MANCHESTER 1824

The University of Manchester

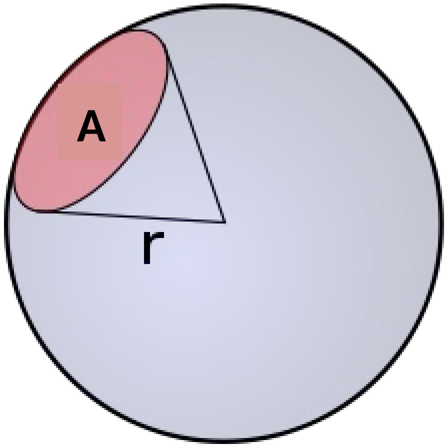
DARA Training Unit 1 2018
Radio astronomy
Lecture 2
Brightness, Flux density, Emission Mechanisms, Radiative Transfer

Professor Peter Wilkinson

Jodrell Bank Centre for Astrophysics

Note 1) this presentation has been put together from more than one presentation given during PW's Manchester lecture course in 2014. Hence narration references to other slides and/or references to material in other lectures are not always correct. I do not think that this greatly affects its usefulness.
2) Slides removed from original presentations to save time for DARA are retained after "THE END"
3) The handwritten slides (referred to as Notes A, B etc) were written in real-time in Manchester lectures by PW so that students had to pay attention and write this material down in their own notebooks in order to achieve a full set of course material– this scheme also mitigated the deadly "Death by Powerpoint" disease and kept students awake.

Solid angles



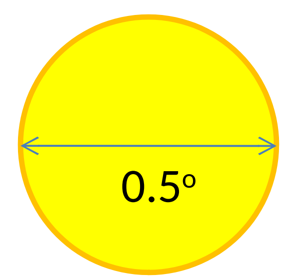
The definition of an angle in radian measure is the *arc of a circle divided by its radius*. If arc=radius → angle=1 radian $=180/\boldsymbol{\pi} = 57.3^{o}$

A solid angle $\boldsymbol{\Omega}$ is the *area* **A** on the surface of a sphere *divided by its radius* **r** *squared.*

$$\boldsymbol{\Omega = A/r^2}$$

Mathematically, the solid angle does not have units, but for practical reasons, we assign virtual units of "steradians" or sterads

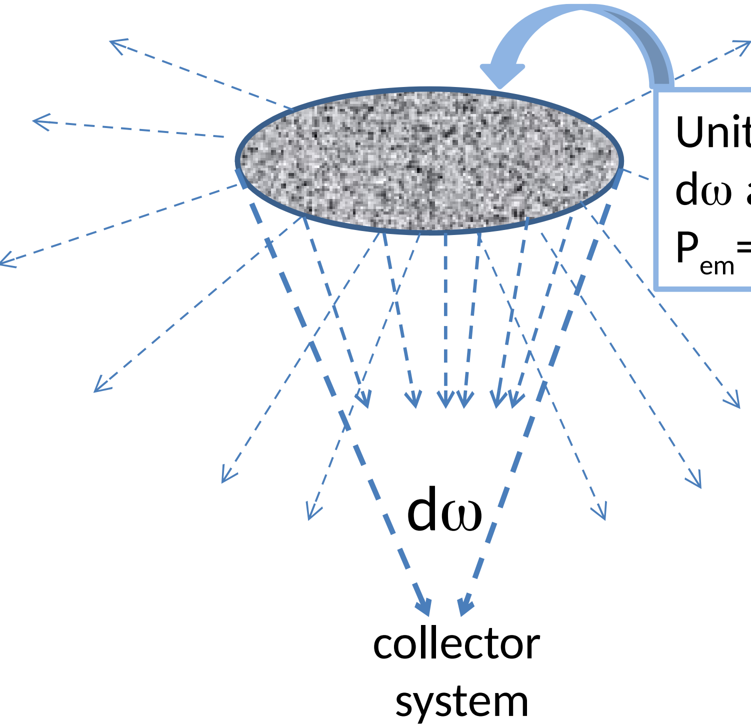
- The surface area of a sphere is $4\boldsymbol{\pi r^2}$ hence its solid angle $\boldsymbol{\Omega} = 4\boldsymbol{\pi}$ sterads
- Consider a "square" area whose sides = **r** → 1 sterad $= (180/\boldsymbol{\pi}) \times (180/\boldsymbol{\pi}) = 57.3^{o} \times 57.3^{o} = 3282.8\ deg^2$ → *a sphere (THE SKY) subtends 41253 square degrees as seen from its centre*
- The Sun and the Moon both have angular diameters ~0.5 degrees → angular area $= (\pi/4).(0.5)^2 = 0.196\ deg^2 = 6 \times 10^{-5}$ sterad



Specific Intensity I_ν

The power *leaving* unit area dA of an emitting region, at a frequency ν per unit frequency interval $d\nu$, into a solid angle $d\omega$ heading towards the collector.
I_ν is a fundamental property related to the physics of the emission process e.g. the Planck radiation law for black bodies: units: watts m^{-2} Hz^{-1} sterad^{-1}

Radio photons leaving an extended emitting region. A few of these photons are heading towards the collector system and constitute an energy flow towards it – the vast majority miss the collector!

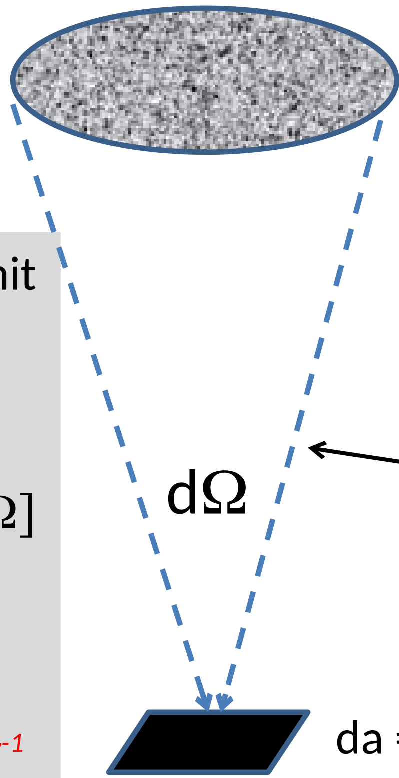


Unit area dA of the apparent surface subtends a solid angle $d\omega$ as seen from the collector system and emits power $P_{em} = I_\nu \, dA.d\nu.d\omega$

Note: astronomical sources are effectively at infinity – so 3-D objects appear as 2-D surfaces and the energy stream can be described in terms of plane waves fronts or rays – as drawn here.

Apparent specific surface brightness B_ν

The power *arriving* per unit area da of the collector system at a frequency ν per unit frequency interval $d\nu$ from a solid angle $d\Omega$ of the emitting region.



Area dA of apparent surface of source
(see previous slide)

reception beam of antenna
(see lecture 3)

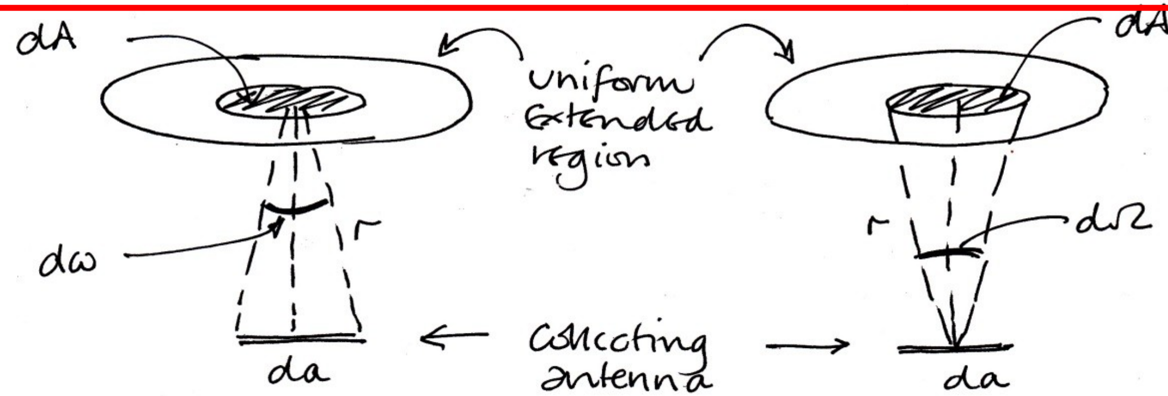
da = unit of area of the collector, receives power $P_{rec} = B_\nu da d\nu d\Omega$

Flux = energy arrival rate per unit area $F = [dE/dt].[1/da]$

Brightness B $= F/\text{solid angle}$
$= [dE/dt.da].[1/d\Omega]$

B_ν = B per unit frequency
$= dE/dt.da.d\Omega.d\nu$
watts m^{-2} sterad^{-1} Hz^{-1}

For extended sources I_ν & B_ν are the same



$$\left.\begin{aligned} dP_{rec} &= B_\nu \, da \, d\nu \left(\frac{dA}{r^2}\right) \leftarrow d\Omega \\ dP_{emit} &= I_\nu \, dA \, d\nu \left(\frac{da}{r^2}\right) \leftarrow d\omega \end{aligned}\right\} \text{ just from definitions}$$

$$\rightarrow \quad dP_{rec} \equiv dP_{emit}$$

" 1 m² of sky radiating into a 1 deg² antenna beam ≡
1 m² of antenna radiating into 1 deg² of sky "

$\rightarrow$ $\boxed{B_\nu \equiv I_\nu}$ $\therefore$ often used interchangeably!

We use: B_ν = the observable surface brightness which translates to I_ν $\therefore$ physical conditions in source for extended sources.

B_ν is conserved regardless of distance

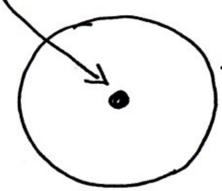
$$dP_{rec} \propto \frac{1}{r^2} \cdot r^2$$

$\frac{1}{r^2}$: inverse square law

r^2: larger area of source intercepted

$$\boxed{\text{a small radiotelescope (large beam) collects as much power as a large radiotelescope (small beam)! FOR AN EXTENDED SOURCE ONLY}}$$

- SOURCES SMALLER THAN RECEPTION BEAM



$\Rightarrow$ Cannot measure B_ν (can't tell size) $\rightarrow$ only averaged B_ν across source:

define: flux density $S_\nu = \int_{\Omega_{source}} B_\nu(\Omega)\, d\Omega$ $\quad W m^{-2} Hz^{-1}$

(sterad gone)

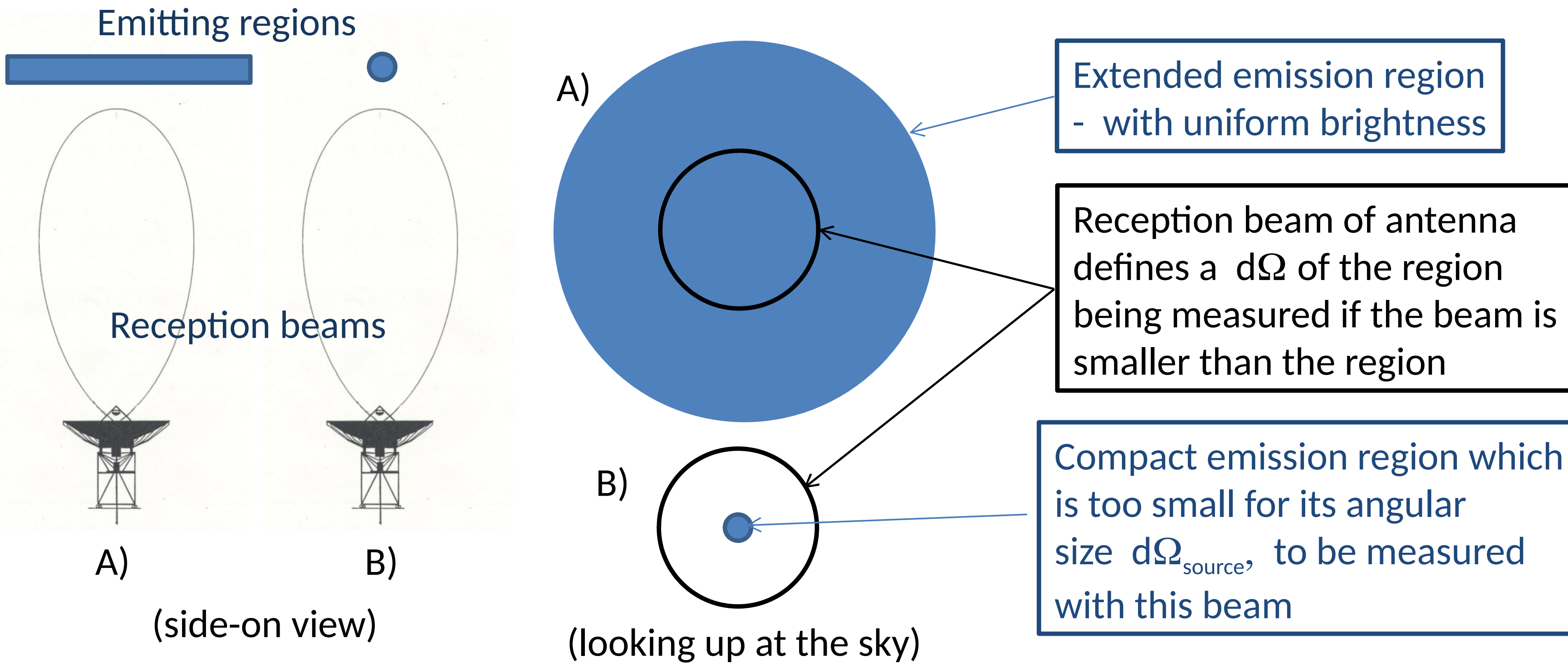
units: Jansky (Jy) $\equiv 10^{-26}\, W m^{-2} Hz^{-1}$

since B_ν is independent of distance, S_ν decreases with $r \propto 1/r^2$ due to smaller area

$$\boxed{\text{Sources get smaller not less bright}}$$

physics at source

Beam sizes and source sizes



Case A) receiver detects the incoming power in watts; we know the bandwidth $d\nu$;
we know the collecting area of the antenna in m^2; we get $d\Omega$ from the beam itself
→ *everything required to define the brightness B_ν of the region within the beam*

Case B) same as A) except that we cannot measure $d\Omega$
→ *cannot define the brightness of the emitting object*
→ *have to use the more limited information provided by the flux density S_ν*

Breakdown of $I_\nu \equiv B_\nu$ identity

1) Absorption of photons by material within the beam (see radiative transfer – this lecture)

2) Scattering of photons out of the beam (rare in radioastronomy)

3) Change of frequency: in the expanding universe the frequencies (photon energies) all fall by the same factor $(1 + z)$

[The integral of B_ν over ν is $\alpha\ \nu^4$ $\rightarrow$ the surface brightness of a galaxy at a redshift z compared with that of a similar galaxy nearby, falls by $(1+ z)^4$]

4) Diffraction effects: so far assumed geometrical optics (ray tracing). In reality the clean reception beam will have "sidelobes" pointing in different directions (see Lecture 3) towards regions most likely with different brightnesses (e.g. the surrounding ground!) . *Only if all the sidelobes are embedded in regions of the same brightness can we claim to have measured the intrinsic I_ν from a measurement of B_ν.*

Assessing the effect of the radiation entering via the sidelobes is basic challenge when measuring the total power coming from the sky with a radio telescope.

