.977T_{IN}$ (2.3% loss of signal)
  → $(1-\eta)T_{ATTEN}$ = 290 (1-0.977) = 6.7K (unwanted addition to T_{system})
  → signal to-noise ratio: $[0.977T_{IN}/(T_{system} + 6.7K)] \equiv [T_{IN}/(T_{system} + 6.7K) \times 0.977]$
  → can describe overall result by two effects on T_{system}
- Note that by cryogenic cooling the attenuator to 20K its noise contribution would fall to 0.5K but attenuation effect may stay same – depends on change in resistive losses

LESSON: SEEK TO AVOID LOSS BEFORE 1ST LOW NOISE AMPLIFIER !!

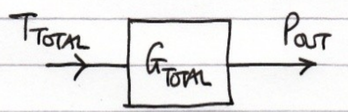
Effect of power gain through receiver on noise

- AMPLIFIER ACTS AS IF RESISTOR OF TEMP. T_{AMP} WAS AT INPUT, THEN MAGNIFIED BY GAIN
- POWER GAIN : $P_{OUT}/P_{IN} = G_{AMP}$

EG. POWER FLOW THROUGH 3 AMPLIFIERS



TREATING SYSTEM AS ONE.....



$$G_{TOTAL} = G_1\, G_2\, G_3$$

T_{TOTAL} = EFFECTIVE TOTAL INPUT TEMPERATURE WHICH WOULD GIVE RISE TO P_{OUT}

$$G_{TOT}\,T_{TOT} = G_1G_2G_3T_1 + G_2G_3T_2 + G_3T_3$$

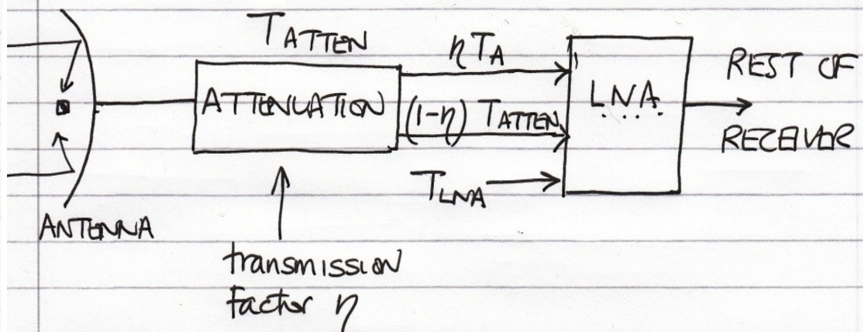
$$\boxed{T_{TOT} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1G_2} \quad \text{etc.}} + \boxed{T_A \text{ INCOMING FROM ANTENNA}}$$

- EFFECTIVELY NOISE FROM 1ST STAGE MULTIPLIED BY G_1 "SWAMPS" REST OF STAGES
- IF $G_1 \sim 30\,dB$ (FACTOR 1000) CAN USUALLY IGNORE NOISE CONTRIBUTIONS FROM OTHER STAGES – EVEN IF (USUAL) MUCH NOISIER

- MUST TAKE LOSSY COMPONENTS INTO ACCOUNT

(i) ATTENUATION BEFORE 1ST LOW NOISE AMPLIFIER (L.N.A.)

Eg. – TRANSMISSION LINE OR CABLE LOSS
– FILTER TO EXCLUDE INTERFERENCE SIGNALS



$$T_{TOTAL,\,INPUT} = \underbrace{\eta\, T_A}_{\text{reduced signal}} + \underbrace{(1-\eta)\,T_{ATTEN}}_{\text{added noise}} + \underbrace{T_{LNA}}_{\text{added noise}}$$

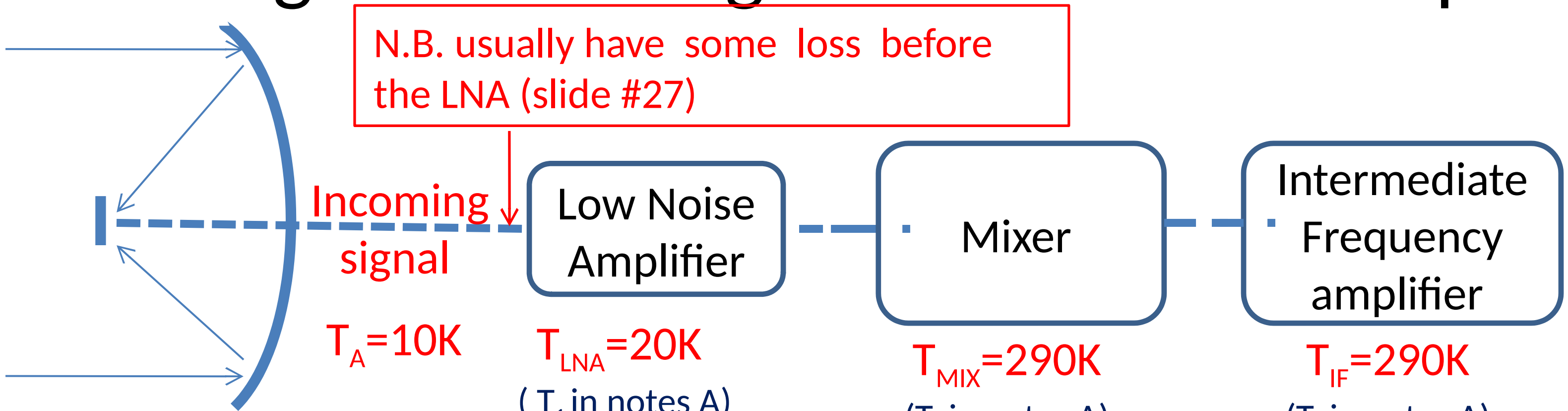
→ 0·1 dB attenuation → $\eta = 0{\cdot}977$
- 2·3% loss IN SIGNAL
- 6·7K added noise

as in slide 26

See workshop examples
Note if the pre-LNA loss is higher effects can be serious

(ii) CONVERSION LOSS IN MIXER

Tracing noise through receiver: an example



T_A=10K

T_{LNA}=20K (T_1 in notes A) G_{LNA} =100

T_{MIX}=290K (T_2 in notes A) G_{MIX} =0.25

T_{IF}=290K (T_3 in notes A) G_{IF} =100

Applying formula for cascaded stages (additional notes A):

T_{sys} = 10K +20K + 290/100 + 290/(100 x 0.25)

T_A T_{LNA} T_{MIX}/ G_{LNA} T_{IF}/ (G_{LNA} x G_{MIX})

T_{sys} ~ 10K +20K + 3K + 12K = 45K

$T_A/T_{receiver}$ = 10/35 → signal-to-noise ratio = 0.29

Note the significant contribution from the second stage IF amplifier → need more gain before the lossy mixer e.g. if G_{LNA} =1000 (30 dB as in additional notes A)

T_{sys} ~ 10K +20K + 0.3K + 1.2K = 31.5K (13.5K better)

$T_A/T_{receiver}$ = 10/21.5 → signal-to-noise ratio = 0.47

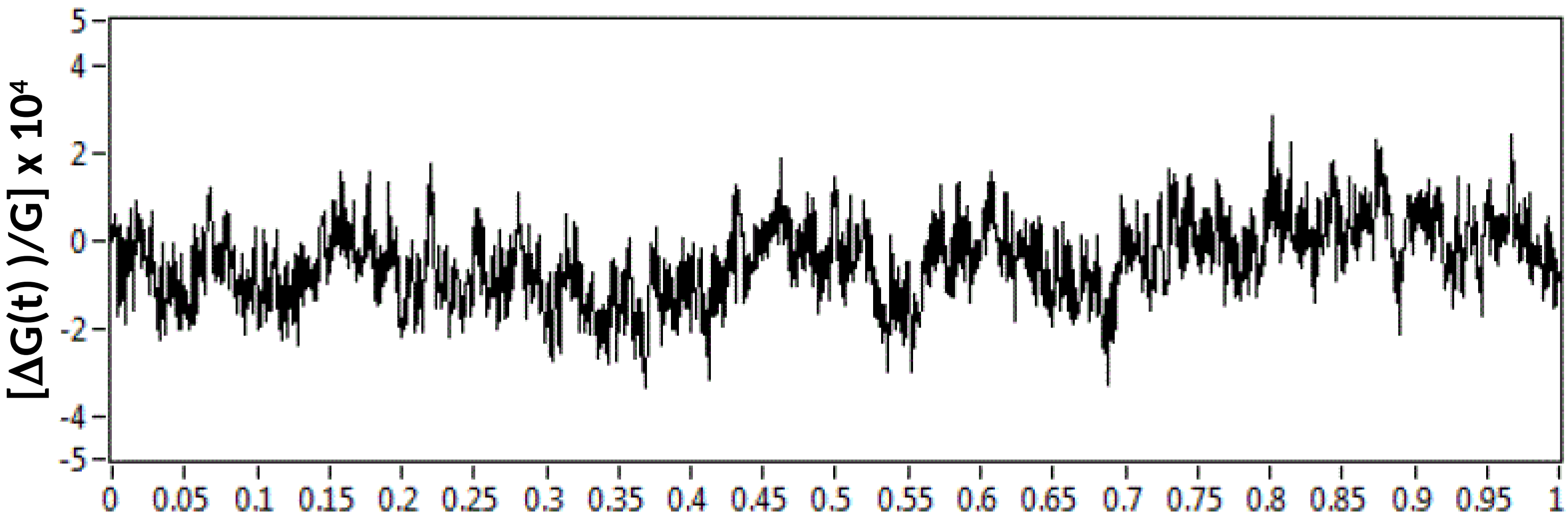
Typical T_{system} "noise budgets"

Emission	30 GHz	1.4 GHz (Hydrogen Line)	1.4 GHz (Hydrogen Line)
T_{CMBR}	3K	3K	3K
T_{STRAY}	7-10K	7-10K	7-10K
$T_{ATMOSPHERE}$	>10K (sea level)	~1K (sea level)	~1K (sea level)
$T_{BACKGROUND}$	<0.1K	2-3K (Galaxy)	2-3K (Galaxy)
$T_{RECEIVER}$	15K (cryo cooled)	3K (cryo cooled)	28K (uncooled)
$T_{ATTENUATION}$	2K	2K	2K
TOTAL	37-40K	17-21K	~45K

For the Planck spacecraft achieved 10-11K since only contributions from $\mathbf{T_{CMBR}}$ and $\mathbf{T_{RECEIVER}}$ and super-low noise Manchester LNAs gave $\mathbf{T_{RECEIVER}}$ ~ 8K .

(1st Planck all-sky image in Lecture 1)

Receiver gain variations: "1/f" or "flicker noise"



D'Cruze: MSc Dissertation Manchester (2012)

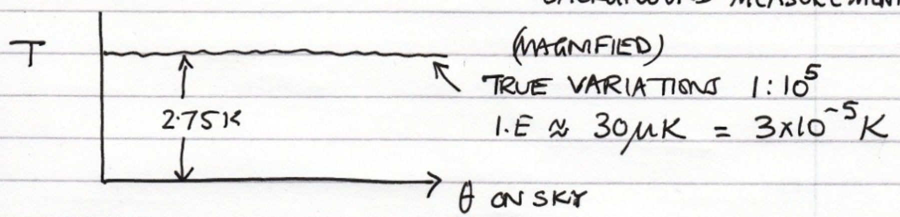
Arbitrary timescale →

Receiver power gain fluctuations $[\Delta G(t)\,)/G]$ typical of "1/f" or "flicker" or "pink" noise. In contrast to thermal "white noise" there is more fluctuating power on longer timescales i.e. variations at lower output frequencies. The statistical excursions from a short-term mean value increase with time (inversely with frequency hence "1/f") and come to dominate the receiver output fluctuations for long integration times. Note :

- The term "*noise*" is wrong: we are actually talking about *gain variations or fluctuations*
- The *voltage gain* variations α $1/\sqrt{f}$ (α $\sqrt{t}$) → *power gain* variations $[\Delta G(t)/G]$ α $1/f$ (α t)
- Typically $[\Delta G(t)/G] < 10^{-4}$ on the timescales of interest.
- N.B. receiver gain variations can mimic real sources or obscure them !

