The University of Manchester

MANCHESTER 1824

DARA Training Unit 1 2017
Lecture 6

A very brief introduction to interferometry

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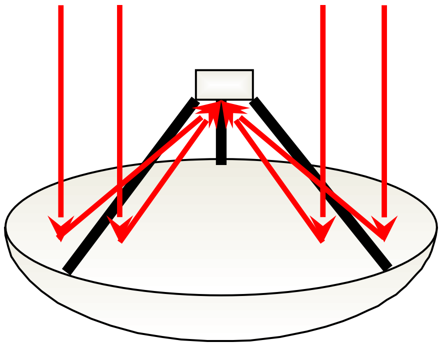
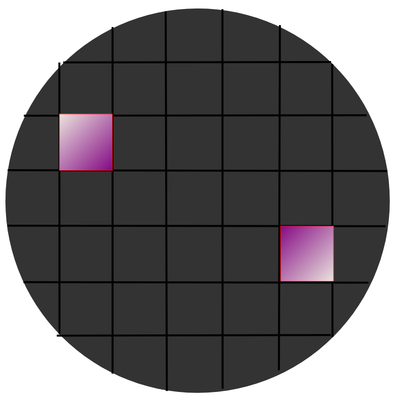
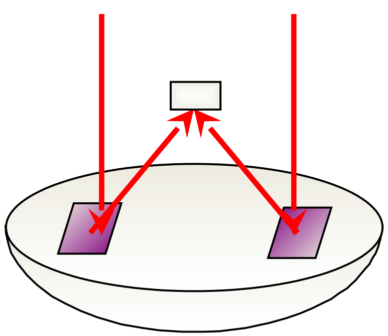
Interferometers – as parts of filled apertures



Conventional telescopes reflect the light of a distant object from a parabolic surface to a focus. *The shape of the surface differentially delays the signals arriving across the aperture so as to bring them to the point focus- and everything works at the speed of light!*



The reflecting surfaces do not need to be part of the same surface. Suppose cover up most of the surface of the mirror….

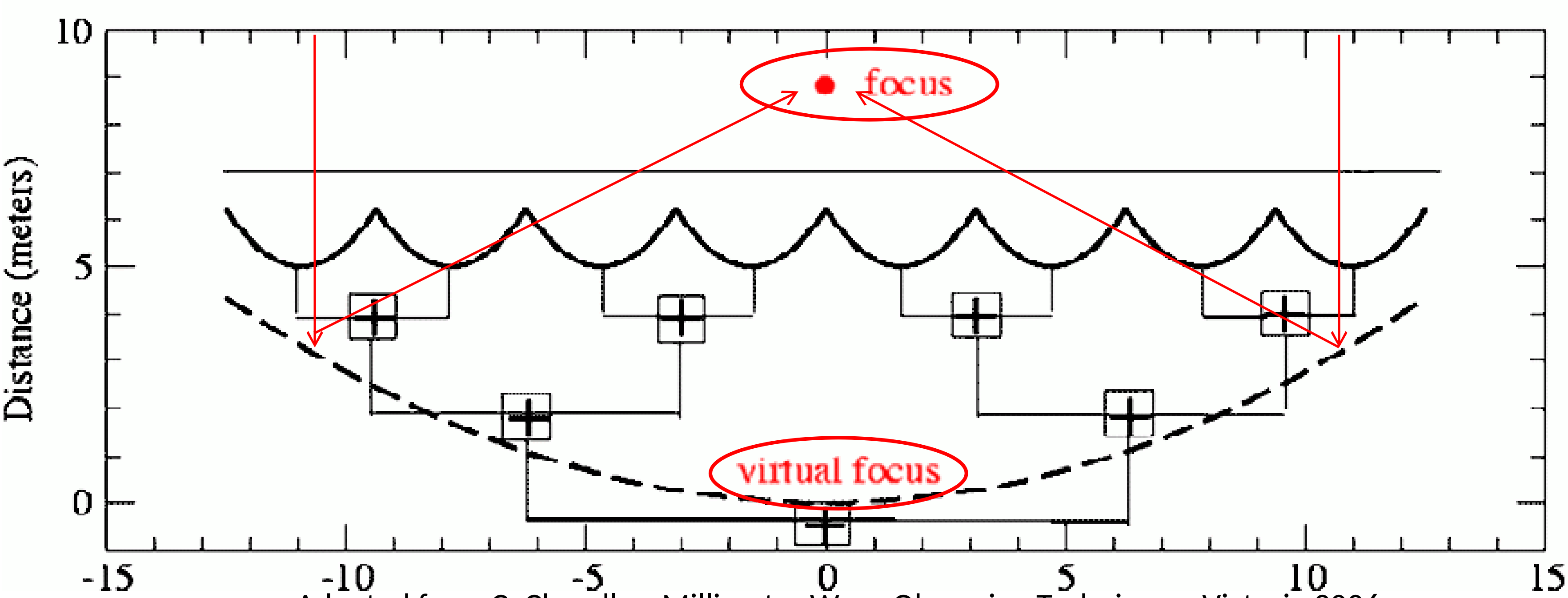


Can still combine the radiation from the uncovered sections to create an image of the distant object, <u>if</u> arrange the path lengths to the focus to be the same and then combine the results from many separately uncovered sections. Can thus *synthesise* an aperture.

from Malcolm Longair

Mimicking a filled aperture with a phased array

- Split aperture into an array of small collecting elements
- Signals at each point in the aperture are (electronically or via cables) brought together in phase at the antenna output (the virtual focus) and *added* together
- Like a dish, an array connected like this forms an *instantaneous* single beam pointing upwards (as drawn)
- This is *"direct"* imaging with an interferometer array

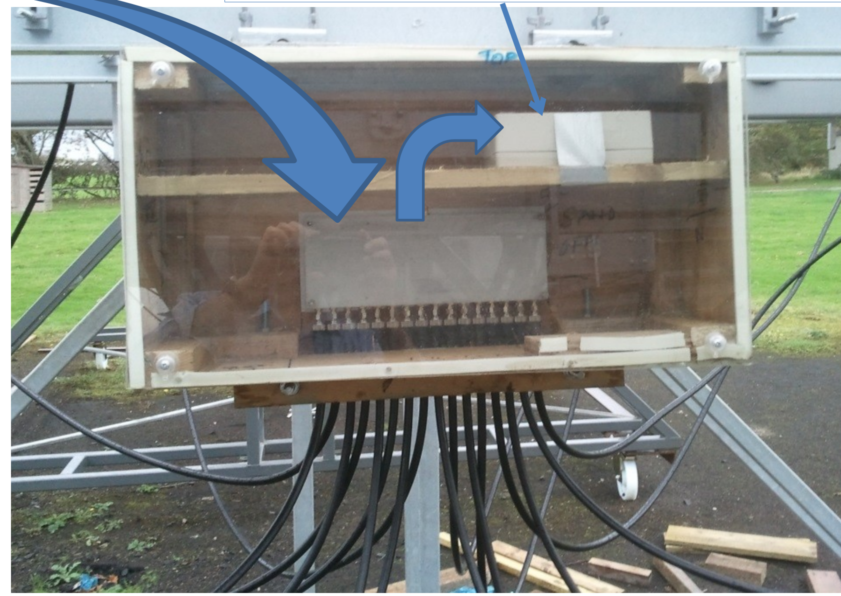


Adapted from C. Chandler: Millimeter Wave Observing Techniques, Victoria 2006

Manchester University Student Telescope (MUST) - a phased array

Low noise amplifier – then out to rest of receiver (not shown)

One of four flat frames each carrying 16 Yagi antennas as the collecting elements.



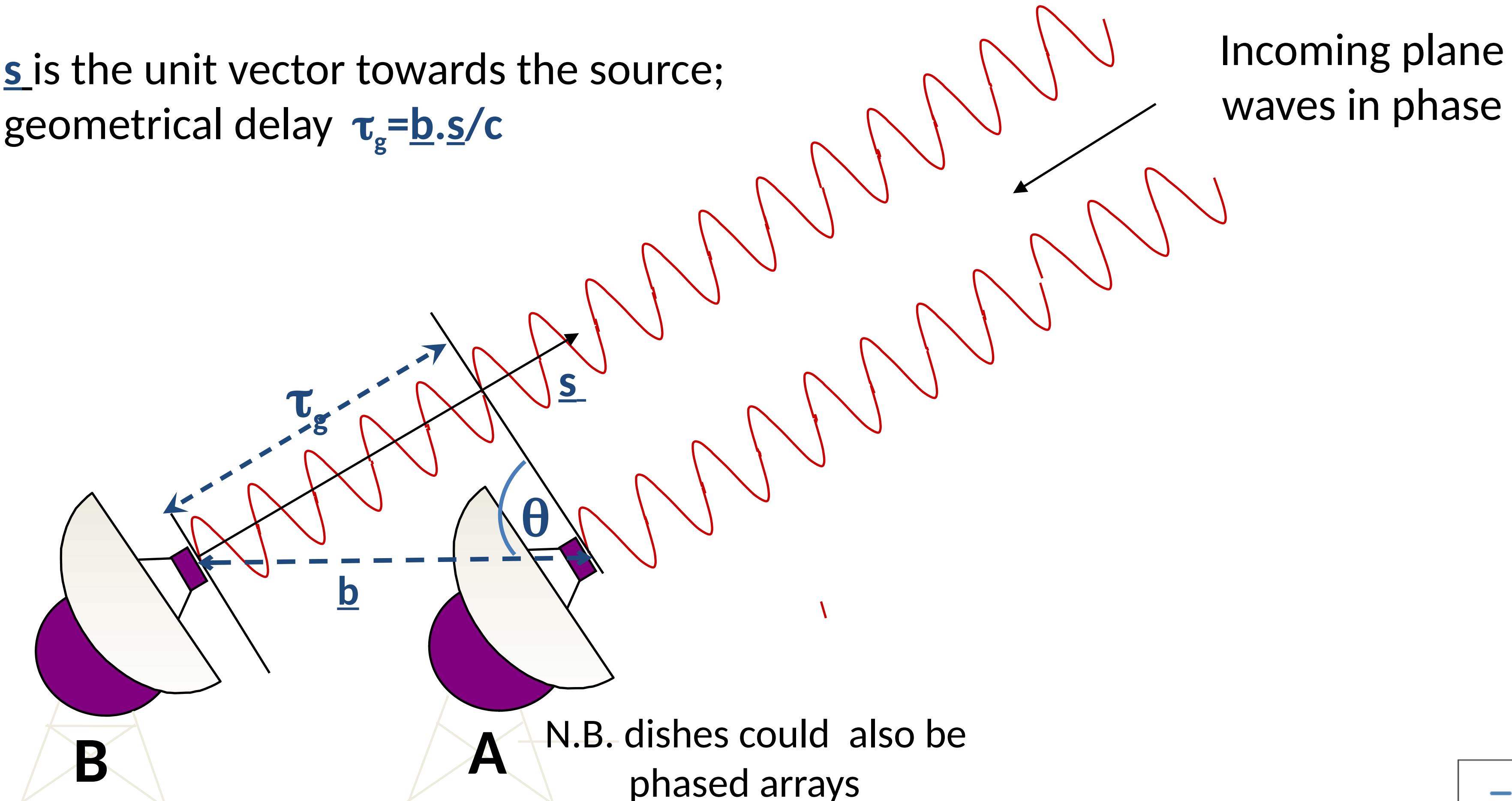
As in slide 3: each Yagi is connected with an equal length of cable to a central combiner – the virtual focus of each frame.

Each frame array acts like a single dish antenna forming a single beam perpendicular to the plane of the frame

Need for delay compensation

When the pointing direction is at an angle θ to the "baseline" $\underline{b}$ between two antennas the signals must travel the same electrical distance before they are combined → add the path delay τ_g to the signal from **A.**

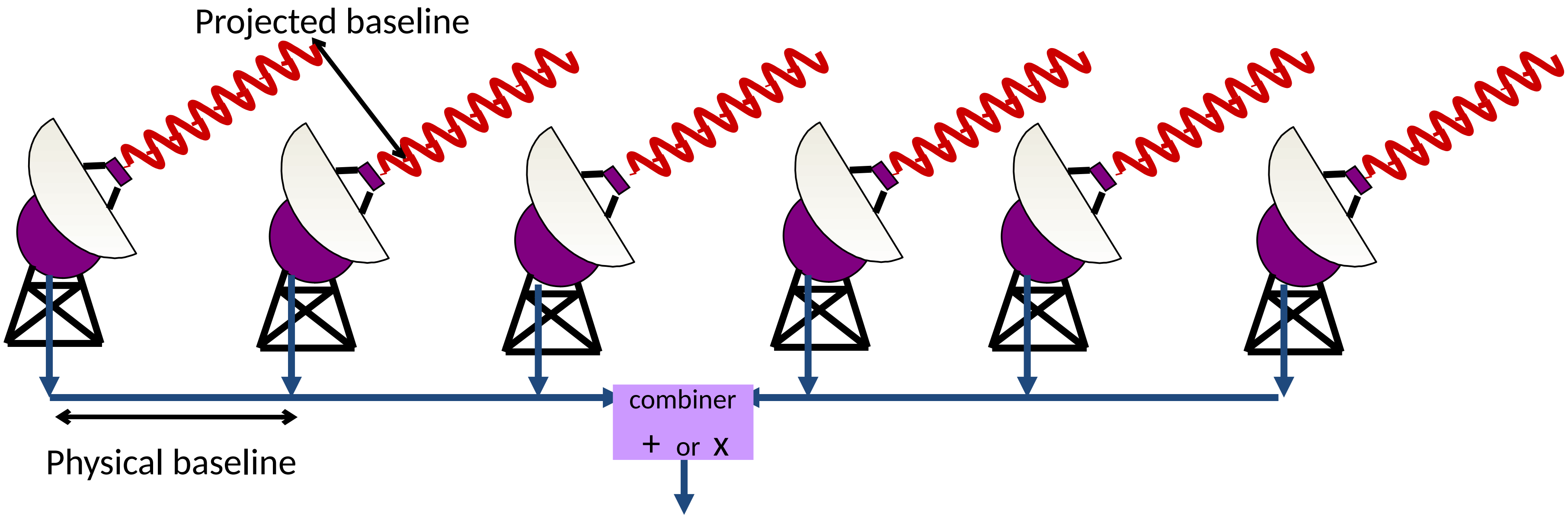
$\underline{s}$ is the unit vector towards the source;
geometrical delay $\tau_g = \underline{b}.\underline{s}/c$



Multiple baselines

- Each pairing of antennas is called a “baseline”; for N telescopes there are N(N-1)/2 baselines
- Effective length of the baseline is the projection perpendicular to the source direction
- Each baseline measures a complex Fourier component of the source at any particular instant
  - More Fourier components → more detailed image.
- Short baselines – low angular resolution - provide coarse structure.
- Long baselines – higher angular resolution - provide the fine detail

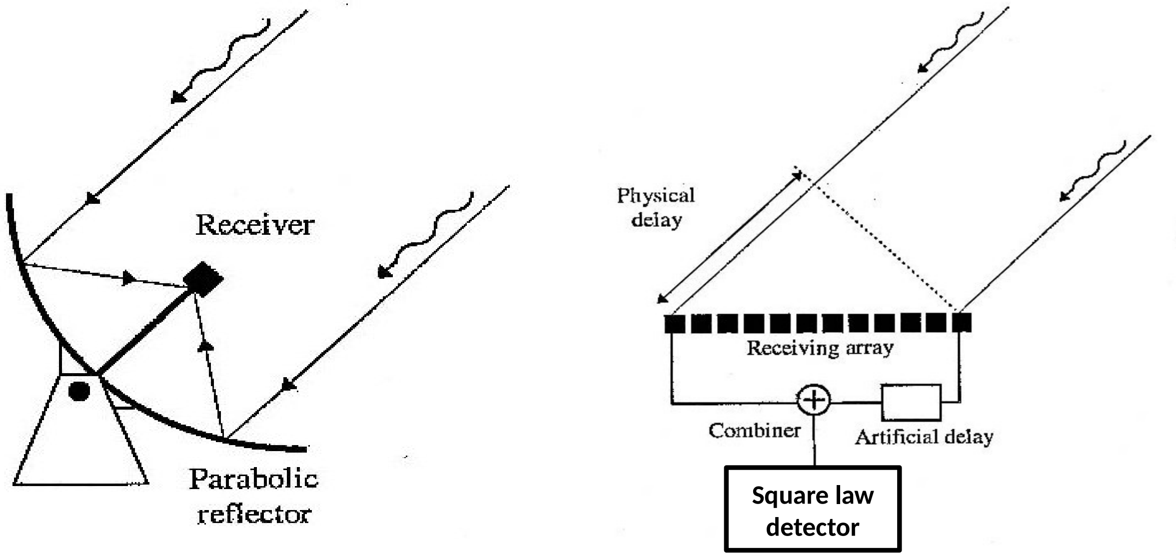
A 6-antenna interferometer → 15 baselines. The central “combiner” takes in all the telescope data streams. Inside it the 15 independent pairs are *either* added or multiplied and integrated (correlated).



Adapted from Malcolm Longair

“Aperture arrays”

Some applications (radio astronomy, remote sensing systems; radar etc) require many instantaneous, *electronically steerable*, beams formed from the addition of the signals from *many small* collector elements forming the phased array. These are also called “aperture arrays” since they constitute the whole receiving aperture. They do not require mechanical pointing as does a dish.



The equivalence between physical optics and electrical beam-forming.

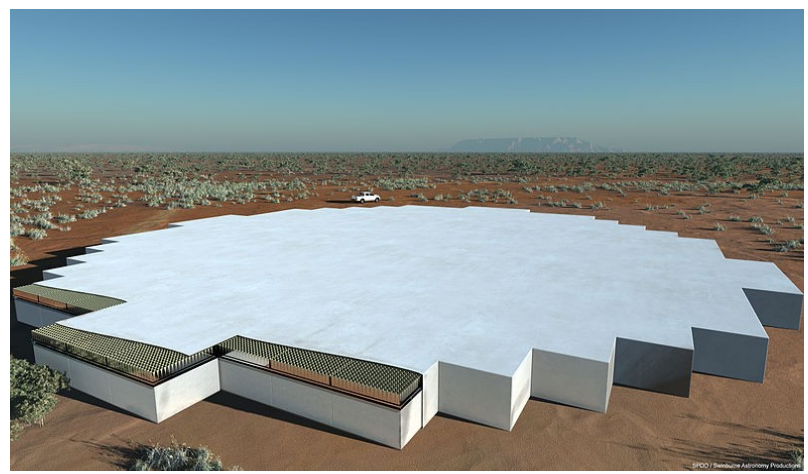
All elements (not just 2 as shown) are *co-added* with an appropriate phase/delay *gradient* across the array for the chosen pointing direction. The sum is then square-law detected. *Different pointing directions can be simultaneously achieved electrically with different phase gradients – just needs more electronics*. In practice the arrays are 2-D – as on next slide.

“Aperture arrays” - examples

Phased array radar

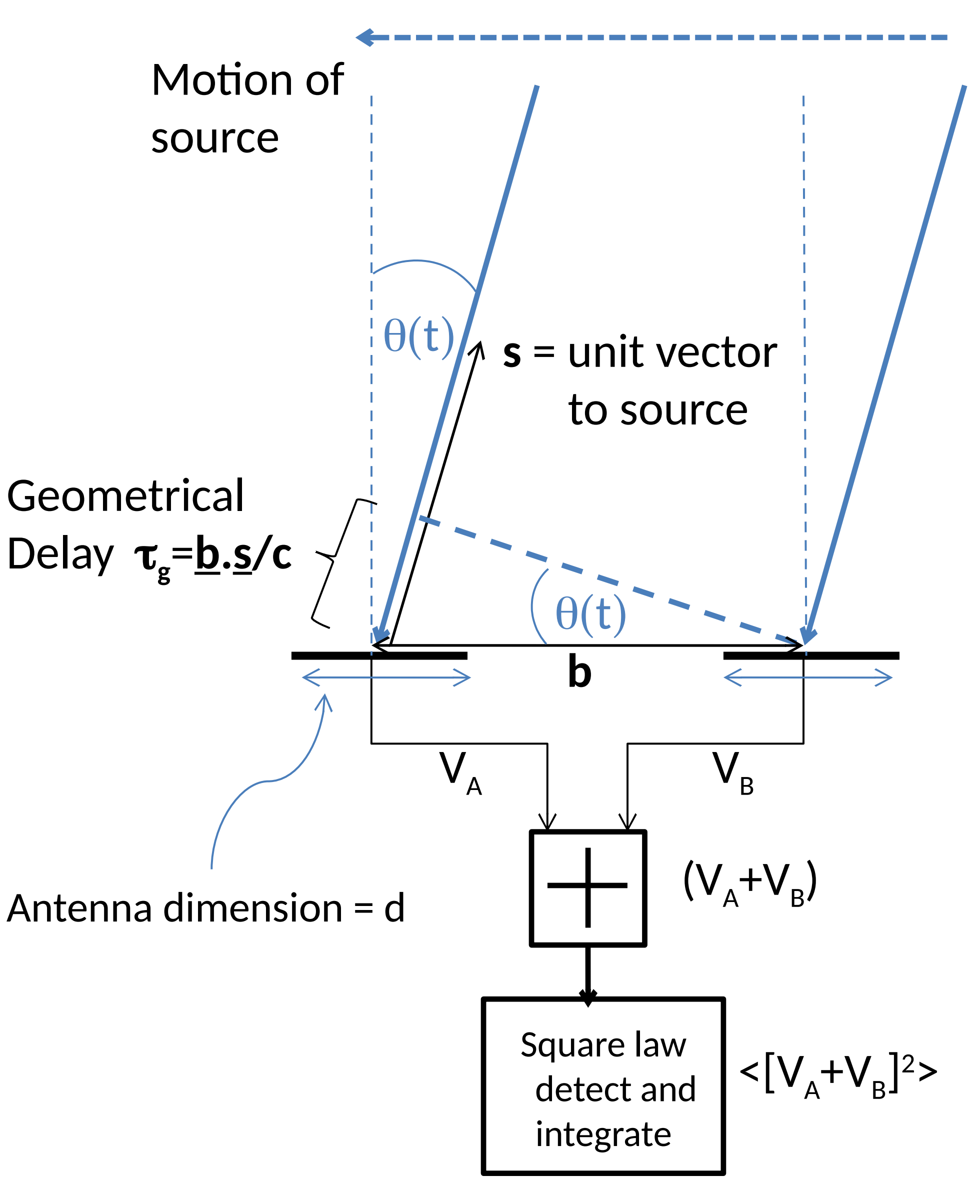
http://en.wikipedia.org/wiki/Phased_array#mediaviewer/File:PAVE_PAWS_Radar_Clear_AFS_Alaska.jpg

Prototype dense aperture array for SKA



Future dense “aperture array” in SKA

2-element “adding" interferometer- observing the zenith



Two antennas of equal dimension d *at fixed locations and pointing direction (here vertical)* & separated by a baseline vector **b**.

A distant source of quasi-monochromatic (very narrow band) plane waves passes overhead along the direction of the baseline. At any moment **s** is the unit vector pointing at the moving source.

The purpose of the antenna, receiver and associated electronics is to convert the incoming electric field to a voltage, *preserving its amplitude and phase (i.e. behaving linearly) even though the signal has been through heterodyne stages*(*not shown*).

In the receiver, the outputs from two antennas are *added* and the sum is square-law detected.

In this basic arrangement the two signal paths are equal, fixed, lengths of cable which restricts us to looking at only a relatively small angle in the sky around the zenith (see later).

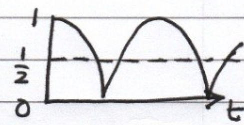
Adding Interferometer

Output Power $R \propto \left\langle \left[V_A \cos\omega(t-\tau_g) + V_B \cos\omega t + v_{REC,A} + v_{REC,B} \right]^2 \right\rangle$

$R \propto \left\langle \left[V_A^2 \cos^2\omega(t-\tau_g) + V_B^2 \cos^2\omega t + 2V_A V_B \cos\omega t \cos\omega(t-\tau_g) + v_{REC,A}^2 + v_{REC,B}^2 \right] \right\rangle$

Cross terms (no long-term phase relationships) $\rightarrow 0$ when averaged over many $\times\, \tau_{coh}$

Simplifying remaining terms (i) $\left.\begin{array}{l} \langle \cos^2\omega(t-\tau_g) \rangle \\ \langle \cos^2 \omega t \rangle \end{array}\right\} = \frac{1}{2}$

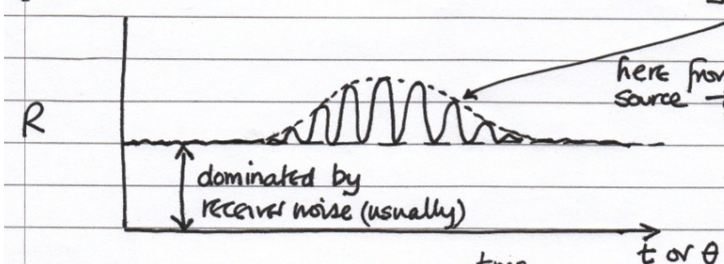


(ii) $\langle \cos\omega t \cos\omega(t-\tau_g) \rangle = \frac{1}{2} \langle \cos\omega\tau_g + \underbrace{\cos(2\omega t - \omega\tau_g)}_{\text{oscillates} \rightarrow 0 \text{ on averaging}} \rangle$

$(\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)])$

$\therefore\ R \propto \frac{1}{2} \left\langle \left[V_A^2 + V_B^2 \right] + \left[v_{REC,A}^2 + v_{REC,B}^2 \right] + \left[V_A V_B \cos\omega\tau_g \right] \right\rangle$

[antenna noise powers usually small cf receiver] [receiver noise powers, usually dominate] [interference fringes vary with τ_g]



(see also next two slides)

Disadvantages

since responds to total power from antennas & receivers

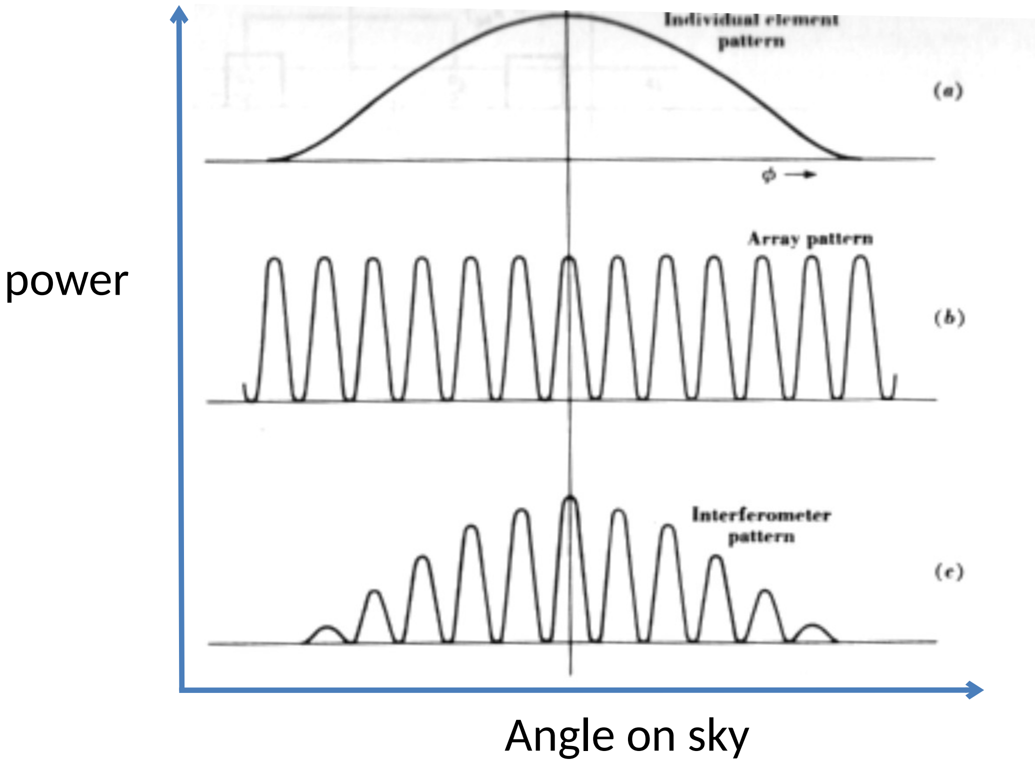
i) Sensitive to receiver gain variations

ii) atmosphere optical depth variations add varying noise

Note: Phase-switched interferometers not included

Two antenna adding interferometer -
point source passing through the beam pattern