Radio emission mechanisms

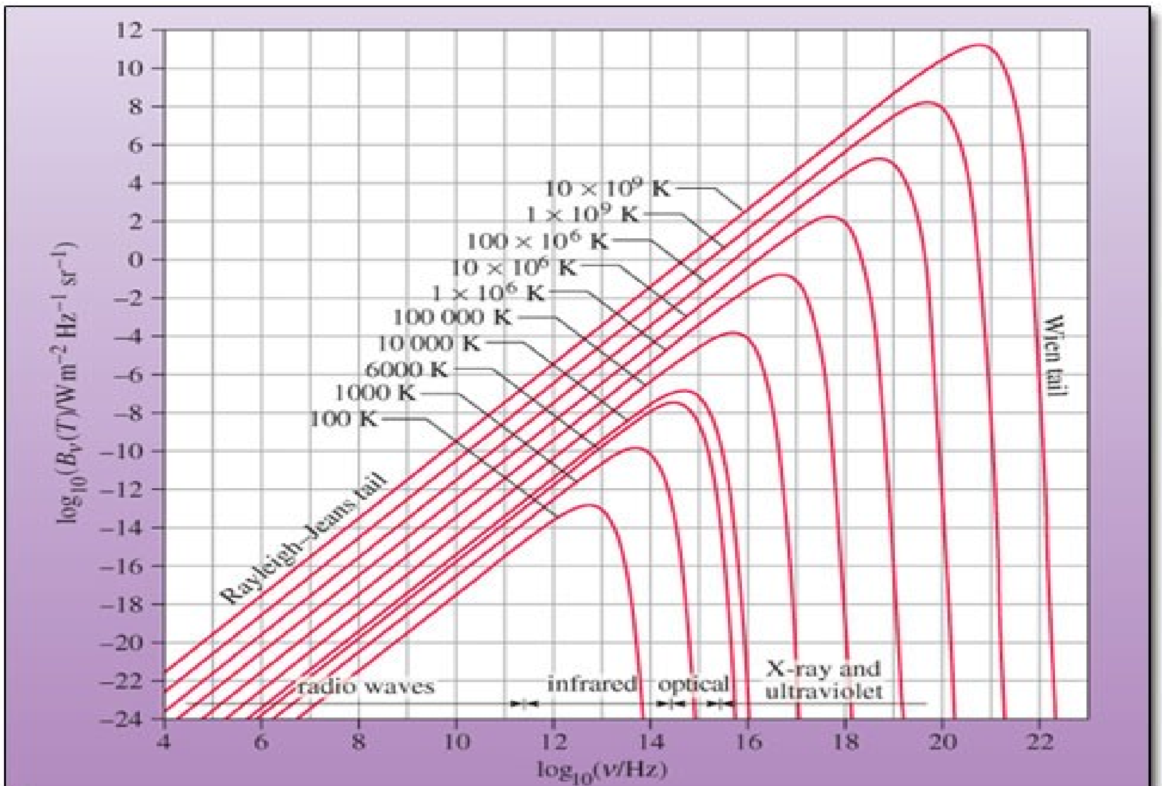
EM radiation is emitted by charged particles when they are accelerated – dominated by electrons since they are the lightest charged particle.
Thermal emission: the emitting particles are in equilibrium or quasi-equilibrium with the radiation produced & about as many photons emitted as absorbed. The radiation depends on the physical temperature of the emitting object: includes *blackbody radiation, "free-free" emission* from an ionized gas, and *spectral line emission. Random processes* → *emission is unpolarised*

"Non-thermal "emission: arises from a particle distribution that is far from thermal equilibrium (i.e. non-Maxwellian) - no classical temperature. A power law distribution of *relativistic* energies produces *synchrotron radiation*; a non-thermal (non- Boltzman) population of molecular energy levels produces amplified line emission from *masers* in space.

NOTE:
Magnetic fields produce *preferred directions* → *often partially polarised* (synchrotron always, some maser lines depending on physical conditions)

Black-body Radiation

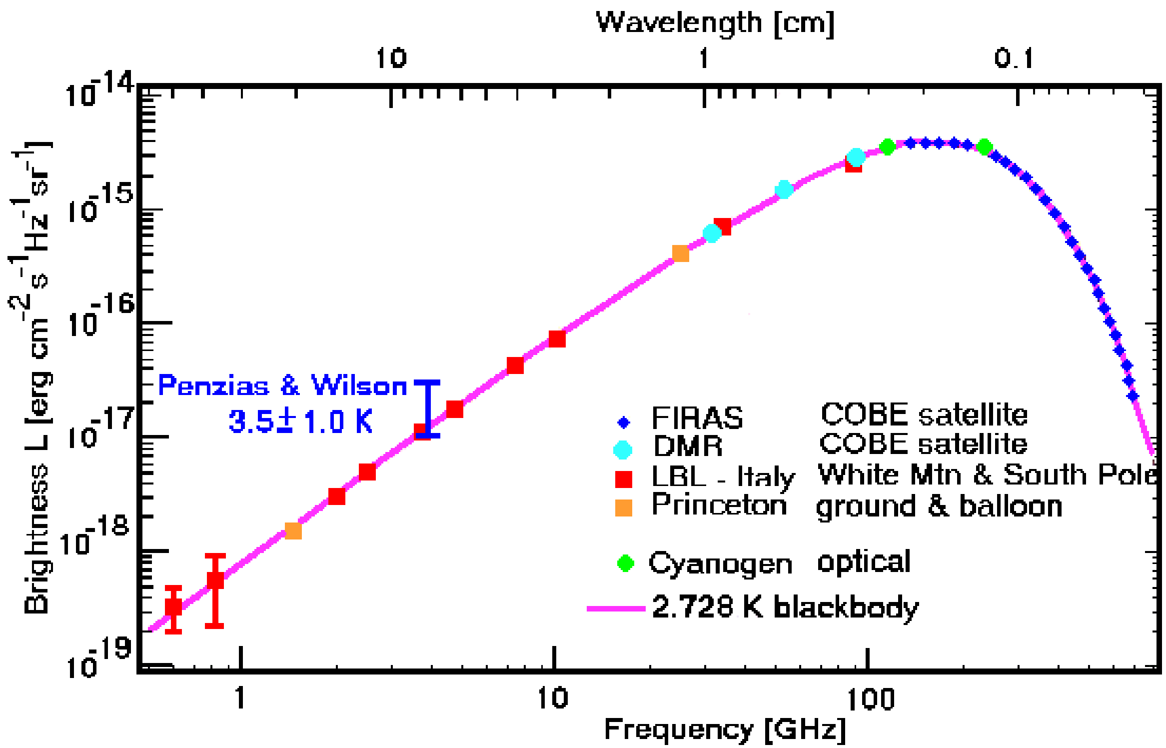
For perfectly radiating (or perfectly absorbing) surfaces- "black-bodies" - the emitted power output is given by the Planck radiation formula *which depends only on the physical temperature T.* The total energy radiated *per unit area per unit time* is $5.7 \times 10^{-8}\ T^4$ Watts (Stefan's Law). All other objects emit less than that of a black body by a factor ε, the dimensionless emissivity (a function of emitted wavelength) hence the spectrum from real objects ("grey bodies") does not follow the perfectly smooth curves given by the Planck Formula.



N.B. radio astronomers usually plot the spectrum as a function of *frequency on a log scale.*

http://dspace.jorum.ac.uk/xmlui/bitstream/handle/10949/957/Items/S381_1_section12.html

The spectrum of the CMBR: a nearly perfect black-body



http://super.colorado.edu/~astr1020/l12body.html

The Cosmic Microwave Background Radiation (T = 2.75 K) from the Big Bang has an almost perfect black body spectrum peaking at 160.2 GHz (λ= 1.9 mm) if you plot the brightness *per unit frequency interval against frequency* (as in the plot above). BUT if you plot brightness *per unit wavelength interval against wavelength* the peak is at λ= 1.06 mm (from usual Wein's Law) corresponding to a frequency of 283 GHz. *The difference is purely mathematical.*

You Tube video (11 mins) on
Black Body Radiation

https://www.youtube.com/watch?v=jbxty6aDfhU

from series:

http://brightstorm.com/science

Now knowing about black bodies…..we can now understand "effective brightness temperature"….

B-B RADIATION & RAYLEIGH-JEANS APPROXIMATION

B-BODY ≡ PERFECT ABSORBER; PERFECT RADIATOR

PLANCK FORMULA GIVES BRIGHTNESS FOR TEMP. T $\Bigg\}$ $B_\nu = \dfrac{2kT\nu^2}{c^2}\left[\dfrac{h\nu/kT}{e^{h\nu/kT}-1}\right]$

Units are: W m^{-2} Hz^{-1} $sterad^{-1}$

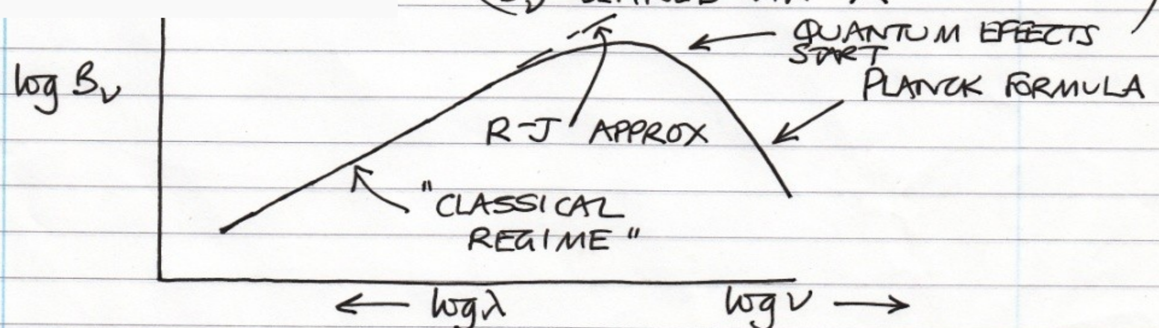
IN RADIO BAND $h\nu << kT$ UNLESS T VERY LOW AND/OR ν " HIGH

$\rightarrow$ [ ] TERM $\longrightarrow$ 1

$$\boxed{B_\nu = \frac{2kT\nu^2}{c^2} = \frac{2kT}{\lambda^2}}$$

Rayleigh - Jeans approximation

(EASIEST TO REMEMBER BUT NOTE B_ν DEFINED VIA λ)



R-J OK IF ν(GHz) << 20 T (in K)

eg. 30GHz & T=3K ⇝ MARGINAL → NOT OK
30GHz & T=100K ⇝ OK TO USE R-J

EXAMPLE: IF AVERAGE T_B OF MOON ~ 210K WHAT IS ITS FLUX DENSITY AT $\begin{cases} \nu = 1.4\,GHz \\ \lambda = 21\,cm \end{cases}$?

(NB 210K CLOSE TO THERMAL TEMP OF SURFACE LAYERS : VARIES AS FUNCTION OF PHASE)

$$S_\nu = \int_{MOON} B_\nu \, d\Omega \qquad B_\nu = \frac{2kT_B}{\lambda^2} \quad \left(\begin{array}{c} B\text{-}B \\ \text{Rayleigh-Jeans} \end{array}\right)$$

$$S_\nu = \frac{2kT_B \, d\Omega_{MOON}}{\lambda^2}$$

$$d\Omega = \pi \left[\frac{0.5^\circ}{2} \cdot \frac{\pi}{180} \right]^2 = 6 \times 10^{-5} \text{ sterad}$$

(See slide 2)) (SAME AS SUN!)

$$S_\nu = 7.9 \times 10^{-24} \ W \, m^{-2} \, Hz^{-1}$$

$$= 790 \ Jy$$

(c.f. 2000 Jy for Cass A (Lecture 1 slide 9)

NB. To collect all power need to receive both polarisations ~~otherwise~~ (X & Y) or (R & L)

Effective Brightness Temperature

Most sources are not B-Bodies – but it's convenient to use them as a reference

Define a "brightness temperature" T_B such that a B-Body at a physical temperature T_B would produce the same brightness B_ν. The measured brightness is expressed as a brightness temperature T_B using the Rayleigh-Jeans approx:

$$T_B = B_\nu c^2/2k\nu^2 \quad (\text{or } B_\nu \lambda^2/2k)$$

Cosmic radio sources exhibit a wide range of brightness temperatures

"Cold"

- Actual B-Body radiation from regions at physical temp of a few K to tens of K
  - Cosmic Microwave Background ~3K
  - Interstellar dust ~tens of K
- Molecular clouds ~tens of K → kT ~1 milli eV i.e. much lower than *electronic* energy levels → correspond instead to *rotational* energies of molecules
  - Molecular lines
- Special case of spin-flip in *atomic hydrogen*

"Warm"

- Free-free emission from astrophysical plasmas in higher-energy environments
    - *- stellar photospheres,*
    - *- tenuous atmospheres around stars (stellar winds)*
    - *- planetary nebulae*
    - *- gaseous nebulae ionised by hot stars*

T_B typically 5,000 – 15,000 K

"Hot"

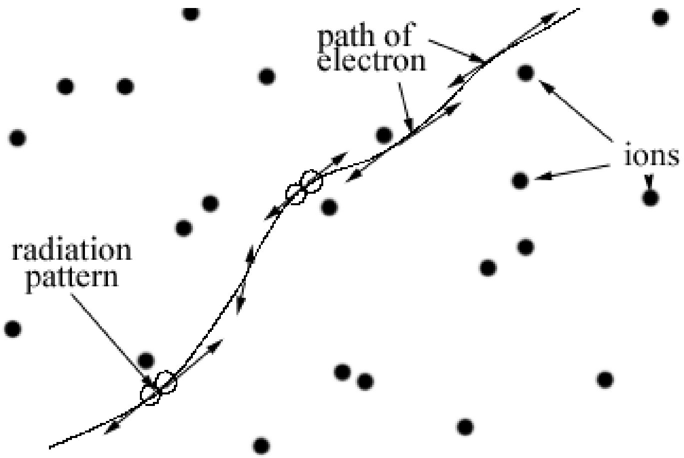
- Non-thermal radiation emitted by relativistic electrons e.g. synchrotron
- T_B can be low in diffuse regions (e.g. slide 3) but in extreme environments
    - *- Supernovae (as in slide 6)*
    - *- Pulsars (see later lectures)*
    - *- Gamma-ray bursters (super-powerful supernovae)*
    - *- Active Galactic Nuclei*

T_B can be 10^5 - 10^{11} K - not a physical temperature in the standard sense

Rule: If $T_B > 10^5$ K radiation is non-thermal , even if the spectrum is not known.

Now to explain the emission mechanisms referred to in previous two slides

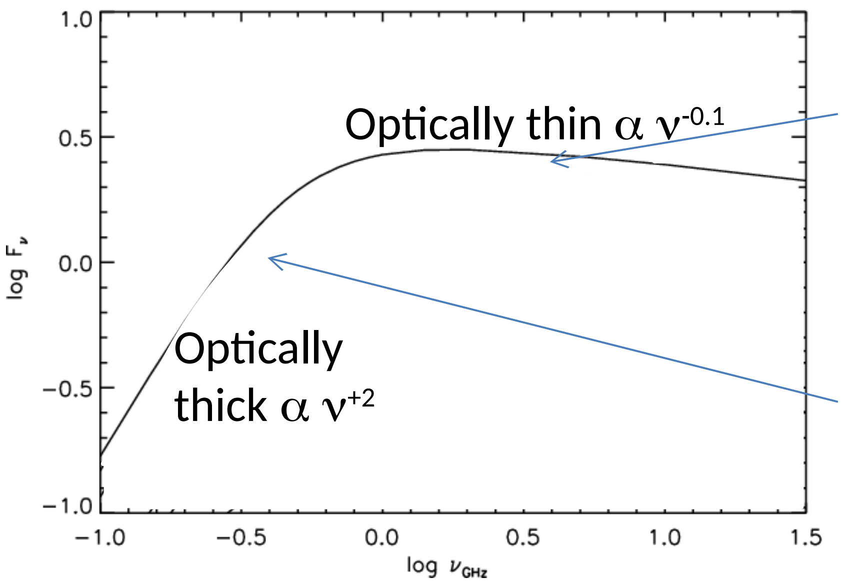
Bremsstrahlung or "free-free" emission



http://web.njit.edu/~gary/728/Lecture2.html

An electron, moving through relatively immobile ions in a plasma, experiences many small accelerations, each one producing a dipolar radiation pattern *along its direction of motion (i.e. perpendicular to its acceleration)*.