GAIN STABILITY IN RECEIVERS

PROBLEM :– VIA AN EXAMPLE : COSMIC MICROWAVE BACKGROUND MEASUREMENTS



- INTEGRATION TIME τ TO DETECT VARIATIONS TO 5σ ?

$$\left.\begin{array}{l} T_{SYS} \sim 40K \\ \Delta\nu \sim 10GHz \end{array}\right\} \text{ TYPICAL FROM GROUND}$$

$$\Delta T_{RMS}\,(1\sigma) = T_{SYS} / \sqrt{\Delta\nu . \tau}$$

2

$\therefore$ for 1σ

$$\tau = \left(\frac{T_{SYS}}{\Delta T}\right)^2 . \frac{1}{\Delta\nu} = \left(\frac{40}{3\times10^{-5}}\right) . \frac{1}{10^{10}} \simeq 180s$$

$\therefore$ for 5σ

$$\tau = 180 \times 25 = 4500\,sec \equiv 1{\cdot}25 \text{ hours}$$

(per pixel !)

- IMPOSSIBLE TO KEEP $G_{RECEIVER}$ CONSTANT OVER THIS TIME

$$\Delta T_{RMS}\,(4500sec) = \frac{40}{\sqrt{10^{10} . 4500}} = 6\times10^{-6}K$$

(THERMAL ONLY) (GAUSSIAN) $\quad \nwarrow (3\times10^{-5}/5)$

$6\times10^{-6}/40 \rightarrow \underline{1{\cdot}5\times10^{-7}}$ OF SYSTEM NOISE

- GAIN DRIFTS WILL DOMINATE GAUSSIAN NOISE, DUE TO
  – { AMBIENT TEMP CHANGES (SLOW) . TRANSISTOR FLUCTUATIONS (RAPID)
  VARIATIONS IN POWER SUPPLY (RANGE OF TIMESCALES) }

o GAIN FLUCTUATIONS CAN LOOK LIKE SIGNAL

$$\underbrace{P_0 + \Delta P(t)}_{\text{varying output power}} = [\underbrace{G}_{\text{constant}} + \Delta G(t)]\; k\, T_{SYS}\, \Delta\nu$$

$\rightarrow$ FALSE SIGNAL $\underset{(RMS)}{\Delta T_{GAIN}} = \left[\frac{\Delta G(t)}{G}\right] T_{SYS}$

fractional change in gain

(SLIDE 3) FLUCTUATIONS TEND TO GROW WITH TIME. TYPICAL OUTPUT INTEGRATIONS (τ) FROM $10^{-3}s \rightarrow 10$s sec
i.e. FREQUENCIES 1 kHz $\rightarrow$ 10s milli Hz

$[\Delta G/G]$ typically $1:10^{-4} - 1:10^{-5}$ on these timescales

o GAIN FLUCTUATIONS & THERMAL NOISE FLUCTUATIONS ARE INDEPENDENT $\rightarrow$ ADD QUADRATICALLY

$$\Delta T^2_{TOTAL} = \Delta T^2_{THERMAL} + \Delta T^2_{GAIN}$$

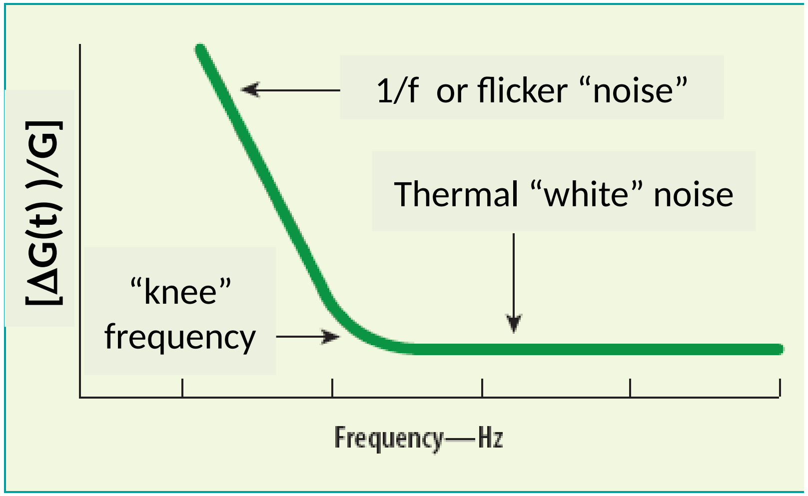
$$= \frac{T_{SYS}^2}{\Delta\nu . \tau} + T_{SYS}^2 \left[\frac{\Delta G(t)}{G}\right]^2$$

$$\rightarrow \boxed{\underset{(rms)}{\Delta T_{TOTAL}} = T_{SYS}\left[\frac{1}{\Delta\nu . \tau} + \left(\frac{\Delta G(t)}{G}\right)^2\right]^{1/2}}$$

o At "knee (corner)" frequency (SLIDE 10) $\Delta T_{THERMAL} = \Delta T_{GAIN}\,(\tau = 1/f_{knee})$

$f_{knee} \sim 100$ Hz for cryocooled amplifiers with high bandwidth $\Delta\nu \sim 10GHz$ since $\Delta T_{THERMAL}$ is very low

1/f noise: power spectra

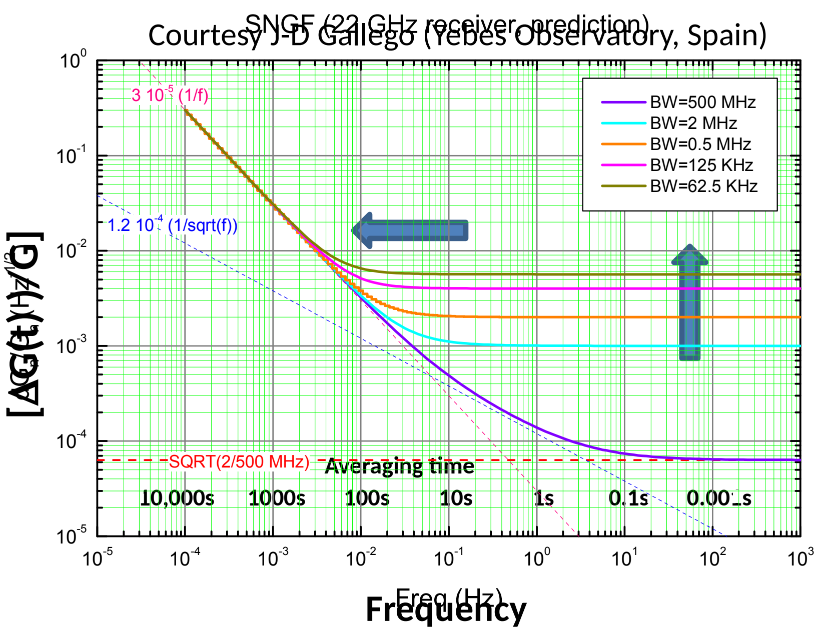


http://mwrf.com/site-files/mwrf.com/files/archive/mwrf.com/Files/30/19392/fig_01.gif

The power spectrum of thermal noise fluctuations (*after SL detection and averaging - typically to tens of seconds to milliseconds*) is constant as a function of time or frequency in the output → called “white”.

The power spectrum of *gain fluctuations rises at lower frequencies* so these fluctuations will be the dominant source of output fluctuations on timescales longer than $1/f_{knee}$ where f_{knee} is the so-called “knee” frequency.

Courtesy J-D Gallego (Yebes Observatory, Spain)



Predicted 1/f power spectra for actual radio astronomy receivers

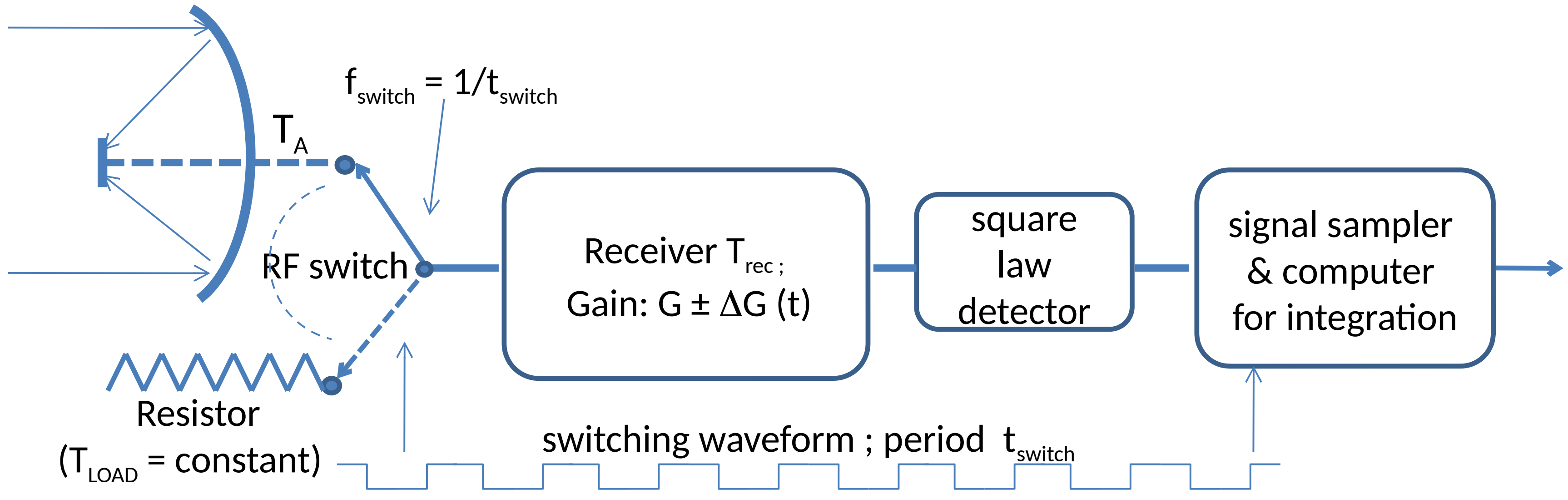
Note that as the thermal noise level (flat part) rises (i.e. receiver is noisier) the knee frequency falls i.e. the white noise is dominant for longer periods over the 1/f noise

Thus f_{knee} changes typically α 1/[white noise level]

N.B. For a pedagogical review of 1/f noise in Nature see: http://arxiv.org/ftp/physics/papers/0204/0204033.pdf