Kraus “Radio Astronomy” Fig 6.14



Primary element beamwidth
= ~λ/d radians

Fringe spacing ≡ wavelength on sky
= λ/b radians

Angular frequency of fringes on sky
= b/λ cycles per radian

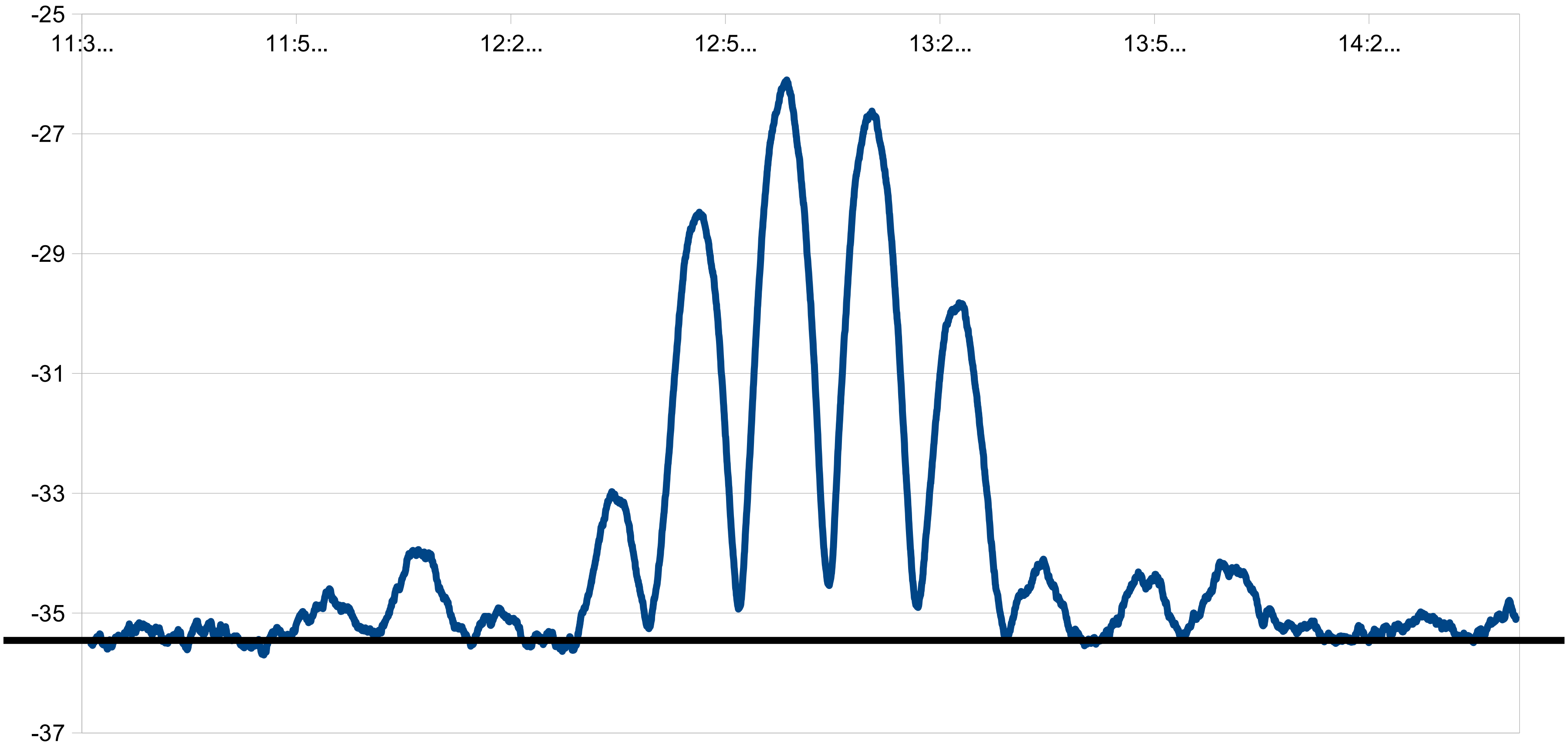
Combination

Note that the noise power from the individual radio receivers has been suppressed these diagrams (Slide 9 bottom) shows a more representative sketch).

The 2-element adding interferometer is a direct analogue of “Young’s slits” in optics.

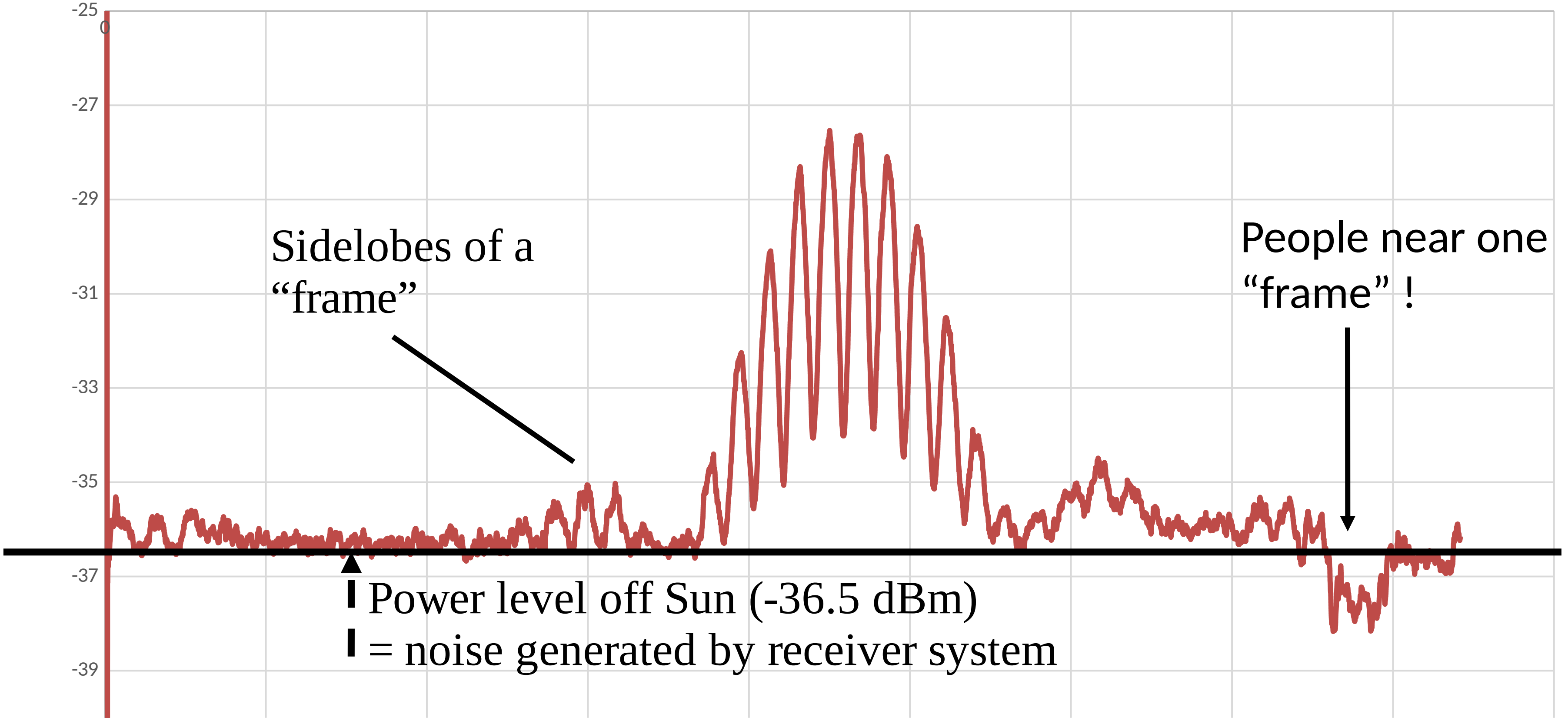
The MUST antennas pointing at the Sun

Single MUST baseline (two “frames”) on Sun -1



Short baseline: fringes well separated
Sun hardly resolved: fringes almost reach “off Sun” level

Single MUST baseline (two “frames”) on Sun -2



Longer baseline: fringes closer together

Sun partly resolved: fringes do not reach “off Sun” level

With many *equispaced* antennas the patterns add up to produce a set of narrow responses "inside" the "primary beam" pattern of a single antenna

Like the Fraunhofer pattern of a diffraction grating

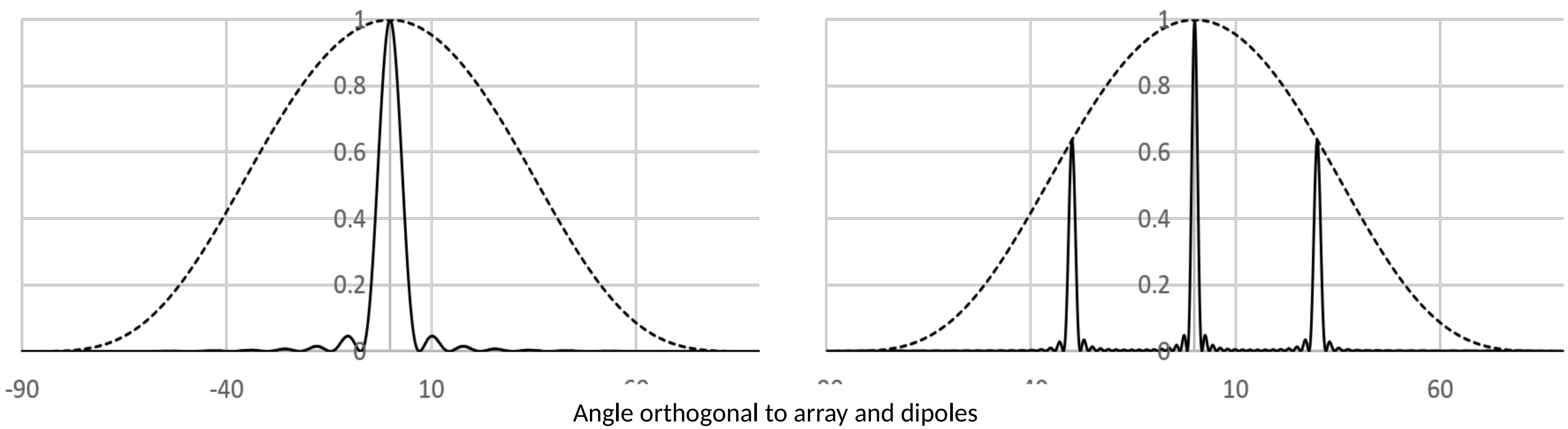
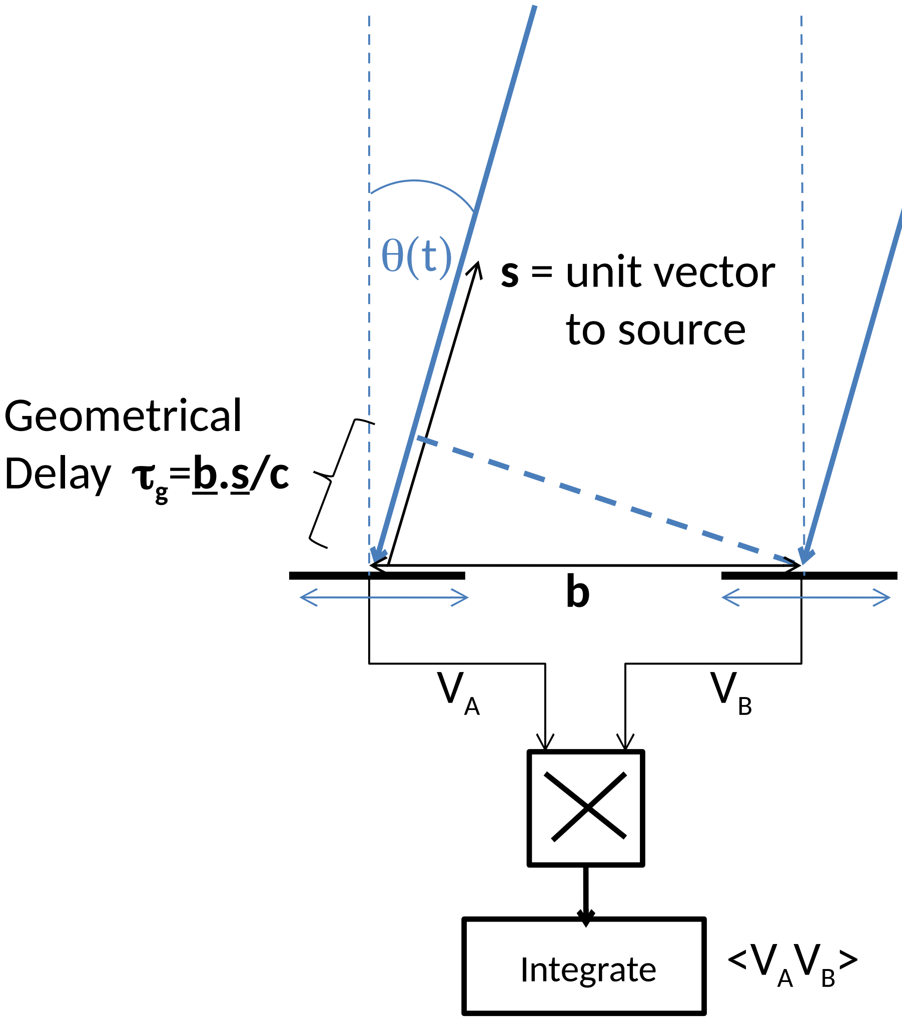


Fig 8.11 Normalised power patterns for uniformly-spaced phased arrays of half-wave dipoles above reflecting screens: left) $\lambda/2$ spacing ; right) 2λ spacing. The dotted line is the power pattern of an individual dipole plus reflector.

The central beam can be moved around by means of different delay/phase gradients across the array (look back at slide 6) – alternatively many beams cab be formed in different directions with independent sets of electronics and phase gradients

Adding phased arrays are ideal for detecting and locating pulsars & transient sources – they provide an instantaneous response with better positional information than from a single antenna – if all the beam-forming electronics can be afforded !

Multiplying Interferometer



Geometry as in slide 9 – but now *multiply* the voltages and integrate:

Referring back to algebra in slide 9 - only the V_A & V_B terms, which both come from the source itself, correlate. All other terms “wash out” after integration (check this argument out…)

$$<V_A V_B> = <V_A \cos\omega(t-\tau_g)\ V_B \cos\omega t>$$

$$R \ \alpha\ ½ < V_A V_B \cos\omega\tau_g>$$

Main advantage:

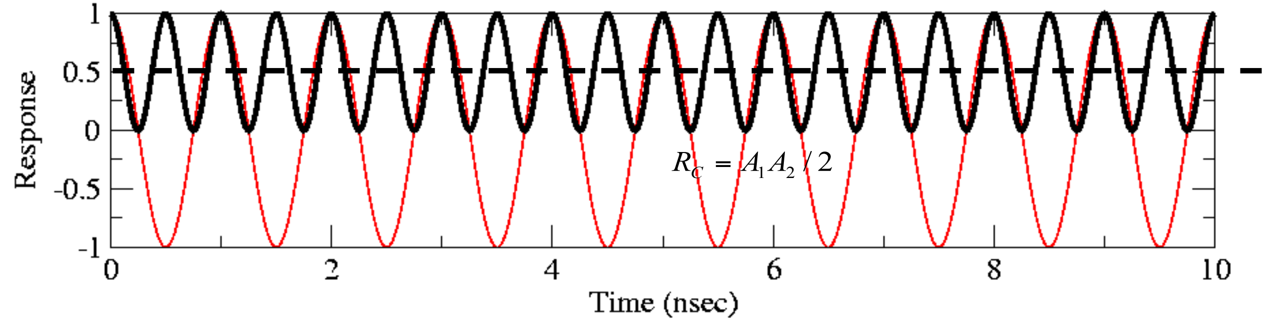
Total power terms don’t correlate so output is insensitive to receiver gain variations and to variations in atmospheric noise.

Signal Multiplications- pictorially

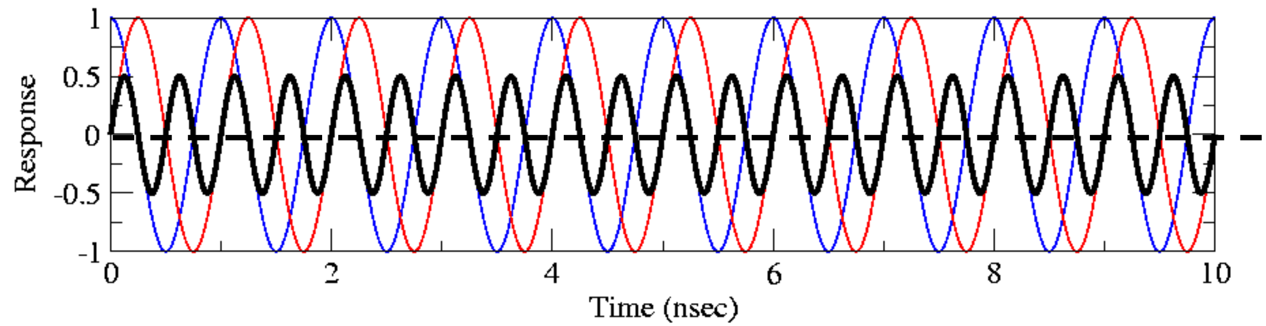
Two sinusoidal RF input voltages shown: red (V_A) and blue (V_B) with different relative phase shifts due to τ_g

- Solid black: $V_A V_B = V_A \cos\omega(t-\tau_g).V_B \cos\omega t$
- Dotted black: $\langle V_A V_B \rangle = 1/2[V_A V_B \cos\omega\tau_g]$

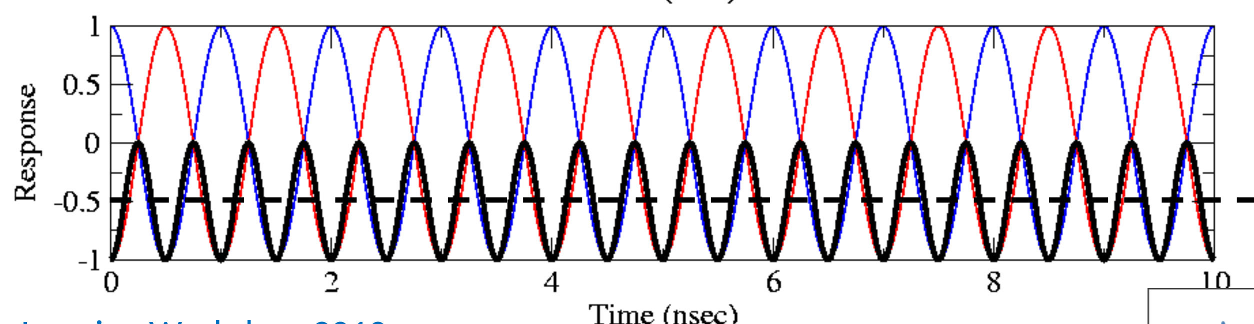
In Phase:
(can't see the blue trace)



Quadrature Phase:

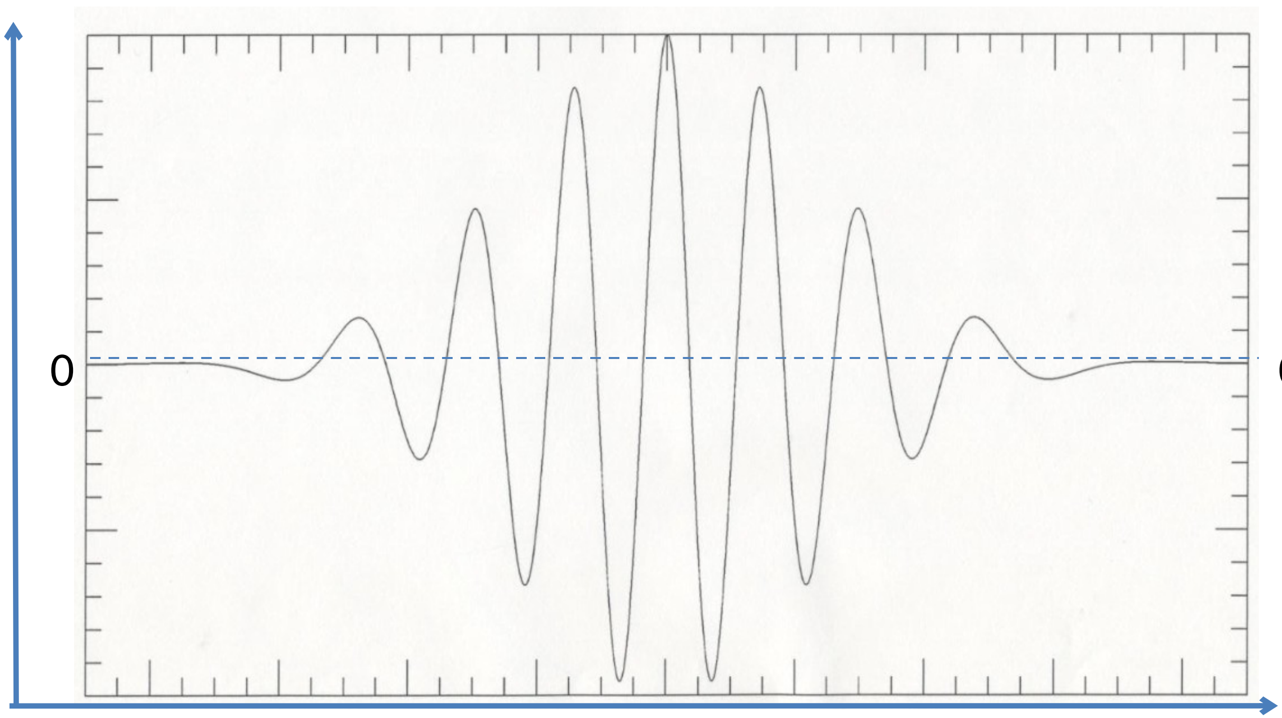


Anti-Phase:



After R.Perley12th NRAO Synthesis Imaging Workshop 2010

Multiplying interferometer fringe pattern

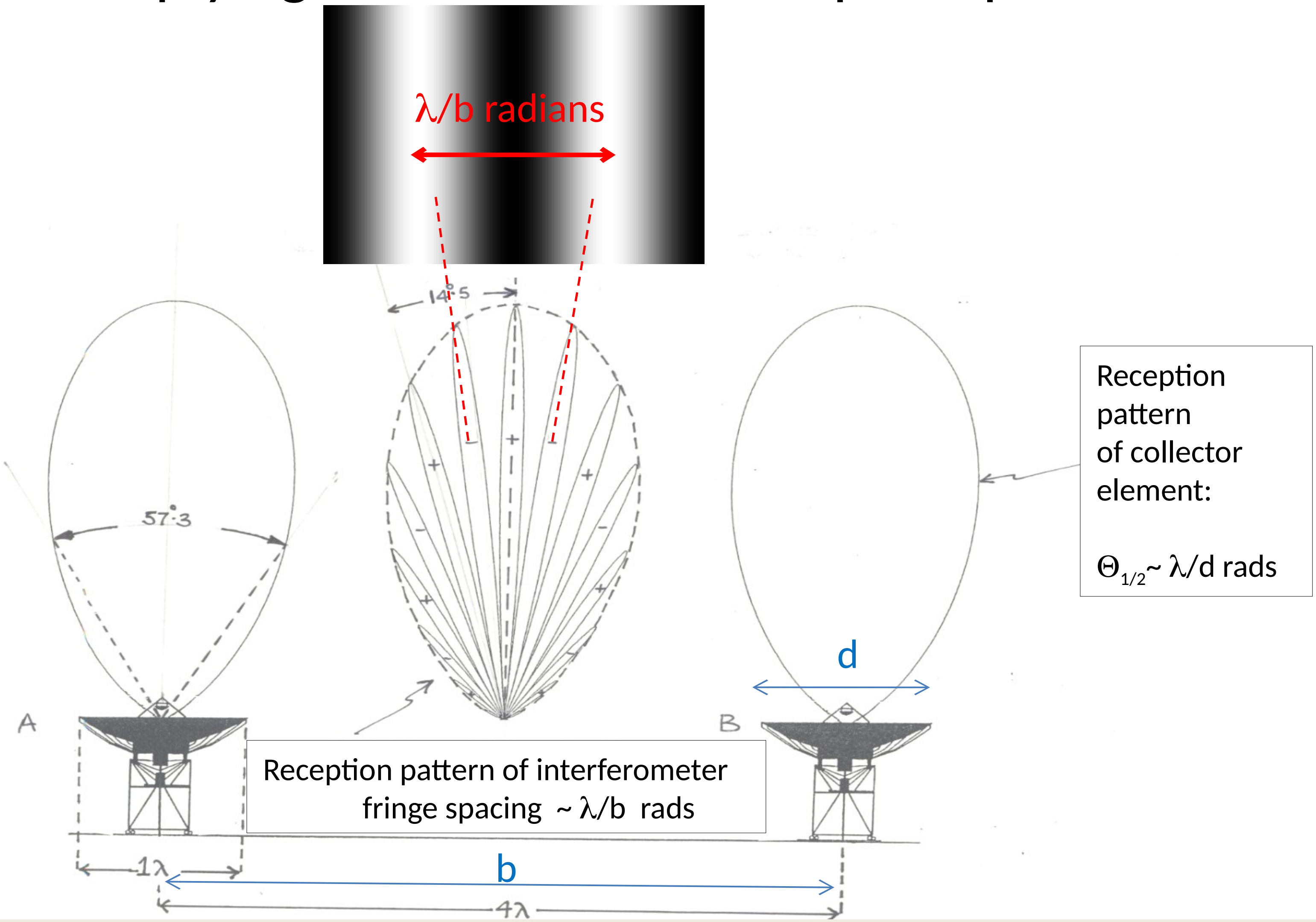


Primary element beamwidth
= ~λ/d radians

Fringe spacing ≡ wavelength on sky
= λ/b radians

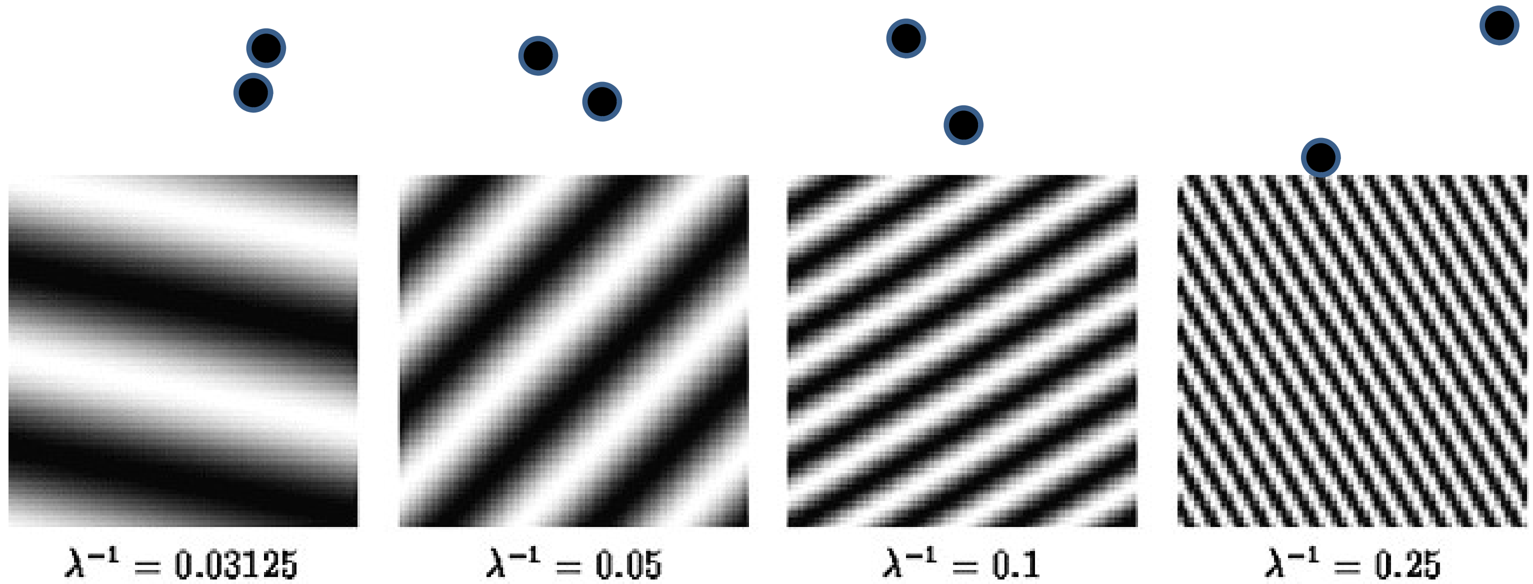
Angular frequency of fringes on sky
= b/λ cycles per radian

Multiplying interferometer reception pattern



Period or wavelength of fringes on sky = λ/b 1/4 radian as illustrated

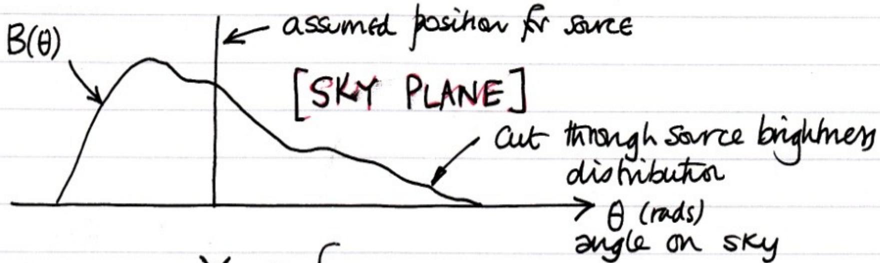
Angular frequency of fringes = b/λ 4 cycles per radian as illustrated



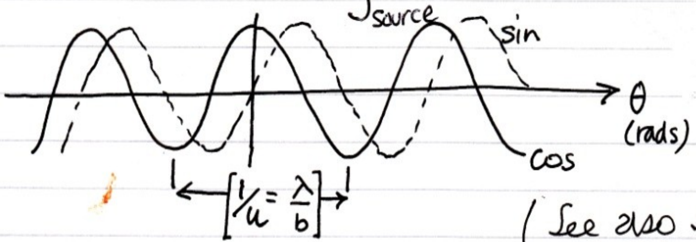
The separation B and observing wavelength λ of the two antennas defines the angularor spatial frequency of the corrugations λ/B - the further apart and the shorter the wavelength the finer the corrugations. The orientation of the two antennas with respect to the source defines the orientation of the corrugations - the line along their maxima is at right angles to the projected baseline.