(spectrum of a population of electrons is a complicated calculation!)
http://www.cv.nrao.edu/course/astr534/FreeFreeEmission.html



Spectrum: (flux density vs frequency ν; log-log scale)

At **high frequencies** the plasma becomes transparent to its own radiation and emits less strongly; the flux density falls α $\nu^{-0.1}$ (much slower than the fall off in the black-body curve).

At **low frequencies** the plasma becomes opaque to its own emission and we only see its surface layer – hence it **acts like a black body** with the same spectral slope ~ 2 (α ν^{+2} i.e. Rayleigh Jeans Law – next lecture).

You Tube video (16 mins) deriving
Thermal Bremsstrahlung /Free Free emission

https://www.youtube.com/watch?v=78fxc_44WZQ

from series:

https://casper.berkeley.edu/astrobaki/

Sources of “free-free” emission

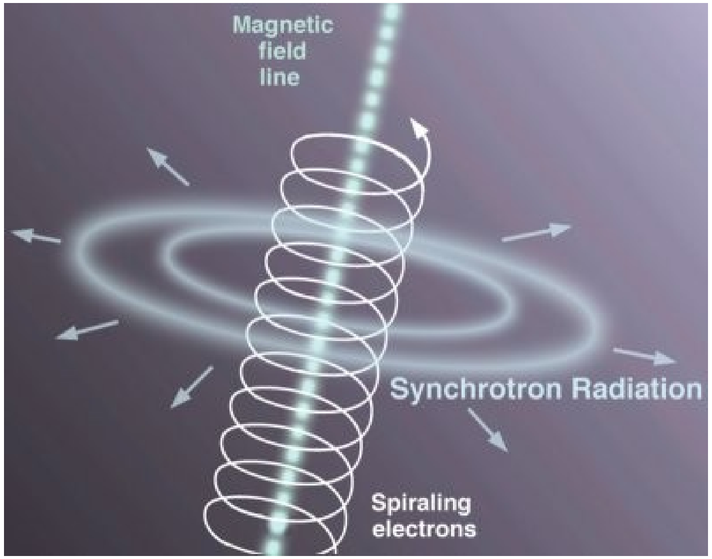
http://www.uv.es/jrtorres/index6_archivos/image003.png

Free-free is the dominant continuum emission in thermal plasmas
e.g. “planetary nebulae” – hot (up to 15,000K) ionised plasma shells surrounding hot stars in the *late stages of their evolution*
.

E.g. “HII regions” – interstellar gas also ionised by hot stars but in this case ones recently formed from the gas cloud.
(this one is the Orion Nebula in Orion’s Belt)

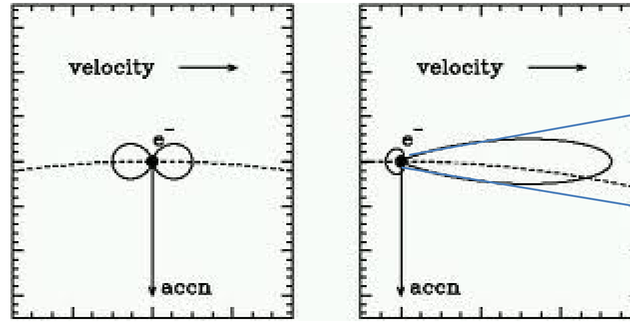
Synchrotron radiation

http://astronomyonline.org/Stars/Images/ChrisFlynn/SynchrotronRadiation.jpg

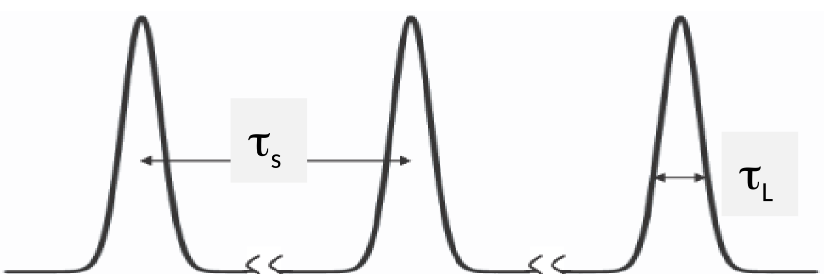


Electrons *spiral* around magnetic field lines → radiation

http://www.astro.utu.fi/~cflynn/astroII/l4.html



angle = $2/\gamma$



Gyro-radiation;
non-relativistic:
pattern is dipolar

$\omega_G = eB/m$

$\nu_G = \omega_G / 2\pi$

$\tau_G = 2\pi / \omega_G$

Synchrotron radiation;
relativistic: pattern changed due to rel. aberration plus light travel time effects

$\omega = eB/\gamma m$

$\tau_s = \gamma \tau_G$ **(v. slow)**

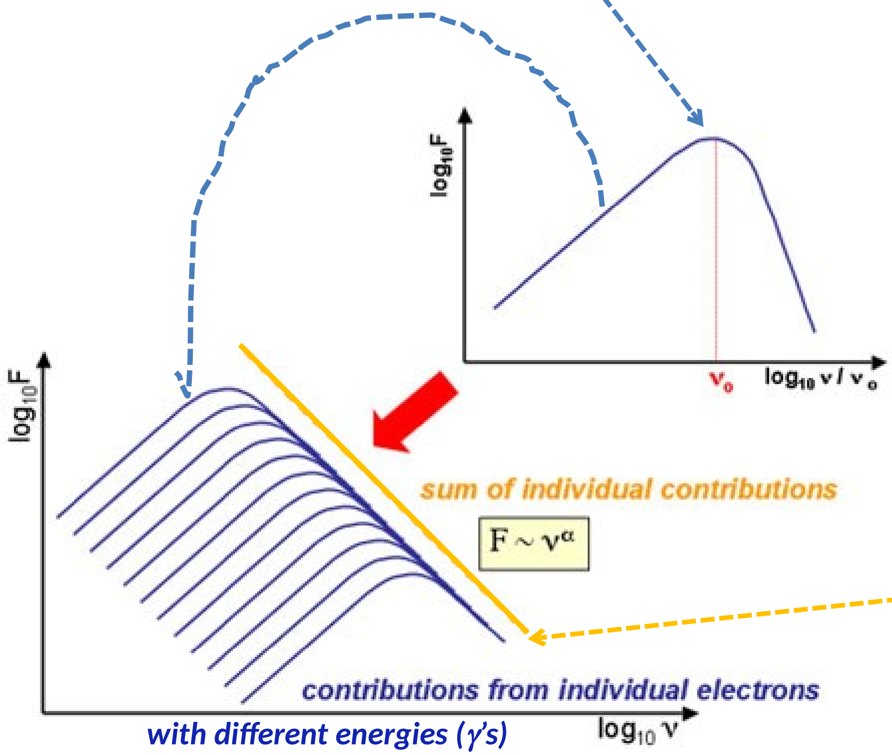
$\tau_L \sim \gamma^{-3} \tau_s$ **(v. fast)**

Synchrotron radiation is a *very spiky series of widely spaced very narrow pulses*. For an electron with Lorentz factor $\gamma \sim 10^4$ in a magnetic field B ~5 μG (5 x 10^{-10} T; (typical of the interstellar medium in our Galaxy) → $\tau_s \sim 10^3$ s and $\tau_L \sim 10^{-10}$ s

For further details see: http://www.cv.nrao.edu/course/astr534/SynchrotronSpectrum.html

Synchrotron radiation

The frequency spectrum from each electron is given by the Fourier transform of the time series – series of sharp pulses, many harmonics contribute → spectrum covers a very broad band (a messy calculation) – but the frequency of the peak is $\nu_0 \sim 1/\tau_L$ (remember $\Delta\tau\Delta\nu$~1 true for all waves). For the specific parameters on the previous slide $\nu_0 \sim 10^{10}$ Hz i.e. 10 GHz



Single electrons : *with different energies in a power law distribution (see next slide)* radiate at a different characteristic frequencies (corresponding to value of γ)

Final spectrum (flux density plotted here) is a sum of all the individual contributions - typically it produces a **power law spectrum (next slide)** and the radiation is **linearly polarised** (see later lecture)

http://astronomy.swin.edu.au/cosmos/S/Synchrotron+Emission

You Tube videos (17 &12 mins) deriving

Synchrotron Power

https://www.youtube.com/watch?v=-0dm-x1oxpE

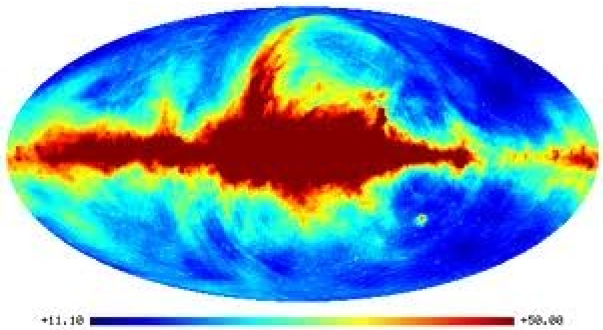
Synchrotron Spectral Shape

https://www.youtube.com/watch?v=BBJBvhOwBKI

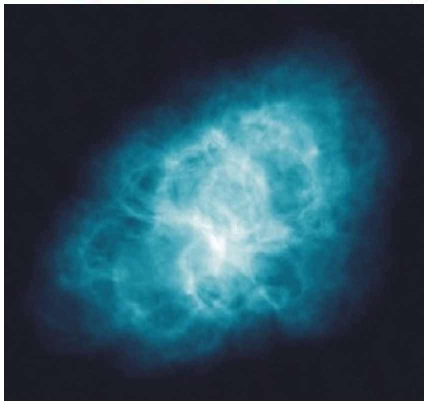
from series:

https://casper.berkeley.edu/astrobaki/

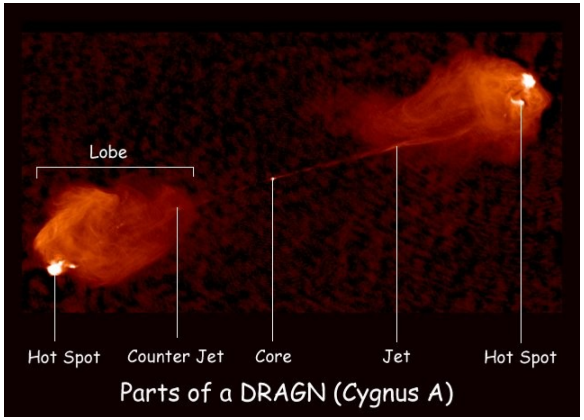
Synchrotron Sources



The Milky Way: electrons and magnetic fields
In interstellar space produce the most intense emission along the plane of our galaxy.



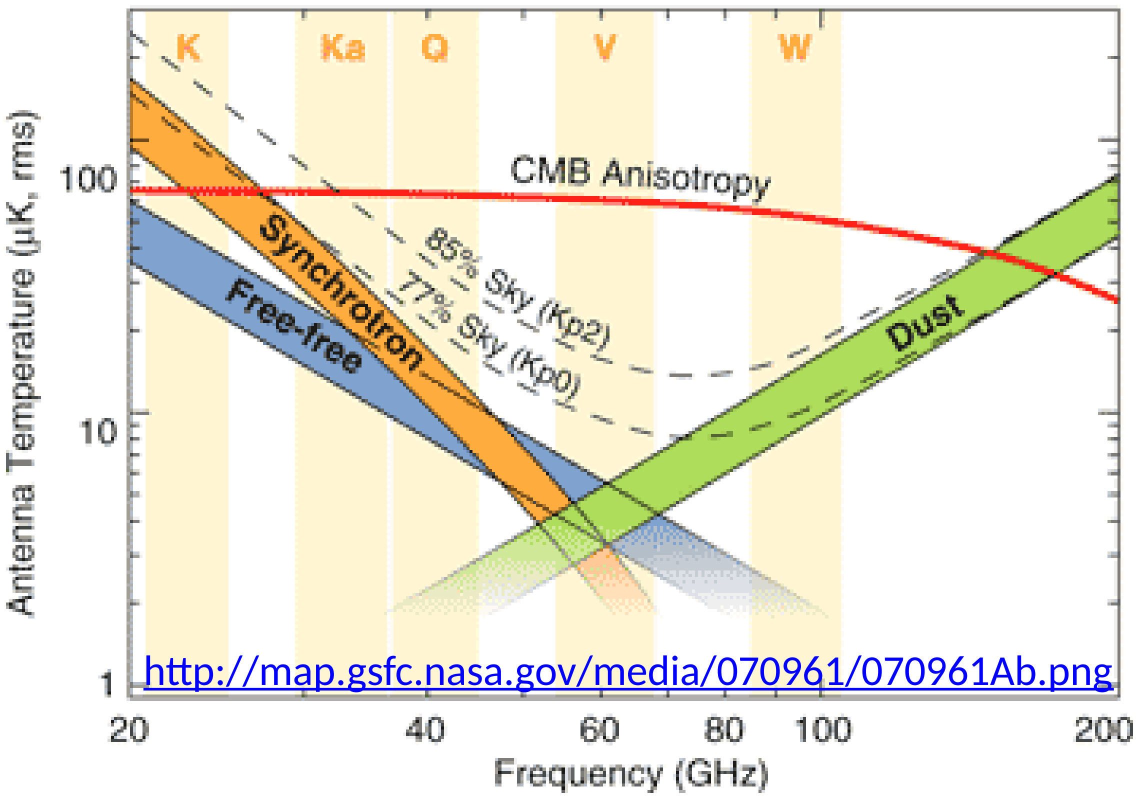
Supernova remnant: e.g. the Crab Nebula– remnants of a star which blew up in 1054 AD. Detection of linear polarization showed that *synchrotron* was the radio emission mechanism.



Extragalactic Radio Source: e.g. Cygnus A a black hole
In the centre of an active galaxy is the energy source for Collimated jets of relativistic plasma which inflate twin "radio lobes" on opposite sides of the galaxy.

http://www.cv.nrao.edu/~abridle/dragnparts.htm

Components of the diffuse Galactic radio emission



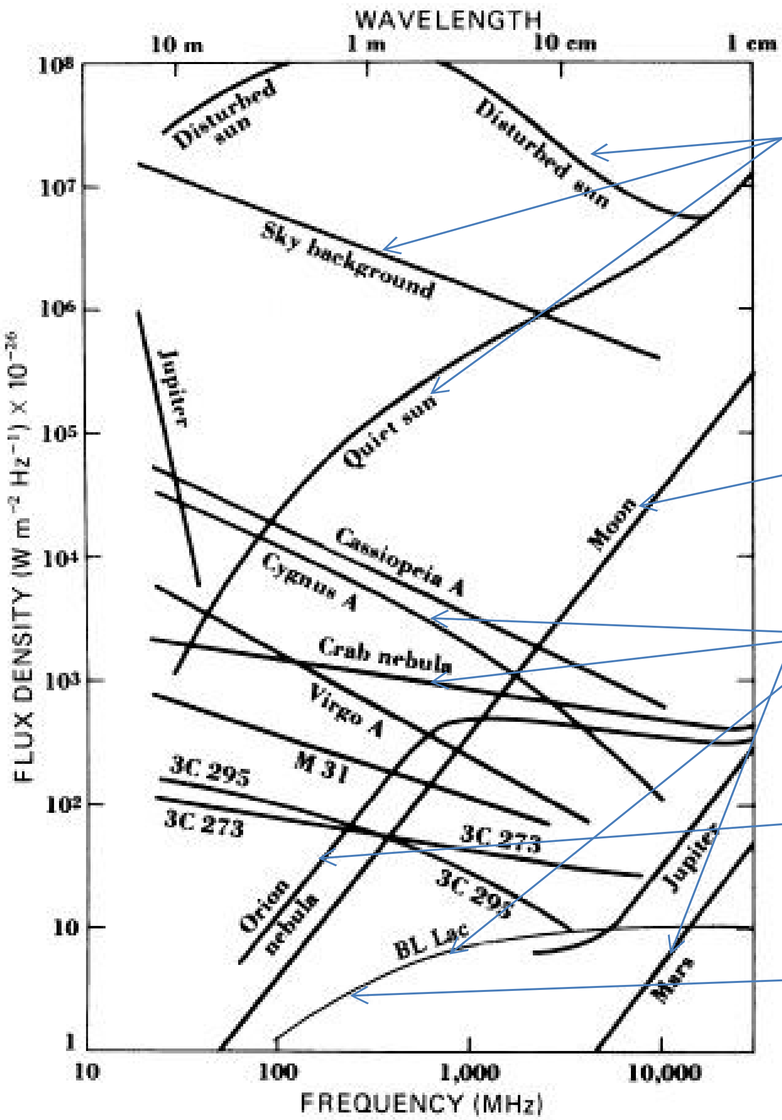
http://map.gsfc.nasa.gov/media/070961/070961Ab.png

Note the spectrum is plotted in terms of antenna temperature T_{ANT} (this lecture) which is calibrated with respect to a black body. The Rayleigh- Jeans approximation for a black body gives $B_\nu \propto 2kT\nu^2/c^2$ (this lecture) therefore $T \propto Bc^2/2k\nu^2$.