The Dicke switch receiver

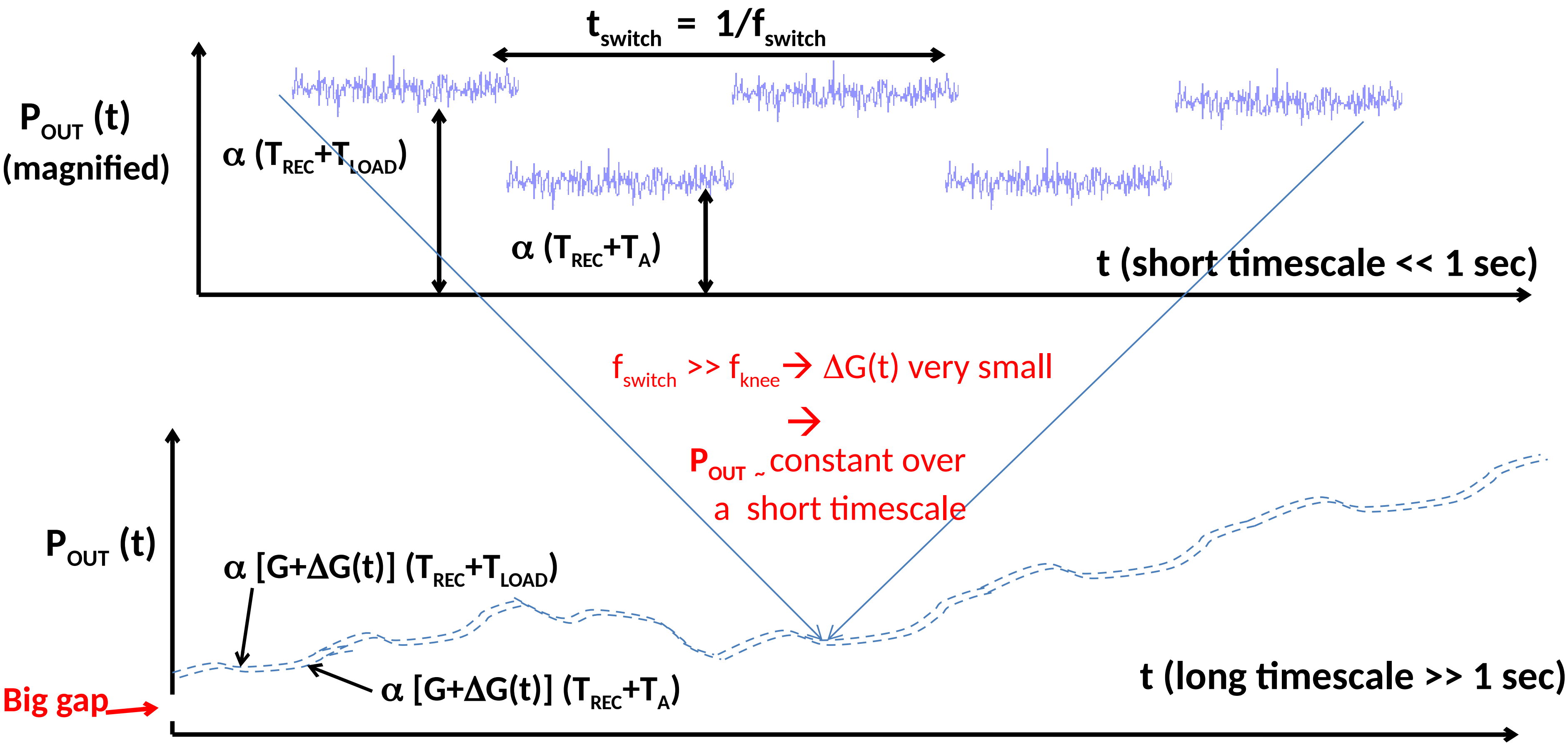
If ΔG (t) fluctuations were very slow (e.g. minutes) then you could calibrate the entire system by repeatedly pointing the antenna at a reference region of known brightness more frequently than this. However this is never the case for receiver "1/f noise" (fluctuations too fast) → have to adopt a different strategy *by building the reference system into the receiver itself* – one of first approaches was the Dicke-switch.



On short enough timescales $t_{switch} << 1/(f_{knee}$ the gain variations ΔG (t) in the receiver are small (think of the fast shutter speed on a camera to "stop " any motion). The philosophy is therefore to compare the output of the receiver against a constant power source – a "load" resistor at a fixed temperature (T_{LOAD}) – on short timescales. This is done by fast switching of the receiver input between the antenna and the resistor load and averaging the *difference* between them. *Thus we measure not* $<T_A>$ *but* $<(T_{LOAD} - T_A)>$ *- see also next slide.*

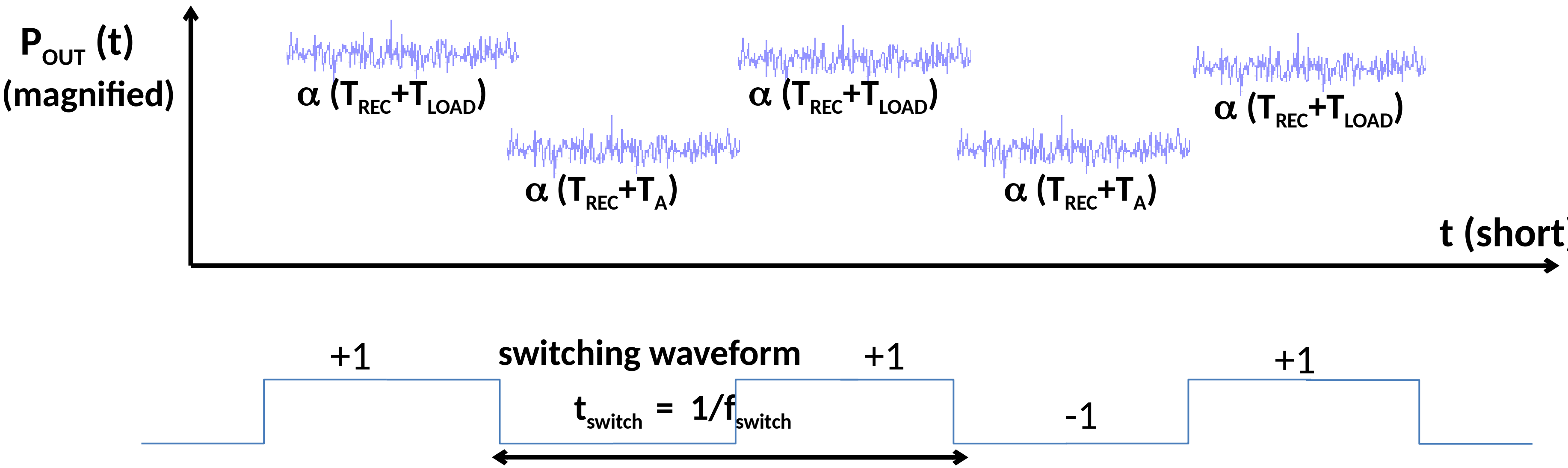
Power as function of time in Dicke switch - 1

(Note: P_o suppressed & arbitrarily drawn with $\mathbf{T_{LOAD}} > \mathbf{T_A}$)



- The mean difference $(\mathbf{T_{REC}+T_{LOAD}})$ - $(\mathbf{T_{REC}+T_A})$ = $(\mathbf{T_{LOAD}-T_A})$ rather than $\mathbf{T_{sys}} = (\mathbf{T_{REC}+T_A})$ is measured on timescales faster than that over which $\Delta G(t)/G$ is significant
- Note that the $\mathbf{\Delta G(t)/G}$ relative gain variations are enormously exaggerated here by suppressing the P_0 part of $P_{OUT}(t)$ - *they are typically 1:10^{-4} to 1:10^{-5}*

Power as function of time in Dicke switch - 2



- System is arranged to form continuously

$$+1(T_{REC}+T_{LOAD}) \quad -1(T_{REC}+T_A) \quad = \quad [T_{LOAD}-T_A]$$

and to average this difference for the specified integration period → $\langle T_{LOAD}-T_A \rangle$

Thus, on the timescale of t_{switch}

No switching: $\delta P(t)\ \alpha\ \ (\Delta G(t)/G)\ [T_{REC}+T_A]$ (top right in slide)

With switching: $\delta P(t)\ \alpha\ \ (\Delta G(t)/G)\ \{[T_{REC}+T_{LOAD}] - [T_{REC}+T_A]\} = (\Delta G(t)/G)\ [T_{LOAD}-T_A]$

Advantages & Disadvantages of Dicke switch

Advantages:

i) the big offset of T_{REC} is automatically subtracted out;

ii) the *relative improvement* in stability on the timescale t_{switch} = $\boxed{[\mathbf{T_{REC}} + \mathbf{T_A}] / [\mathbf{T_{LOAD}} - \mathbf{T_A}]}$

iii) *if* $\boldsymbol{T_{LOAD} - T_A = 0}$ *there is a perfect balance between antenna temperature and load temperature the receiver is perfectly insensitive to gain variations on timescales longer than* t_{switch} *and the output changes only if* $\boldsymbol{T_A}$ *changes to alter the balance → very important to equalise* $\boldsymbol{T_{LOAD}}$ *and* $\boldsymbol{T_A}$ *as closely as possible* (see Examples 11&12).

Disadvantage:

i) Reduced signal-to-noise ratio - why?

- Antenna is connected to the target and load for only half time → hence $\mathbf{T_A}$ data channel has √2 higher noise ; $\mathbf{T_{LOAD}}$ data stream has the same noise as $\mathbf{T_A}$ channel
- Since $(\mathbf{T_{LOAD}} - \mathbf{T_A})$ is the difference between two noisy signals each with √2 greater noise → result is √2 x √2 = 2 x higher noise (basic data analysis)

$$\Delta T\ (\mathrm{rms}) = 2T_{sys} / \sqrt{(\tau_{avg}\,\Delta\nu)}$$

THE DICKE-SWITCH RADIOMETER EQUATION

- Twice the thermal noise level of a basic radiometer – but often worth it because of much greater gain stability !

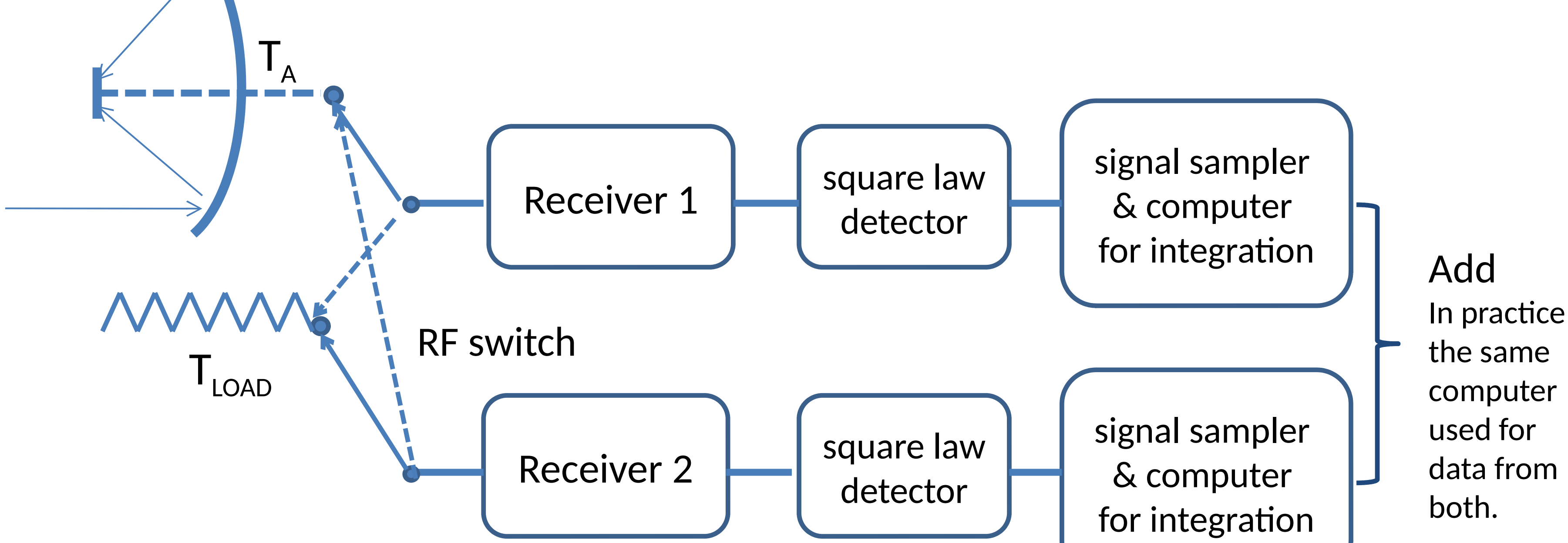
Left out various other receiver configurations in 2017
as beyond Unit 1 scope