When a source is observed at one instant of time these two properties are fixed. The output of the interferometer then tells us how alike the radio source is to the spatial frequency or corrugation. This is given by the amplitude. The phase then tells us how much to shift the corrugation sideways in order to best fit the actual sky brightness distribution.

Acknowledgement I.Morison



$\times$ & $\int_{\text{source}}$



(See also SLIDES 31 & 34)

$$R_{\cos}(u) \propto \int_{\text{Source}} B(\theta)\cos(2\pi u\theta)\, d\theta$$

[power out as function of baseline in λs]

$$R_{\sin} \propto \int_{\text{Source}} B(\theta)\sin(2\pi u\theta)\, d\theta$$

} simplified actually 2-D integral $\int d\Omega$

Note: $u = \frac{b}{\lambda}$ = # wavelengths in baseline (a number)

≡ # of fringe cycles per radian on sky also slide 24

$\sin\theta \to \theta$: small angle approximation

$\therefore u\theta \equiv$ fraction of a fringe period

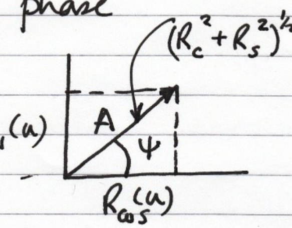
Both R_c & R_s needed since $B(\theta)$ is not even (symmetrical)
∴ can't be built up out of $\sum$ cosines

Define the complex fringe visibility function $A\, e^{i\psi}$ for a given baseline b or equivalently its length expressed as a number of RF wavelengths u ($u = b/\lambda$)

Magnitude = fringe amplitude $\quad A = \left[R_{\cos}^2(u) + R_{\sin}^2(u)\right]^{1/2}$

fringe phase $\quad \psi = \tan^{-1}\left[\frac{R_{\sin}(u)}{R_{\cos}(u)}\right]$

i.e. in vector form



- ψ is the shift of that Fourier component with respect to the assumed position for the source centroid (in radians)
- it is measured in terms of the fringe period (1 period = 2π radians)

Recapitulation

- The integration of the cosine pattern over the source picks out the even part of the brightness distribution; the sine pattern picks out the odd part.

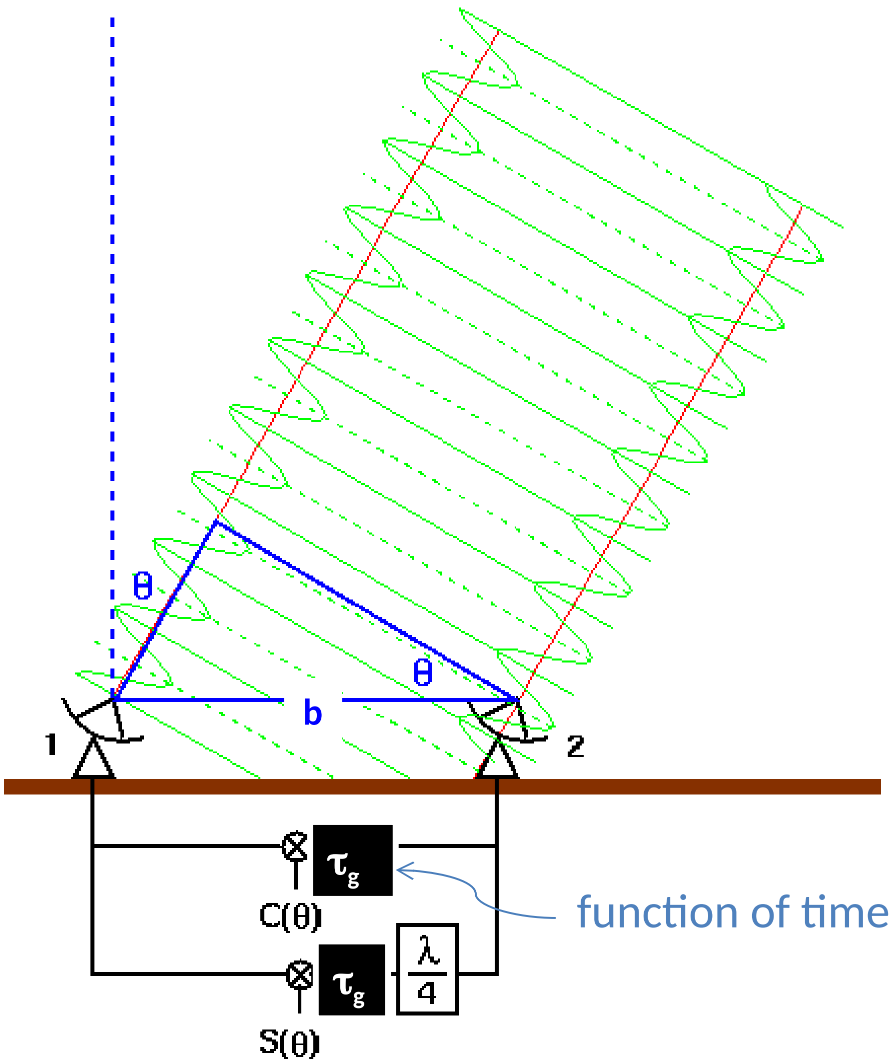
- For a single baseline at any given time these make up a single complex Fourier Component (FC) of the sky brightness distribution– depending on the length of the baseline and its orientation with respect to the source.

- The complex fringe "visibility" $V = Ae^{i\psi}$ is for a particular baseline and source structure
  - Amplitude: $(R^2_{sin} + R^2_{cos})^{1/2}$
  - Phase $\psi = \tan^{-1}(R_{sin}/R_{cos})$
  - ψ is the starting point of a FC with respect to an assumed position on the sky – it is measured in terms of the period of the fringe pattern measuring that FC (1 period = 2π rads).

- Sufficient knowledge of the fringe visibility function (as derived from many interferometer baselines) can provide us a good image of the source brightness……..(see next lectures!)

Tracking interferometer: sine and cosine fringes

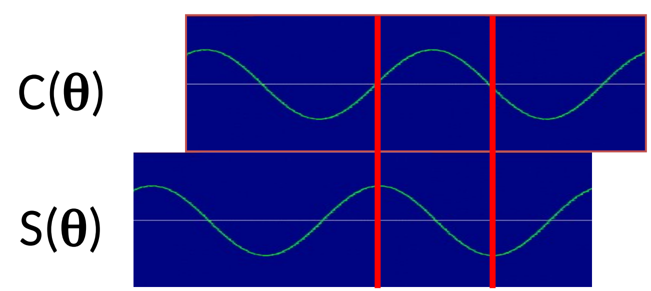
http://dsnra.jpl.nasa.gov/pkg/encyclopedia-0.1/html/encyclopedia/interferometer/interferometer.html



A *tracking* correlation interferometer (with two multiplier receiver chains). The same *variable* time delay $\tau_g(t) = \underline{b}.\underline{s}/c = b.\sin\theta(t)/c$ is inserted in both.

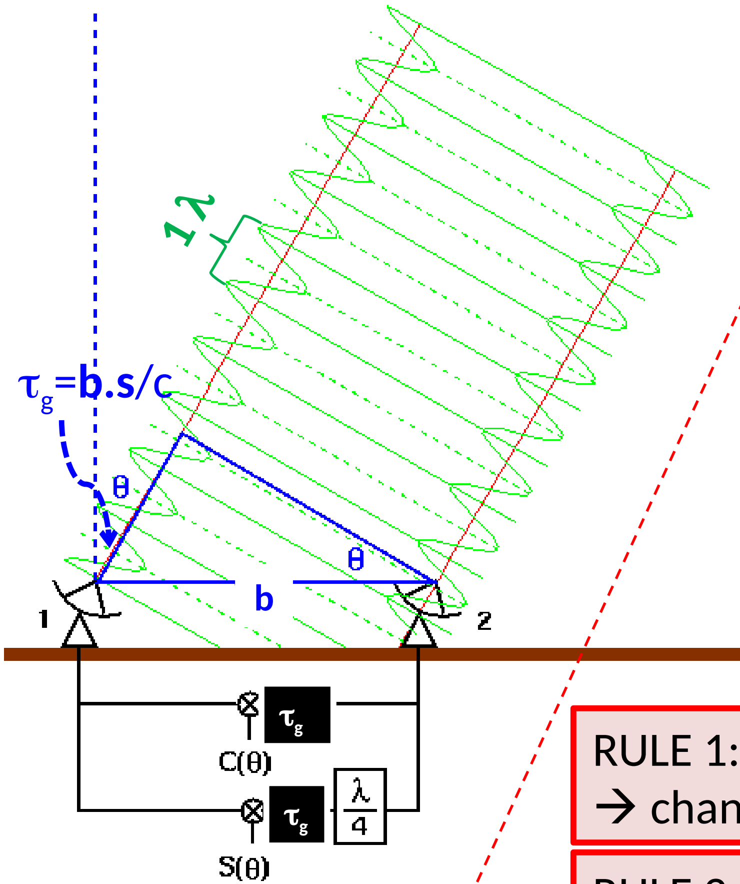
As a source moves across the sky the required delay changes. If the delay tracks perfectly the source will remain at exactly the same place relative to the fringe pattern (next slide). (The antennas – in this case dishes - also track the source).

Note that in one of the chains an *additional* delay equivalent to $\lambda/4$ i.e. to a phase difference of $\pi/2$ is included – this produces a sine fringe pattern S(**θ**) shifted by ¼ of a fringe period; the other chain produces the cosine fringe pattern C(**θ**).



Interferometer phase shifts

(**Refer back to slide 20** and ignore motion of the source)



$V_AV_B\cos\phi$ and $V_AV_B\sin\phi$

$\tau_g \equiv b\sin\theta/\lambda$ wavelengths = # of RF cycles

$\equiv b\theta/\lambda$ (for $\theta << 1$ radian)

$\phi = \omega\tau_g = 2\pi\,[b\theta/\lambda]$ radians = total phase difference

Change "sky" angle to $\theta\pm\Delta\theta$ (rads) → interferometer phase changes to $\phi\pm\Delta\phi$ (rads)

i.e. $\Delta\phi = 2\pi\,[b\Delta\theta/\lambda]$ rads

Thus if $\Delta\theta = \lambda/b$ = one fringe (1 angular cycle) on sky

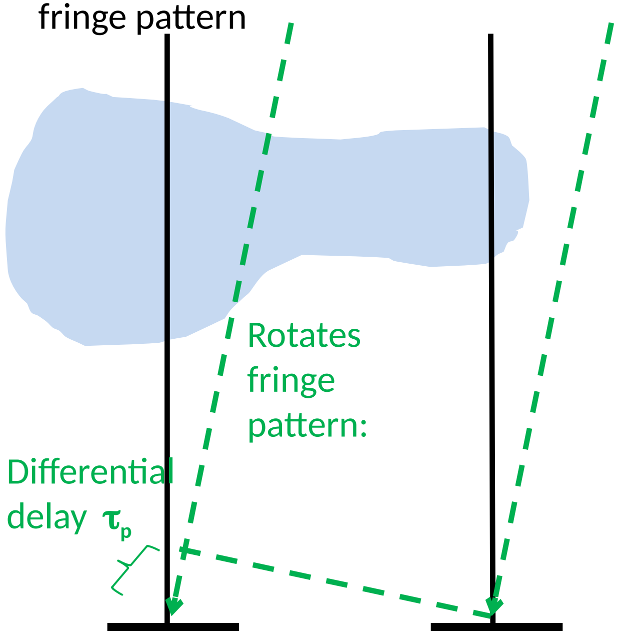
→ $\Delta\phi = 2\pi$ rads = 1 cycle of output phase

RULE 1: Change source position θ on sky by 1 fringe spacing (cycle) → change interferometer output by 1 cycle.

RULE 2 : Change delay τ_g by one RF cycle (λ) in travel time → change position of fringe pattern on sky by one fringe spacing (cycle).

http://dsnra.jpl.nasa.gov/pkg/encyclopedia-0.1/html/encyclopedia/interferometer/interferometer.html

Effect of optical depth fluctuations



Variations in *optical depth* of the troposphere

1) produce changes in signal *absorption and noise emission* → both the correlated signal $<V_A V_B>$ and noise terms ($<v^2_{noise,a} + v^2_{noise,b}>$) vary.

- *Adding* interferometer : both terms affect the mean power output.
- *Multiplying* interferometer : only the reduction in the correlated term by absorption affects the *visibility amplitude*.

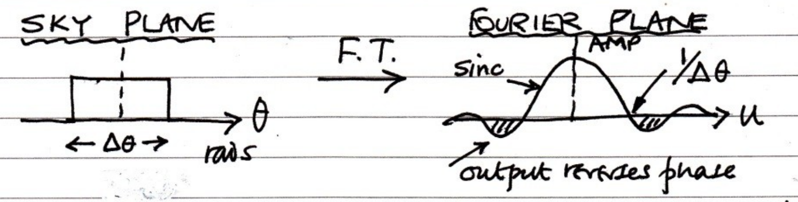
2) changes the refractive index (also involves ionosphere below few GHz) and hence *the propagation delays* to each antenna . The delay depends strongly on frequency, like the optical depth. Differential delays affect the *visibility phase*.

If the antennas are separated by more than the correlation length of the atmospheric fluctuations (a few km in the troposphere) → differential delay τ_p → rotation in the fringe pattern → shift in *the* output (visibility) phase . (See Slide 25 RULE 2 and Examples 21+22 Question 3)

As the wind blows across the beams the propagation delay fluctuates in time, the fringe pattern fluctuates and, unless this can be calibrated, the interferometer is said to be "phase unstable" and cannot make good positional measurements or make good images – [slide 30].

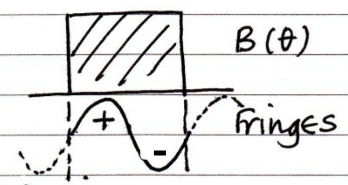
Additional Notes "A"

Simple sources & their visibility amplitudes



$B(\theta)$: BRIGHTNESS DISTRIBUTION $\qquad$ $V(u)$: COMPLEX VISIBILITY F^N

- **At 1st zero**

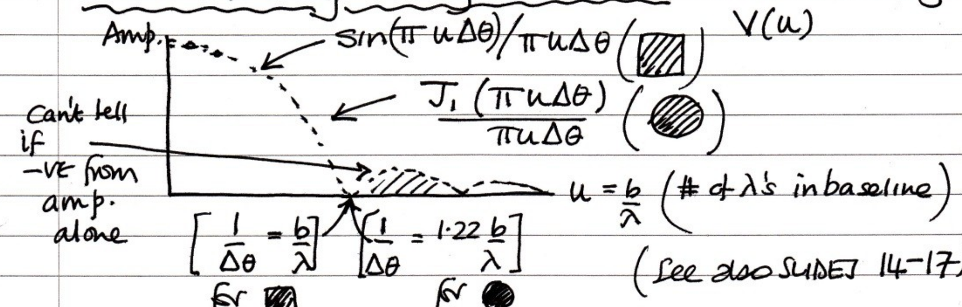


'+' cancels '-' $\quad \leftarrow \frac{\lambda}{b} \rightarrow \quad$ when one fringe fits across source

$$\therefore \ \Delta\theta = \frac{\lambda}{b} \quad \text{OR} \quad \frac{1}{\Delta\theta} = \frac{b}{\lambda} = u$$

{u = # of cycles rad^{-1} for that fringe spacing}
{u = # of λ's in baseline producing that fringe}
(Fraunhofer diffrac^n & small angle approx)

- **Increase |b| by moving one antenna** $\rightarrow$ 1-D cut through $V(u)$



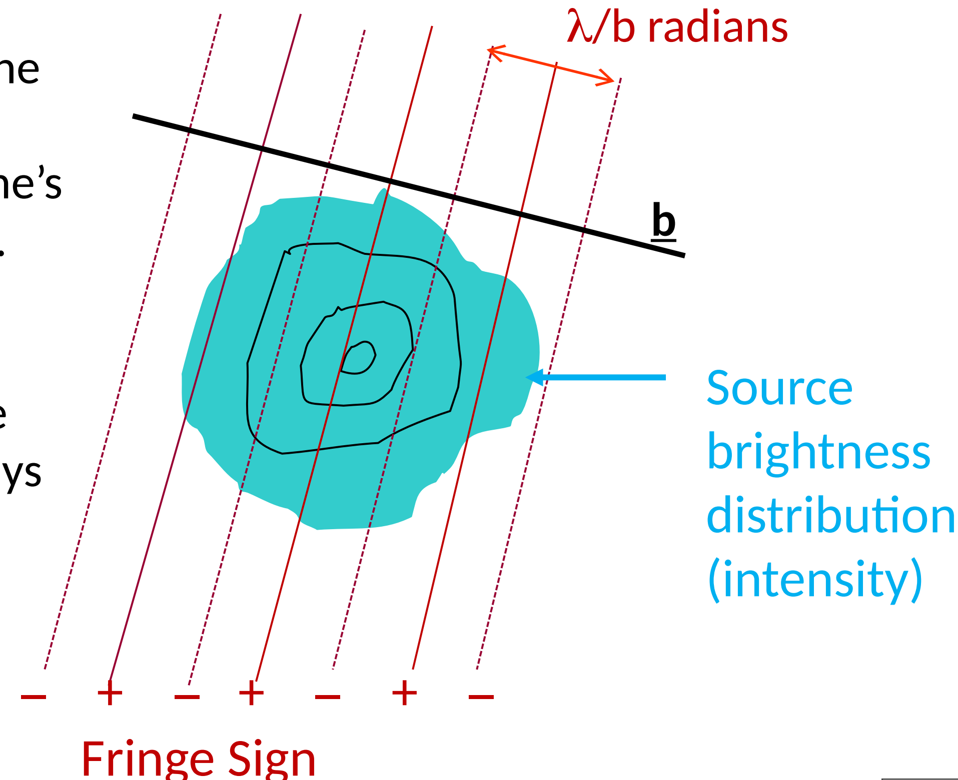
(See also SLIDES 14-17)

Imaging: basic action of a correlation interferometer

- The correlator can be thought of "placing" a cosinusoidal and sinusoidal (not shown) fringe or "spatial coherence" pattern (see later) , of angular period λ/b radians, onto the sky.

- The overall effect is to multiply the source brightness distribution by the cosinusoidal and sinusoidal (not shown) patterns, and to integrate the results (of each separately) over the sky.

  - Fringe orientation: perpendicular to the baseline vector **b** .
  - Fringe spacing: set by projected baseline's (slide 10&18) length & the wavelength.
    - long baselines: close-packed
    - short baselines: widely-separated
  - Fringe phase: known if source/baseline geometry known and propagation delays calibrated.
  - Physical location of baseline does not matter if source is in the far field (see later)



After R. Perley NRAO 12th Synthesis Imaging Workshop 20

The interferometer visibility function & extended sources

The complex fringe"visibility" V(u) = A(u) $e^{i\psi(u)}$ is for a particular baseline vector (whose length measured in wavelengths = u) and source structure

- Amplitude: $(R^2_{sin} + R^2_{cos})^{1/2}$
- Phase $\psi = \tan^{-1}(R_{sin}/R_{cos})$

A single Fourier Component of the brightness distribution

- ψ is the starting point of that Fourier Component with respect to an assumed position on the sky – measured in terms of the period of that FC (period = 2π rads).

V is a continuous function of the source structure and the baseline vectors.

- Each observation of the source with a given baseline vector provides one measure (sample) of V
- Sufficient knowledge of V (numbers & range of samples) provides an estimate of the source brightness – after taking the Fourier Transform (see later) .
- The phases enable the FCs to be correctly "lined- up" to produce a correct reconstruction (**see next slide**).

The visibility phase ψ is a more difficult (see this lecture) yet more critical measurement for imaging than the visibility amplitude (see next slide).

HOWEVER

If ψ not measureable can still get structures of simple sources from amplitude alone.

The importance of phase in imaging

Prof. V.M. Gadre (VMG)

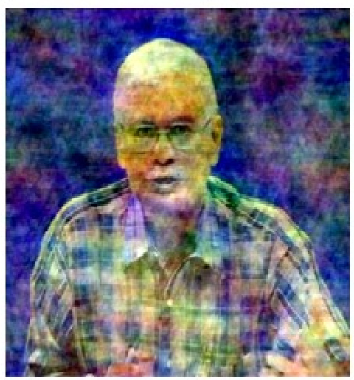
$A_1e^{i\psi_1}$

Prof. Somnath Sengupta (SSG)

$A_2e^{i\psi_2}$

Magnitude of VMG, Phase of SSG

$A_1e^{i\psi_2}$



Magnitude of SSG, Phase of VMG

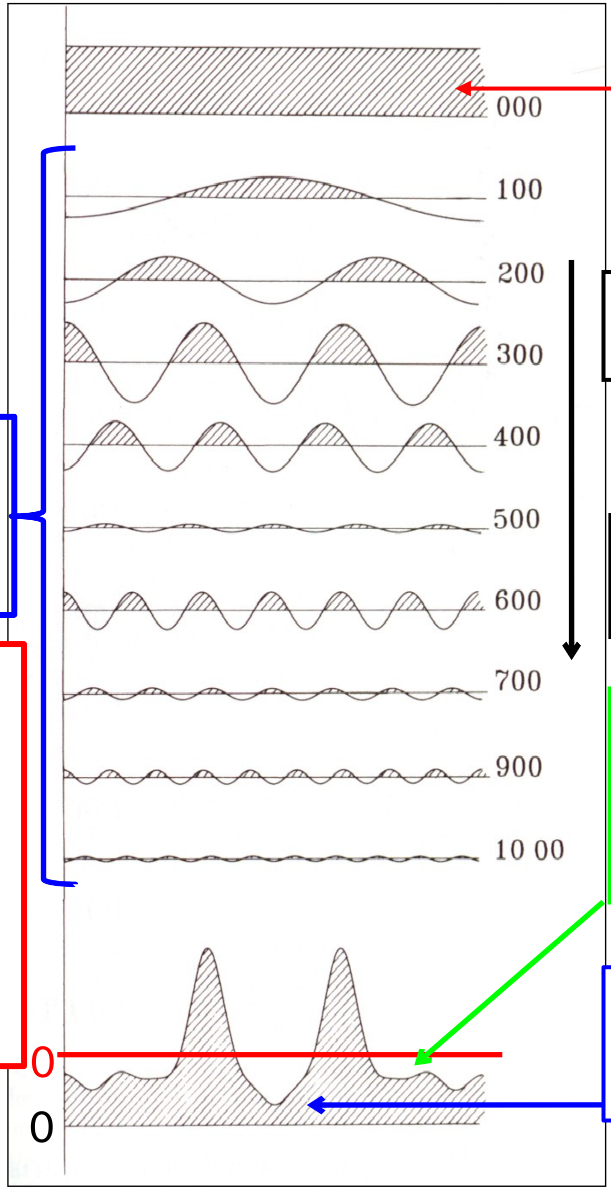
$A_2e^{i\psi_1}$

http://www.mathworks.com/matlabcentral/fileexchange/43645-demonstration-showing-importance-of-phase-in-images

Swapping the Fourier phases of images shows the importance of the phase.