Thus the slope of a power law in a logarithmic plot of T(ν) vs. ν = -2 (slope in brightness or flux density) . Thus the “free-free” slope -0.1 (in slide **18** is now -2.1. The “synchrotron” slope was -0.7 (in slide **23**) is now -2.7.

Note: *only part of the complete spectrum of the free-free and dust components are plotted*. The dust spectrum peaks off to the right of the graph (corresponding to black-body radiation with T~60K) while the peaks of the free-free and synchrotron spectra are off to the left of the graph. The free-free radiation at frequencies above a few GHz comes from optically-thin regions and hence has a falling power law spectrum rather similar to the non-thermal synchrotron radiation. *Synchrotron radiation is, however, partially plane polarised whereas free-free emission, being thermal in origin, is randomly polarised. This is the best route to separating their contributions.*

Spectra of typical radio sources. (Adapted from Kraus, J. D., *Radio Astronomy*, McGraw-Hill Co., 1966.)



complex mixture of several

black/ grey body

synchrotron

free-free

synchrotron self absorption - but not as sharp as $\nu^{2.5}$

Radio spectra of different cosmic objects emitting broad-band continuum radiation (*not spectral lines*)

http://hera.ph1.uni-koeln.de/~heintzma/k1/A_RadioSp.htm

Spectral lines from atoms and molecules are powerful diagnostics of the physical and chemical conditions in emitting clouds in interstellar space

SPECTRAL LINES FROM ATOMS

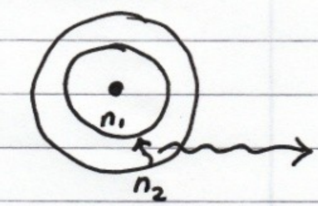
(i) RECOMBINATION LINES

next slides

OUTERMOST ELECTRON IN ALL ATOMS IS "HYDROGENIC" (WHY?)

→ BOHR ATOM IS OK

n = 1st quantum number



FREQUENCY $\nu = R.c\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

speed of light

RYDBERG CONSTANT VARIES WITH NUCLEAR MASS M ($R\uparrow$ AS $M\uparrow$) : EFFECT OF "REDUCED MASS"

HYDROGEN $\nu = \underbrace{3.28805\times10^{15}}_{R.c.}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ Hz

NAME CONVENTION : H 109α

much bigger than for usual calculations

ATOM

lower n

$n_2 - n_1 = 1$

$n_2 - n_1 = 2 \rightarrow$ "β" etc

Eg. $\nu(\mathrm{H}109\alpha) = 3.29\times10^{15}\left[\frac{1}{109^2} - \frac{1}{110^2}\right]$

$= 5.0089\times10^{9}$ Hz $\equiv 5.0089$ GHz

(ii) THE HYDROGEN LINE :

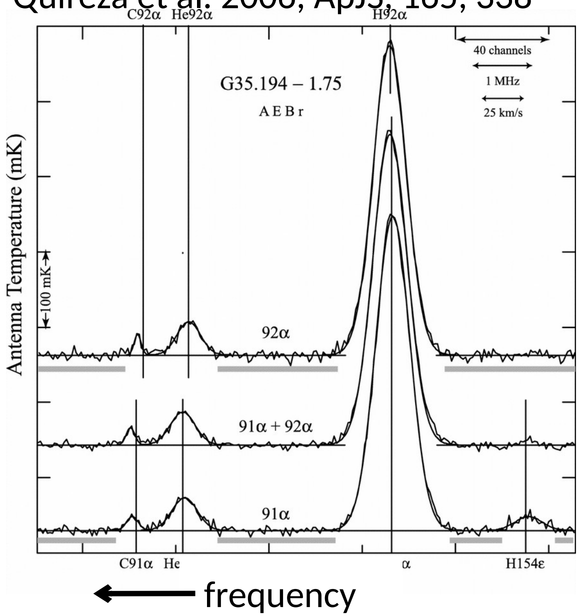
neutral or atomic hydrogen

$\nu(\mathrm{HI}) = 1.42040575177$ GHz !!

$\lambda \sim 21.1$ cm

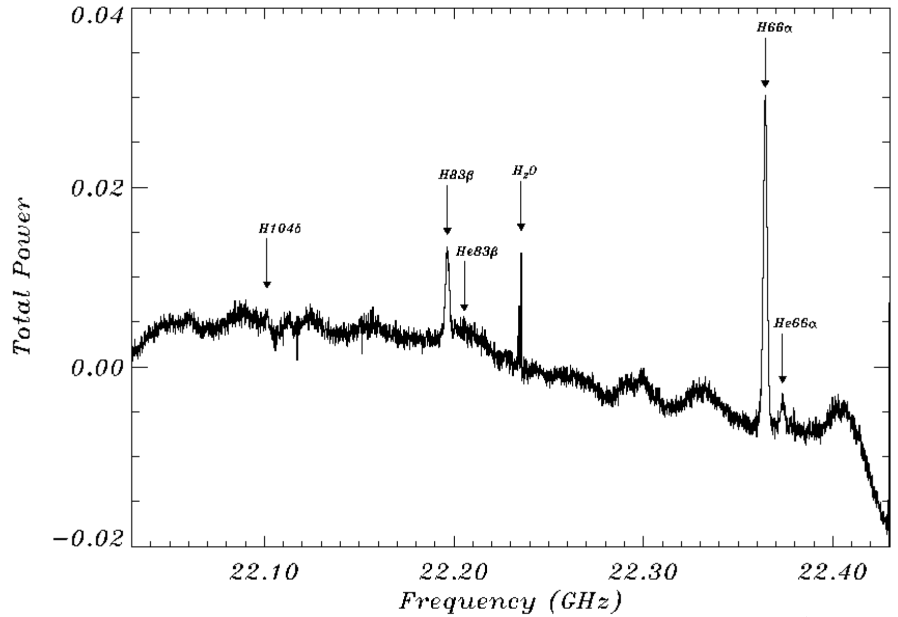
Atomic recombination lines

Quireza et al. 2006, ApJS, 165, 338



Spectrum showing Hydrogen, Helium and Carbon lines.
Note the marker indicating 1 MHz and 25 kHz

http://dsnra.jpl.nasa.gov/Ops_planning/line_sources/Recomb_lines/index.html

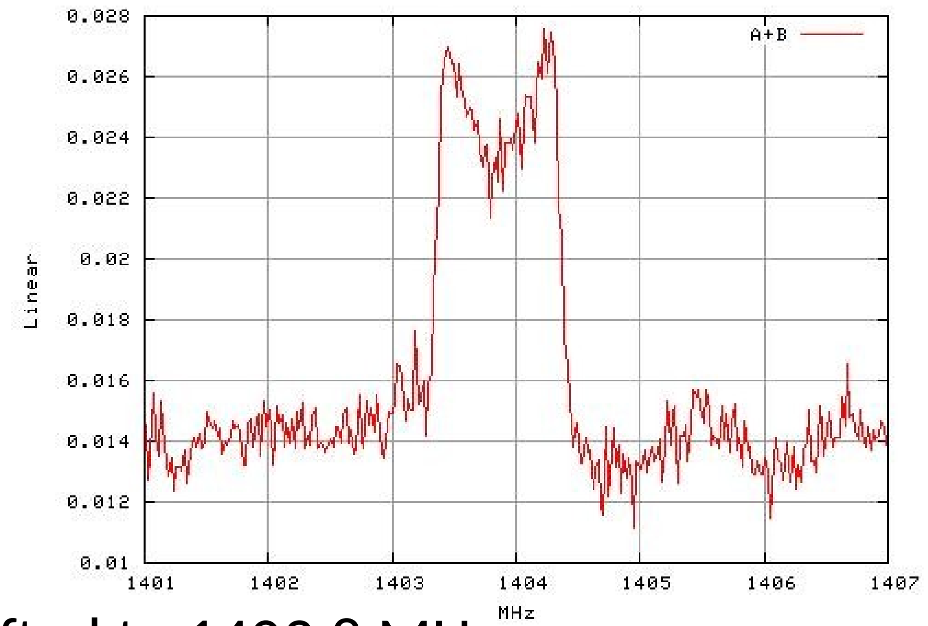


Much wider spectral scan at a higher frequency (22 GHz) showing several recombination lines. Note that the “H_2O “ line is a narrow maser line – not a recombination line. The broad “wiggles” in the spectrum are due to instrumental effects – not broad spectral lines.

Line intensity ratios and line widths are diagnostic of internal energetics of emitting clouds i.e. microscopic - electron speeds (“temperatures”) and macroscopic - bulk motions

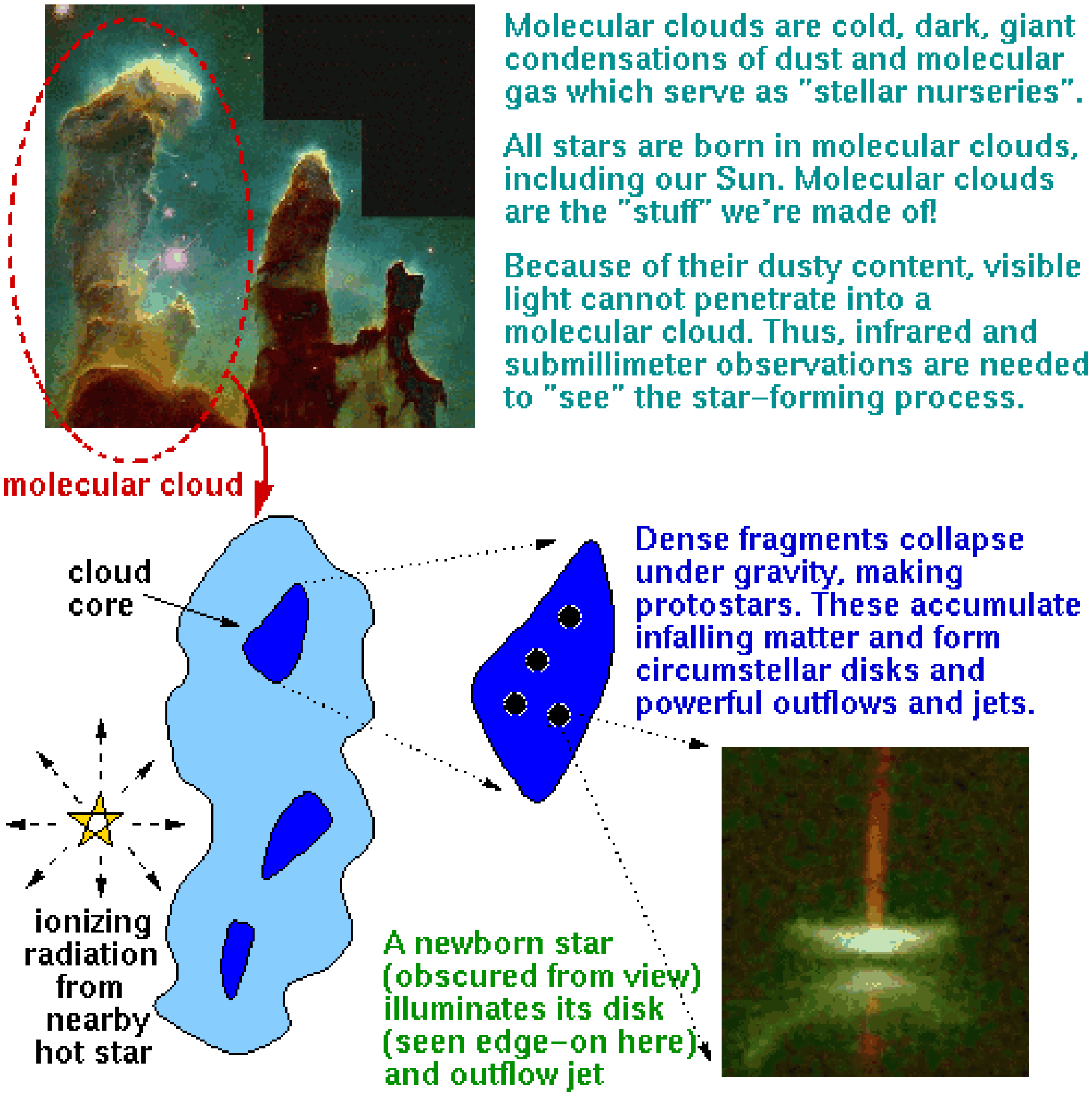
Note: radius of hydrogen ground state = 0.5×10^{-10} m and increases as n_2^2 hence radius of highest n-state yet identified (n=776) is 1/30th of a millimetre!

21cm line from atomic hydrogen in the galaxy ugc1291



- 1420.4 MHz line doppler shifted to 1403.8 MHz
  - dν/ν = v/c → object vel=3507 km/sec
  - Hubble's law: ~70km/sec for each megaparsec (3.26 x10^6 light years).
  - 3507/70 * 3.26 x10^6 =163 Million light years away (time it took the light to get to us)
  - The galaxy is about 1 MHz "wide" (about 210 Km/sec). The galactic rotation velocity at the edge of the hydrogen disc is ~100 km/s.