These and many other detailed slides about receivers have been left in beyond “THE END”

END

Variants: Double-Dicke switch receiver

a 2nd receiver is also connected to the antenna and the load



RF switch position 1: receiver 1 connected to antenna
receiver 2 connected to load

RF switch position 2 : reverse situation

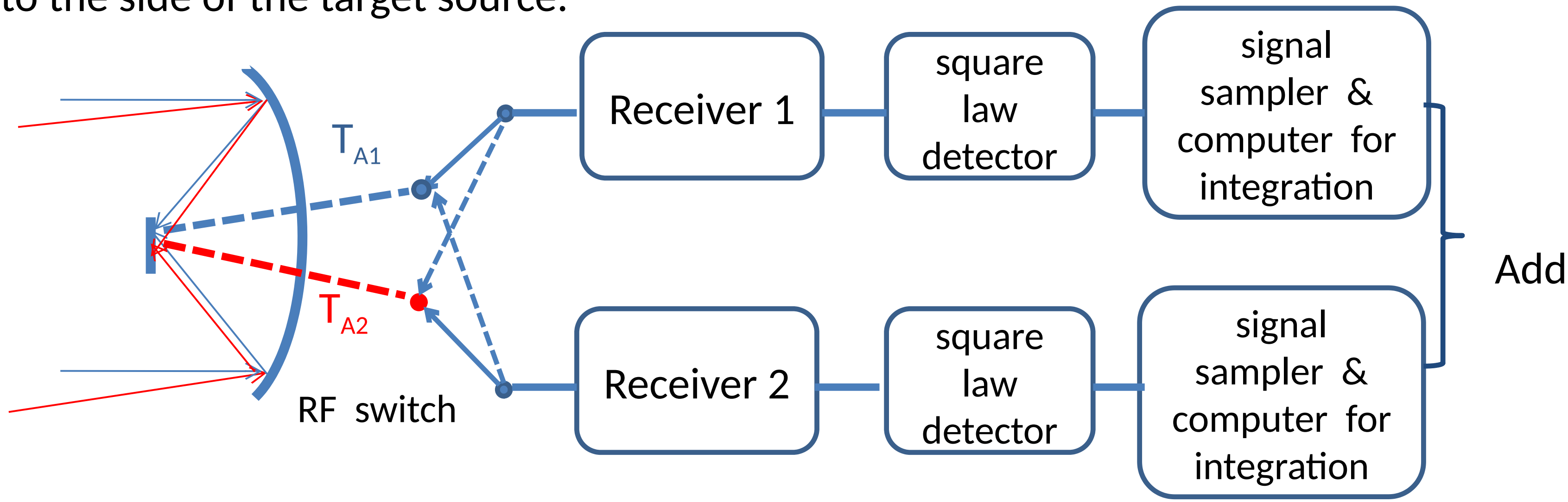
The outputs of the two receivers are added to increase the signal-to-noise ratio. Since one receiver is always looking at the target source one factor of √2 from the single Dicke is gained back

$$\Delta T\ (\mathrm{rms}) = \sqrt{2}\,T_{sys} / \sqrt{(\tau_{avg}\ \Delta\nu)}$$

DOUBLE DICKE-SWITCH RADIOMETER EQUATION

Variants: using sky as reference load

The double Dicke configuration again but with the resistor load replaced by an adjacent path to the side of the target source.



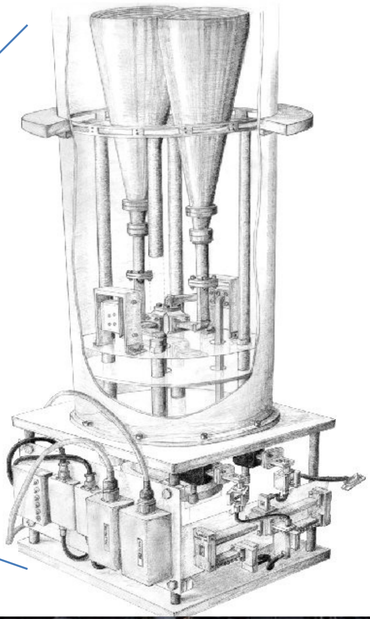
The inputs T_{A1} and T_{A2} are drawn from the same antenna equipped with two closely-spaced receiving elements at the focus – see slide 17. These receive radiation from closely-spaced paths through the atmosphere (*in the near field*) to two closely-spaced patches on the sky *(in the far field)* - see slides 18-20. In general T_{A1} and T_{A2} will be very similar and hence in terms of the reduction of gain variations and in *thermal noise terms* this architecture produces results similar to the double-Dicke (in the case when $T_{LOAD} \sim T_A$). Thus:

$$\Delta T\ (\mathrm{rms}) = \sqrt{2}\,T_{sys} / \sqrt{(\tau_{avg}\,\Delta\nu)}$$

TWIN-BEAM RADIOMETER EQUATION

OCRA twin-beam receivers

32-m diameter radio telescope of Nicolas Copernicus University Torun, Poland on which the OCRA receivers are mounted



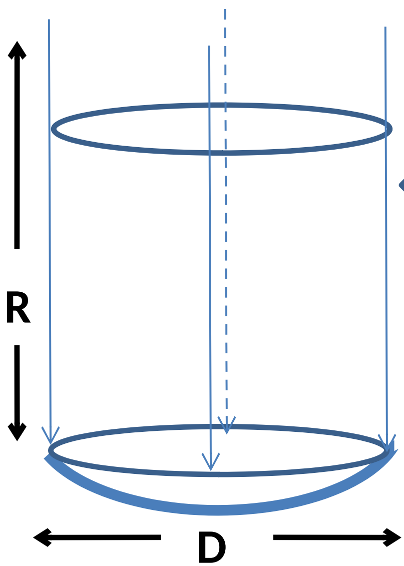
"One Centimetre Receiver Array" (OCRA) operates at a centre frequency of 30 GHz. The receivers are based on the twin 180º hybrid correlation approach architecture (next lectures).

Top) drawing of interior of two-beam prototype "OCRA-p" – the two "feed" horns sit on either side of the optical axis. The horns are 85mm across at their mouth.

Bottom) photograph of OCRA-p inside its cryostat. The horns, first amplifiers and 1st hybrid are all cooled to <20K to reduce receiver noise.

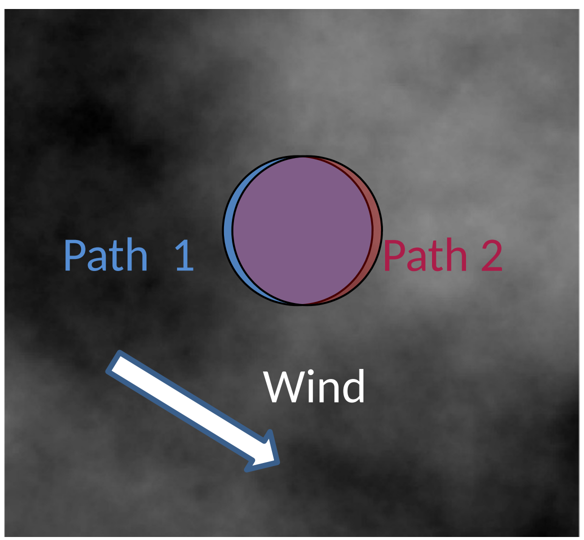
N.B. The small red tube is for piping dry nitrogen into space behind the white" Gore-Tex" window to reduce condensation in the beam path

Passage through the varying atmosphere



In contrast to the far field (Fraunhofer) antenna reception patterns, in the *near field* (Fresnel region $R < 2D^2/\lambda$) the reception pattern approximates to a cylinder equal to the telescope diameter D **(see lecture 3 slide 13 bottom picture)**

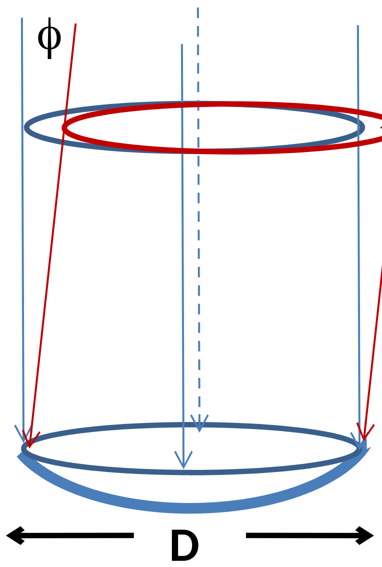
For a large antennas (D > 15m) and at frequencies above ~10 GHz (λ < 0.03 m) the near-far transition is more distant than ~15km (often much more) hence the path through the atmosphere is all within the near field.

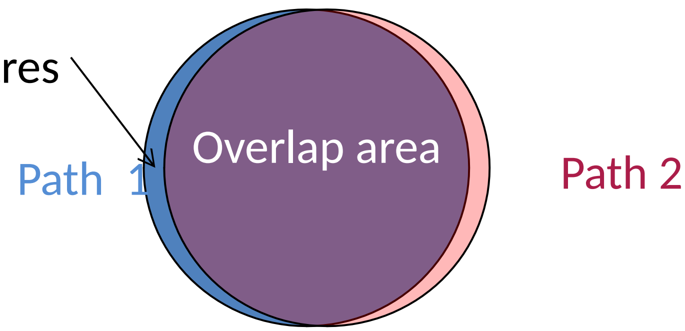


The atmospheric transmission varies. The picture is a representation of variations in the transmission η of the *clear* atmosphere. The transmission fluctuates in space and time and hence both the attenuated source emission i.e. ηT_{source} and the noise emitted by the atmosphere $T_{atmos}(1 - \eta)$ also fluctuate.

The wind (direction arbitrary) blows patches of different transmission across the ~cylindrical reception paths. In the twin-beam system the two paths largely overlap and there is a high degree of correlation between them → differencing the outputs greatly reduces the effect of the *both* the receiver gain variations *and* the transmission variations (although not quite perfectly since the overlap is not complete) . *Dicke-switch systems have only one path through the atmosphere and so the transmission variations are not reduced.*

Two beams through atmosphere (near field)





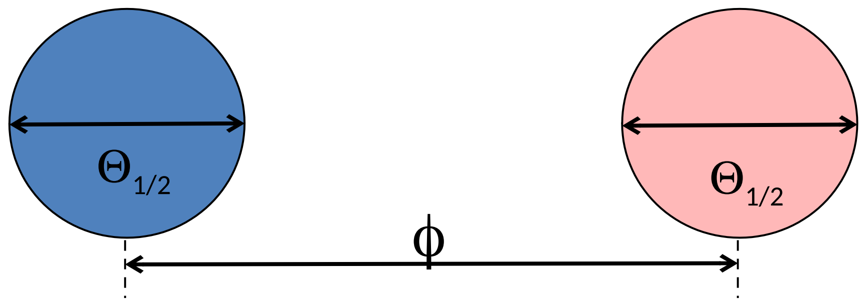
- 2km is the effective atmospheric depth for H_2O absorption (see Lecture 2)

- Typically ϕ <0.1 degrees→ $\delta < 2000 \times 0.1 \times \pi/180 \sim 3.5$ m

→ beams overlap to better than 3.5m

For same atmosphere the *fractional* overlap increases as D increases - since δ only depends on ϕ → get greater reduction in atmospheric fluctuations with bigger telescopes

Two beams above atmosphere (in far field)