LESSON: Phase errors → errors in "lining up" the Fourier Components → image distor

Essence of Fourier synthesis: “indirect” imaging



Average brightness in source region is not available from a correlation system. The “zero-baseline” Fourier Component can’t be measured – two telescopes can not be top of each other (Lectures 23&24)

Shorter baselines→ low angular frequency Fourier Components

Longer baselines→ high angular frequency Fourier Components

The Fourier Components don’t all start at the same position – i.e. they have different phase shifts

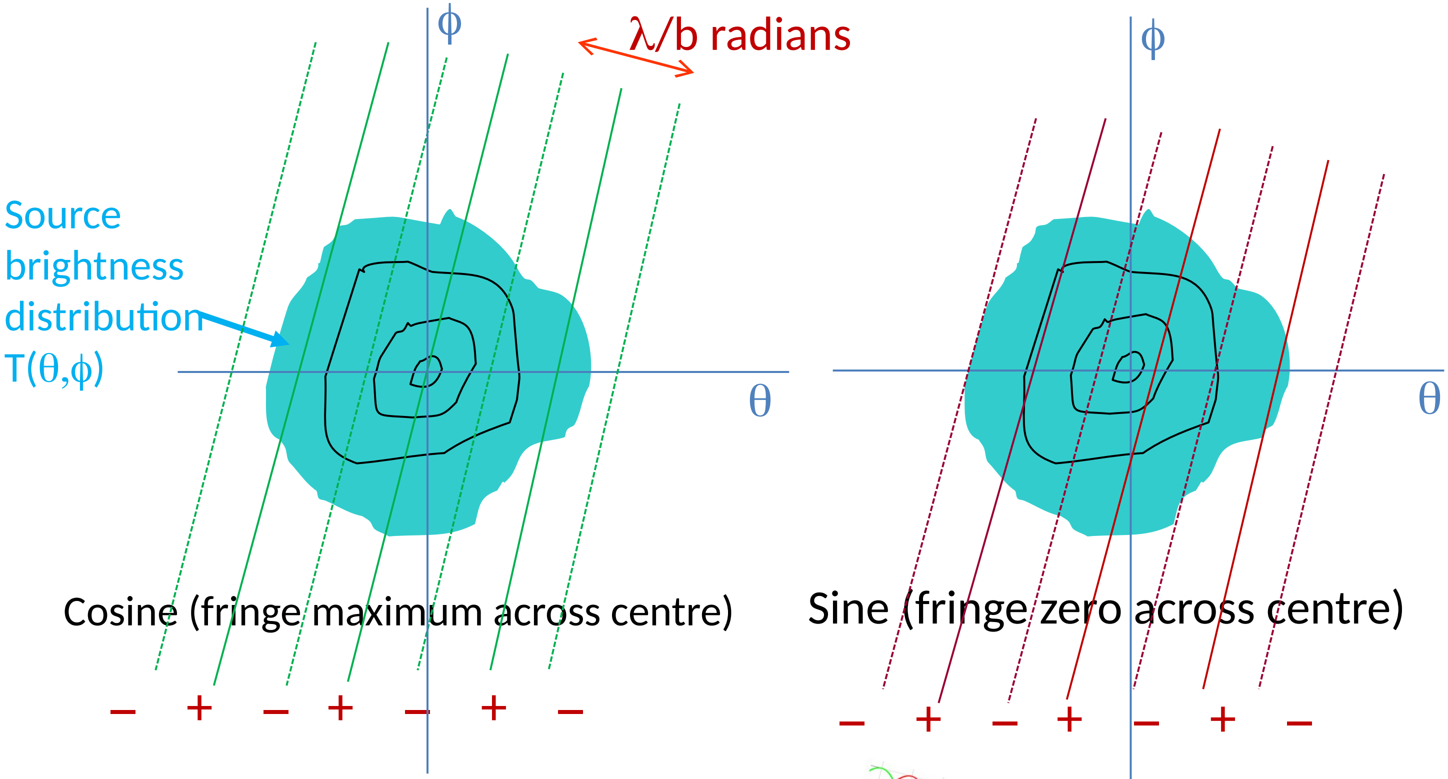
Lack of zero-baseline Fourier Component → synthesised image goes negative in places *even though the sky intensity is always positive.* Another effect of not collecting all the Fourier Components (Lectures 23+24)

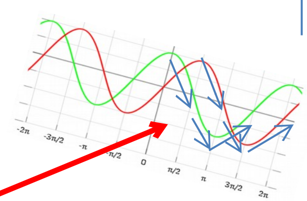
Image “synthesised” by adding all the Fourier Components. Not collecting all the Fourier components → unwanted fluctuations

Fourier Components must be correctly aligned i.e. relative *phases* measured accurately, otherwise a distorted image results (slide 30)

Extension of slide 27

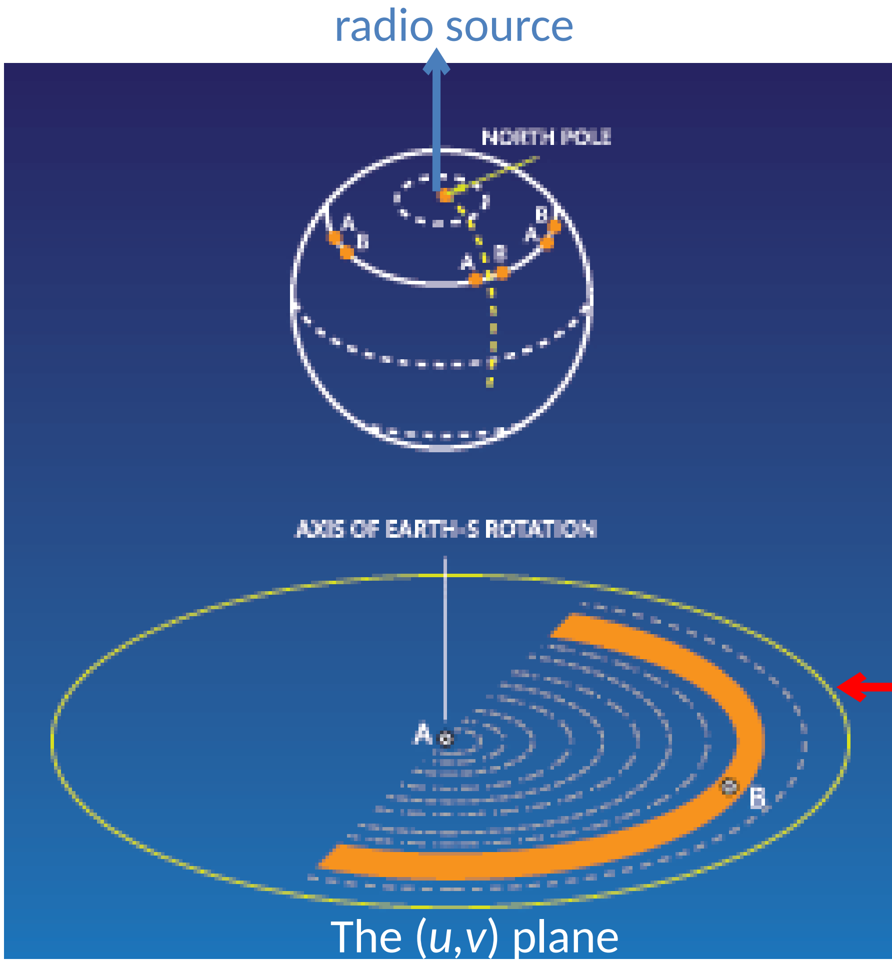
Cosine and Sine Fringes in sky plane





At any instant, when the projected E-W and N-S components of the projected baseline are $u = b_x/\lambda$ & $v = b_Y/\lambda$ wavelengths. The phase term: $2\pi[u\theta+v\phi] \equiv 2\pi[(b_x/\lambda)\theta+(b_Y/\lambda)\phi]$ varies according to the angular position on the sky (θ, ϕ) tracing out the cosine and sine fringe patterns.

Earth rotation and the (*u*,*v*) plane



http://www.almaobs.info/en/outreach/newsletter/181-newsletter-5

Aperture synthesis interferometers for radio astronomy use the rotation of the Earth to increase the number of baseline orientations.

Shown here are two antennas arranged on an East-West baseline. *For clarity, the radio source is assumed to be in the direction of the north celestial pole i.e. along the Earth's rotation axis.*

As the Earth rotates the baseline between A & B changes angle with time as viewed from the radio source. As seen from A, B completes a circle around it over a 24h day, moving anti-clockwise. Note: only the *relative* location of A&B matters.

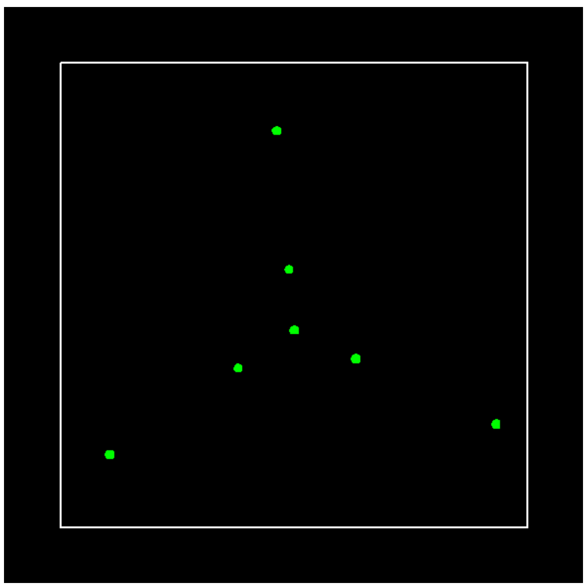
The baseline vector, projected in the direction of the source, can be separated into two orthogonal directions, referred to as "u" and "v" with units of wavelengths (i.e. angular frequency in cycles per radian). These define the (*u*,*v*) plane.

For other source directions i) the circle compresses into an ellipse
ii) the source may not be visible for 24h and hence the ellipse is incomplete

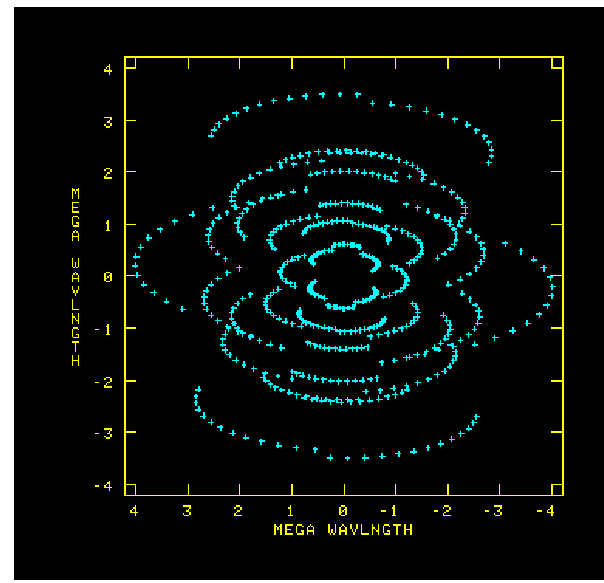
(u,v) plane coverage

- A single baseline provides one sample of $V(u,v)$ per integration
- Build up coverage in the (u,v)-plane by:
  - having lots of antennas → N(N-1)/2 baselines
  - waiting for the Earth to rotate to provide changing *projected* baselines
  - physically relocating the antennas between sets of observations (usually not possible)
  - some combination of the above

Telescope locations:



(u,v) coverage for 6 hour track:

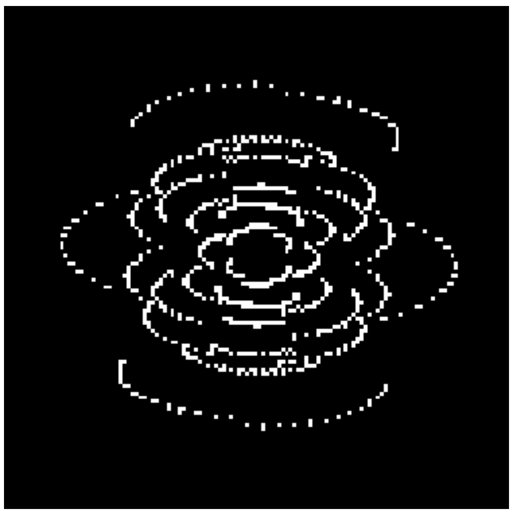


Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

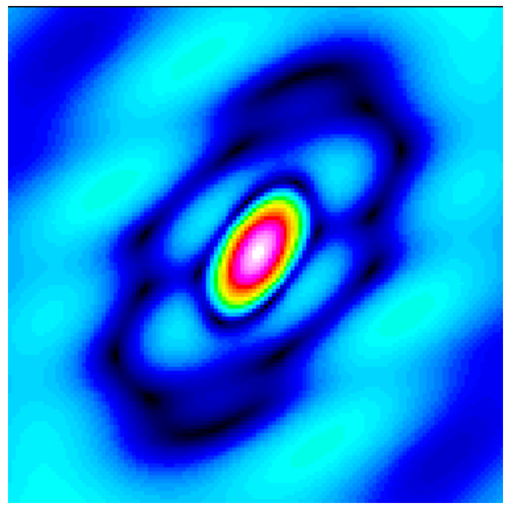
The (*u*,*v*) coverage samples the Visibility Function

[(*u*,*v*) coverage]	x	[$V(u,v)_{true}$]	=	$V(u,v)_{measured}$
Sampling function		FT of source		Measured visibilities

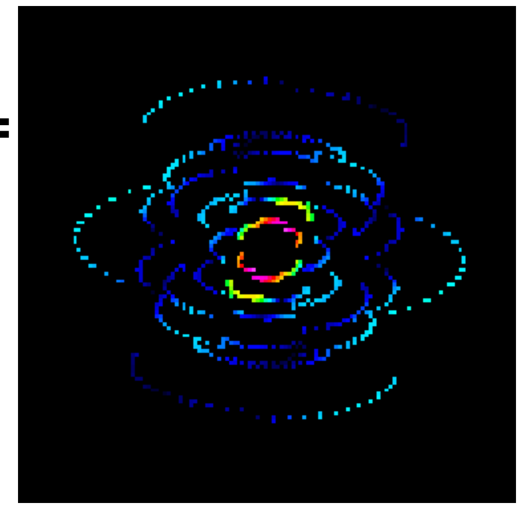
(*u*,*v*) coverage $V(u,v)_{true}$ $V(u,v)_{measured}$



X



=



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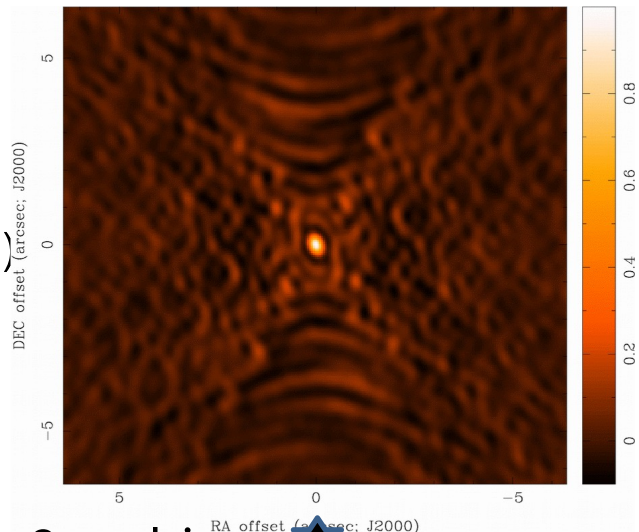
Because the Visibility Function for a *real* distribution (i.e. just positive or negative numbers) is "Hermitian" each measurement results in *two* points in the (*u*,*v*) plane, one for baseline 1-2, the other for baseline 2-1, etc.. Thus Earth rotation traces out two arcs per baseline. *Note that there is no new spatial information from the second arc - it does not double the resolution! - but by enforcing Hermitian symmetry the resulting map, after Fourier Transformation, is forced to be entirely real.*

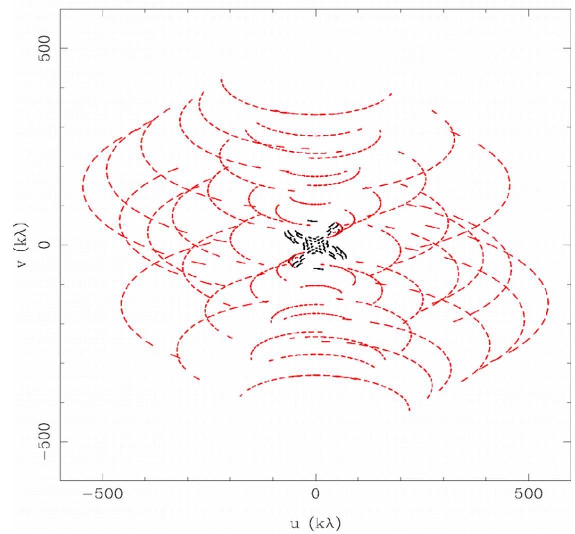
"Dirty Beam" and "Dirty Map"

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

S(θ,ϕ)
(Dirty Beam)



F.T.

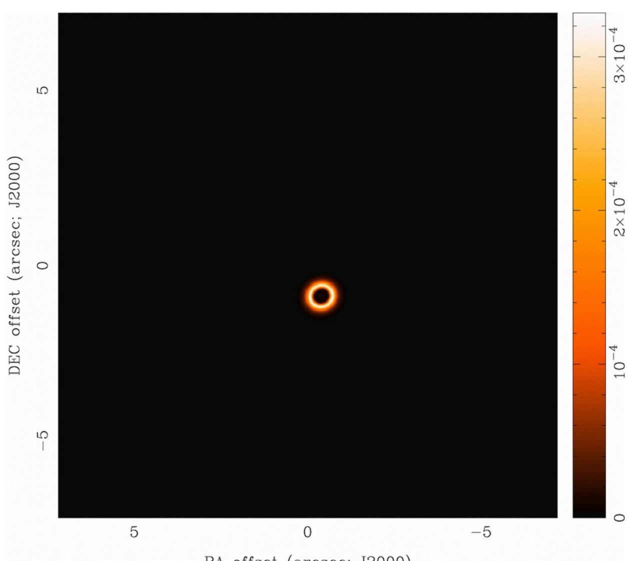


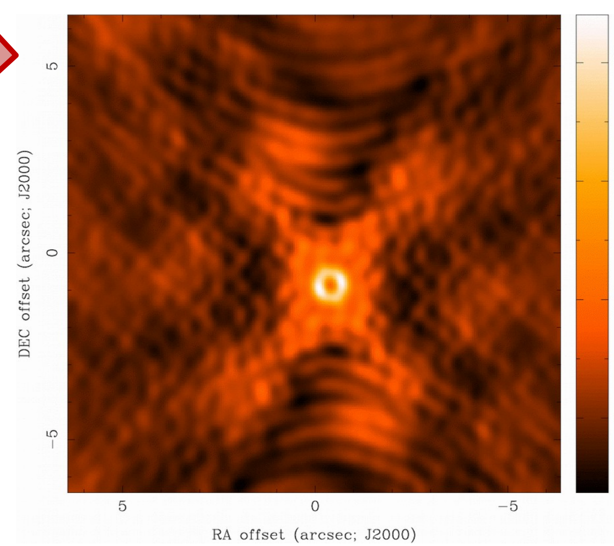
S(u,v)

sampled
(u,v) plane

Convolving

T(θ,ϕ)
(true sky)





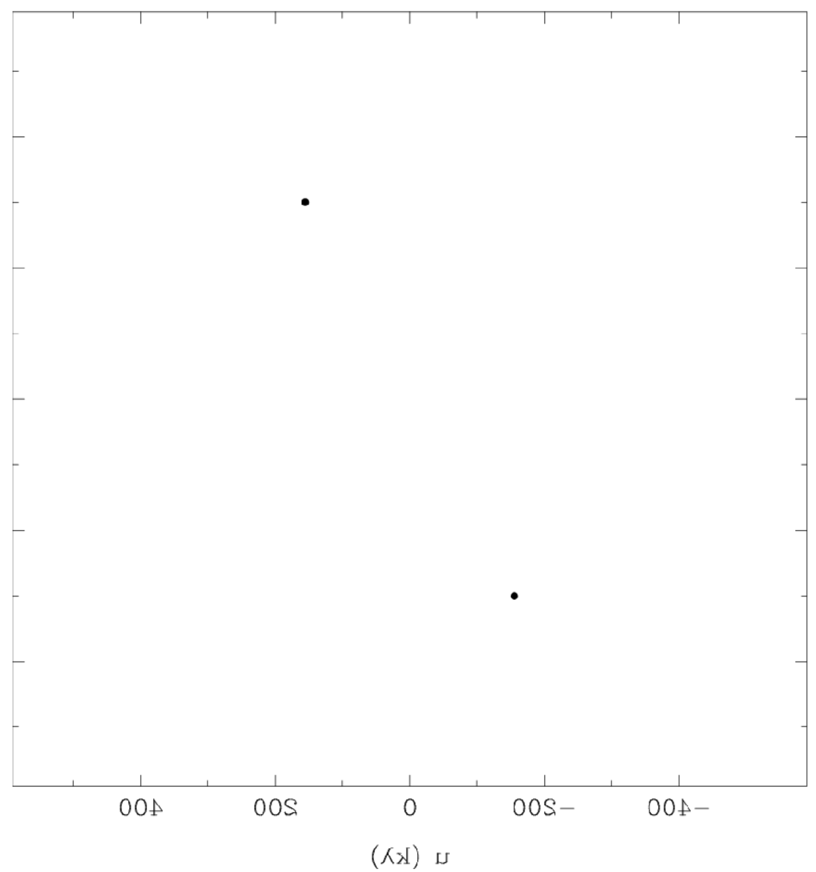
T^D(θ,ϕ)
(Dirty Map)

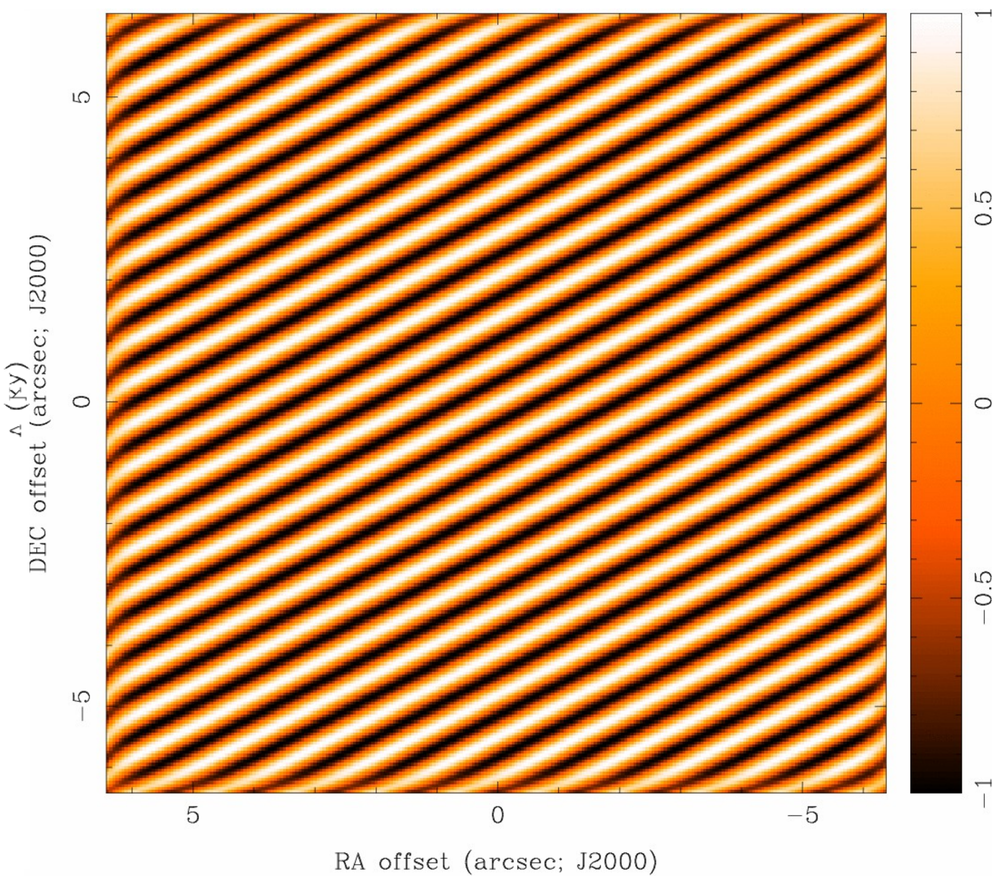
(contrast
enhanced)

Dirty Beam Shape and N Antennas

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

2 Antennas





Dirty Beam Shape and N Antennas

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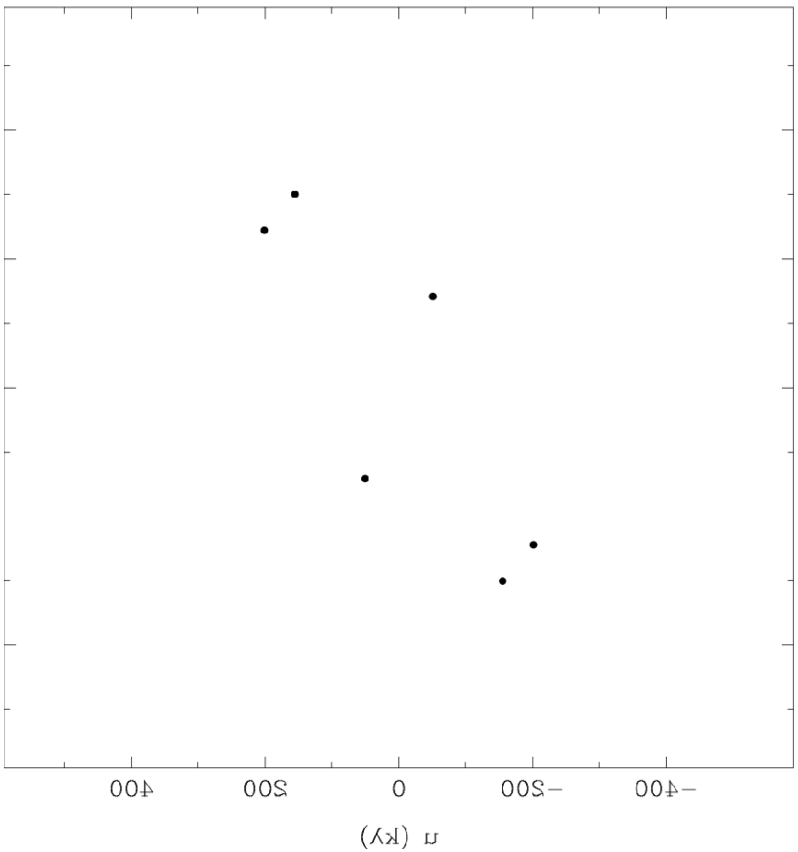
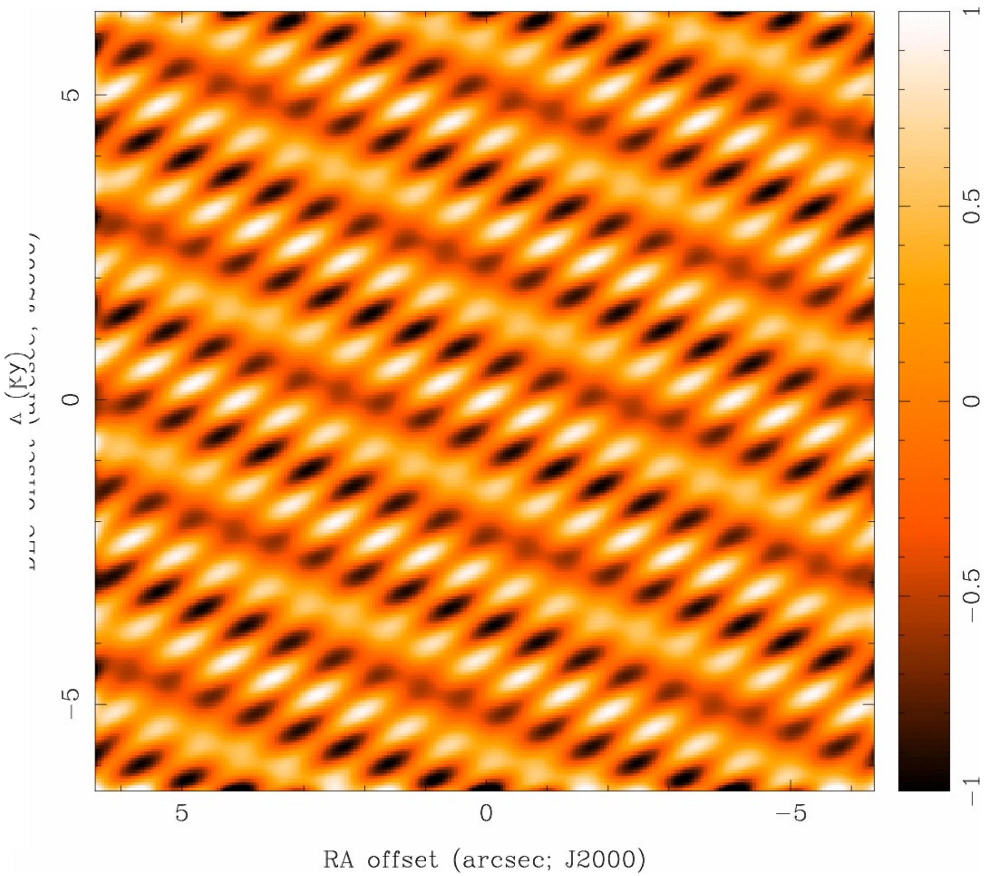
3 Antennas