Molecular line radiation



Quantized rotational transitions from molecules with dipole moments give rise to spectral lines in the radio band

Nearly 180 molecular species have now been identified including the simplest amino acid glycine

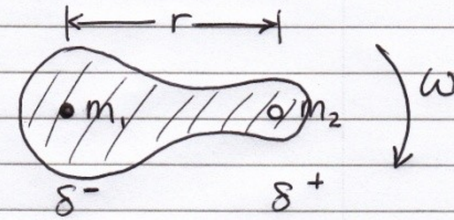
Their study has become new subject

ASTROCHEMISTRY

LINES FROM MOLECULES

REQUIRE 'POLAR' MOLECULES WITH ASYMMETRIC ELECTRON DISTRIBUTIONS →

eg SCHEMATIC DIATOMIC MOLECULE



ROTATION → ACCELERATION → RADIATION

CLASSICALLY:

$$\left\{\begin{aligned} \nu_{ROT} &= 1/\text{PERIOD} \\ E_{ROT} &= \tfrac{1}{2} I \omega^2 \\ E_{ROT} &= L^2/2I \end{aligned}\right.$$

$$\omega = 2\pi\nu$$
$$I = \mu r^2$$
$$\mu = m_1 m_2 / (m_1 + m_2)$$
$$L = I\omega$$
$$\hbar = h/2\pi$$

QUANTUM:

$$L = J\hbar$$

J ← ROTATIONAL QUANTUM NUMBER

$$E_{ROT} = \frac{J(J+1)}{2I}\hbar^2$$

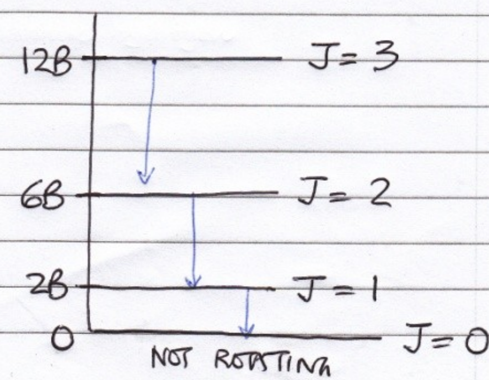
(NO NEED TO PROVE)

ONLY SOME ROTATION RATES ALLOWED & SELECTION RULE $\Delta J = \pm 1$ (PHOTON ANGULAR MOM $= \hbar$)

ENERGY LEVEL DIAGRAM

$$E_{ROT} = B\, J(J+1)$$

B ← $\hbar^2/2I$



You Tube Videos explaining

“Rigid Rotor” model of a molecule

https://www.youtube.com/watch?v=1PNmximc-PI

Rigid Rotor Energy Levels

https://www.youtube.com/watch?v=6eE_RJBUgPM

Diatomic Molecular Energy Levels (ignore vibrations)

https://www.youtube.com/watch?v=WG0HUGZYyvk

Radio emission mechanisms - summarised

- ***Black-body radiation (thermal)*** – rapidly varying motions hence accelerated charges
  - from all terrestrial objects; planetary surfaces ; dust in interstellar space

- ***Bremsstrahlung or free-free emission (thermal)***
  - from ionised plasmas – in experimental fusion reactors (tokamaks); plasma in interstellar space ionised by ultra-violet radiation from stars

- ***Atomic and Molecular Lines (thermal)***
  - from many species in tenuous gas in interstellar space and giant molecular clouds

- ***Synchrotron radiation (non-thermal)***
  - relativistic electrons spiralling around magnetic fields in interstellar space; supernova remnants ; active galaxies
  - related emission from pulsars (gyrosynchrotron)

- ***Maser emission from specific molecular species (non-thermal)***
  - from around stars – pumping by infra-red radiation into non-Boltzmann distribution

PROPAGATION EFFECTS

Material between emitting region and receiver can also affect the incoming radiation via:

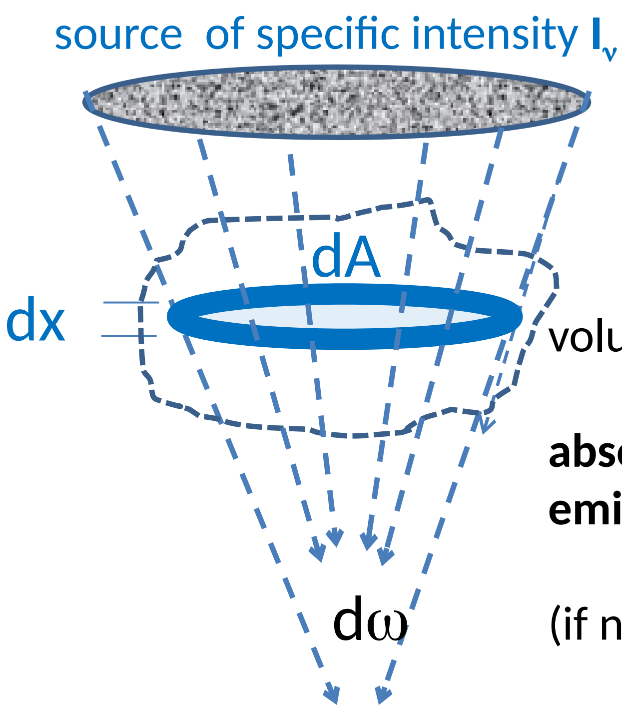
- **absorption** - interaction with intervening ionized plasma (the free-free process) or quantised rotations of gas molecules with permanent dipole moments – especially in the Earth's atmosphere O_2 and $H_2$0
- **emission** - if the intervening material absorbs energy it must also re-emit energy (Kirchoff's Law) - in equilibrium the rate of emission = the rate of absorption
- **scattering** - by small scale (comparable to wavelength) variations in refractive index e.g. fluctuations in water vapour and rain drops or non-uniformities in the interstellar ionised plasma
- **polarisation** - caused by reflection, or by anisotropy in propagation conditions for orthogonal states of polarisation e.g. in the magnetized interstellar plasma

Scattering effects are generally small in the radio band. We focus on absorption & emission in the atmosphere - with discussion of some polarisation effects in later lectures.

Absorption and re-emission in semi-transparent media

(Look back at slide 3 on specific intensity)

Radiation from background source passes through an intervening medium, e.g a cloud of gas, which can both absorb incoming photons and emit and absorb its own photons. Ignore the detailed *microscopic* physics and describe the effects in terms of *macroscopic* absorption and emission coefficients . We ignore scattering out of the beam – a good assumption in the radio band.



volume element dV = dA.dx within intervening gas cloud

absorption → decrease in specific intensity along path
emission → increase in specific intensity along path

(if neither then specific intensity is constant along path)

The optical depth τ

The optical depth $\tau = \kappa_\nu x$ (dimensionless) is a "summary" parameter telling us how opaque a region is to radiation passing through it – regardless of how the opaqueness arises.

κ = fraction of intensity lost per metre of travel (as a function of frequency ν)

x = number of metres of travel through the aborbing regions

$\tau = 0$ → medium is completely transparent

$\tau = \infty$ → medium is completely opaque

Radiative Transfer: the Main Result

$$I_\nu = I_\nu(0)\, e^{-\tau} + I_\nu^{BB}[1 - e^{-\tau}]$$

total source medium

In the Rayleigh Jeans regime: all specific intensities can be summarised by a brightness temperature : either an actual physical temperature of a Black Body or an equivalent black body temperature (This lecture).

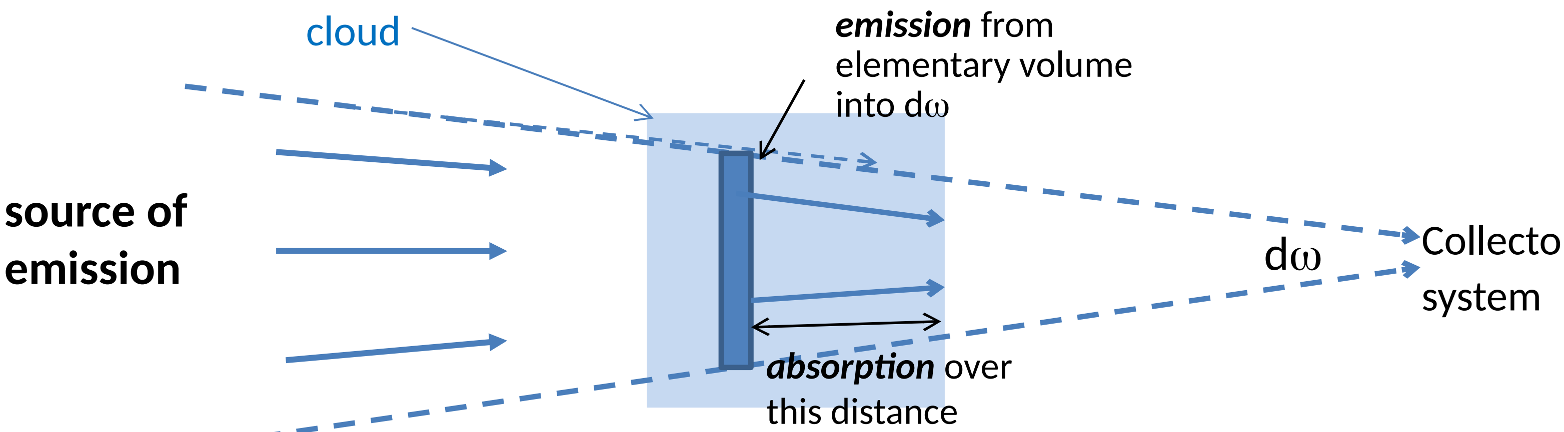
This enables the above equation to be written simply, for a uniform cloud and a given optical depth in terms of observed brightness temperature :

$$\boxed{T_{obs} = T_{source} e^{-\tau} + T_{cloud}[1 - e^{-\tau}]}$$

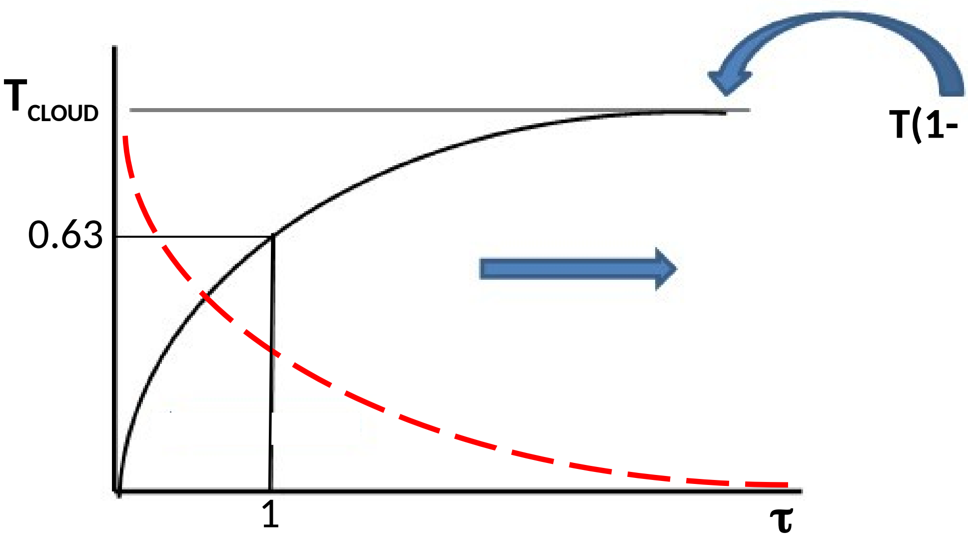
$\tau=0$	$T_{obs} = T_{source}$ (perfectly transparent to source)
$\tau=0.1$	$T_{obs} \sim 0.9T_{source} + 0.1T_{cloud}$ (source dominates – see cloud in absorption)
$\tau=1$	$T_{obs} = 0.37T_{source} + 0.63T_{cloud}$ (cloud dominates – see cloud in emission)
$\tau=\infty$	$T_{obs} = T_{cloud}$ (cloud is perfectly opaque – only see its surface)

Isolated cloud in LTE at temperature T_{CLOUD}

Redrawing slide 36 (side view)



<u>Effect of increasing τ</u>



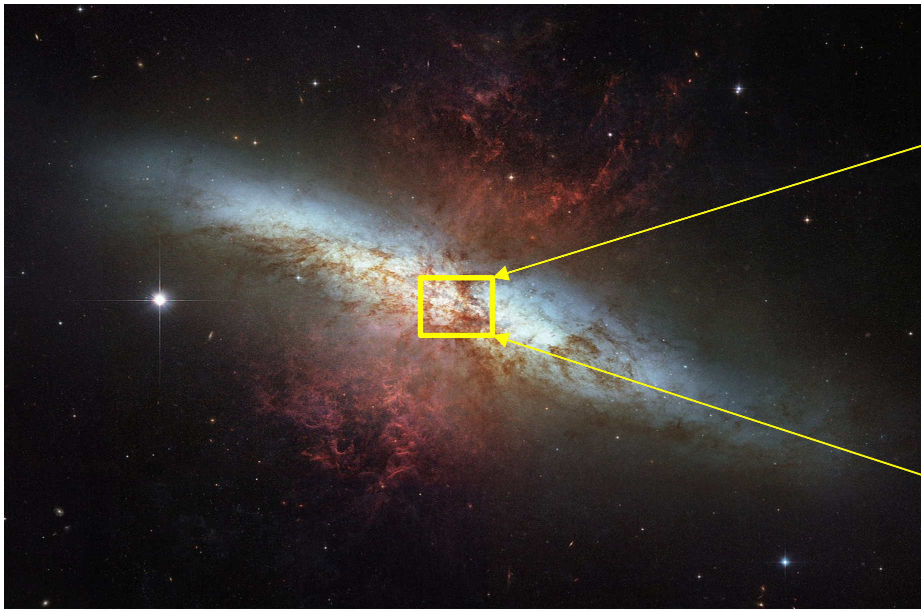
:self emission

$T(1- e^{-\tau})$→ if $\tau=\infty$ cloud is opaque and see only cloud surface → cloud therefore radiates at its intrinsic temperature T_{CLOUD}

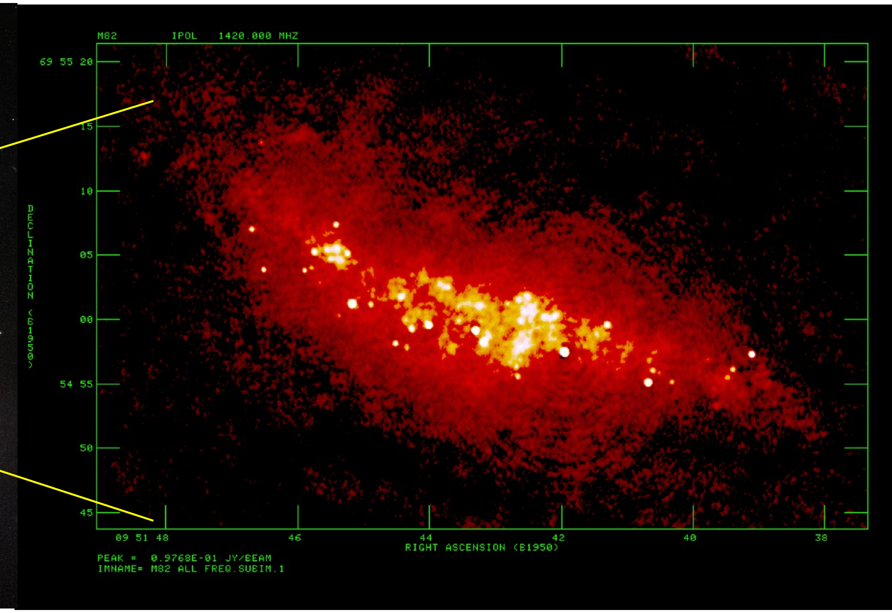
:source emission

x $e^{-\tau}$ as in slide 43

Intervening gas can absorb radiation from background source



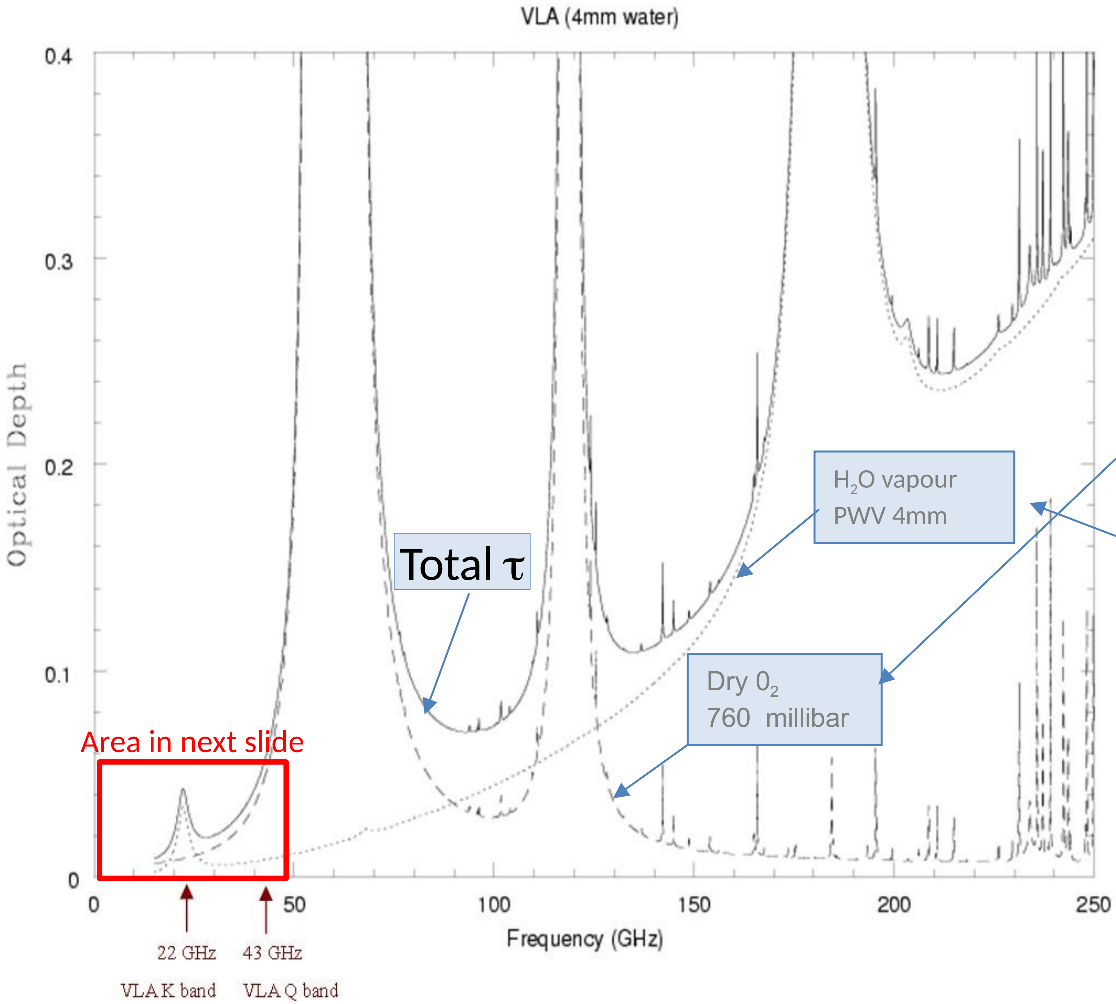
HST image of M82 starburst galaxy



MERLIN+VLA radio image of central Region of M82

Spots in radio image are supernovae remnants. Many of their radio spectra “turn over” at low frequencies due to “free-free” absorption by the large amount of ionised gas in the central region of the galaxy

Zenith optical depth τ (i.e. vertically through the atmosphere) vs. frequency in the radio band



N.B. this plot is for site at altitude 2000m NOT at sea-level !