The overlapping "quasi-cylindrical " near field patterns separate into the classical Fraunhofer beams with $\Theta_{1/2}$ ~ (1.15 ± 0.05) λ/D - *these beams do NOT overlap*. Consider two cases:

i) **Astronomical (not atmosphere) emission in both beams is very similar**

- Constant background → $[T_{A1} - T_{A2}]$ → 0 so no astronomical signal results ?
- Taking the difference between the beams has "differentiated" the sky brightness temperature distribution - hence *this method is insensitive to slowly varying T_B variations*
- Note: Double Dicke does not differentiate sky since differences against loads only

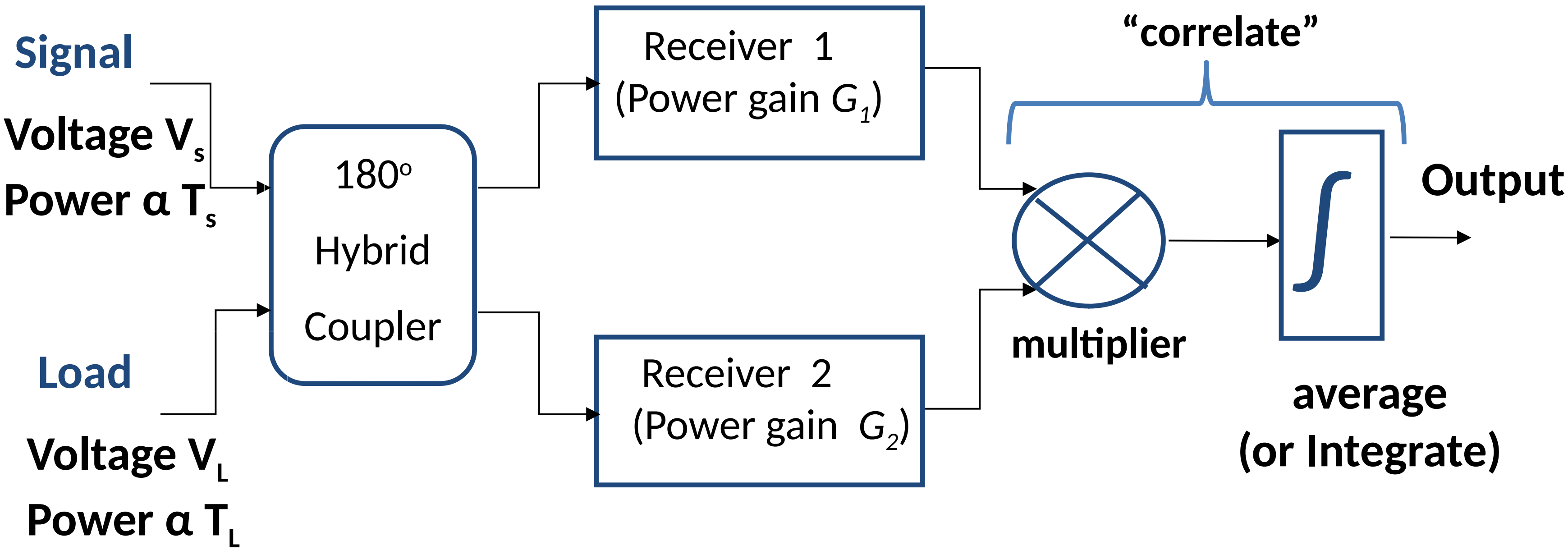
ii) **Astronomical background emission is different in each beam**

- Compact source in one and not the other
- System *ideally suited for surveys for compact sources* (as is Double Dicke)

Big advantage of twin-beam using sky as reference load over Double Dicke is subtraction of atmospheric opacity fluctuations

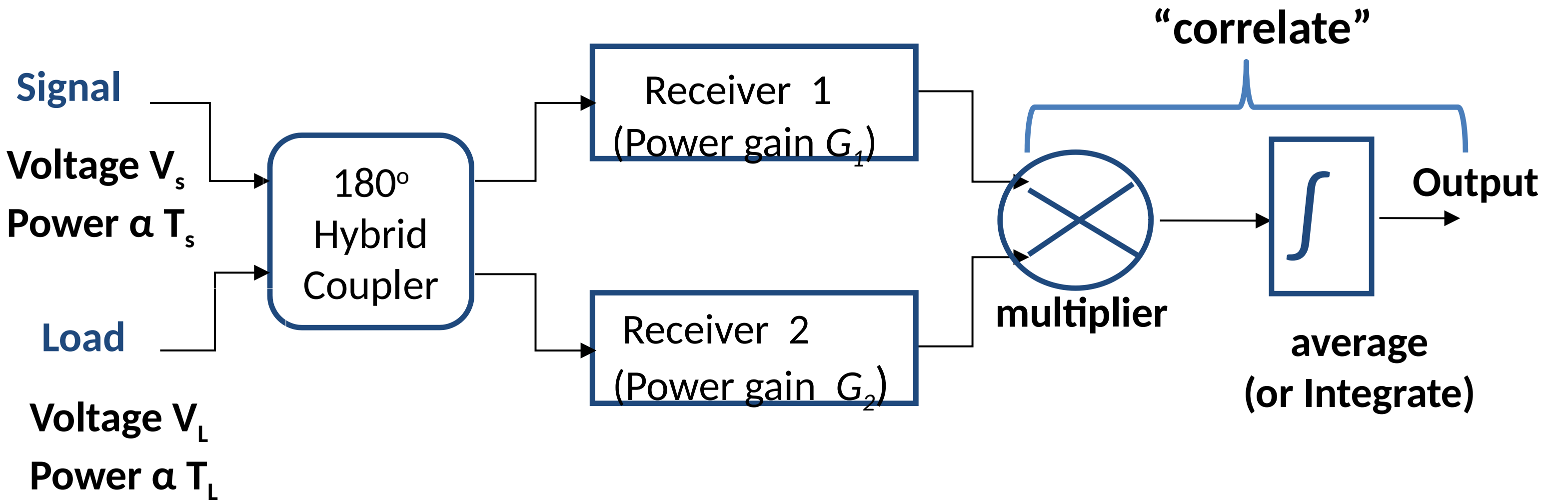
Correlation Radiometers

(another way to combat receiver gain variations)



- Replace *the lossy* RF switch in Dicke-type receivers (which comes before the LNAs and hence adds noise to the system) with a "hybrid coupler".
- Many different types of couplers – one (called a "rat race" !) is described in slide 3.
  - Splits the *powers* at the inputs equally between the two outputs (**T_S/2; T_L/2** at each)
    → input *voltages* $V_S/\sqrt{2}$ & $V_L/\sqrt{2}$ to each output (see next slide).
  - Reverses the phase (hence by 180°) of *one* of the input *voltages* at one of the outputs compared to the other (see next slide and notes A)

Correlation radiometer: simplest input signals



Choose two input voltages *from noise* to be: $\mathbf{V_S = A_s\cos(\omega_1 t)}$ **and** $\mathbf{V_L = A_L\cos(\omega_2 t)}$

(we want to illustrate uncorrelated signals so we choose different frequency components)

The voltages going into the multiplier are:

$$\mathbf{V_1 = \sqrt{G_1}[A_s\cos(\omega_1 t) + A_L\cos(\omega_2 t)]/\sqrt{2}} \quad \textbf{(the } \Sigma \textbf{ channel)}$$

$$\mathbf{V_2 = \sqrt{G_2}[A_s\cos(\omega_1 t) - A_L\cos(\omega_2 t)]/\sqrt{2}} \quad \textbf{(the } \Delta \textbf{ channel)}$$

Multiplying these voltages together :

$$\mathbf{\sqrt{G_1 G_2}\,[A_s^2\cos^2(\omega_1 t) - A_L^2\cos^2(\omega_2 t)]/2} \quad \text{(the cross terms cancel in this simple case)}$$

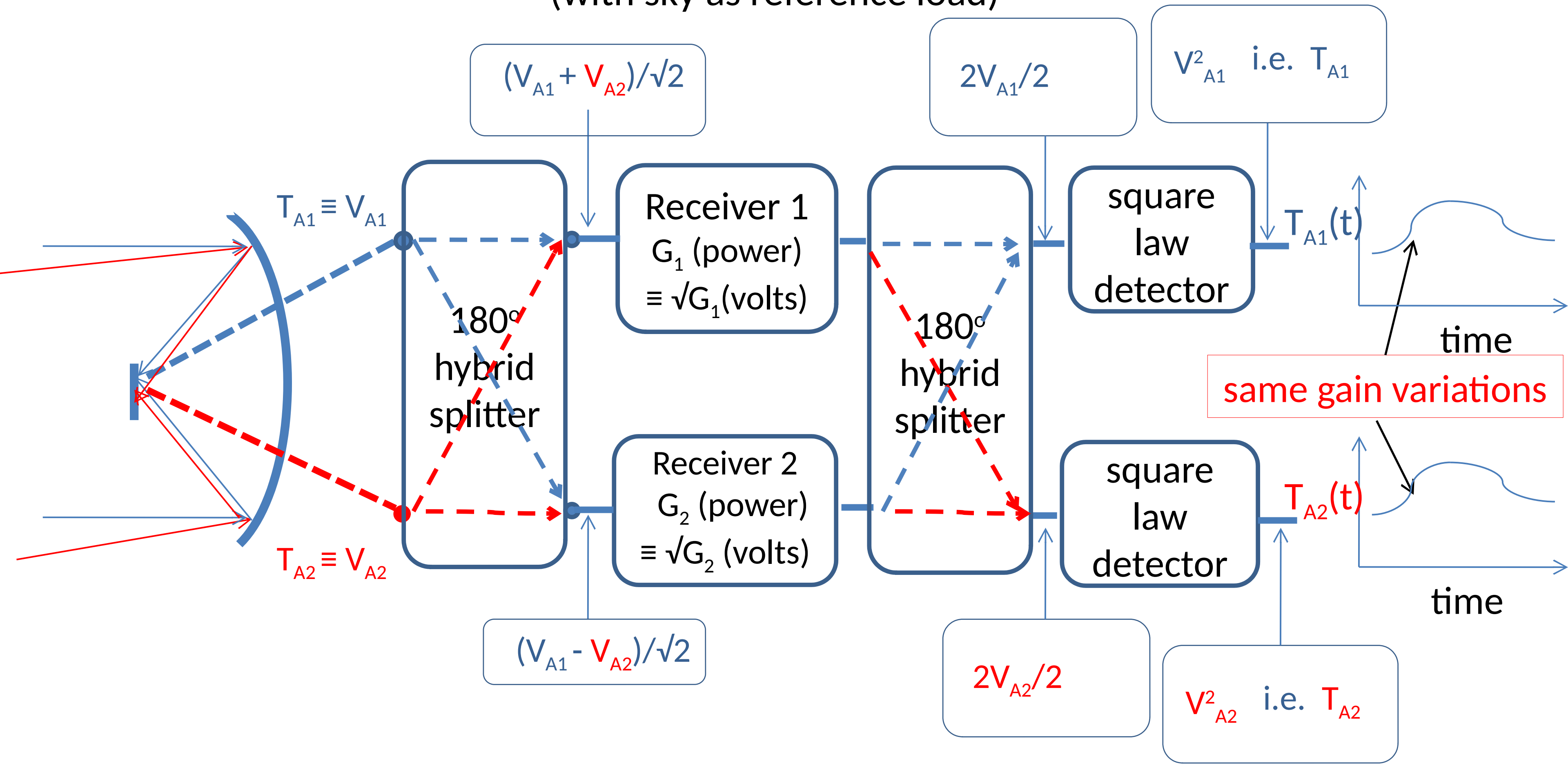
Using the trigonometrical identity $\cos^2\theta = ½\,[1+\cos 2\theta]$

$$\mathbf{\sqrt{G_1 G_2}\,[½\,A_S^2(1+\cos(2.\,\omega_1 t) - ½\,A_L^2(1+\cos(2.\,\omega_2 t)]/2}$$