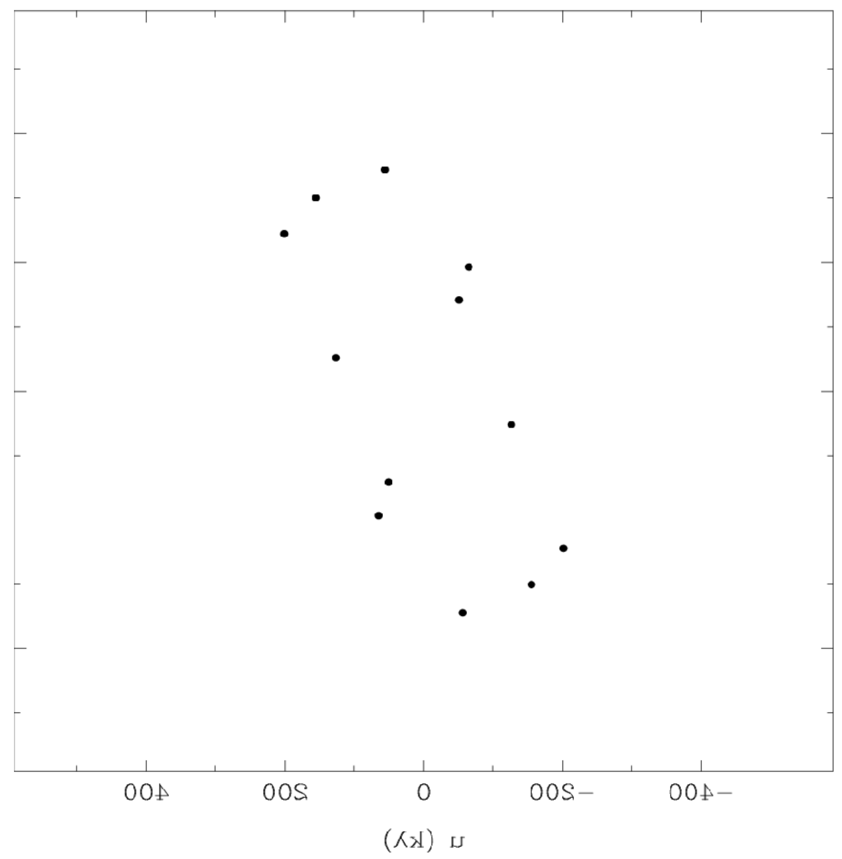


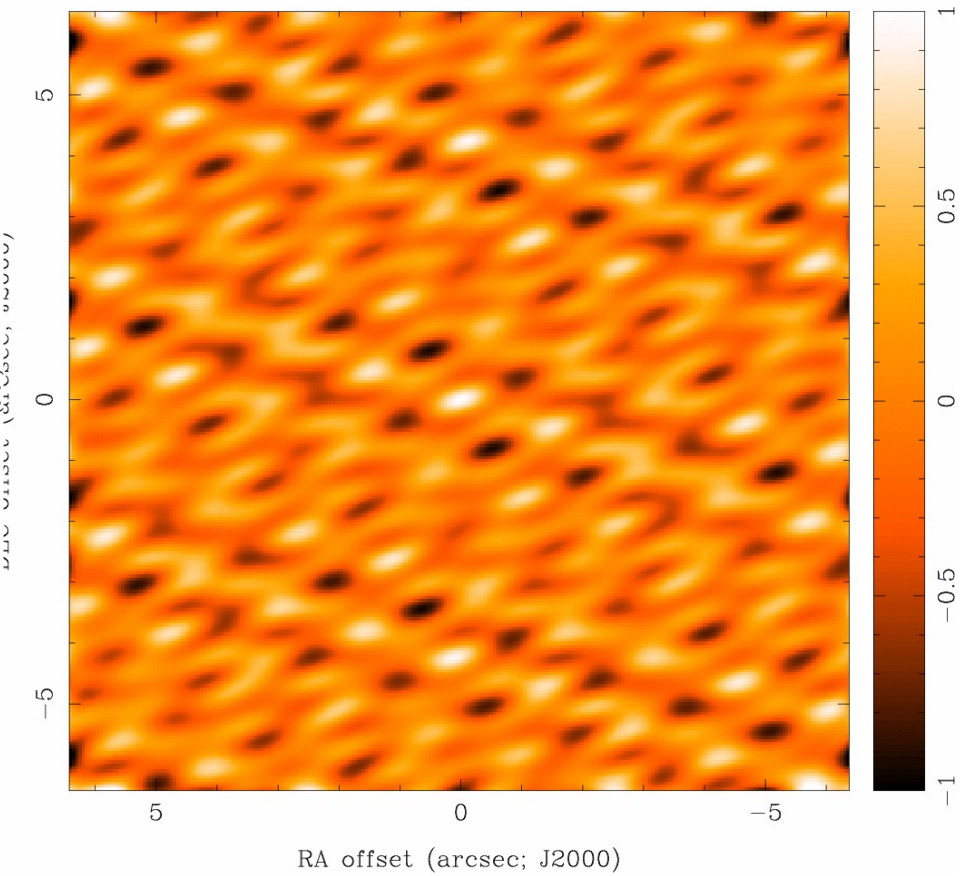


Dirty Beam Shape and N Antennas

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

4 Antennas

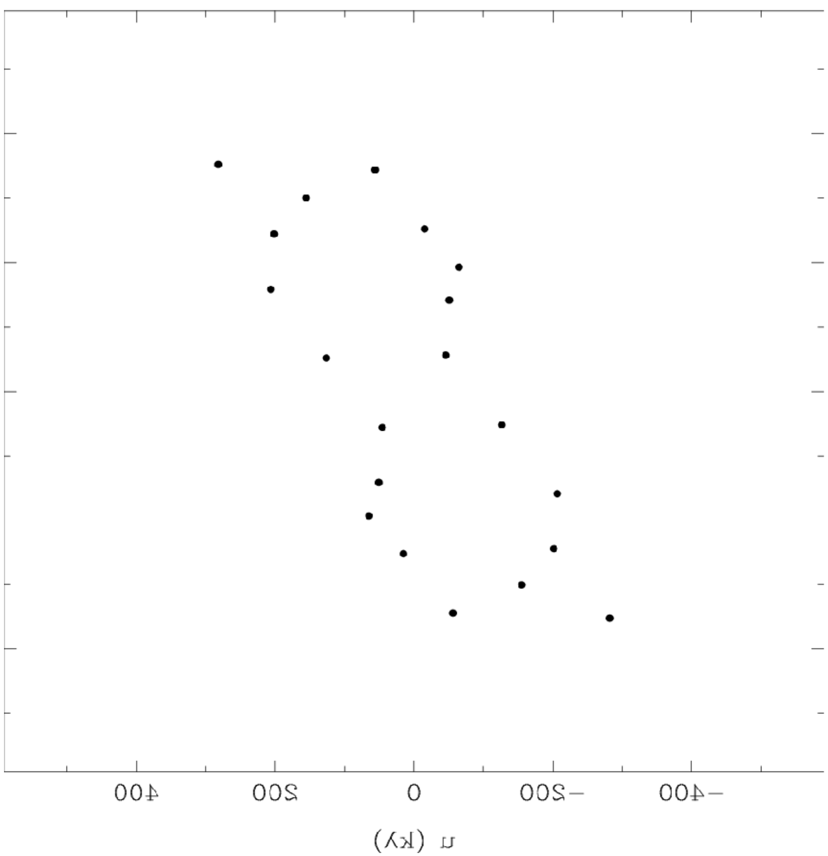


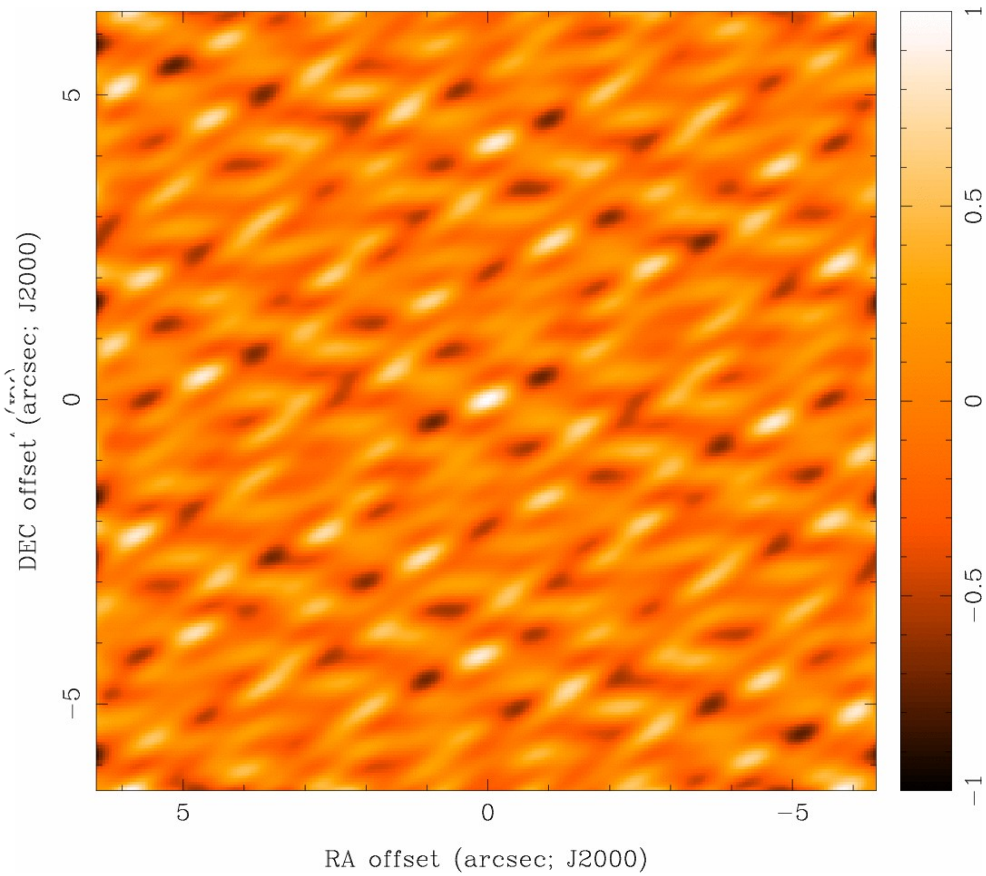


Dirty Beam Shape and N Antennas

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

5 Antennas

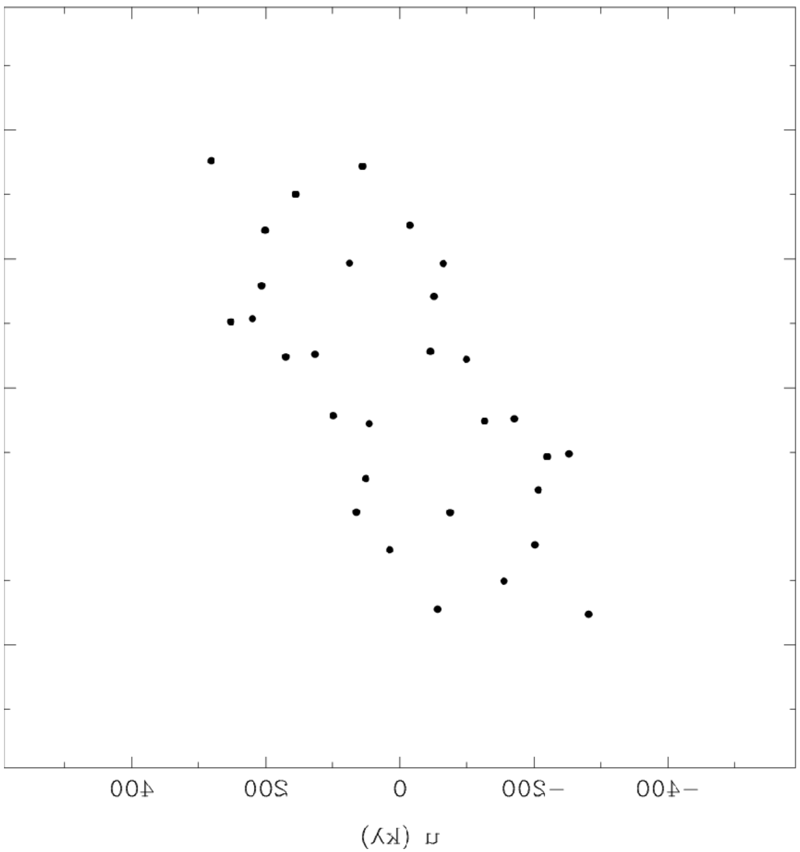


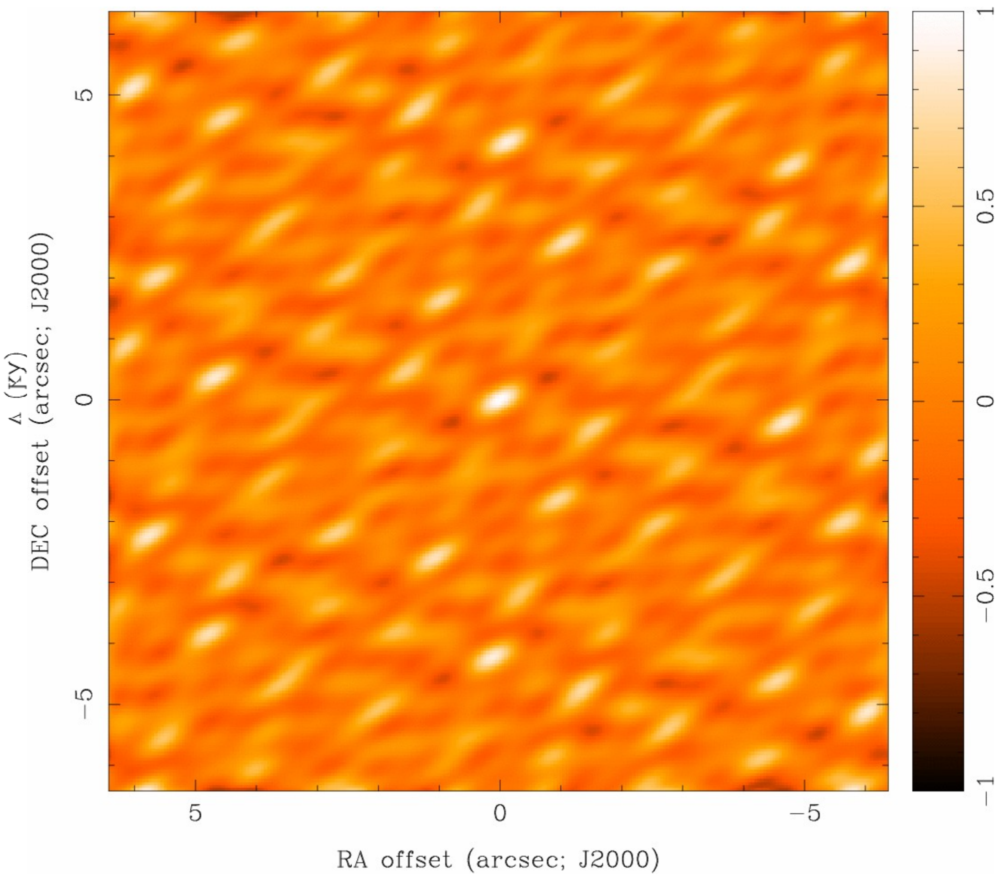


Dirty Beam Shape and N Antennas

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

6 Antennas





Dirty Beam Shape and N Antennas

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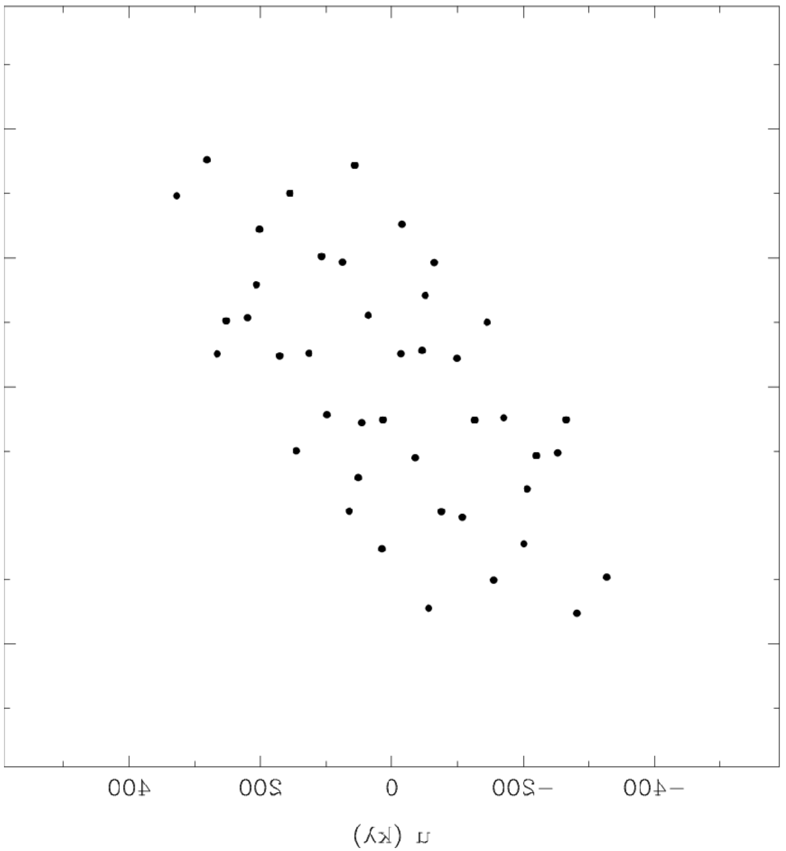
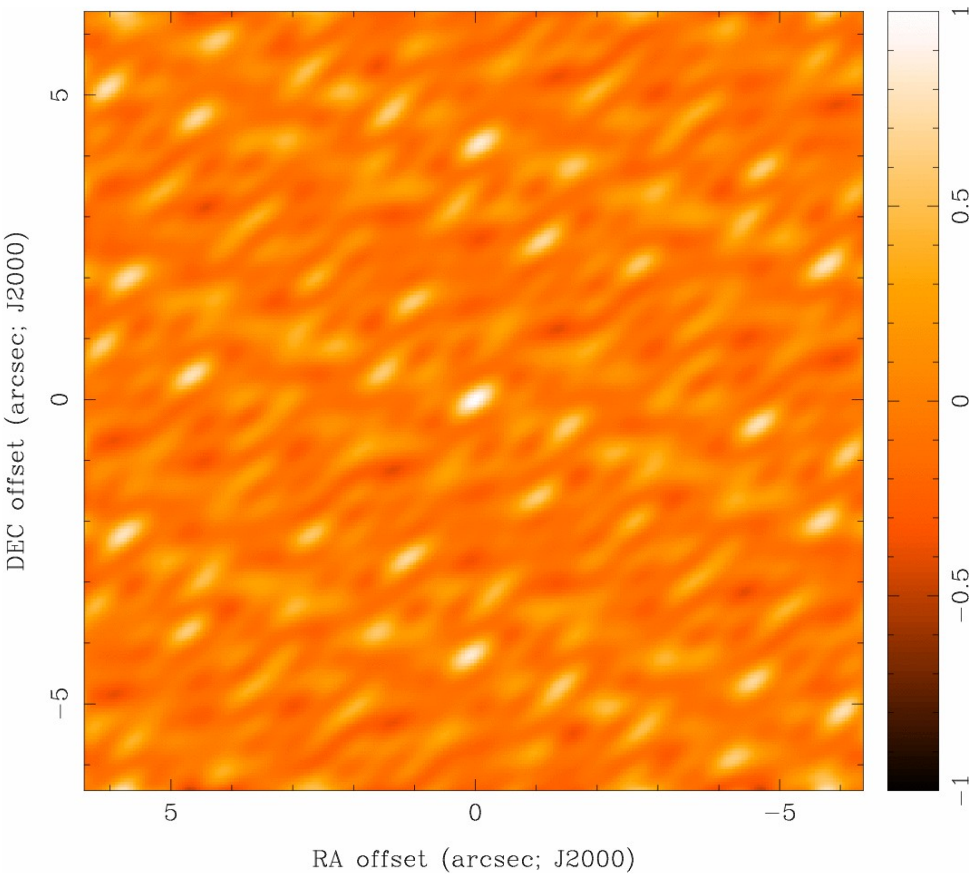
7 Antennas





Dirty Beam Shape and N Antennas

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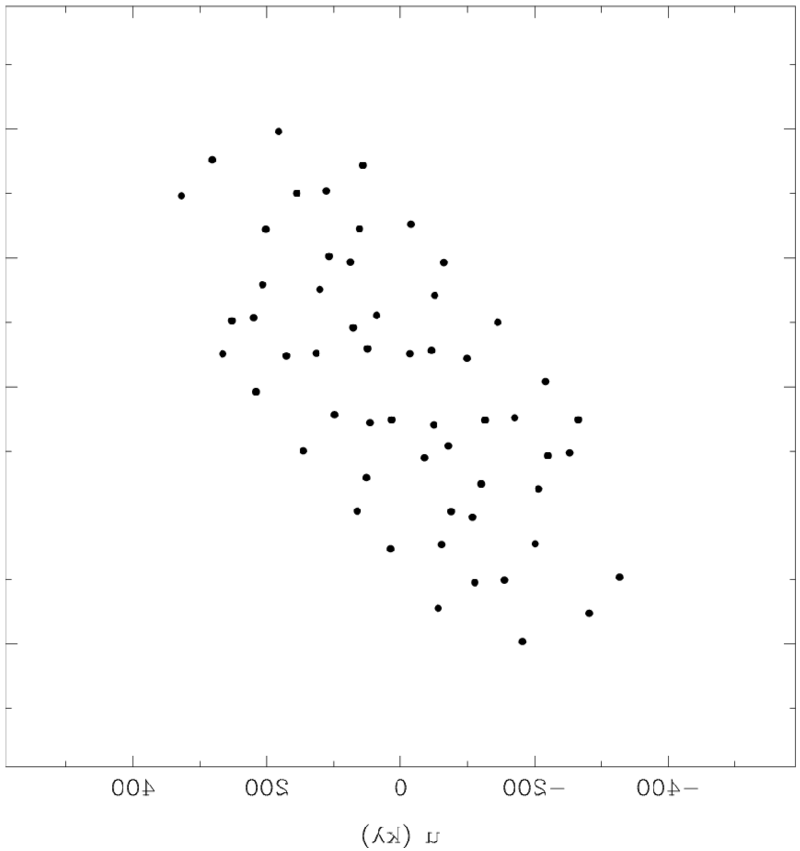
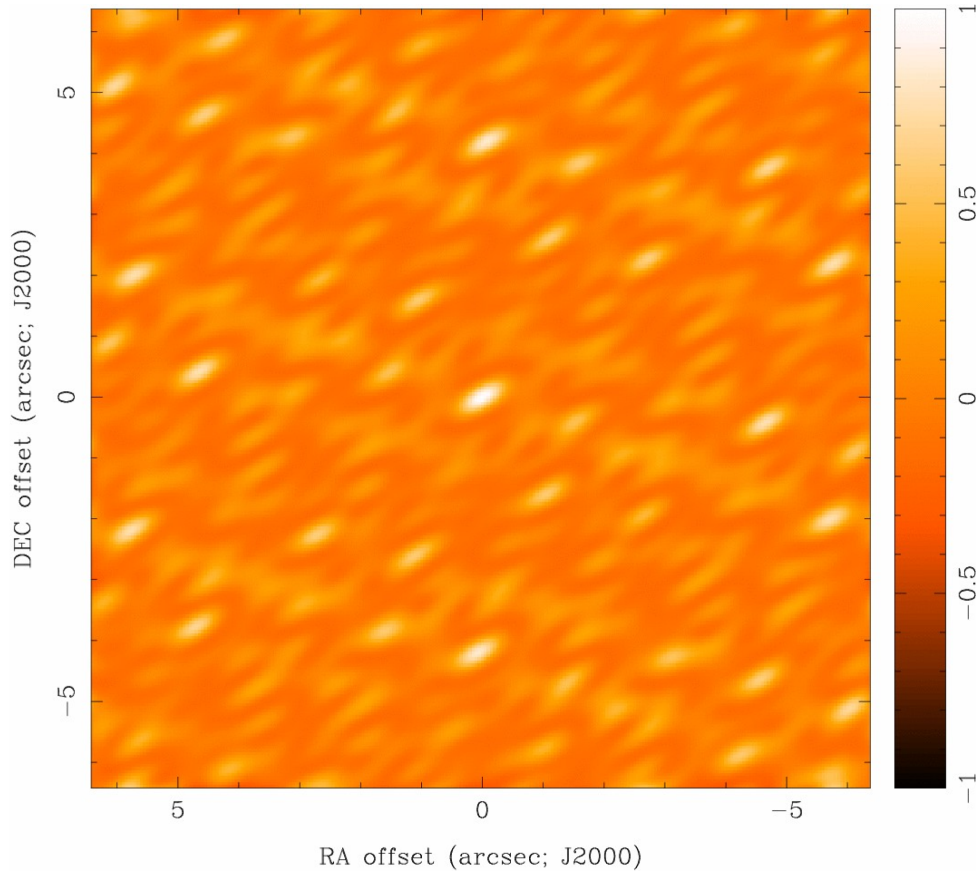
8 Antennas