The optical depth is due to a combination of absorption from rotational transitions in water vapour and oxygen molecules - water dominates except in the 0_2 lines

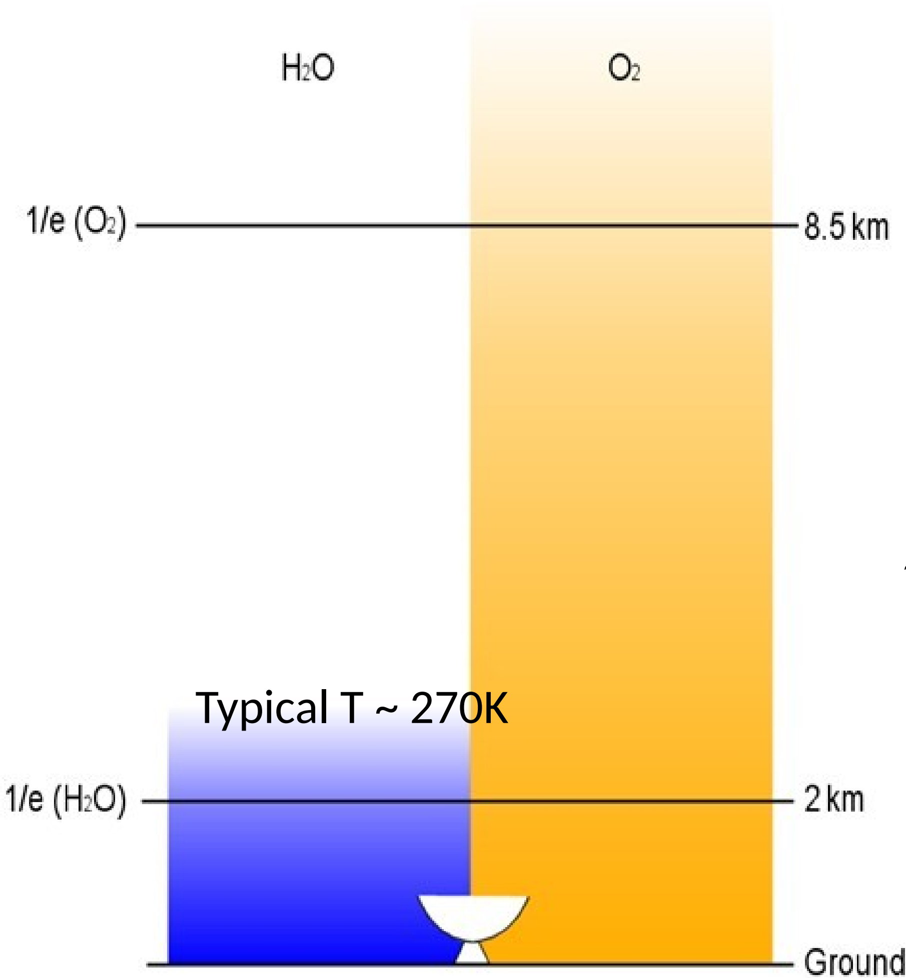
This plot is for an altitude of ~2000m above sea level → pressure is 76% of sea level (SL) → O_2 density = 76% of SL

Site is a dry desert in SW USA hence precipitable water vapour (PWV) at 4 mm is much lower than in Manchester (PWV 20-30 mm)

Effect: at 30 GHz (λ = 1cm) the total optical depth t ~ 0.02 (Manchester τ ~0.05 → 0.1)

At frequencies below the 22 GHz H_2O absorption the optical depths are very low and very good observations near Manchester are possible.

Scale heights and temperatures of O_2 & H_20



Courtesy S.Lowe Ph.D thesis 2006

The pressure or density of atmospheric constituents falls off exponentially with height above sea level . The "scale height" is that at which the density or pressure of an atmospheric constituent falls to 1/e ~37% of its sea-level value.

- For oxygen ~8.5 km
- For water vapour ~2 km

High altitudes → reduced H_20 and O_2 absorption
e.g. At 5000m in Chilean Andes at 40 GHz

τ (H_20) ~ 0.002
τ (0_2) ~ 0.01 Atmospheric T_B~3K

At balloon altitudes (30-40km)
τ <<0.01 Atmospheric T_B~0 K

Estimate temperature for the dominant constituent (H_2O) via the "wet adiabatic lapse rate" which can vary by 10-20% but typically is ~6.5 K per kilometre of height (verify on your next long haul flight). At the 2km scale height of H_20:

$$T_{atmosphere} \sim T_{surface} - 13\text{ K}$$

e.g. typically 270K at temperate latitudes

Summary of effects of the troposphere on radio astronomy

- The opacity is dominated by rotational transitions in O_2 and H_2O molecules –

- The opacity (and hence the optical depth) changes with:
  1. frequency – for the radio/mm-wave region)
  2. altitude
  3. time
  4. angle to zenith or horizon

- The lower atmosphere "hurts" observations twice - by reducing the signal and by adding noise. (Exactly like loss in attenuating components in a receiver)

Additional Propagation Effects not covered

Propagation effects due to ionized plasma in the Interstellar Medium (and Ionosphere):

- Faraday Rotation – added effect of magnetic field – extragalactic radio sources & pulsars
- Dispersion – important for pulsars
- Scattering – fluctuations in density in ISM (as in atmospheric twinkling) – affects pulsars

Summary - 1

- Brightness is defined from the power coming into the reception system from a solid angle cone onto a square metre of the reception antenna per Hz. *The definition is the same regardless of the emission mechanism.*
- The intensity radiated by the source and the brightness associated with the reception are often approximately the same and people (incorrectly) often don't distinguish between them.
- We add the word "specific" for the brightness/intensity at a specific frequency.

Summary -2

- Brightness/ intensity is related to the physics at emission and does *not* change with distance (except where the effect of the expanding universe needs to be taken into a/c).
- What changes with distance – to give the inverse square law – is the solid angle of the source we observe *not* its intrinsic brightness.

- Different radiation mechanisms give rise to sources of different brightnesses.

Summary - 3

- To inter-compare brightnesses we refer back to that of a black body which gives us a well-understood “gold standard” for comparison.
- At radio wavelengths the Planck radiation equation simplifies to the Rayleigh-Jeans equation which makes life easier.
- When we observe sources which are small compared with the telescope’s beam we can only measure their flux density which is a less informative quantity than the brightness.

Summary – 4

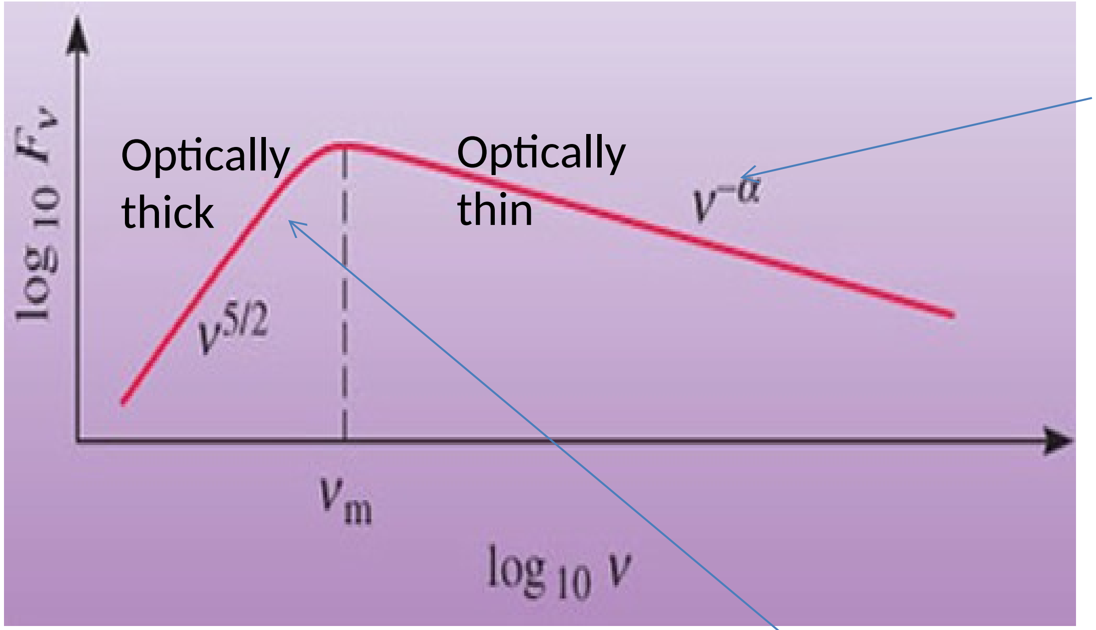
- When we observe sources through absorbing media we lose some noise power from source and gain some noise power from the absorber – all quantified via the (simplified, but sufficient) equation of radiative transfer
- Frequency–dependent absorption by intervening gas can change the observed radio spectrum.
- Earth's atmosphere becomes an increasing problem at higher observing frequencies and forces mm-wave (>100 GHz) telescopes to high altitude sites.

End of Lecture 2

Deleted slides from longer original version of this lecture

Synchrotron spectrum

http://openlearn.open.ac.uk/mod/oucontent/view.php?id=398792&section=7.8



Slope of combined emission spectrum depends on the energy distribution of the relativistic electrons. This is *typically a power law* i.e. $N(E)\,dE = N_0E^{-s}\,dE$

→ "flux density" $F_\nu \;\alpha\; \nu^{-(s-1)/2}$

→ the typical "spectral index" $\alpha \sim 0.7\text{-}0.8$

At lower frequencies *source becomes opaque to its own radiation and we see only the surface layers.* The surface brightness is defined as = $2kT/\lambda^2$ (Rayleigh Jeans Law – this lecture) where "**T**" is a typical relativistic electron kinetic temperature. *Synchrotron self-absorption* sets in at ν_m (N.B. ν_m not the same as ν_o for single electron) when the brightness becomes comparable to "**T**".

The optically thick spectral slope ($\nu^{5/2}$ not same as for a black-body since this is non-thermal radiation) *is not seen in practice* since sources are non-uniform → mixtures of spectra with different "turnover frequencies". *The optically thin power law spectrum is the characteristic*

Radiative Transfer - 1

κ_ν = absorption coefficient → fraction of incident power lost passing through unit distance

→ units are m^{-1}

→ Power *removed* from beam $= \kappa_\nu I_\nu dA d\nu d\omega.dx$ (look back at slide 3)

(Note: using same dA for emitting source and absorbing element;
specific intensity is distance independent – see this lecture;
& is defined by a solid angle only; see also the previous slide)

ε_ν = emission coefficient → specific intensity added per unit distance through medium

→ units are W m^{-3} Hz^{-1} $sterad^{-1}$

→ Power *added* to beam $= \varepsilon_\nu dA d\nu d\omega.dx$

→ Overall change in power = emission – absorption

the increment in specific intensity

$$\underbrace{\cancel{dI_\nu dA d\nu d\omega}}_{\text{Slide 3}} = \varepsilon_\nu dA d\nu d\omega.dx - \kappa_\nu I_\nu dA d\nu d\omega.dx$$

$$\boxed{\frac{dI_\nu}{dx} = \varepsilon_\nu - \kappa_\nu I_\nu \text{ : EQUATION OF RADIATIVE TRANSFER (ERT \#1)}}$$

self generated and will also be absorbed

incoming absorbed

Radiative Transfer - 2

- **Case of absorption of radiation from source only**
  (but remember absorption always implies emission)

$$dI_\nu /dx = -\kappa_\nu I_\nu$$ (should remind you of equation for radioactive decay)

Integrating over the absorbing path starting from $x=0$ $I_\nu = I_\nu(0)$ and *assuming a uniform medium* you can easily show….

$$\boxed{I_\nu = I_\nu(0)\, e^{-\kappa_\nu x} = I_\nu(0)\, e^{-\tau}}$$

The optical depth $\tau = \kappa_\nu x$ (dimensionless but measured in nepers or dB) is a "summary" parameter telling us how opaque a region is to radiation passing through it – regardless of how the opaqueness arises.