Time averaging → the oscillating terms reduce to zero - thus the output voltage is

$$\mathbf{V_{OUT} = \sqrt{G_1 G_2}\,[A_S^2 - A_L^2]/4 \text{ i.e. } \alpha\ [T_s - T_L)}$$

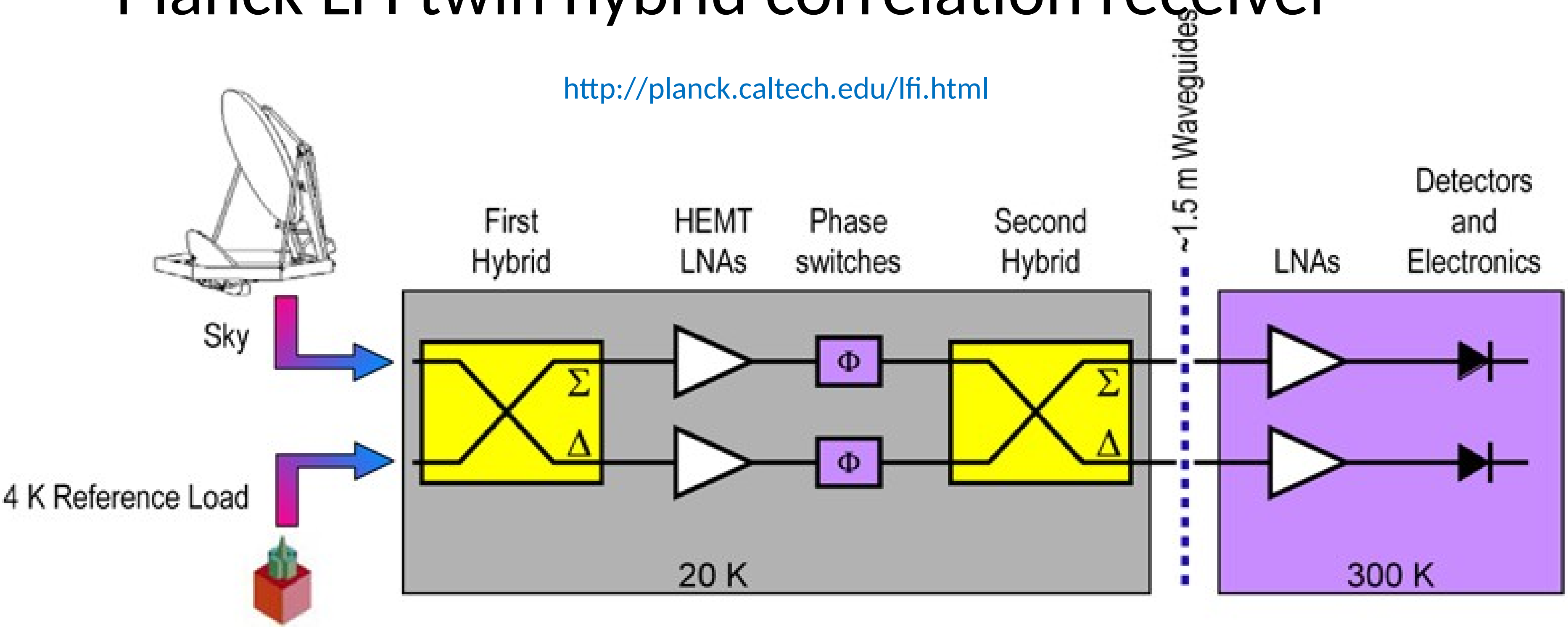
Twin-hybrid correlation receiver
(with sky as reference load)



The voltage outputs V_{A1} and V_{A2} (alternatively V_{LOAD}) , have passed through both receivers and hence are subject *to the same* gain fluctuations. Hence, after square law detection the powers (T_{A1} and T_{A2} or T_{LOAD}) vary together and the difference T_{A1} - T_{A2} → zero if the receiver inputs are exactly balanced. When there is different emission in one of the beams (near field atmosphere or far field sources), this will appear in the difference signal. This is the OCRA architecture (Slide 39)

Planck LFI twin hybrid correlation receiver

http://planck.caltech.edu/lfi.html

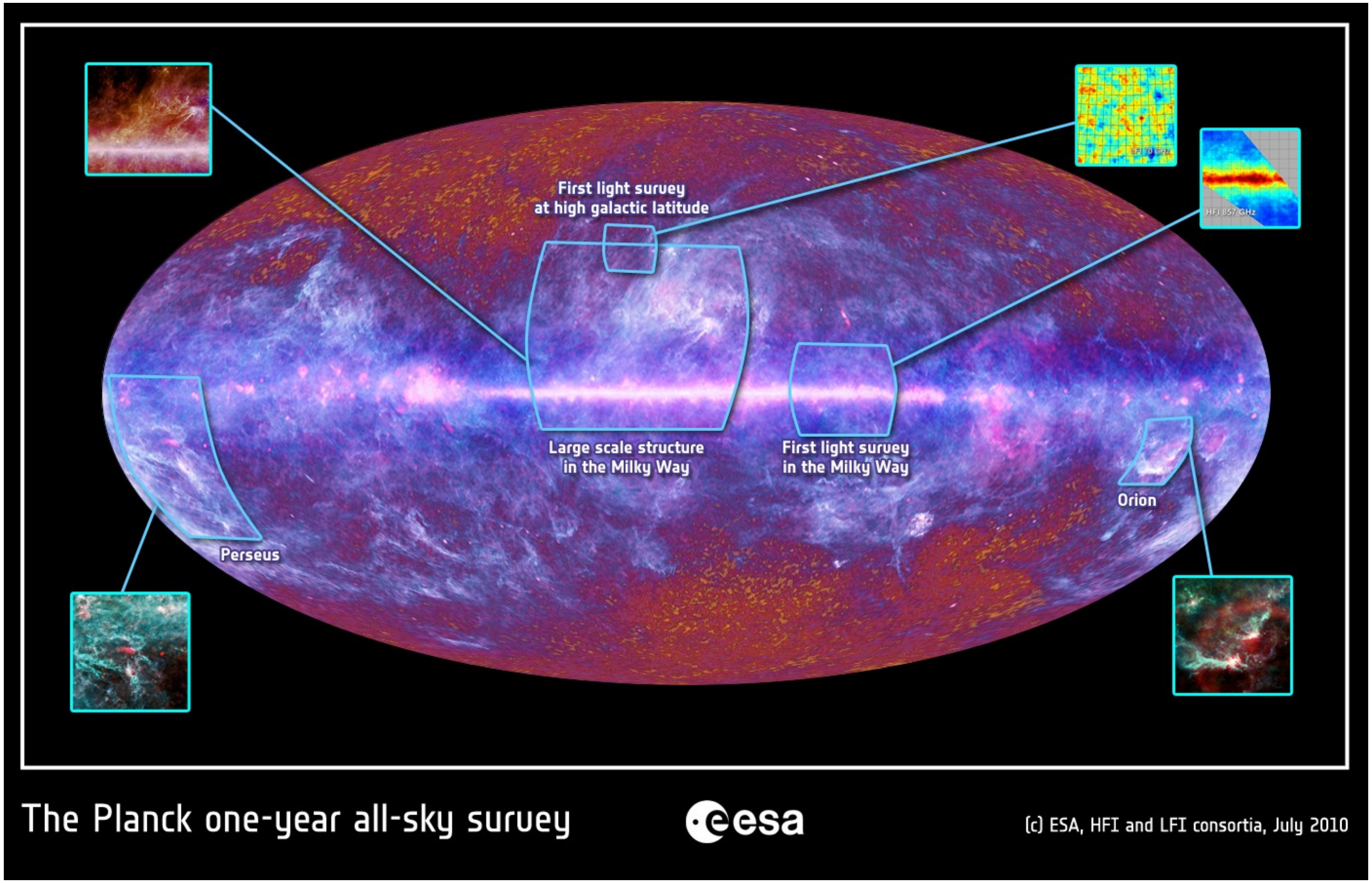


- "LFI" involved receivers at 30, 44 and 70 GHz. It used thermal load as the reference since goal was to map very extended emission and so avoided "differencing" the sky. *Major parts of the 30 and 44 GHz receivers were built in Manchester (Jodrell Bank).*

- Ideally the load would have been at the same temperature as the CMBR 2.75K to balance the receiver and hence maximise the reduction in 1/f receiver gain fluctuations. The load was actually at 4K.

- The results are still very good; f_{knee} after the correlation and subtraction scheme was reduced from ~100 Hz to ~40 mHz (t ~ 25 secs) an improvement of 2500. Small non-modelled imperfections in component performance prevented a greater improver

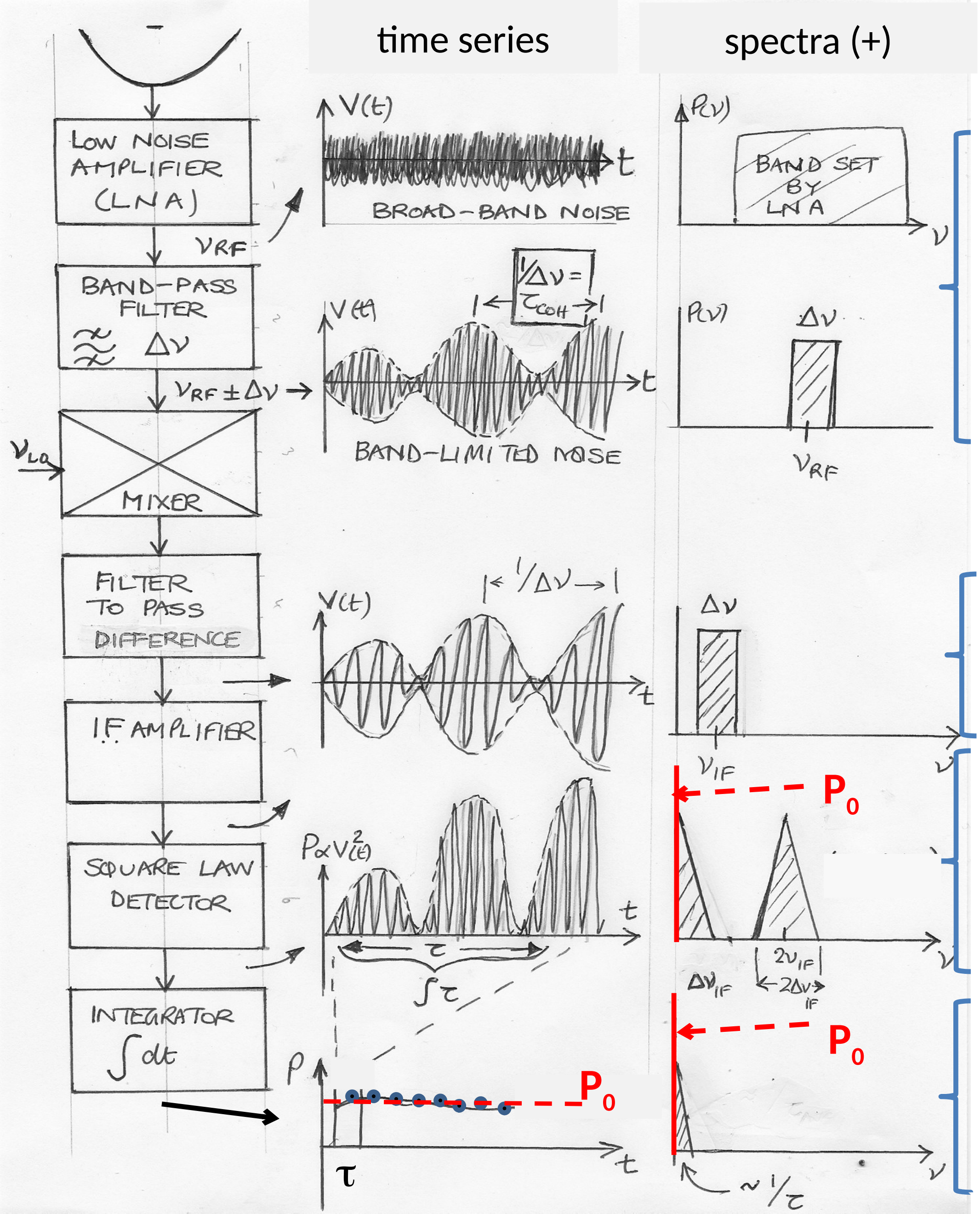
First results from Planck surveys



Multiple frequencies (9) allowed dissection of the contributions from the different emission mechanisms described in Lectures1+2 and slides 3&4 lectures3+4): i) free-free from ionized plasma; ii) synchrotron from relativistic electrons and magnetic fields ; iii) thermal from dust grains at 30-60K. Correlation receivers used in LFI for the three lower frequencies 30,44 and 70GHz. Higher frequencies used superconducting bolometers as detectors which act really like radio thermometers!