Dirty Beam Shape and N Antennas

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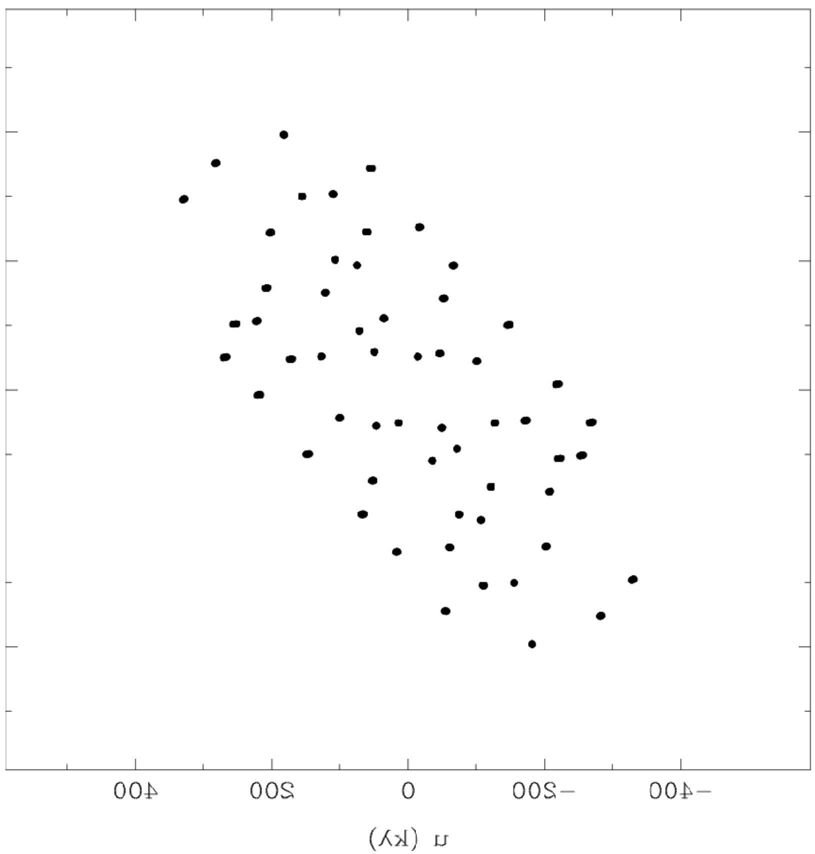
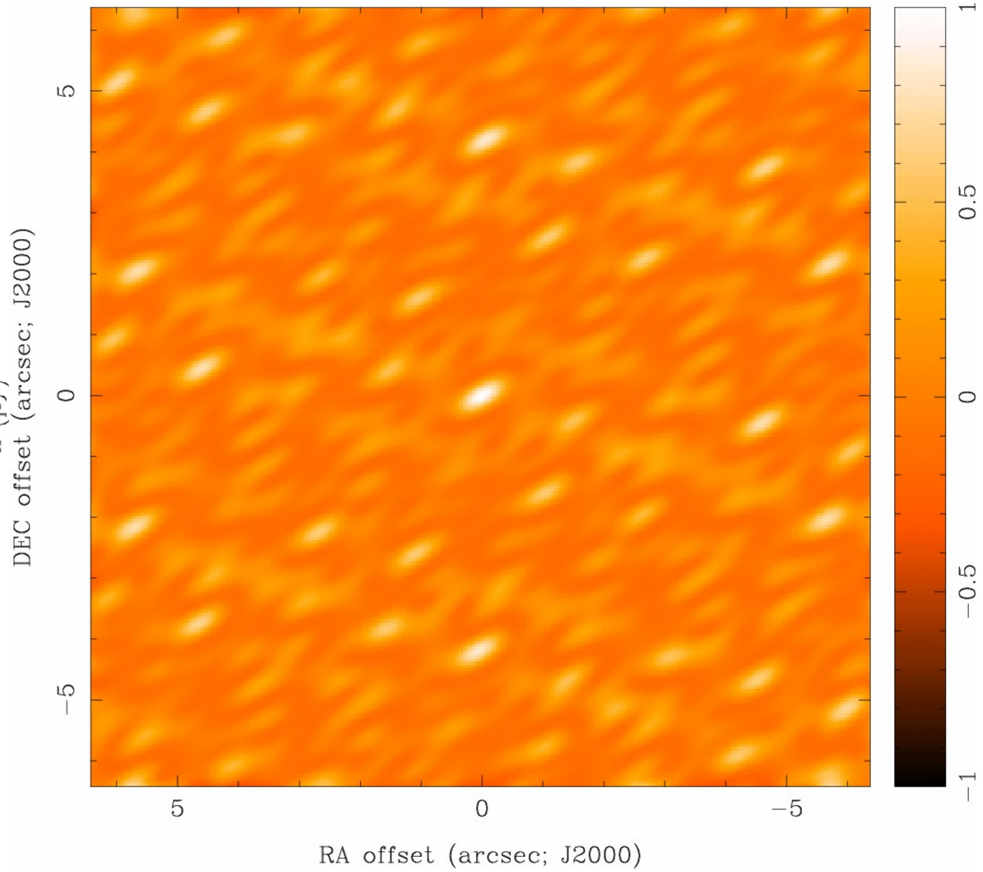
8 Antennas x 6 Samples





Dirty Beam Shape and N Antennas

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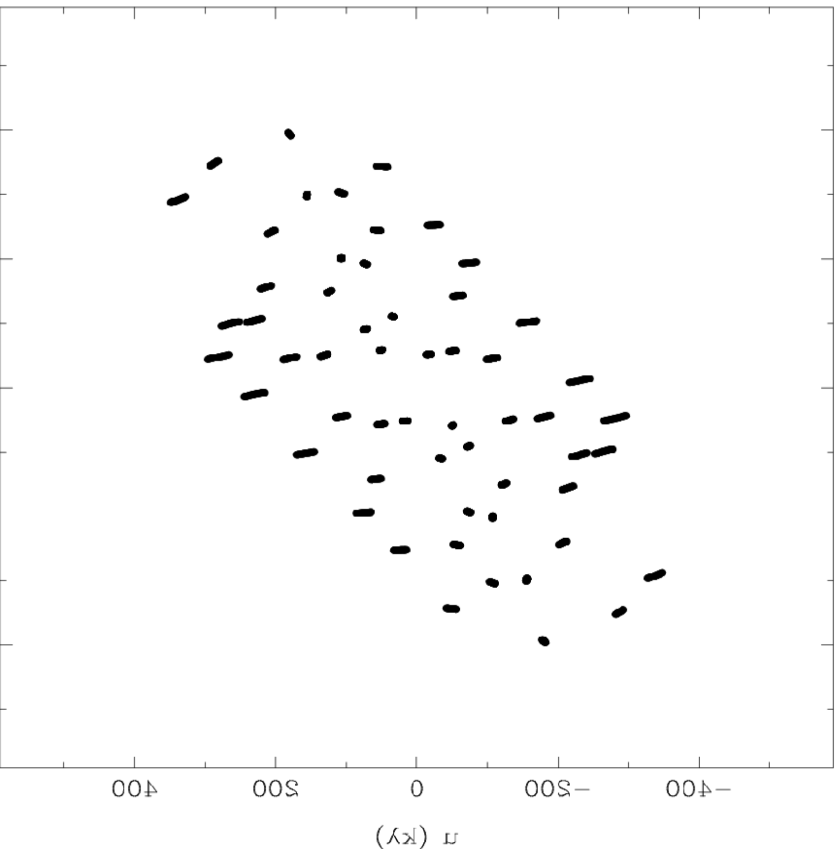
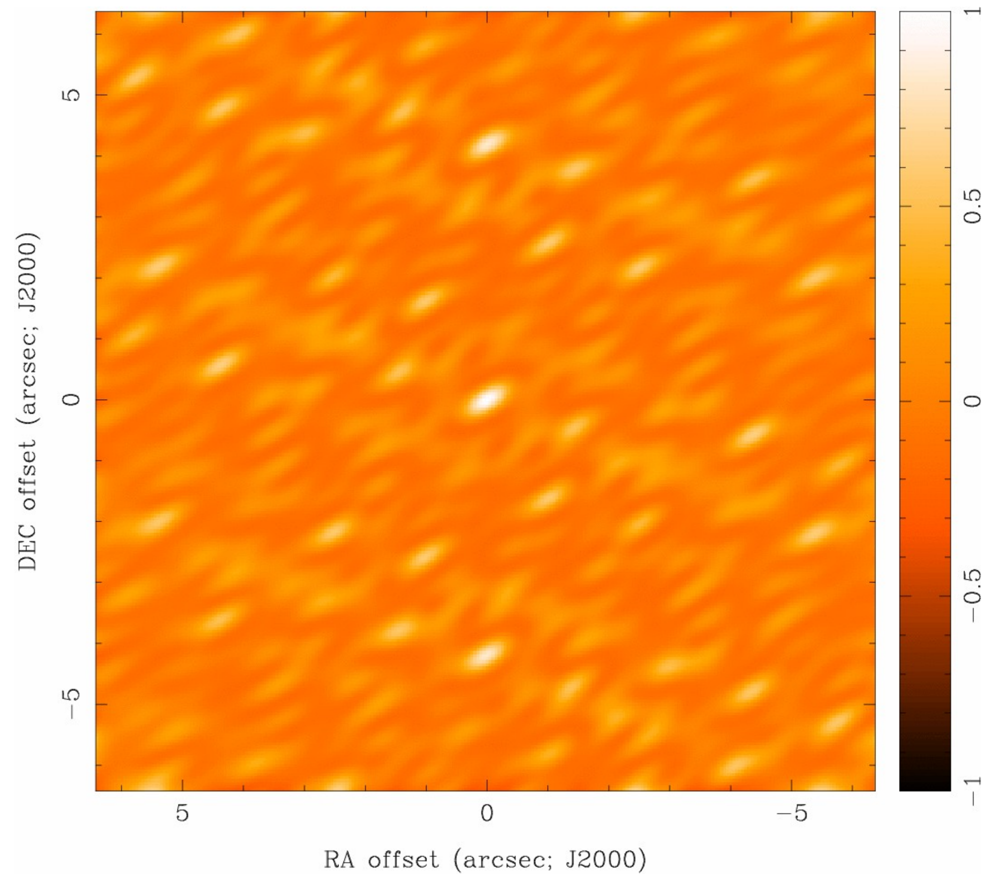
8 Antennas x 30 Samples





Dirty Beam Shape and N Antennas

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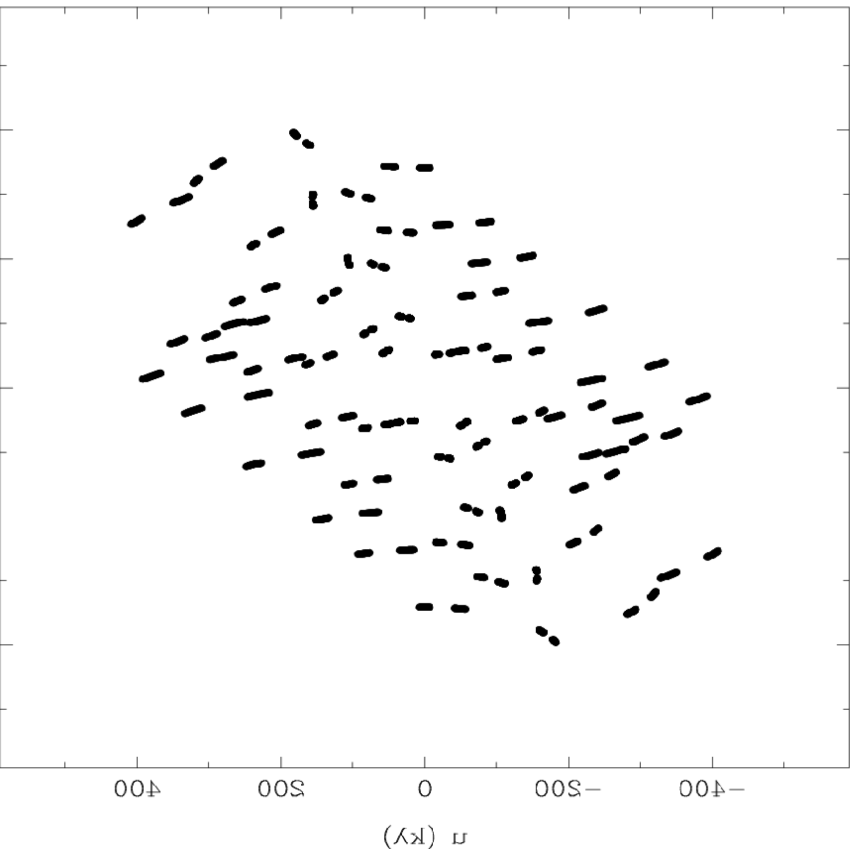
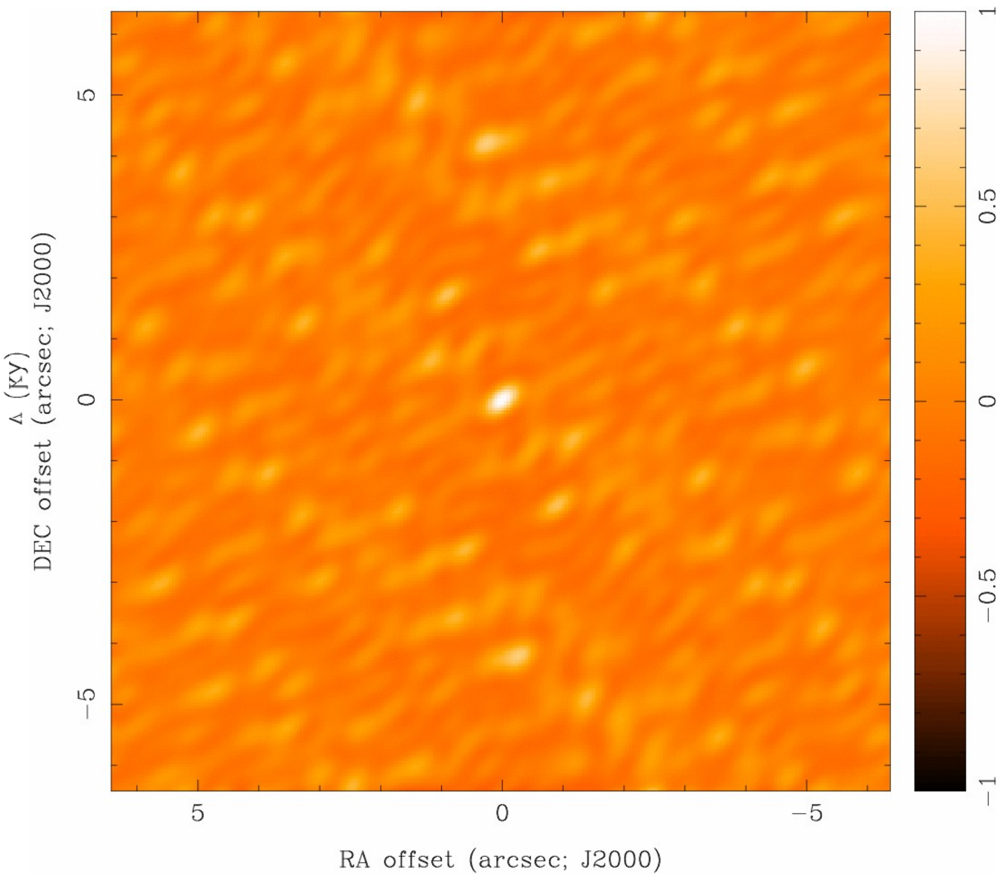
8 Antennas x 60 Samples





Dirty Beam Shape and N Antennas

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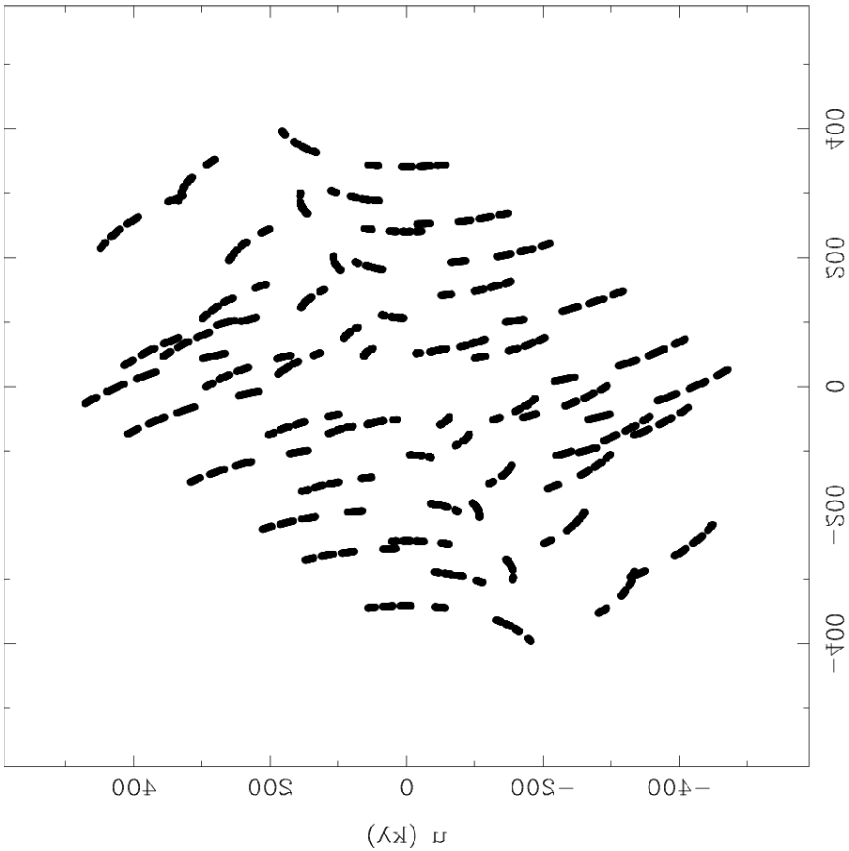
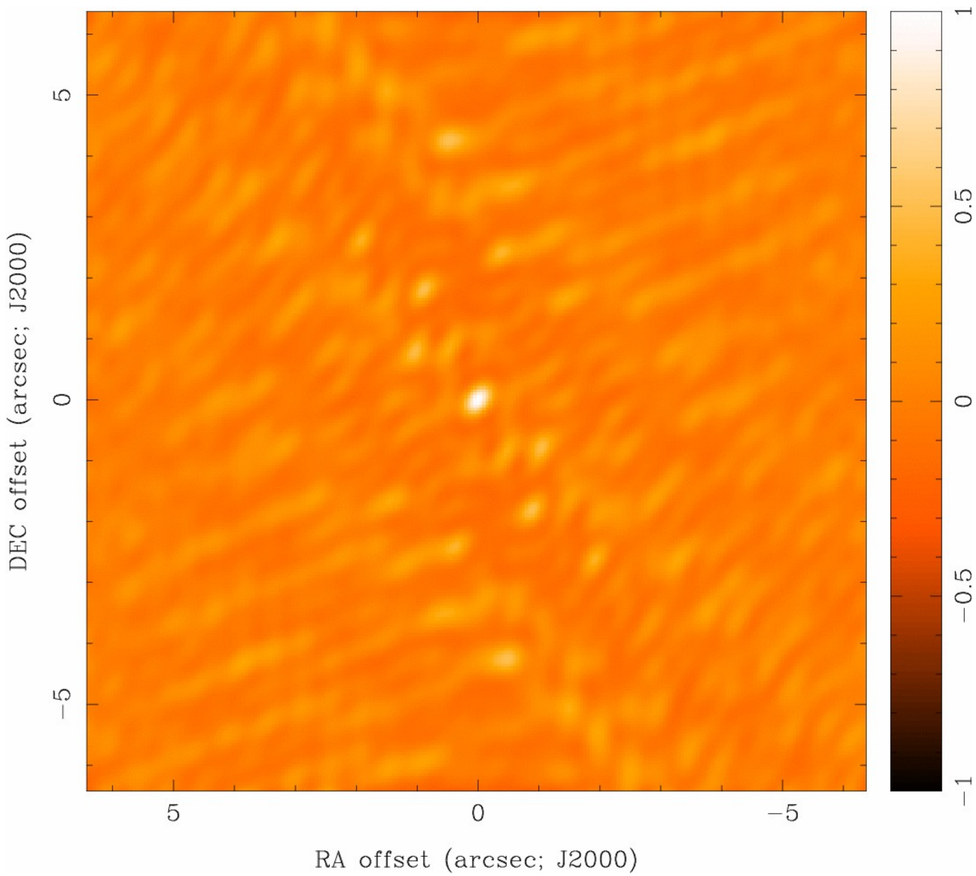
8 Antennas x 120 Samples





Dirty Beam Shape and N Antennas

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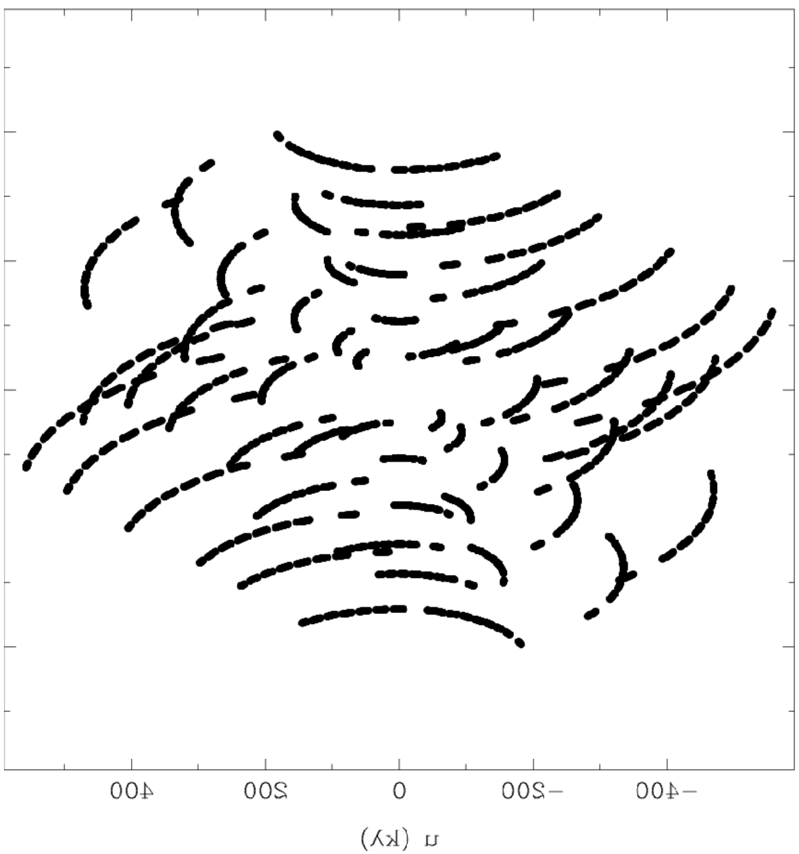
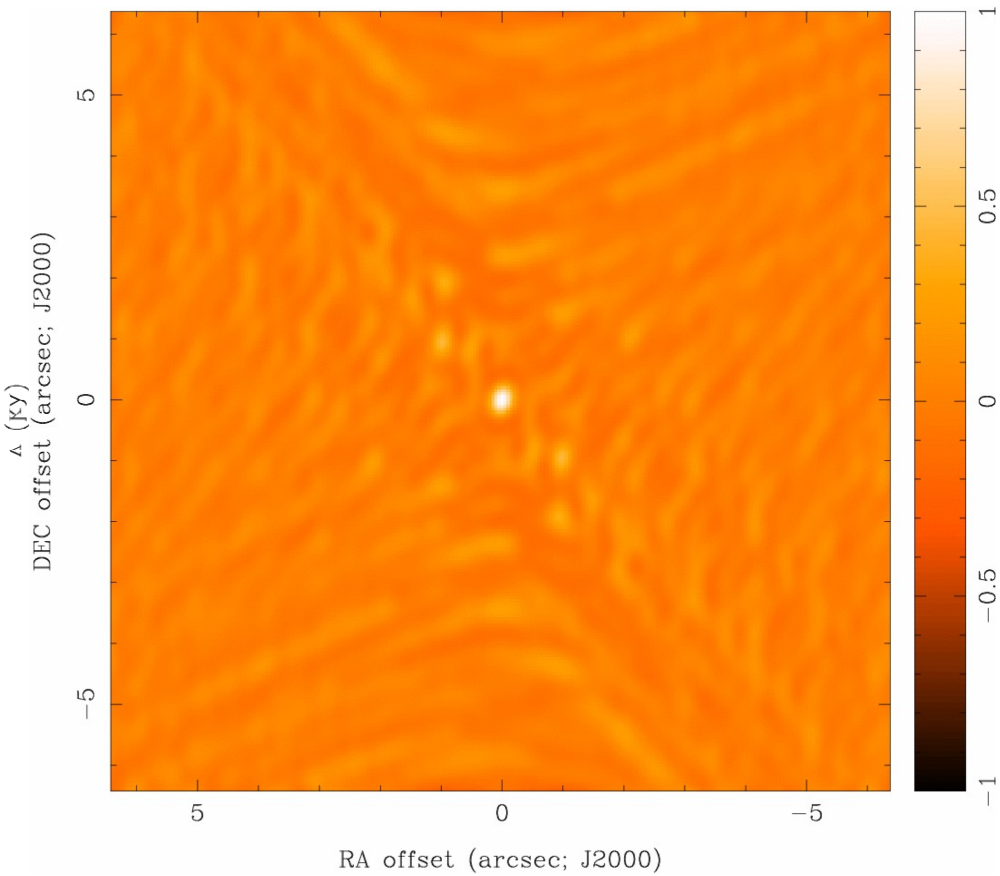
8 Antennas x 240 Samples





Dirty Beam Shape and N Antennas

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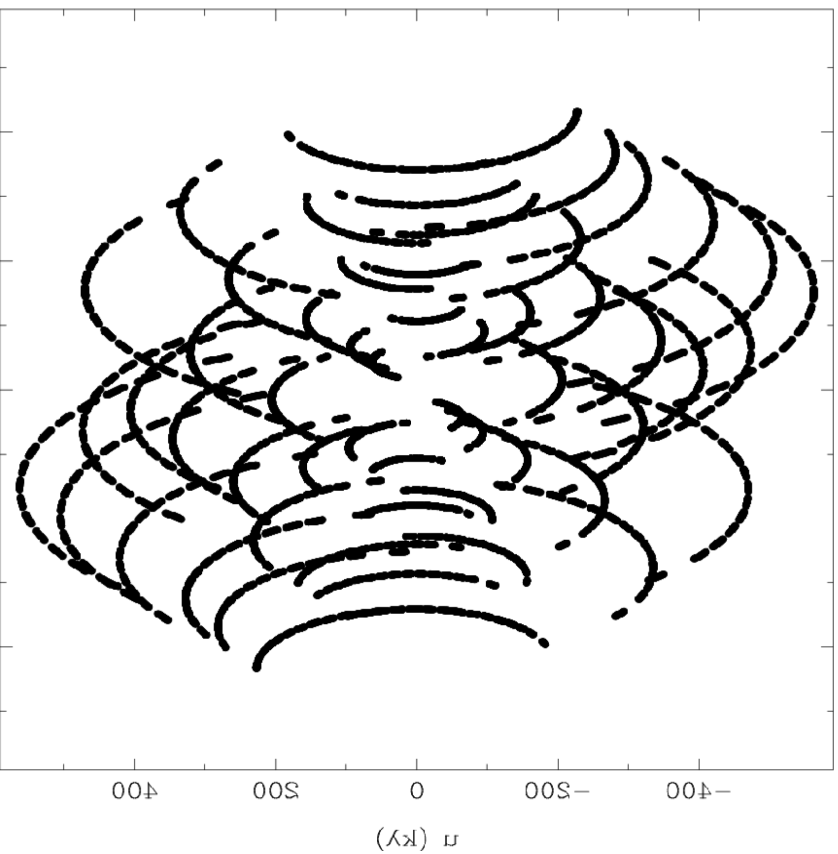
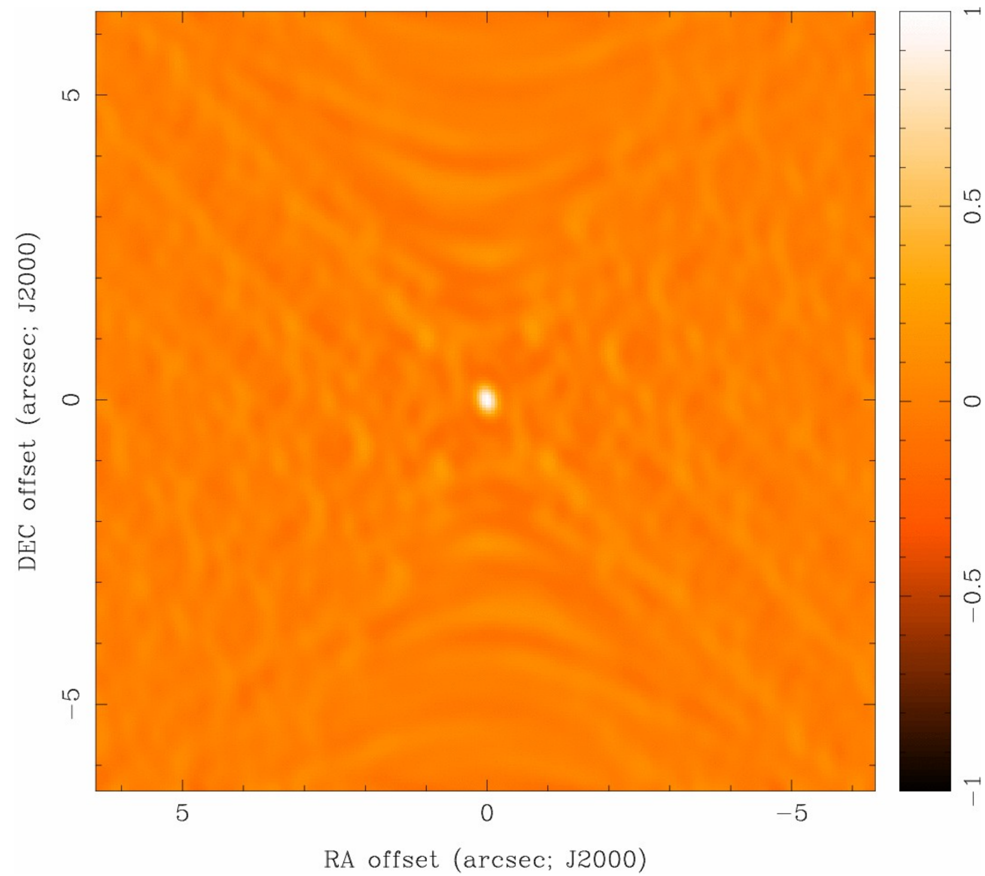
8 Antennas x 480 Samples





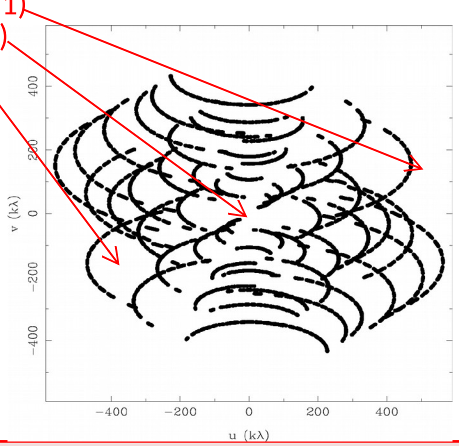
Dirty Beam & missing Fourier Components

N.B. same as previous slide but with contrast enhanced better to show the sidelobes in the synthesised beam

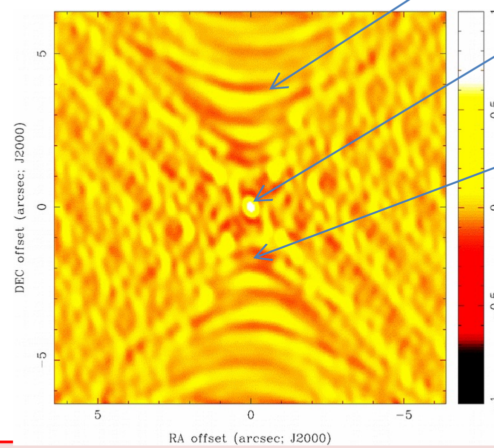
The F.T. of the baseline coverage (u,v plane) is the "Dirty Beam" or the "point spread function" (psf).