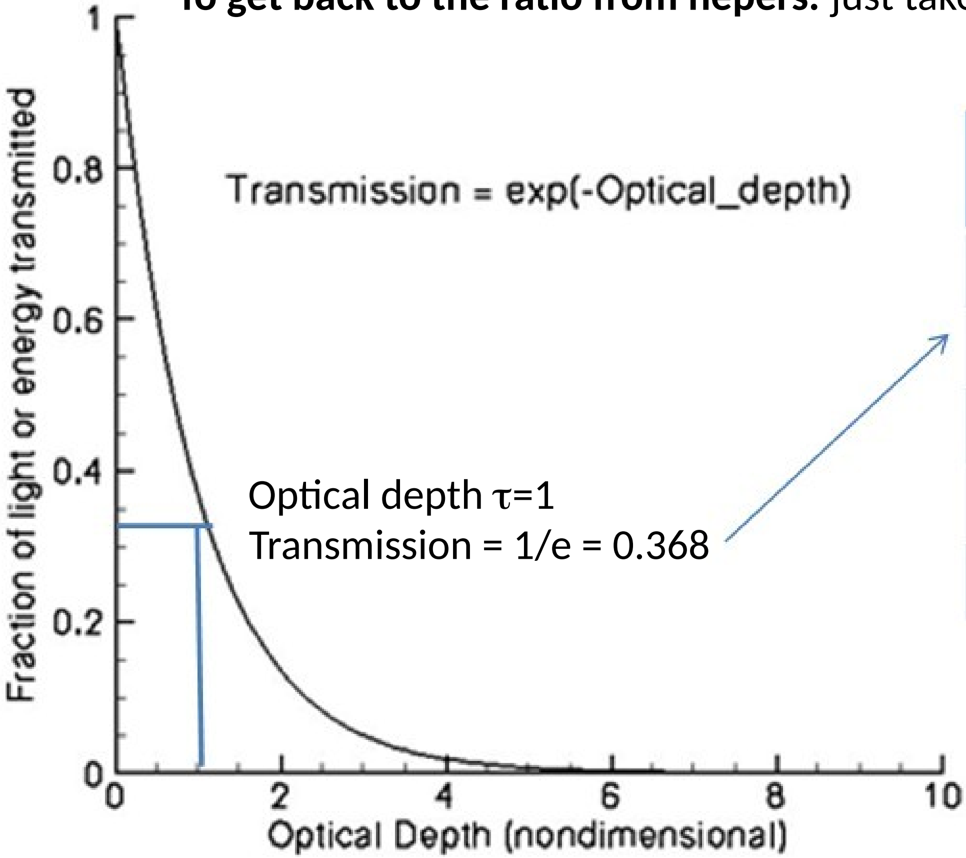
$\tau = 0$ → medium is completely transparent

$\tau = \infty$ → medium is completely opaque

Optical Depth τ (in neper & dB)

Ratio in dB = 10 $\log_{10}(P_{out}/P_{in})$
Ratio in nepers = $\ln(P_{out}/P_{in})$ - *Neper after Napier, less commonly used than dB's.*

To get back to the ratio from dB: ÷10 and take the antilog e.g. -13dB → $10^{-1.3}$ = 0.05
To get back to the ratio from nepers: just take the exponential → convenient for optical depth



http://www.daviddarling.info/encyclopedia/O/optical_depth.html

(Observed/ Initial)	Attenuation nepers (= τ)	Attenuation deciBels (dB)
1	0	0
1/e = 0.368	1	4.343
0.1	2.303	10
0.01	4.605	20
0.001	6.908	30

Alternatively
- the intensity is at the -30db level of the original

OR
- there Is a loss of 30dB

Radiative Transfer - 3

- Back to ERT#1: Divide through by κ_ν and recognise $d\tau = \kappa_\nu dx$

$$\frac{dI_\nu}{d\tau} = \left[\frac{\varepsilon_\nu}{\kappa_\nu}\right] - I_\nu \quad \text{: EQUATION OF RADIATIVE TRANSFER \#2}$$

$$\frac{dI_\nu}{d\tau} = S_\nu - I_\nu \quad \text{The “Source Function” } S_\nu \equiv \left[\frac{\varepsilon_\nu}{\kappa_\nu}\right]$$

S_ν (units are specific intensity): a measure of how photons are removed and replaced by new photons; contains all the physics of the medium. Potentially complicated e.g. scattering; stimulated emission, non-uniformity, mixtures of processes etc . Keep it simple with uniform medium.

- Solve ERT#2: x $e^{+\tau}$ (integrating factor) & integrate by parts

$$\rightarrow \quad \mathbf{I_\nu(\tau) = I_\nu(0)\, e^{-\tau} + S_\nu[1 - e^{-\tau}]}$$

After passing through material with *uniform* “Source Function” with optical depth τ.

- 1st term: attenuated source radiation (slide **47**)
- 2nd term: radiation from within cloud and attenuated by the optical depth from that point to the receiver system.

Radiative Transfer - 4

- Special case: uniform medium in thermodynamic equilibrium (TE) with radiation field at a single physical temperature → Black Body conditions

ERT#1: $dI_\nu / dx = 0$ (for a BB the radiation field does not change: everywhere the rate of emission = the rate of absorption)

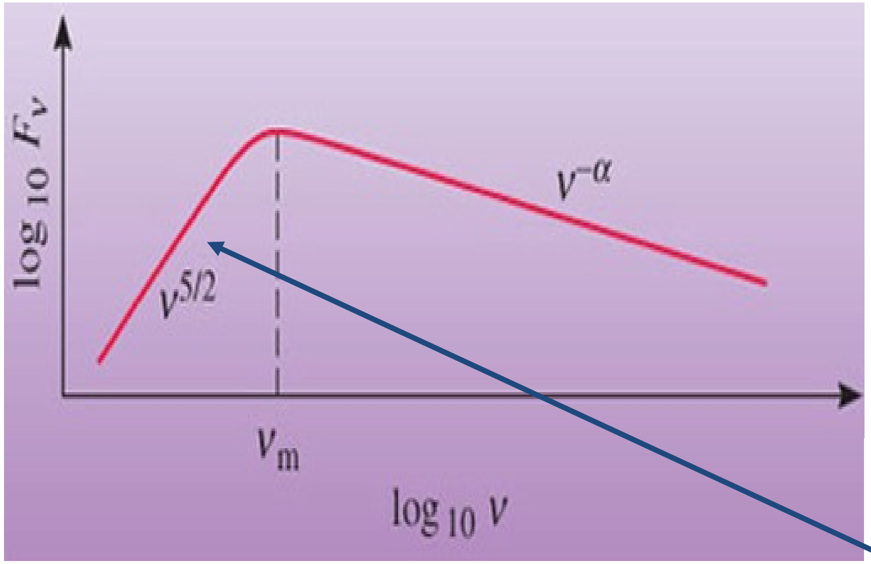
→ $\varepsilon_\nu = \kappa_\nu I_\nu^{BB}$ thus $S_\nu = I_\nu^{BB}$ → **S_ν is the Planck function**

(Kirchoff's Radiation Law – for a B-Body at a single physical temperature)

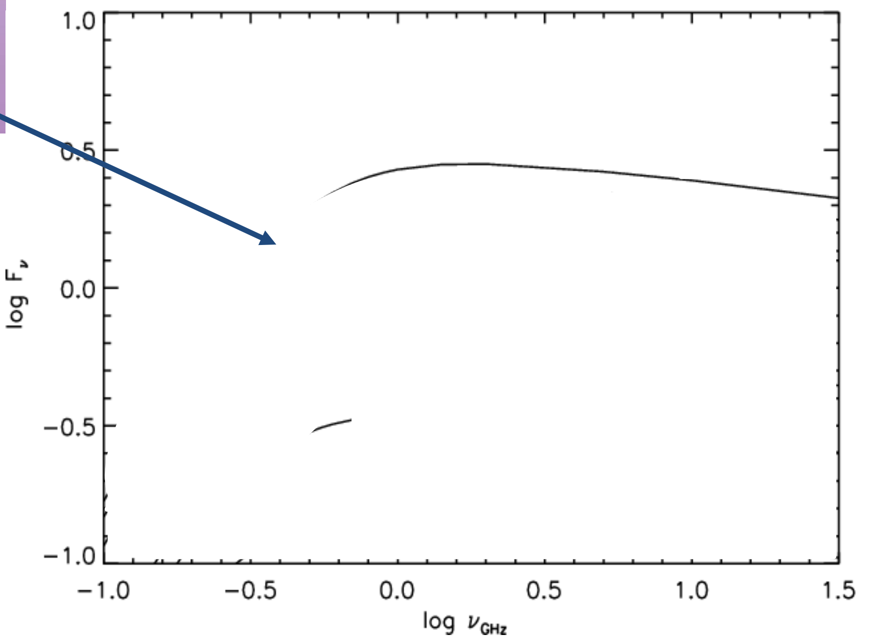
Same reasoning applies even if medium is not in equilibrium with the radiation field – *if it can be described by a single temperature*. These conditions are called "local thermodynamic equilibrium" (LTE).

Example: the Earth's troposphere is approximately at T ~ 260 K but is not in equilibrium with the incoming solar radiation at ~5800K.
The troposphere is mostly heated from below by i-r radiation the Earth it absorbs only a fraction of the radiation – it is "optically thin".

Radio sources absorb their own radiation

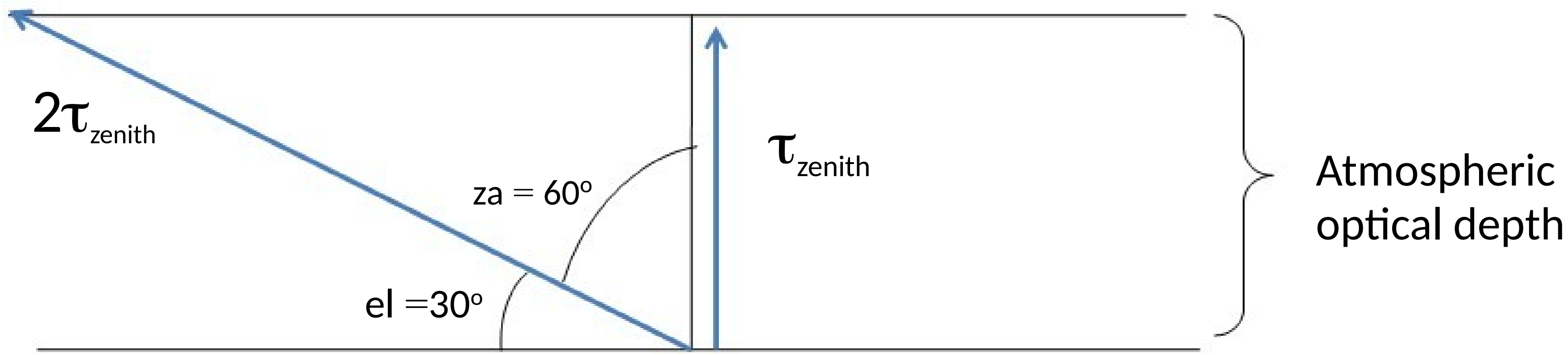


synchroton



free-free

Optical depth away from zenith



Observations at larger angles from zenith or equivalently lower elevations involve looking through greater amounts of the atmosphere and hence the optical depth increases.

za= angle from zenith; el = elevation above horizon

$\tau_{el} = \tau_{zenith} / \sin(el) \rightarrow \tau_{el} = \tau_{zenith} \operatorname{cosec}(el)$

$\tau_{za} = \tau_{zenith} / \cos(za) \rightarrow \tau_{za} = \tau_{zenith} \sec(za)$

$\left.\begin{array}{l} za = 60^o \\ el = 30^o \end{array}\right] \rightarrow$ twice the optical depth at the zenith

Practical result: tip the telescope to za=60^o i.e. el=30^o and the change in T_{sys} is the zenith contribution of atmospheric noise due to its optical depth and temperature.

Additional Slides on Spectral Lines

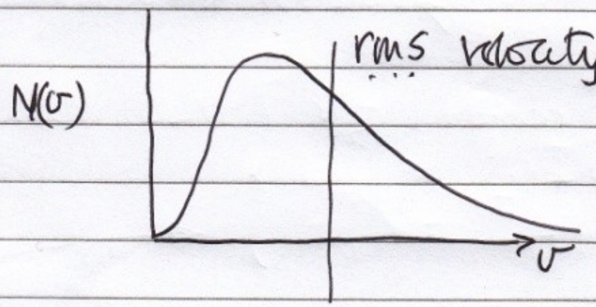
→ LARGER MOLECULES → LARGER I → LOWER ν's
SMALLER " → LOWER HIGHER ν's

eg CARBON MONOXIDE (CO) $\nu_{J=1-0} = 2B = 115\,\text{GHz}$ (lowest)

(ABUNDANT IN SPACE) $\nu_{J=2-1} = 4B = 230\,\text{GHz}$

RELATIVE INTENSITY OF LINES ⟶ T_{CLOUD} ; WHY?

EXCITATION TO HIGHER J → K.E. OF COLLISIONS (MUST BE HIGH ENOUGH)



$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$

Maxwellian distribution

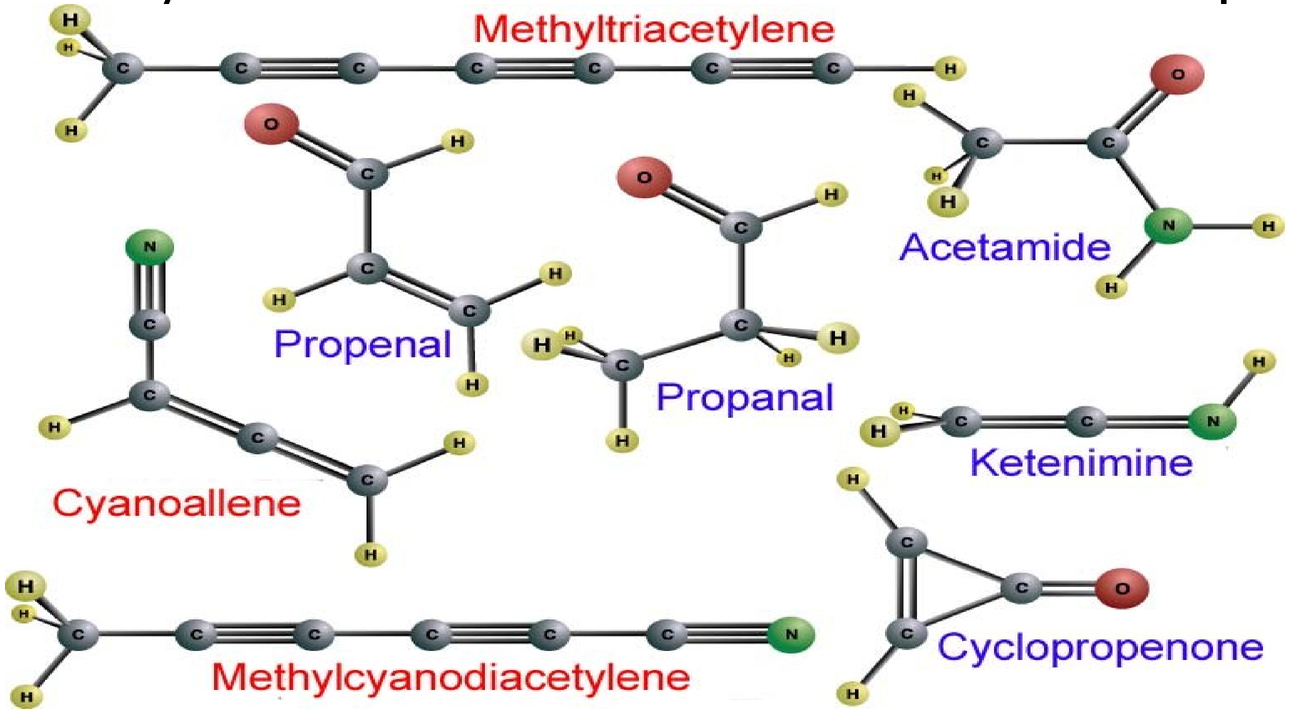
TO FIRST ORDER : $\frac{3}{2} kT_{CLOUD} \approx E_{J(\text{HIGHER LEVEL})}$

Eg. IN 'COLD' INTERSTELLAR CLOUDS $T_{CLOUD} \simeq \{10K - 30K$

TEND TO SEE EMISSION FROM HEAVIER MOLECULES BUT CAN EASILY EXCITE CO!

Thermal line radiation

Nearly 190 molecules detected in interstellar space.