Dicke-switched and correlation receivers compared

	balanced load as reference	"cold" sky as reference
Double-Dicke (Single-Dicke is the same but with √2 higher noise)	$\Delta T_{rms} = \sqrt{2}\, T_{sys} / \sqrt{(\Delta\nu .\ \tau)}$ • greatly reduced "1/f noise" • sensitive to smooth extended emission • affected by atmospheric fluctuations	$\Delta T_{rms} = \sqrt{2}\, T_{sys} / \sqrt{(\Delta\nu .\ \tau)}$ • greatly reduced "1/f noise" • not sensitive to smooth extended emission • greatly reduced atmospheric fluctuations
Correlation	$\Delta T_{rms} = \sqrt{2}\, T_{sys} / \sqrt{(\Delta\nu .\ \tau)}$ • greatly reduced "1/f noise" • sensitive to smooth extended emission • affected by atmospheric fluctuations • hybrids less lossy than RF switches → T_{sys} lower	$\Delta T_{rms} = \sqrt{2}\, T_{sys} / \sqrt{(\Delta\nu .\ \tau)}$ • greatly reduced "1/f noise" • not sensitive to smooth extended emission • greatly reduced atmospheric fluctuations • hybrids less lossy than RF switches → T_{sys} lower



Basic "heterodyne" receiver: for measuring total power P_o

The passive collector element at the antenna focus and the low noise amplifier usually pass a wide range of frequencies . This may contain unwanted signals. A narrower range of frequencies $\Delta\nu$ is selected by a filter (at the incoming signal frequency ν_{RF}) after the first low-noise amplifier → $\nu_{RF} \pm \Delta\nu/2$.

After mixing: centre frequency has been translated down and only one sideband (in this case the lower) has selected; note that $\Delta\nu$ remains the same.

After square law detection; output power spectrum as in slide 11: *Note the number of coherence times averaged to get P_o is much greater than shown. If τ_{AVG} = 1 sec ; $\Delta\nu$ = 100 MHz → 10^8 coherence times!*

After averaging the post-detector output for τ seconds the high frequency terms have been weighted down and only variations $\delta P(t)$ at a slow rate (<1/τ Hz) are passed on.

Further advantages of heterodyne systems

1) Helps to avoid danger of oscillation in the system arising from positive feedback given the enormous RF amplifier gain required from the input to the output of the receiver

e.g. T_{sys} = 100K ; $\Delta\nu$ = 100 MHz → $P_{out} = k\, T_{sys}\, \Delta\nu = 1.4 \times 10^{-13}$ W

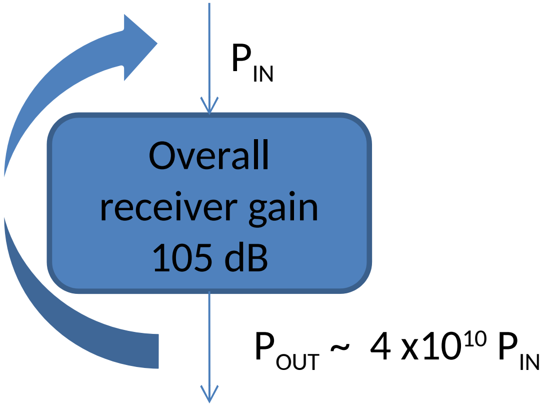
Power for i/p to square law detector typically ~0.01 mW = 10^{-5} W

→ required gain ~ 10^{8} i.e. 80dB *(followed by further >20dB of low freq. amplification)*

Power for i/p to a digitizer typically ~5 mW = 5×10^{-3} W

→ required gain = 3.6×10^{10} ~ 105dB (amplification by $>10^{10}$)

Small leakage of power δP_{OUT} back up the receiver chain can give rise to oscillation if all the gain is at the same frequency.



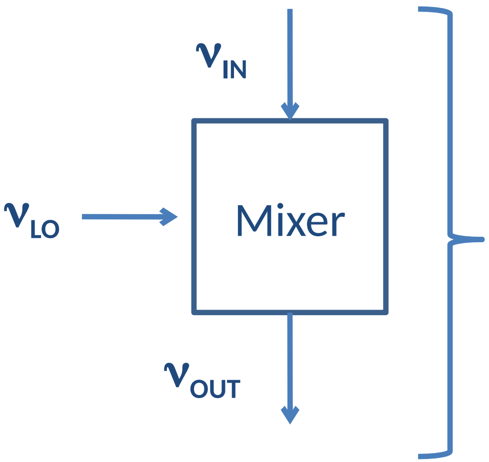
2) Attenuation in signal transport cables is less at low frequencies – see "cable to observatory" **in lecture 3 Slide 5**

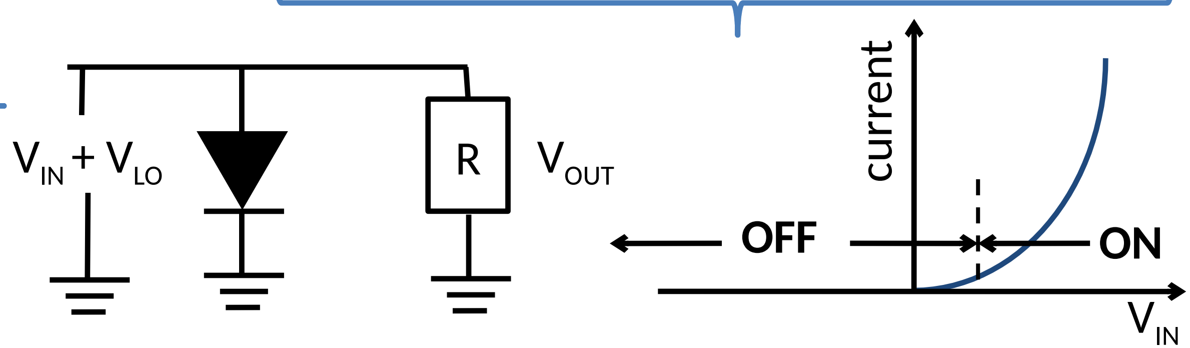
3) Varying the Local Oscillator Frequency ν_{LO} enables the centre frequency of the band under study to be varied – useful for spectroscopic observations – see later

Mixers and multipliers - basics

Any circuit element with a non-linear relation between input and output produces harmonics ($2\nu_{IN}$, $3\nu_{IN}$) etc of a single input frequency ν_{IN} and mixtures (sums and differences) of two inputs. *The instantaneous level of one signal affects the level of the other at the output → non-linear.*

The "Local Oscillator" is a powerful signal at a particular frequency ν_{LO} which modifies the properties of the active element in the "mixer" e.g. the conductance of a diode in a simple rectifier.





The diode is forced "open" and "closed" by the LO signal → the small input signal (V_{IN}) is rectified or "chopped". It then contains a range of Fourier components (frequencies) e.g $n\nu_{LO} \pm \nu_{IN}$ (where n is odd) – illustrated for square wave switching in **slide 20**. Some simple algebra illustrates the basics…

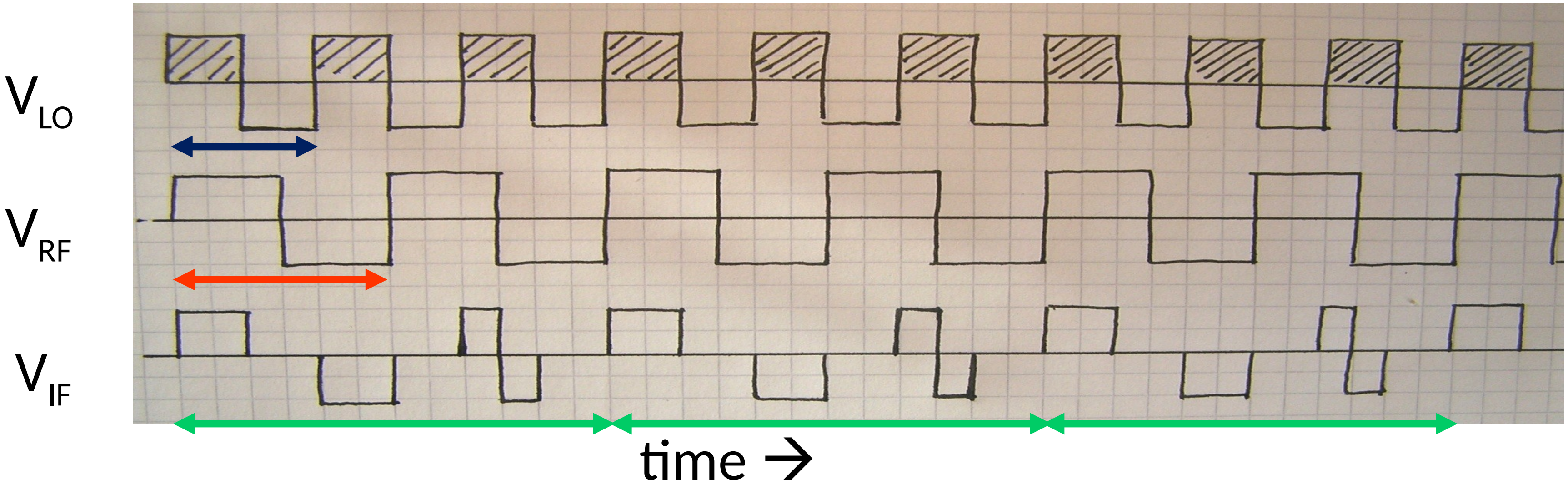
All mixers produce cross products of the form $[V_{IN}\cos(\omega_{IN}t+\phi_{IN})] \times [V_{LO}\cos(\omega_{LO}t)]$

this can be rewritten using trigonometrical identity: $2\cos A\cos B=\cos(A+B) + \cos(A-B)$

→ **$\{V_{IN}V_{LO}/2\}\{\cos[(\omega_{IN}+\omega_{LO})t+\phi_{IN}] + \cos[(\omega_{IN}-\omega_{LO})t+\phi_{IN}]\}$** **$(\omega_{IN}+\omega_{LO})$= "upconversion"**