"missing data" (1) (2) (3)

(u,v plane)



Dirty Beam $S(\theta,\phi)$



sidelobes generated by poor sampling

main lobe of synthesised beamwidth $\sim 1/u_{max}$; $\sim 1/v_{max}$

negative regions – not physical !

aging Workshop Socorro, June 9, 2010

Not all Fourier Components of $T(\theta,\phi)$ are sampled → "missing data

(1) no data beyond u_{max}, v_{max} → unresolved structure → limit on smallest detail in image

(2) no data within u_{min}, v_{min} → failure to measure total power leads to unphysical negative regions
missing short baselines limits largest size features in image
($\sim 1/u_{min}$ x $\sim 1/v_{min}$)

(3) "holes" in (u,v) coverage → sidelobes which confuse the image

There are an infinite number of brightness distributions $T(\theta,\phi)$ ("invisible" distributions") compatible with the sampled $V(u,v)$ since we can put *any* values in the unsampled *u,v* regions. The Dirty Map $T^D(\theta,\phi)$ (called the "Principal Solution") is just the one with $V(u,v) = 0$ at the unsampled points

Recovery of improved images: deconvolution

(Convolution theorem: Lectures 7+8 and these lectures Slides 9&10)

$T(\theta,\phi)$	*	$S(\theta,\phi)$	$\equiv$	$T^D(\theta,\phi)$
true sky distribution	**convolved**	**synthesised beam ≡ "Dirty Beam"**		**"Dirty Map" of sky**
↕ Fourier Transform		↕ Fourier Transform		↕ Fourier Transform
Visibility Function	**X**	**Transfer Function (*u*,*v* coverage)**	**=**	**sampled Visibility Function**

The transfer function filters the Visibility Function:

i) cuts out angular frequencies $>u_{max}$ cycles/radian → resolution is fundamentally limited

ii) only passes finite number of samples – results in a distorted image of the sky ("Dirty Map") full of confusing fluctuations (including negative ones) which limit the science interpretation.

HOW DO WE MAKE A BETTER MAP?

Pulsar dedispersion (**Lectures 17+18 slides 6&7**) is a deconvolution process achieved by reweighting the complex Fourier Components by *multiplying by the inverse of the transfer function* – this "linear" approach (no new frequencies) works fine – since all frequency components are already present.

Interferometric imaging: multiplying by the inverse of the (*u*,*v*) coverage does not work since it is full of unsampled regions. Multiplying by "1/0" produces infinities !→ cannot use "linear methods"

Deconvolution in the sky plane: general principles

- Putting zero at the unmeasured (*u*,*v*) points produces negative regions after Fourier Transforming the data to become the "Principal Solution" image – **so this *must be* wrong**

- We have to find a way to effectively generate "new" information to interpolate or extrapolate into unsampled regions of the (*u*,*v*) plane.

- The fact that we generate "new" Fourier Components → process is "non-linear"

- Aim is to find a sensible model of $T(\theta,\phi)$ compatible with data and <u>also</u>
    - certain *knowledge* about $T(\theta,\phi)$ ***: the sky brightness is always positive***
    - some *assumptions about* $T(\theta,\phi)$ *: e.g.*
        - at high resolution and finite signal-to-noise ratio the sky is relatively empty and contains localised compact sources
        - at low resolution the sky is mostly smooth with only a few point sources

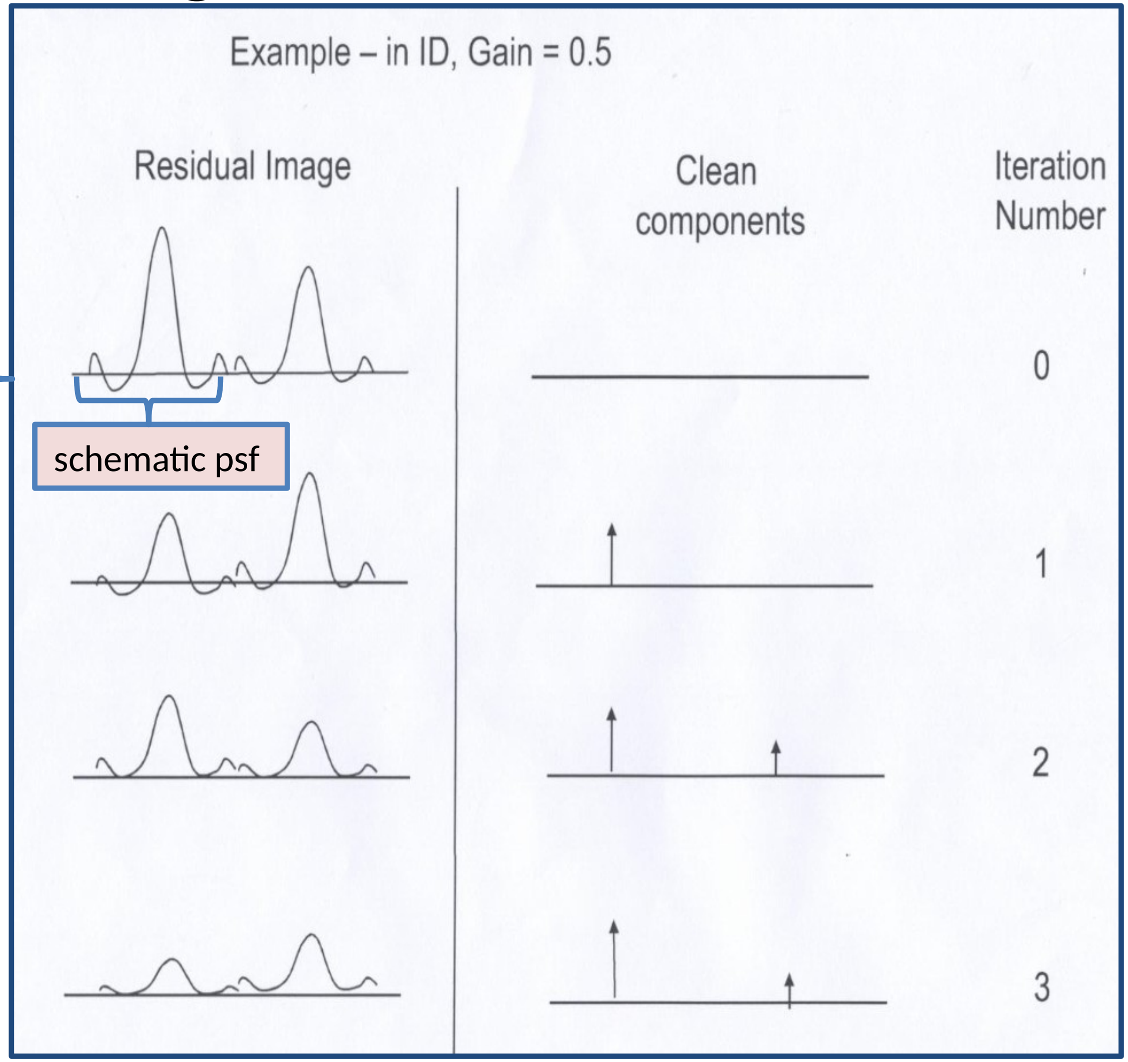
- Different *assumptions* require different deconvolution techniques to get the best results

- Here we describe one commonly-used example (CLEAN) for a simple problem.
    - improving the image of an isolated compact source

Basic CLEAN Algorithm in 1-D

1. Find peak in current map ("*Residual Image*") and identify it as the location of a δ-function convolved with the Dirty Beam (DB).
2. Subtract a fraction ("*Gain*") g<1 times the DB from the map and save a δ-function of flux (g x peak) at this location (a "*Clean component*")
3. Iterate until nothing left but noise in Residual Image
4. Convolve δ-functions by a "clean beam" – often a gaussian having no sidelobes - of same FWHM as main lobe of the DB.
5. Add back the Residual Image

NOT SHOWN



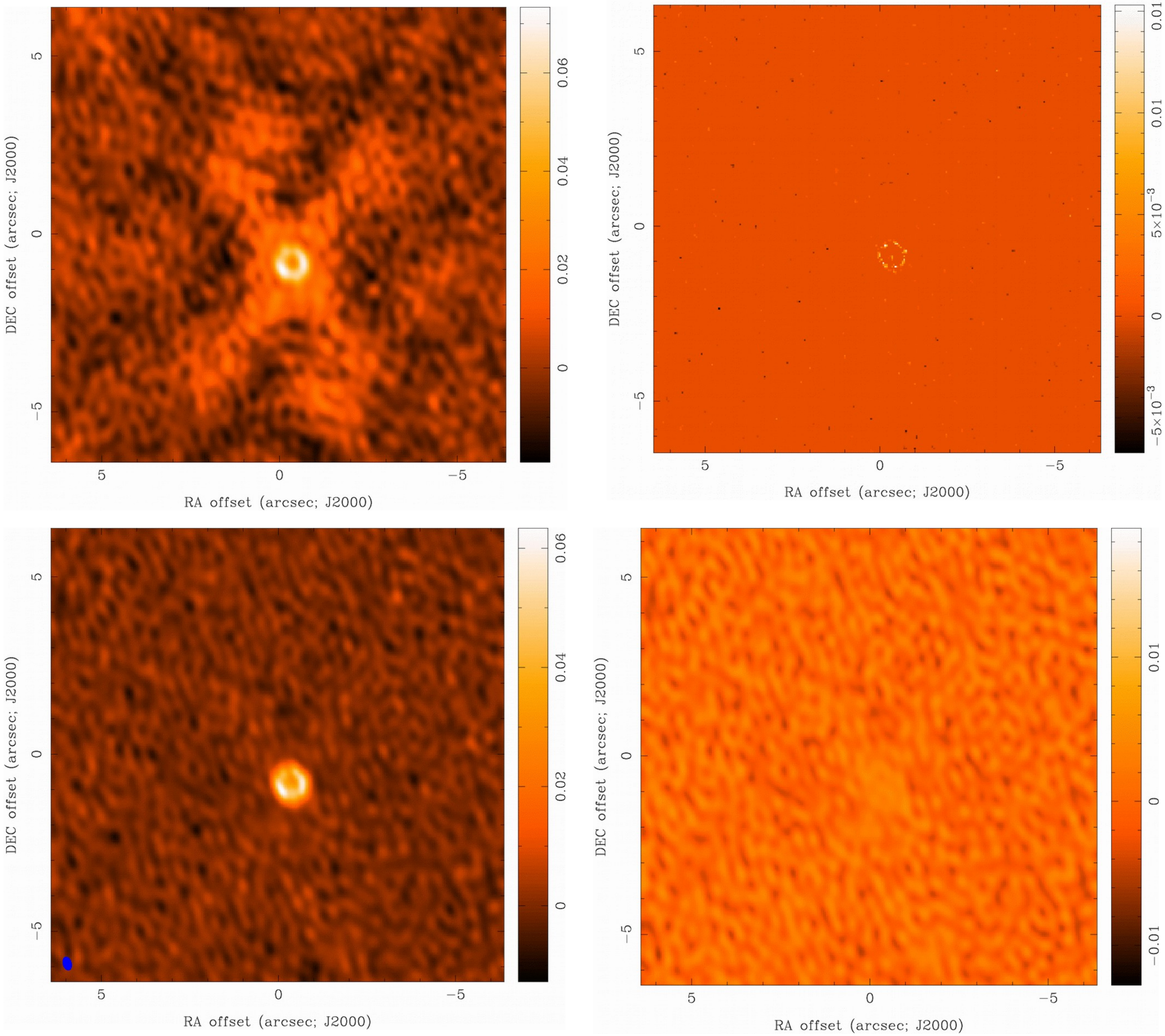
J.Conway http://www.not.iac.es/tuorla2009/local/TurkuLect3.pdf

CLEAN

Adapted from D. Wilner 12th Synthesis Imaging Workshop Socorro, June 9, 2010

$T^D(\theta,\phi)$

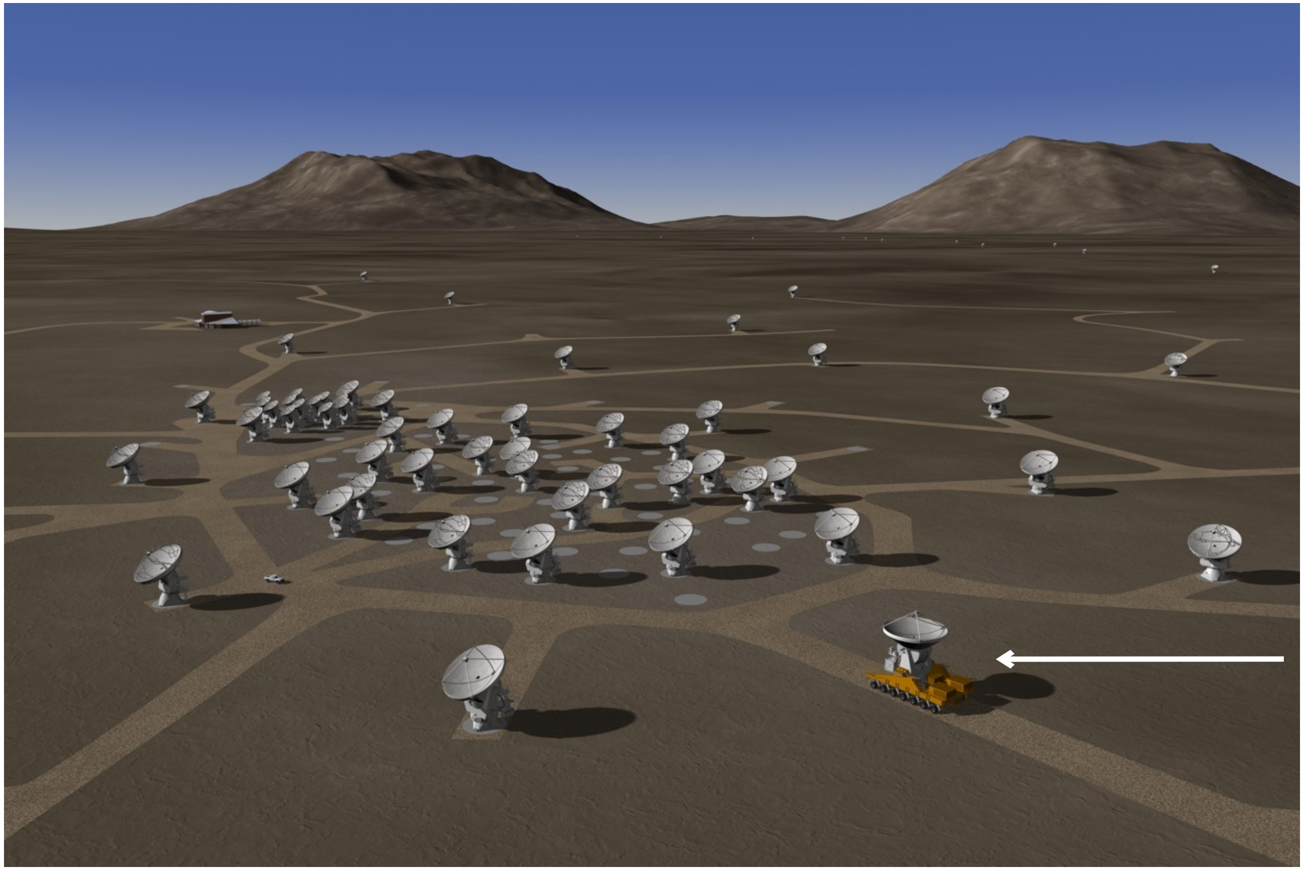
CLEAN model



restored image – is a better estimate of $T(\theta,\phi)$

residual map

The future: ALMA (now fully operational)



Moving an antenna !

The Atacama Large Millimetre Array (ALMA) is situated on the high Andean plain at an altitude of 5000m – to reduce effect of atmosphere at mm to sub-mm wavelengths. ALMA has about 60 telescopes most of which are 12m in diameter and can be moved into new configurations (see one on a moving vehicle in this artist's impression)

The Future: Square Kilometre Array (2020+)

http://www.skatelescope.org/photo/Dishes_overview_compressed.jpg

The SKA is the next generation giant radio telescope – to be located in Australia and South Africa. As shown in this artist's impression one of its main components will be 1000s of 15-m diameter dishes connected as a huge interferometer array. (South Africa).

THE END

Various slides from Manchester lectures not used

Geometrical Delay τ_g & Phase Difference ϕ

- $\boldsymbol{\tau_g = \underline{b} \cdot \underline{s} / c}$ (seconds)

  $\underline{b}$: baseline vector (metres)

  $\underline{s}$: unit vector towards source

  c : speed of light

- $\boldsymbol{\tau_g = b \sin\theta / c}$ (seconds)

- $\boldsymbol{\tau_g = b \sin\theta / \lambda}$ (wavelengths) ≡ # cycles of the RF signal

- $\boldsymbol{\phi = \omega\tau_g = 2\pi \, [b \sin\theta / \lambda]}$ (radians) ≡ total phase difference ϕ

Final Reminder: Main features of the Visibility Function

- *An instantaneous baseline vector* ***b*** *acts as a spatial filter picks out a single complex Fourier Component* $\mathbf{A}e^{i\psi}$. An analogy is a stretched string resonating at its fundamental frequency when sound with a range of frequencies hits it.

- *Measuring the visibility amplitude* ***A or (|V|)*** *is usually straightforward*. As **|b|** increases, **A** decreases – the source is said to be "resolved". When **A** ~ 0, the source is "totally resolved".

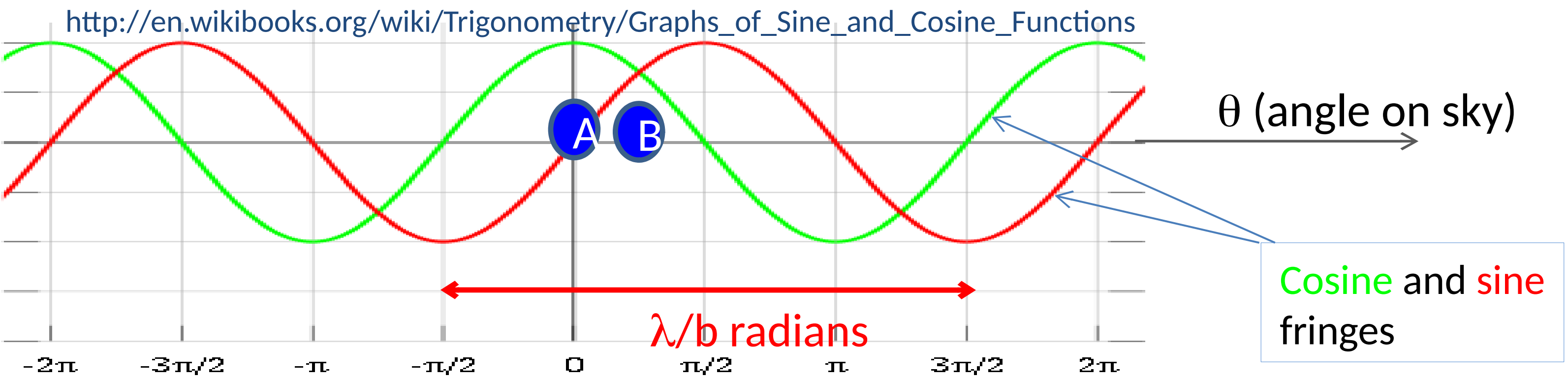
- *Measurement of the visibility phase* ψ *requires a precise knowledge of the geometry of the baseline (with potential to be affected by differential propagation delays)* in order to be able to position the fringes precisely with respect to the source. Each phase is the position of one Fourier Component with respect to an assumed source centroid position. For a point source (i.e. and infinite number of Fourier components all lined up at one point) the phases all give the source position (basis of astrometry).

- *The complex visibility function* ***V*** *is the same thing as the spatial coherence function -the Van Cittert-Zernicke Theorem*. Many different samples of the visibility function enable reconstruction of the brightness distribution of an extended source in the process known as "aperture synthesis" .

- *A correlation interferometer on its own can never measure the true brightness of an extended source* whose size is comparable to, or larger than, the primary beam of the antennas – since it cannot form very short or "zero" baselines (the antennas obstruct each other). It can, however, measure the brightness or the total flux density of a compact source *(next slide)*.

Adapted and revised from R. Perley NRAO 12th Synthesis Imaging Workshop 2010

Astrometry : Geometry – not Fourier



A compact source may lie at different positions *relative to the fringe patterns on the sky.* The fringe spacing and angle on the sky is set by the baseline vector (length and direction) and the τ_g delay settings (exact position of the peak of the cosine and sine fringes – see slide 23 RULE 2).

The interferometer is sensitive to the source position via a phase change $\Delta\phi = \tan^{-1}[R_{sin} / R_{cos}]$ compared with the phase at the expected position of the source – slide 23 RULE 1. For example:

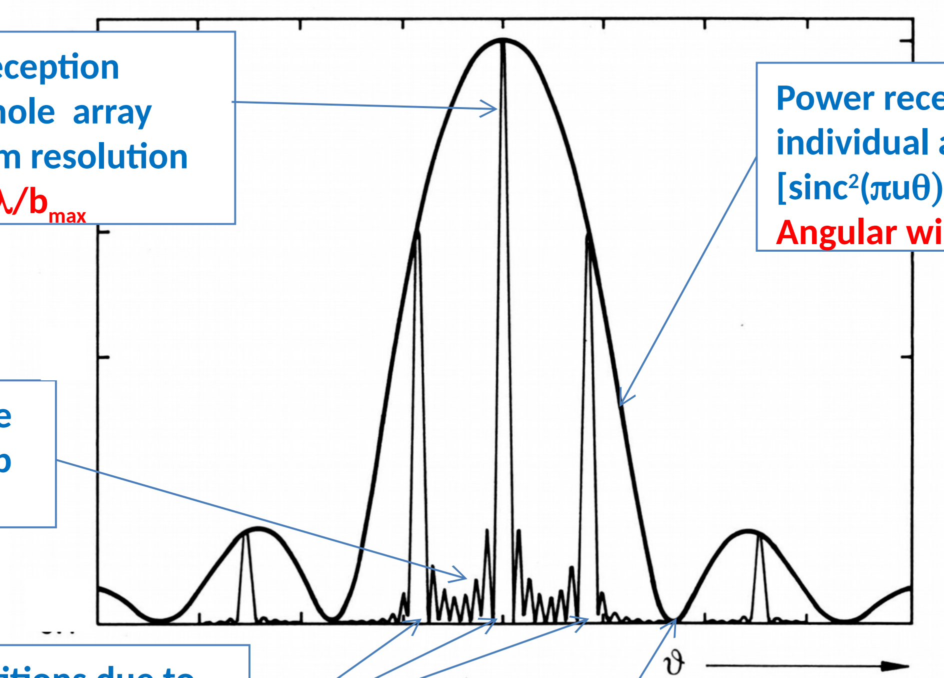
- at A $\Delta\phi$ = 0 rads (or n x 2π) – source where expected; cosine fringe positioned exactly
- at B $\Delta\phi$ = $+\pi/4$ rads; source is 1/8 of a fringe period away from expected position

NOTE:

- As b increases the fringe period λ/b decreases and hence $\Delta\phi$ – which is expressed in terms of fringe periods - gets larger → *longer baselines are more sensitive to position shifts on sky.*
- If the initial position of the source is not known to within 1 fringe period then a single phase measurement (as above) will be ambiguous. Can recover by tracking the source and hence allowing the baseline vector to change.

Phased array pattern produced by an adding beam-former

“Direct” imaging: an instantaneous beam (the “point spread function”) is formed. If a source (e.g a pulsar) is smaller than the main lobe its flux density can be measured. If the source is bigger than the main lobe that its brightness distribution be could be built up by scanning the entire beam across the source – in this case electronically – but just as for a single dish as in Lecture 3 – but this is not the main use of phased arrays.