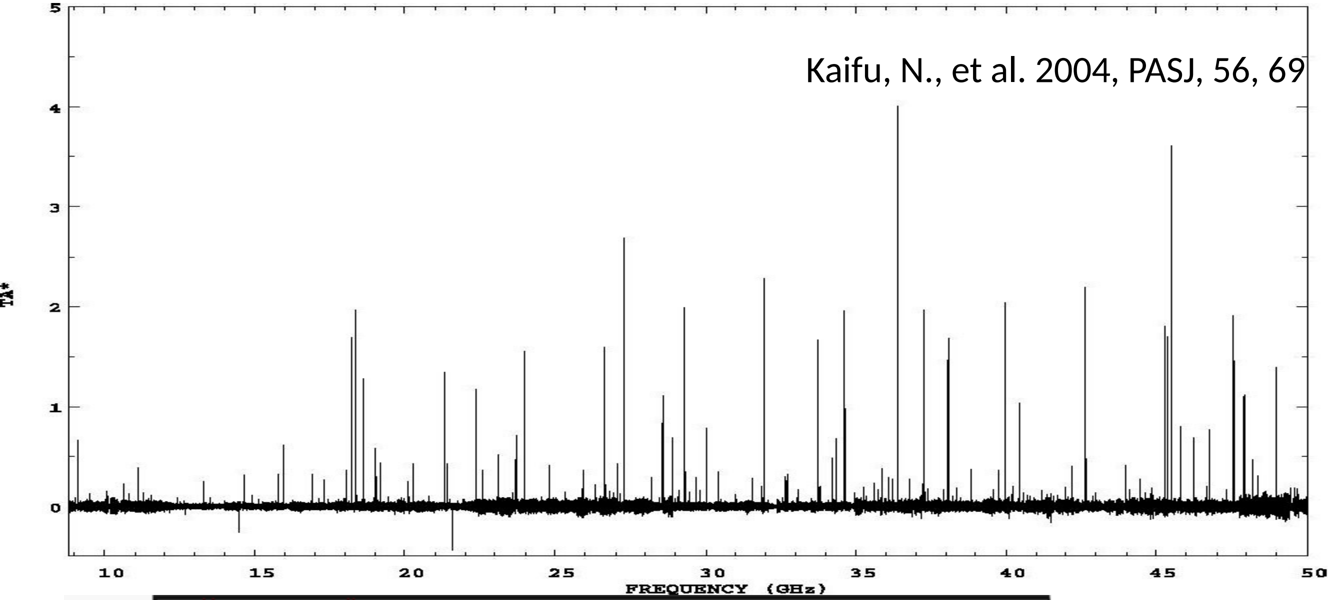


http://www.space.com/scienceastronomy/060808_st_life_molecules.html

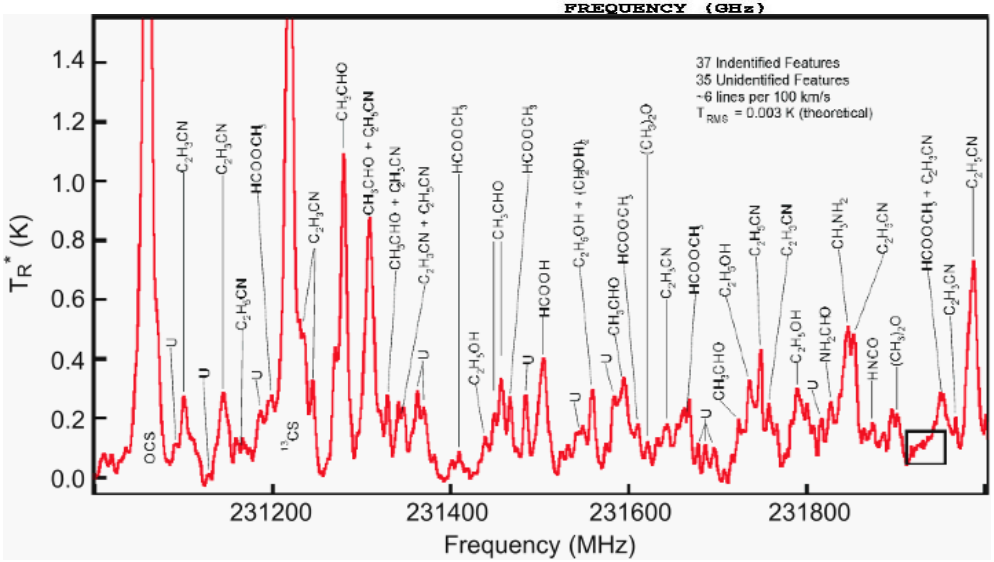
- Many of these are "organic."
  - ➢ Illustrates importance of carbon in chemistry of life.
  - ➢ Are there biological molecules not yet detected? (Amino acids?)
- ➢ Connection to protoplanetary disk chemistry?

http://en.wikipedia.org/wiki/List_of_molecules_in_interstellar_space
http://www.daviddarling.info/encyclopedia/I/ismols.html

There are many radio spectral lines !



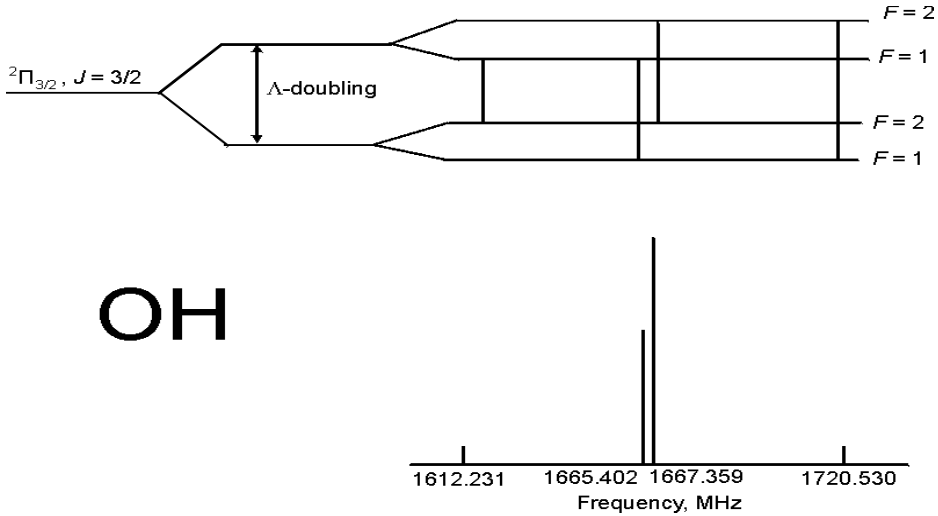
8 to 50 GHz spectrum of a dark molecular cloud TMC-1 containing 414 lines from 38 species - but 117 lines remain unidentified.



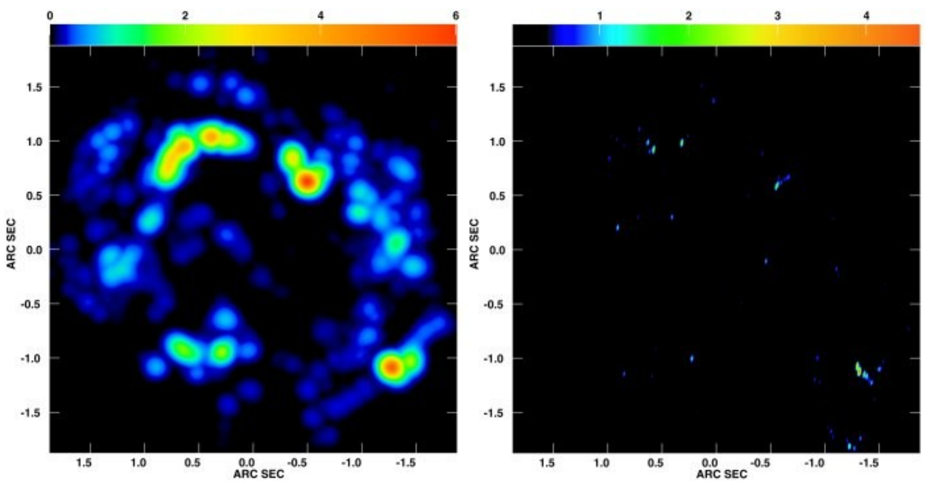
A 231 GHz spectrum of the molecular cloud SgrB2(N) near the Galactic center is completely dominated by molecular lines from known and unknown (U) species (Ziurys et al. 2006, NRAO Newsletter, 109, 11).

http://www.cv.nrao.edu/course/astr534/MolecularSpectra.html

Maser emission



the emission from a maser is stimulated and monochromatic, having the frequency corresponding to the energy difference between two quantum-mechanical energy levels of the species in the gain medium which have been pumped into a *non- thermal population distribution.*



arise in molecular clouds comets, planetary atmospheres, stellar atmospheres and circumstellar rings, or from various conditions in interstellar space.

A circumstellar ring of masers whose diameter is far too small to resolve with optical telescopes

SPECTRAL SHIFTS IN LINES : (N.B. radio astronomy deals in frequency not λ)

$$\Delta\nu = \left| \nu_{EMIT} - \nu_{OBS} \right|$$

$\nu_{EMIT} > \nu_{OBS}$: RECEDE

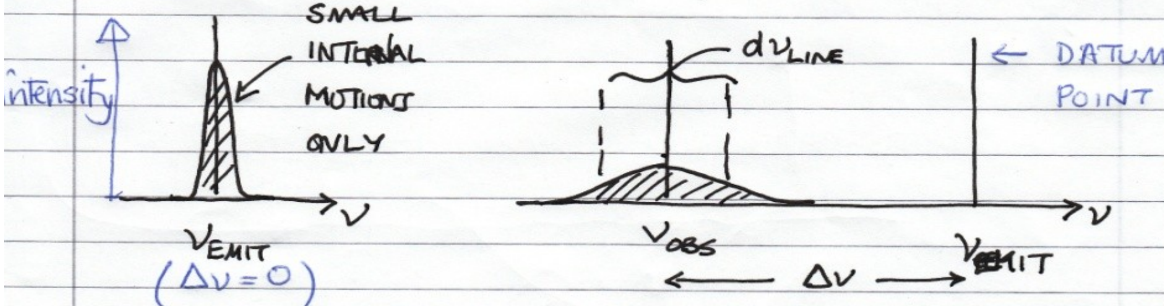
$\nu_{EMIT} < \nu_{OBS}$: APPROACH

(i) DOPPLER SHIFT DUE TO BULK MOTION ALONG LINE-OF-SIGHT

$$V_{BULK} = \left[\frac{\Delta\nu}{\nu_{EMIT}} \right] . c \qquad \text{NON-RELATIVISTIC}$$

(ii) DOPPLER DUE TO INTERNAL MOTIONS WITHIN OBJECT

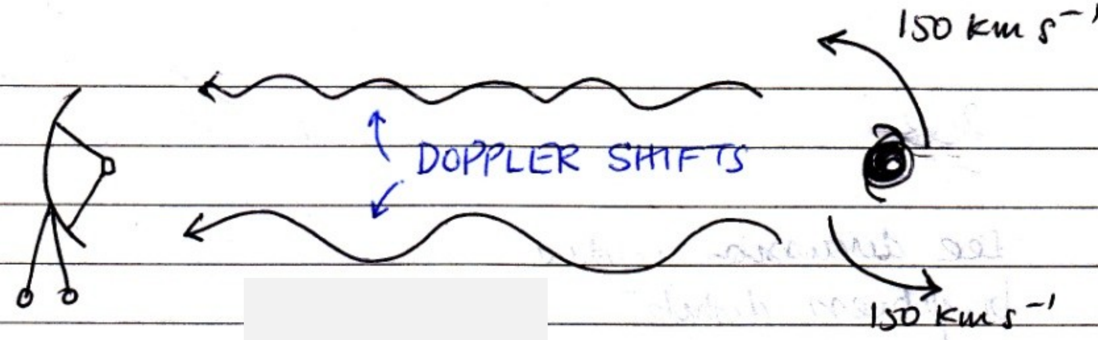
$$V_{INTERNAL} \simeq \left[\frac{d\nu_{LINE}}{\nu_{EMIT}} \right] . c$$



BULK MOTION AWAY + SIGNIFICANT INTERNAL TURBULENCE

WIDE RANGE OF LINEWIDTHS SEEN

0.03	$km\,s^{-1}$	: MASER LINES IN DIFFUSE CLOUDS
10 s	"	: EXTENDED CLOUDS IN GALAXY
100 s	"	: COMBINED EFFECT OVER WHOLE GALAXY (MAINLY ROTATION)



(iii) COSMOLOGICAL EXPANSION

$$\nu_{OBS} = \nu_{EMIT} / (1 + Z)$$

radio astronomy — conventional

$$Z = \text{'redshift'} = \left[\frac{\nu_{emit} - \nu_{obs}}{\nu_{obs}}\right] \equiv \left[\frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}\right]$$

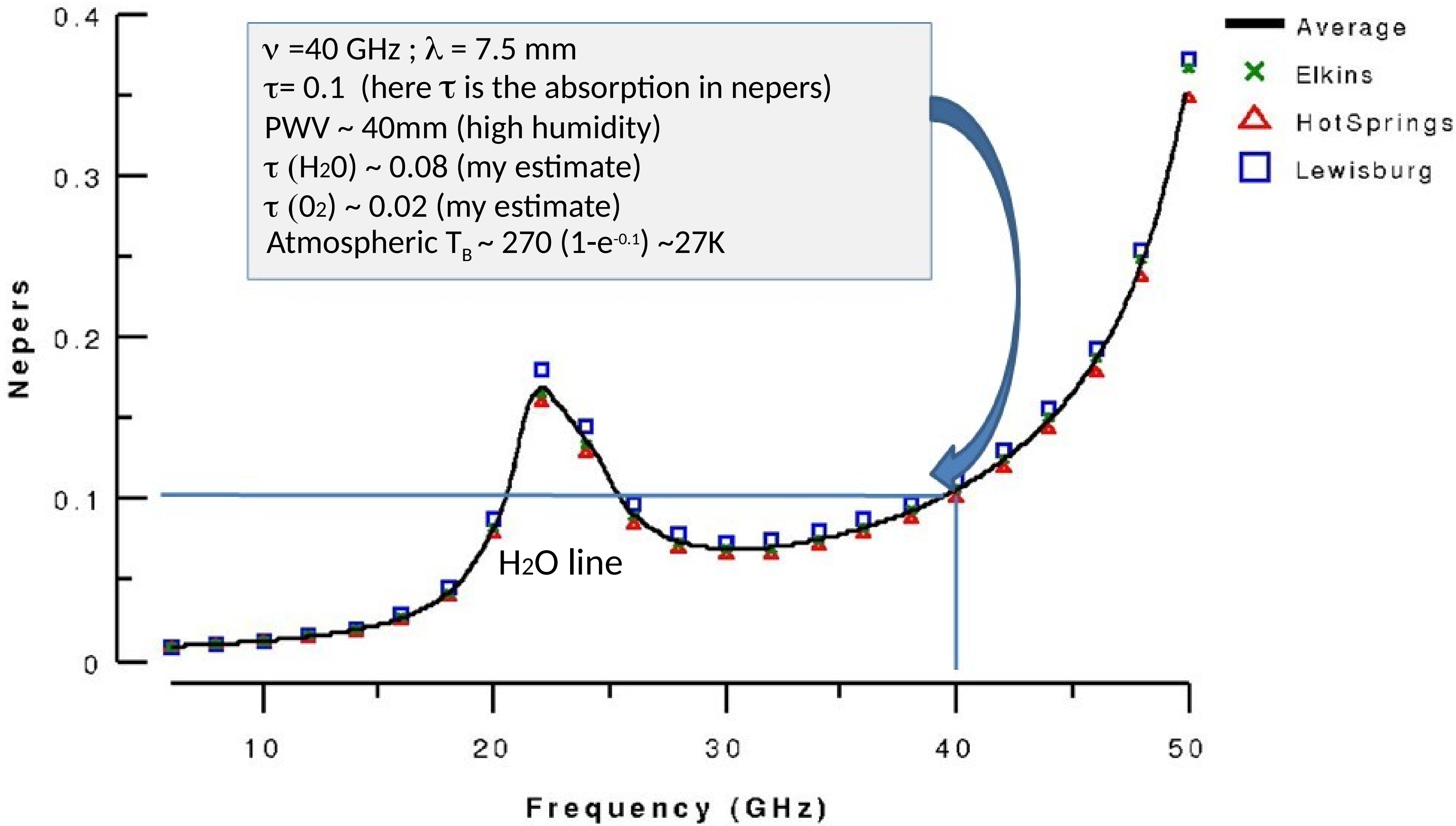
$Z \geq 0$ ie receding due to stretching of space NOT Doppler Effect (through space)

$$(1+Z) = \frac{\nu_{emit}}{\nu_{obs}}$$ = AMOUNT BY WHICH UNIVERSE HAS EXPANDED BETWEEN EMISSION & OBSERVATION

eg. $Z = 1 \rightarrow \nu_{emit} = 2\,\nu_{obs}$ (see Examples)

Zenith optical depth at a low altitude site in summer

http://www.gb.nrao.edu/~rmaddale/Weather/index.html



www.naic.edu/~astro/sdss5/talks/special_concerns_millimeter_ALovell.ppt