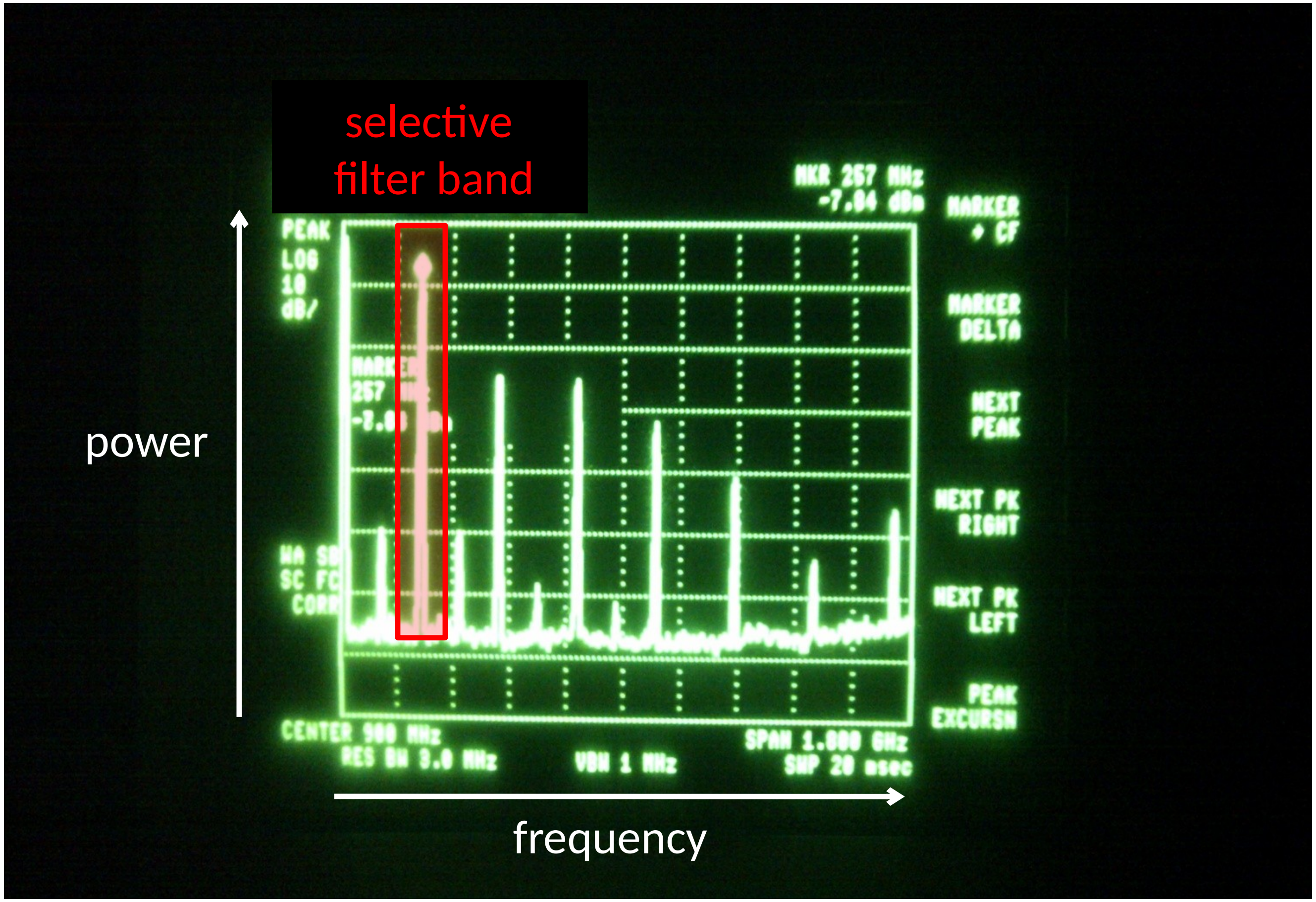
$(\omega_{IN}-\omega_{LO})$= "downconversion"

Mixing (schematic)



- Simply visualised as the action of an “on-off” switch in which the conductance of a diode is controlled by the varying local oscillator (LO) voltage . Here all varying voltages are represented as square waves for simplicity and the LO frequency is shown as higher than the RF frequency – a purely arbitrary choice.
- When the LO voltage (period **4 squares** → n_{LO} =“250 MHz”) is +ve the switch passes the RF signal (period 6 squares → n_{RF}=“166.67 MHz”); when the LO voltage is –ve the RF signal is not passed. This is represented in the intermediate frequency (IF) signal which can be seen to have a *component* with a repetition period of 12 squares → n_{IF} = “83.33 MHz” i.e. the *difference* frequency.
- In realistic mixers the on-off switching is not fully efficient as assumed here and a wide range of harmonics, and mixtures between them, can be generated.

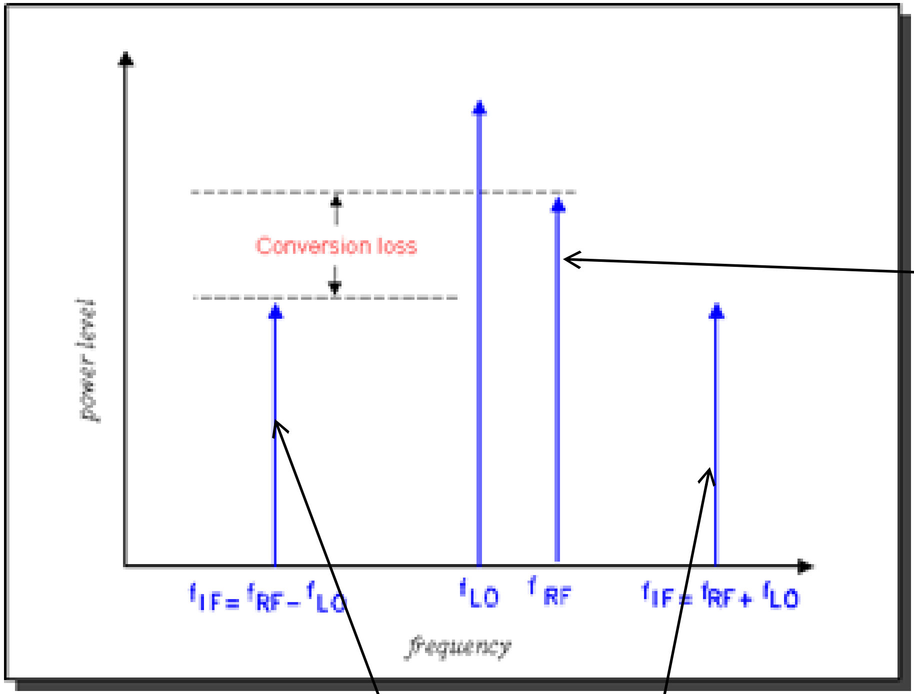
Example output spectrum of a mixer



The desired frequency component has to be selected out by a filter; the power in it is always significantly less that in the original RF signal – this is called the “conversion loss” – typically in the range 6-10dB (next slide).

Conversion loss in mixers

http://na.tm.agilent.com/pna/help/latest/FreqOffset/Conversion_Loss.htm



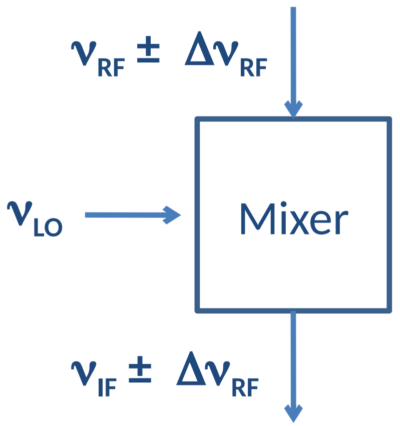
Conversion loss =

$$\frac{\textbf{power at output frequency (IF)}}{\textbf{power at input frequency (RF)}}$$

- Power at output frequency never better than 0.5 since always split power out between the lower and upper frequency bands
- Power is lost in other frequency harmonics (often many) and in resistive losses in diode(s)

  → typically <25% of input power is translated to IF stages: factor >4 loss or >6dB loss

Mixing and sidebands



$\nu_{IF} \pm \Delta\nu_{RF} = (\nu_{RF} \pm \nu_{LO} \pm \Delta\nu_{RF})$

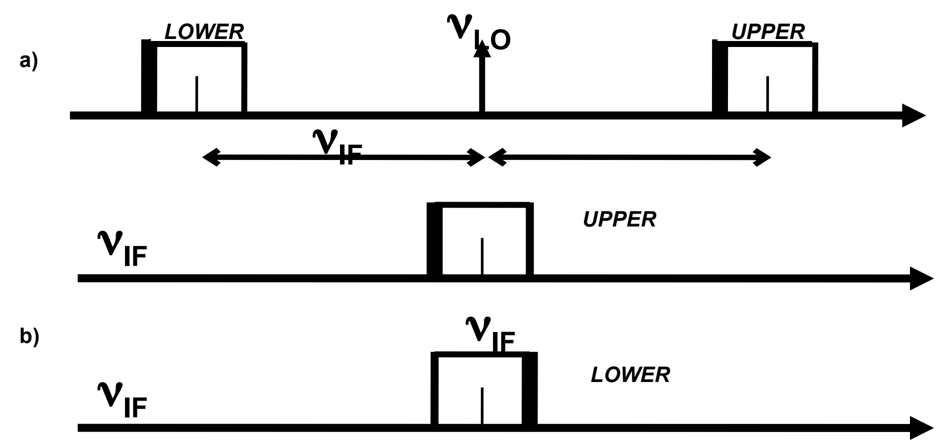
Nearly always select the difference frequency i.e. ν_{IF} is a lower frequency

However there is still a free choice of Local Oscillator frequency:

$(\nu_{RF} \pm \Delta\nu_{RF})$ can be above ν_{LO} : (upper sideband)

OR…

$(\nu_{RF} \pm \Delta\nu_{RF})$ can be below ν_{LO} : (lower sideband)



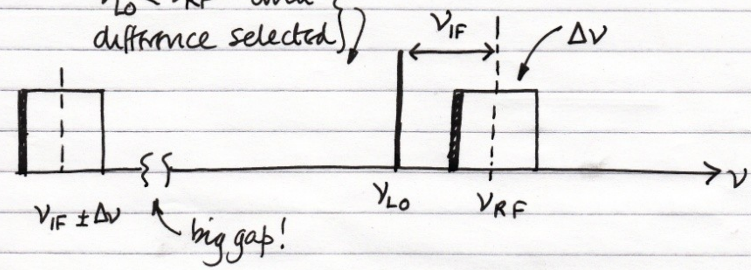
- Usually one wants to produce lower frequencies for further amplification and signal manipulation.
- The boxes represent the two possible frequency bands - the "upper" and "lower" sidebands - each of which would be mixed to the same intermediate frequency (IF)

  $\nu_{IF} = (\nu_{LO} - \nu_{LOWER})$ or $\nu_{IF} = (\nu_{UPPER} - \nu_{LO})$

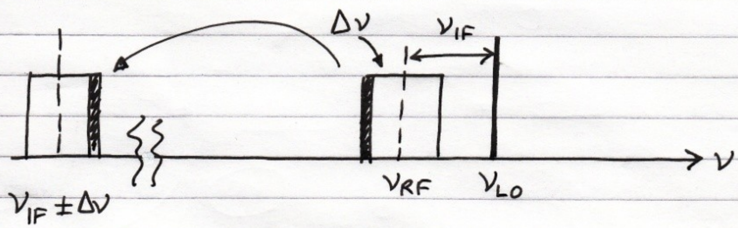
  the thicker side represents the lower "sky" frequency.
- Usually one or the other is filtered out (by a filter after the mixer) - so that both are not mixed together into the IF band. This is important to minimise unwanted "out-of-band" interfering signals.

Pictorially: filter action **(compare with slide 24)**

a) $\nu_{LO} < \nu_{RF}$ and difference selected



b)



$\nu_{LO} > \nu_{RF}$ and difference frequency selected

NB note the frequency reversal in the IF band in case b)

RESULT: After non-linear mixing & then filtering we have a simple translation in frequency for all the input frequency components and the A's and ϕ's of components are also linearly transformed

$A_i \rightarrow A_i'$; $\phi_i \rightarrow \phi_i'$

Linear transformation: retains coherence of the input components *vital for interferometry* - later in course