Main lobe: the reception pattern of the whole array sets the maximum resolution
Angular width ~ λ/b_{max}

Power reception pattern of individual antenna
[$\mathrm{sinc}^2(\pi u\theta)$ for square antenna]
Angular width ~ λ/d

“Sidelobes” due to finite extent of array i.e. sharp cut-off at b_{max}

Periodic repetitions due to periodic spacing
Angular spacing $\Delta\theta = \lambda/b$

First minimum in primary pattern $\theta \sim \lambda/d$

Like the Fraunhofer pattern of a diffraction grating

Point spread function (reception beam) of a phased array

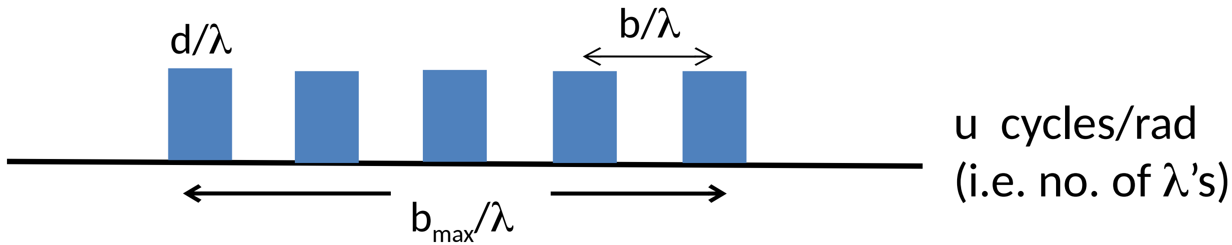
Image = [true brightness distribution] * [point spread function]

CONVOLUTION THEOREM

Point spread function = F.T. [ACF of aperture distribution]
(the reception beam)

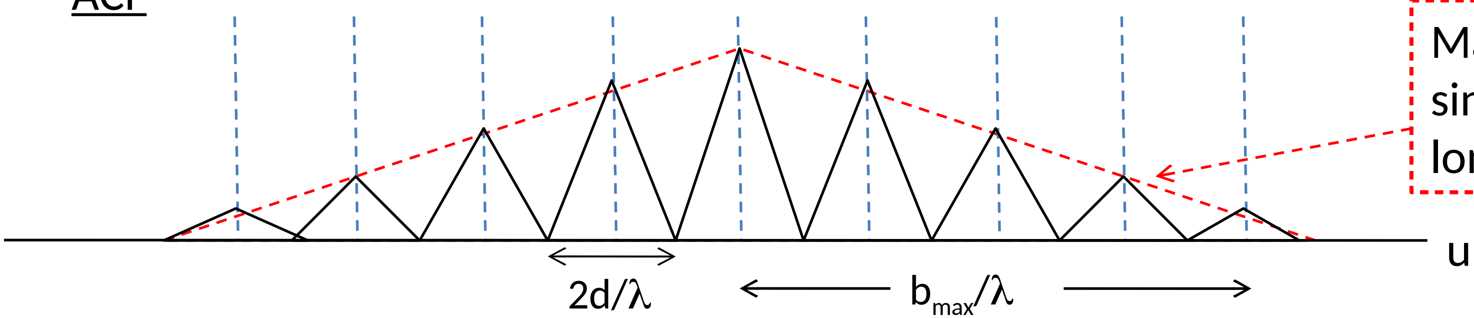
WIENER-KHINCHINE THEOREM

<u>Aperture distribution</u> (equally spaced array with spacing equal to antenna size)



Extension of ideas in lecture 3

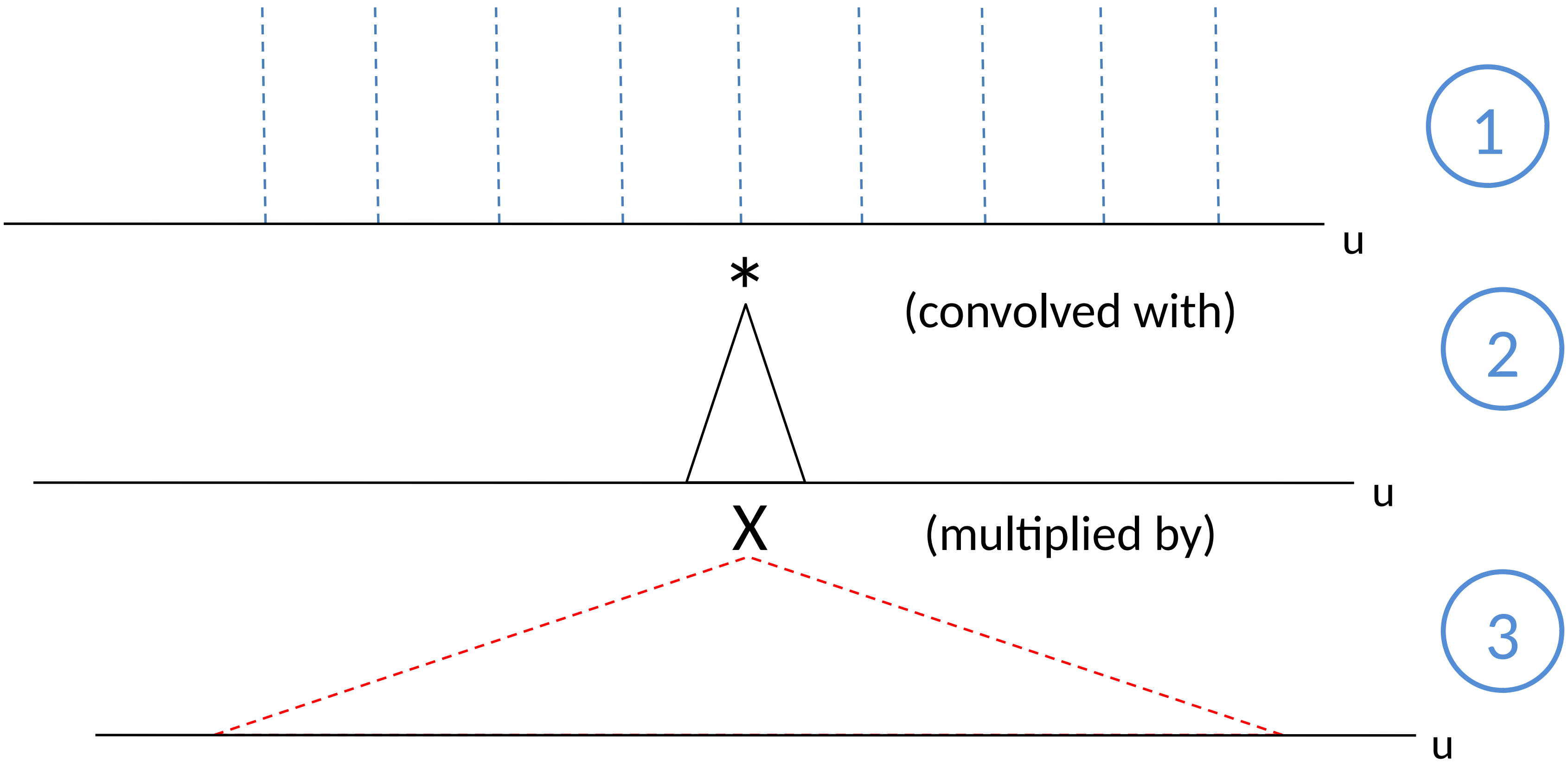
<u>ACF</u>



Magnitude falls off since fewer copies of long baselines

Point spread function (reception beam) of a phased array

ACF can be split up into…



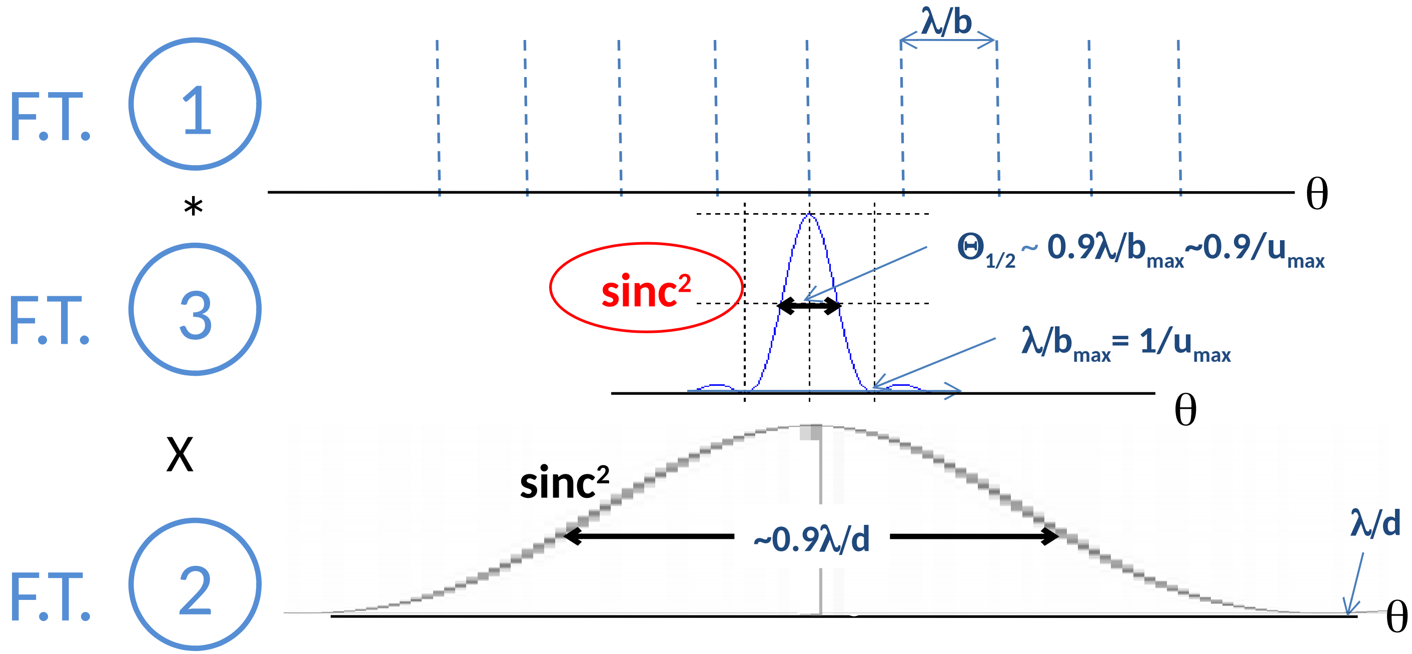
Point spread function (reception beam) of a phased array

Convolution theorem (Lecture 3) works both ways i.e.

$$FT\,[\,f(t) * g(t)] = F(\nu) \times G(\nu)$$

$$FT\,[f(t) \;x\; f(t)] = F(\nu) * G(\nu)$$

Hence the reception beam power pattern is made up as follows:



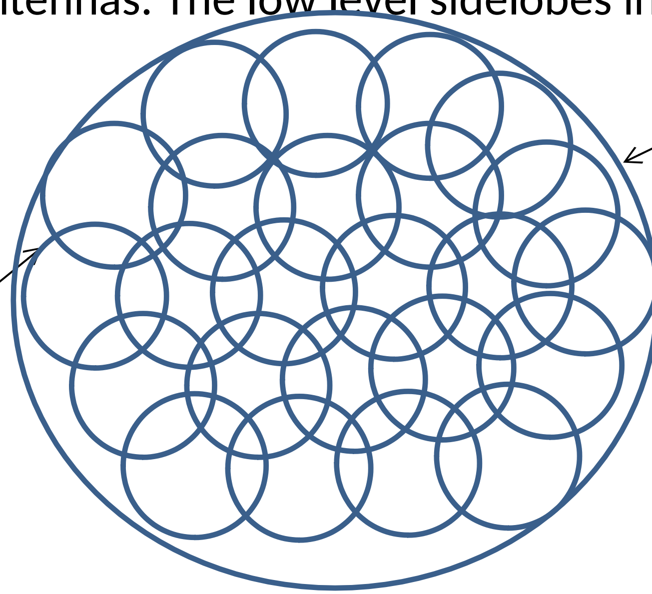
Result in next slide: smallest features in beam come from largest features in aperture
largest features in beam come from smallest features in aperture

Phased array beam structure
(towards the source)

The pattern in slide 14/15 corresponds to a “sparse” array which has large spacings between the collecting antennas giving rise to the large periodic sidelobes on either side of the main lobe

In practical arrays the antennas will be closer together (very close in aperture arrays) and there may be very many array beams (*now dominated by the main lobes*) within the reception pattern of the individual antennas. The low level sidelobes in slide 15 have now been suppressed for clarity.

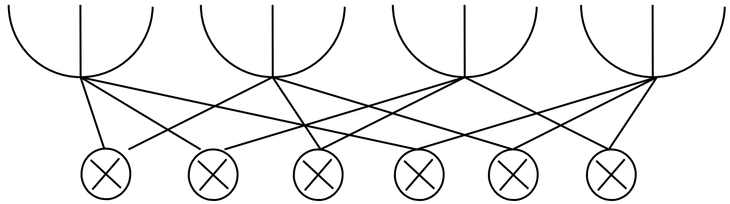
Independent main lobes from the array antennas combined with different phase gradients in 2-D
Angular width ~ λ/b_{max}



Reception pattern of individual antennas
Angular width ~ λ/d

Adding phased arrays are ideal for detecting and locating pulsars & transient sources – they provide an instantaneous response with better positional information than from a single antenna – if all the beam-forming electronics can be afforded !

Multiple correlation interferometer: summarised



- Four collector elements connected together as a correlation array where the outputs from all the elements are multiplied and integrated (i.e. correlated) with all the other elements. There are N(N-1)/2 such correlations – measuring different parts of the "spatial coherence function".

- Each correlation (partial mutual coherence across the incoming wavefront) is a measure of an independent Fourier component at any given time (the Van Cittert-Zernicke Theorem)

- From many baselines (length and orientation) one builds up part of the complete Fourier component spectrum (the "Visibility Function") for that brightness distribution.

- This is **"Indirect imaging"**: since the visibility data are in the Fourier plane and the final image is made by the Fourier transform of the (sampled) complex visibility function.

(Other forms of indirect imaging:

- *X-ray crystallography - can only measure the intensity of the scattered waves so does not have direct access to the Fourier phase as we do in radio interferometry. Has to use tricks to get phase information.*
- *Holography - where the phase information is partially captured via intensity fringes in the hologram formed by adding with a non-scattered reference wave.*

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Effect of missing short baselines

R. Ekers Synthesis Imaging Workshop, Narrabri 2006

Paul Rayner 2001

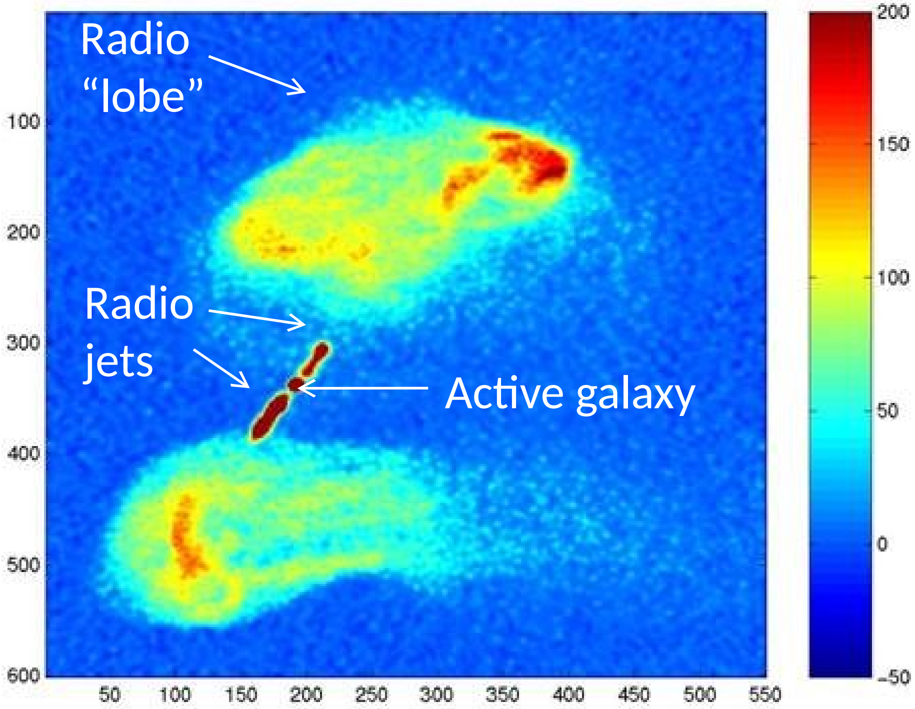


Image from a well-sampled u,v plane

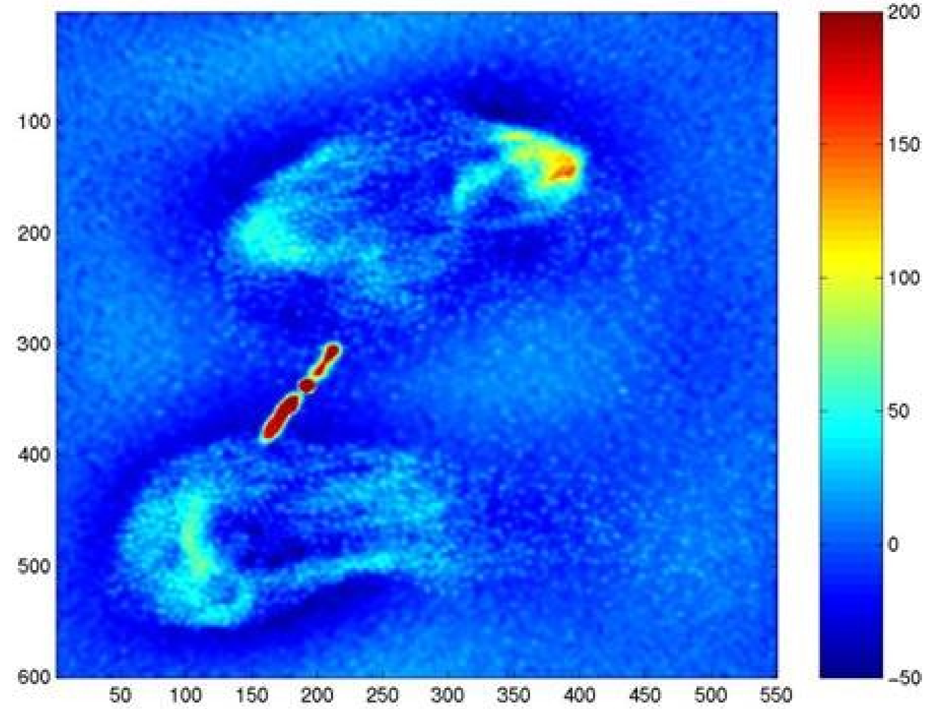
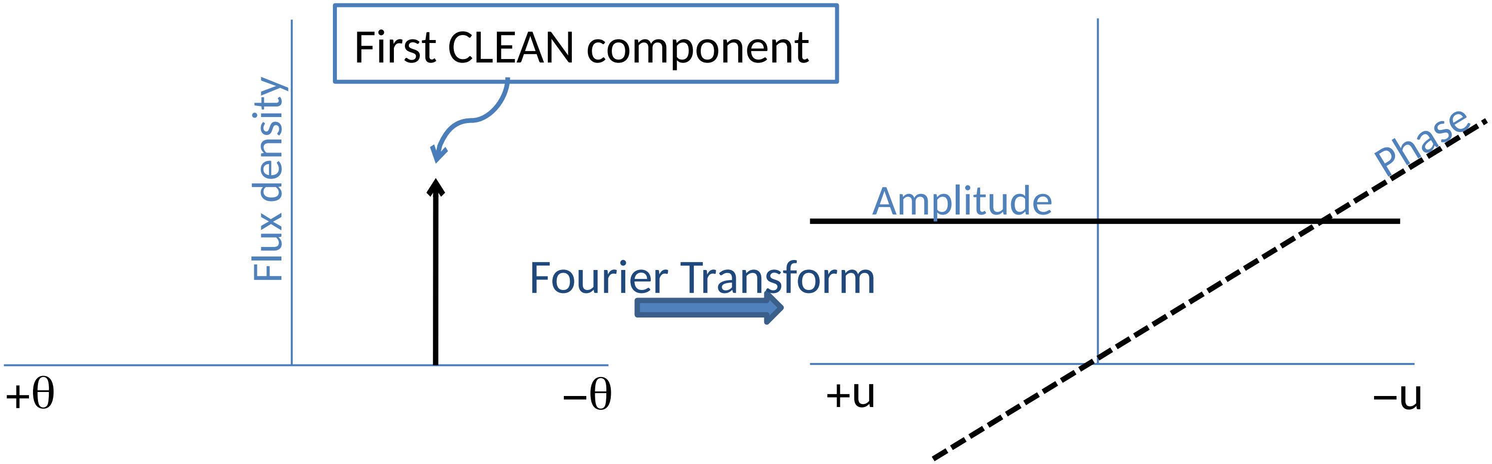


Image involving no short baselines:

Radio interferometer images of a double-lobed radio galaxy

Missing short baselines results in poor sensitivity to low brightness extended structure.
Notice the negative “holes” (darker blue) underlying the bright structure.

The non-linearity of CLEAN



The amplitudes of the Fourier Components of a δ-function are constant at all angular frequencies. For an off-centre δ-function the phases of the FCs vary linearly as a function of frequency (gradient increases as offset increases – this is the so called "shift theorem").

The very first CLEAN component therefore makes predictions about the complex Visibility Function in regions where it has not been sampled. Of course it will not fit the measured data and will be a poor prediction of the unmeasured components, especially if the source, and hence its Visibility Function, has a complicated structure.

As more and more CLEAN components are added their Fourier Transform becomes a better and better fit to the measured data and is likely to be a better prediction of the unmeasured parts of the complex Visibility Function.

Correlation interferometer sensitivities

***POINT SOURCE**: flux density sensitivity ΔS (rms) = a single antenna of area equal to the total effective area of all N antennas added together (=NA_{eff})*

$$\boxed{\Delta S = 2kT_{sys} / NA_{eff} [\Delta\nu.\tau]^{1/2}}$$

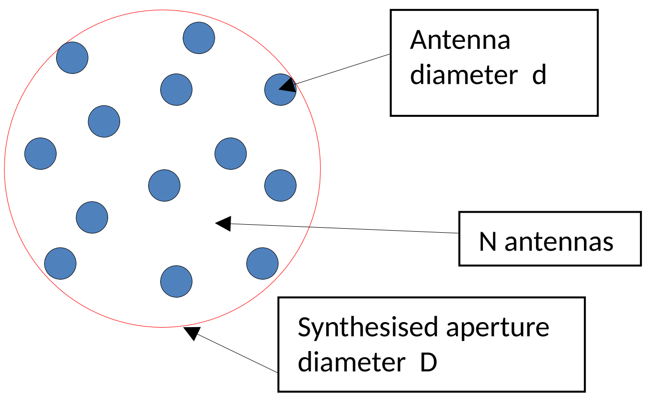
Derivation: $\Delta T = \Delta S.(NA_{eff})/2k$ (lecture Notes A)

$\Delta T = T_{sys}/ [\Delta\nu.\tau]^{1/2}$ (radiometer equation)

***EXTENDED SOURCE**: brightness temperature: ΔT (rms) = radiometer equation x 1/(array filling factor)*

$$\boxed{\Delta T = T_{sys} / [\Delta\nu.\tau]^{1/2} \text{ x } (D^2/Nd^2)}$$

(but can be worse if poor u,v coverage of short baselines)



The array filling factor is the ratio of summed area of all the antennas to the area of the synthesised aperture = N [d/D]^2.

N.B. add loss from digital correlators, sampling and quantizing of the input voltages e.g. 1-bit (two-level) sampling is 0.64 times as sensitive as a perfect analogue multiplier.

Correlation interferometers compared with single apertures

Advantages

1) Higher angular resolution: Set by baseline (antenna separation) – not size of antenna

2) Resistance to "confusion": Higher resolution means less blending of the population of discrete astronomical sources (randomly distributed across the sky) – which limit sensitive single telescopes *(not discussed in lectures* – but see **Lecture1 slides 11&12)**

3) Bigger collecting area possible: Single antennas limited in diameter to ~100m by cost of mechanical structure required to cope with: i) wind loading; ii) gravitational deformation; iii) pointing the beam to ~0.1 FWHM. Many smaller antennas can be connected electronically to produce larger area (SKA)

4) Resistance to uncorrelated signals: i) gain variations in independent receivers; ii) man-made signals iii) atmospheric emission above each antenna

5) Access to the complex electric field: Correlating the electric field at two points gives the phase difference of the electric fields
- in 2-D (far field) gives astrometric capability (positional information to 1/100th of fringe spacing
- in 3-D (near field) gives depth information as well *(research area: not discussed in lectures)*

Correlation interferometers compared with single apertures

Disadvantages

1) Reduced brightness temperature sensitivity by the “filling factor” → less good for imaging smooth low-brightness emission

2) Don’t give a total power response → no access to “zero” baseline - hence always some resolution (i.e. another contribution to point 1)

3) Transfer function is complicated due to missing information in the form of Fourier components → need deconvolution techniques - with no analytic guarantee of final fidelity of the reconstructed image.

Phased arrays compared with correlation arrays

Phased (adding) arrays (SKA-LOW – in part)

- "direct" imaging: form an *instantaneous* power beam like a single antenna
- using independent delay systems can populate the primary beam with many *higher resolution* "array beams"
  - higher sidelobes in array beams arise from unfilled aperture; reducing the maximum baseline to get better *instantaneous* filling limits the resolution for imaging.
  - subject to receiver gain variations

→ well-suited for *real-time* "non-imaging" applications e.g. pulsars and transients too small to resolve and for which precise amplitude calibration is less important.

→ not well-suited to high resolution imaging

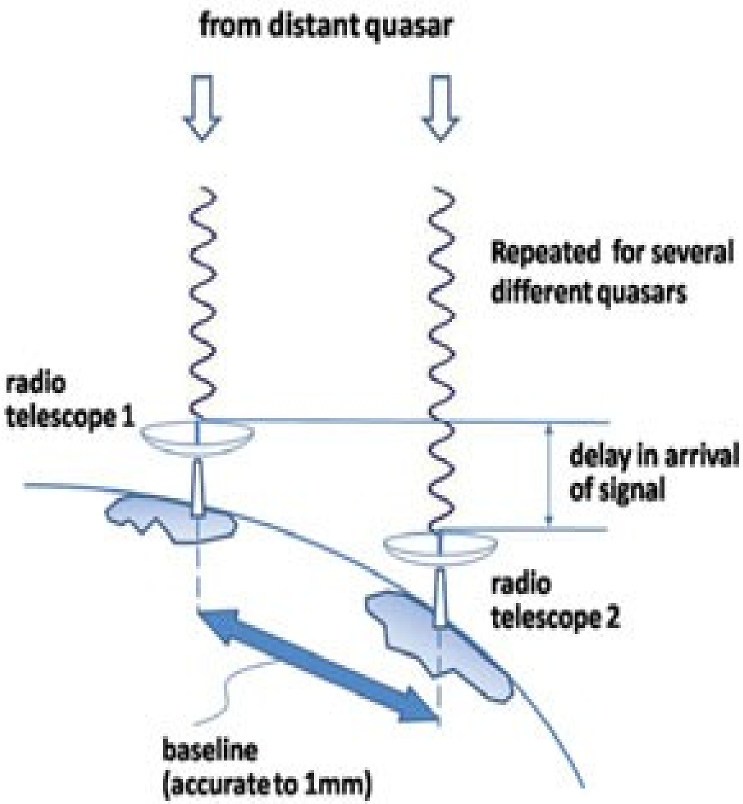
Correlation (multiplying) arrays (SKA-MID)

- "indirect" imaging: capture N(N-1)/2 Fourier Components of the sky brightness
- can correct (to an extent) the sidelobes resulting from the unfilled aperture by *off-line* deconvolution techniques which effectively add F.C.s to fill in the gaps.
- can use Earth rotation to change projected baseline vectors and can combine data from more than one array taken at different times *as long as the source does not vary.*
  - can use long baselines but no access to zero or even very short baselines so *cannot* image all the low brightness features within the primary beam.

→ well-suited for high resolution imaging of *discrete* sources within the primary beam

→ not suitable for observing pulsars and transients

Results from interferometric positional measurements



http://www.linz.govt.nz/geodetic/standards-publications/geodetic-notices/2009/0710-vlbi-in-new-zealand

Precise knowledge of the baseline vector b allows positions of astronomical sources to be measured.

e-MERLIN : $|b|$ = 206 km; λ=0.05m (5 cm; 6 GHz)
source shift: $\Delta\theta$~5 milliarcsec (0.1 x Hubble ST resolution)
~ 0.005/206250 radians

$\Delta\phi = 2\pi\,[b\Delta\theta/\lambda] = 2\pi\,(0.005/206265)(206000/0.05)$
~ 0.2π rads = 36^{o} of phase shift – easy !

Very Long Baseline Interferometry = "VLBI" trans-national and transcontinental baselines

• Observations of the position of quasars defines the International Celestial Reference Frame to ~0.1 milliarcsec

• Locations of radio transmitters in space e.g. Cassini mission to Jupiter's moon Titan - tracked the Huygens probe down to the surface and measured atmospheric winds.

ALTERNATIVELY: Precise knowledge of source positions → "geodesy " allows baseline vector to be measured → motions of Earth. All done with "VLBI"

- Determine positions of widely-spaced antennae to accuracy of few mm.
- Measure tectonic motions of crustal plates 10 – 100 mm /year
- Measure subtle variations in Earth's rotation rate