DARA
DEVELOPMENT IN AFRICA
WITH RADIO ASTRONOMY

DARA Basic Training

NAMIBIA-BOTSWANA 2019

UNIT 1: INTRODUCTION TO ASTROPHYSICS

Windhoek, 7 –18 January 2019

Stellar Structure

- Equations of stellar structure:
  - hydrostatic equilibrium
  - mass conservation
  - energy generation
  - energy transport
  - convective transport
- Solutions and time evolution

Hydrostatic Equilibrium

- In equilibrium upward and downward forces on a mass element balance

$$F_p(r+dr)+F_g=F_p(r)$$

$$F_p(r+dr)-F_p(r)=-F_g$$

$$P=\frac{F_p}{A}\text{ so}$$

$$P(r+dr)dA-P(r)dA=-\frac{GM(r)dm}{r^2}$$

- Mass $dm = \rho(r)dV = \rho(r)dAdr$

$$P(r+dr)dA - P(r)dA = -\frac{GM(r)\rho(r)dAdr}{r^2}$$

$$\frac{P(r+dr)-P(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

and hence

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

ie pressure gradient = gravity

Mass Conservation

- Mass of spherical annulus element

$$dM(r) = \rho(r)dV = \rho 4\pi r^2 dr$$

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

Energy Generation

- Introduce $\varepsilon(\mathrm{r})$ = amount of energy produced per unit time per unit mass ($\mathrm{Wkg^{-1}}$)

$$L(r+dr)-L(r)=\varepsilon(r)dM(r)$$

$$L(r+dr)-L(r)=\varepsilon(r)\rho(r)4\pi r^2dr$$

and hence

$$\frac{dL}{dr}=4\pi r^2\varepsilon\rho$$

Energy Transport

- Intensity of radiation:
  - I_ν = energy crossing unit area and entering a unit solid angle per unit time per unit frequency interval ($\mathrm{Jm^{-2}s^{-1}Hz^{-1}st^{-1}}$)
  - Flux of radiation

    f_ν = amount of energy crossing a unit area per unit time per unit frequency interval ($\mathrm{Jm^{-2}s^{-1}Hz^{-1}}$)

$$f_\nu = \int_{4\pi} I_\nu \cos\theta d\omega$$

Transfer Equation

- Opacity κ_ν = fractional reduction in intensity per unit path length (m^{-1})
- Emissivity j_ν = amount of energy emitted per unit volume per unit time per unit frequency interval into unit solid angle (Jm^{-3}s^{-1}Hz^{-1}st^{-1})

so

$$I_\nu(s+ds) - I_\nu(s) = -\kappa_\nu I_\nu ds + \frac{j_\nu dV}{dA}$$

Now $$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

i.e. change = extinction + emission

- Optical depth τ_ν is defined as

$d\tau_\nu = \kappa_\nu ds$ (dimensionless)

- so for pure extinction

$dI_\nu = -I_\nu d\tau_\nu$ and $I_\nu = I_0 e^{-\tau_\nu}$

- and for pure emission (optically thin case)

$I_\nu = \int j_\nu ds$

Local thermodynamic equilibrium

- In stellar interiors gas is close to LTE

and so

$$j_\nu(r) \approx \kappa_\nu B_\nu(T(r)) \quad \text{(Kirchoff's Law)}$$

and

$$I_\nu(r) \approx B_\nu(T(r)) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$

Radiative Transport Equation

- It can be shown that with reasonable approximations integrating the transfer equation over all solid angles and frequencies leads to

$$\frac{dT}{dr} = -\frac{3\kappa_r L}{64\pi\sigma r^2 T^3}$$

Convective Transport

- It can be shown that the interior of a star will be unstable to convection if

$$\frac{dT}{dr} > \frac{T}{P}\frac{dP}{dr}\frac{\gamma - 1}{\gamma}$$

where γ is the ratio of specific heats.

Hence convection is caused by :

1) Steep temperature gradients

2) γ close to unity, e.g. a change of ionization state

- No good theory for convective transport

- Now have four differential equations
- Plus three linear equations

$$\kappa_r = \kappa_r(\rho, T, \mu) \qquad \varepsilon = \varepsilon_0(\mu)\rho T^{\nu} \qquad P = \frac{\rho R T}{\mu}$$

where μ is the mean molecular mass (composition)

- Total of seven equations in seven unknowns

$P, M, L, T, \rho, \kappa_r, \varepsilon.$

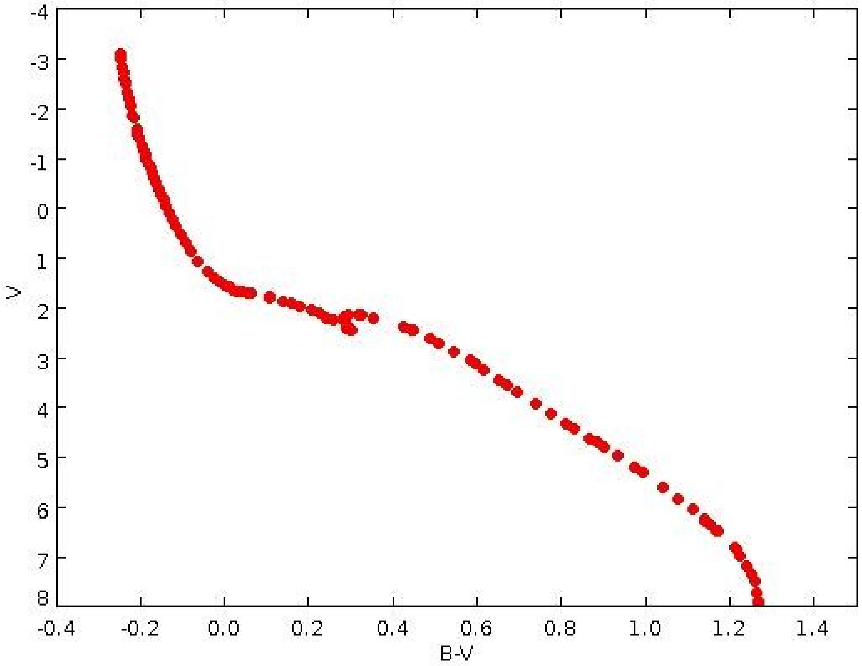
- Plus boundary conditions that set the mass

of the star and the composition

- These can be solved numerically to

give the structure of the star

Theoretical Main Sequence



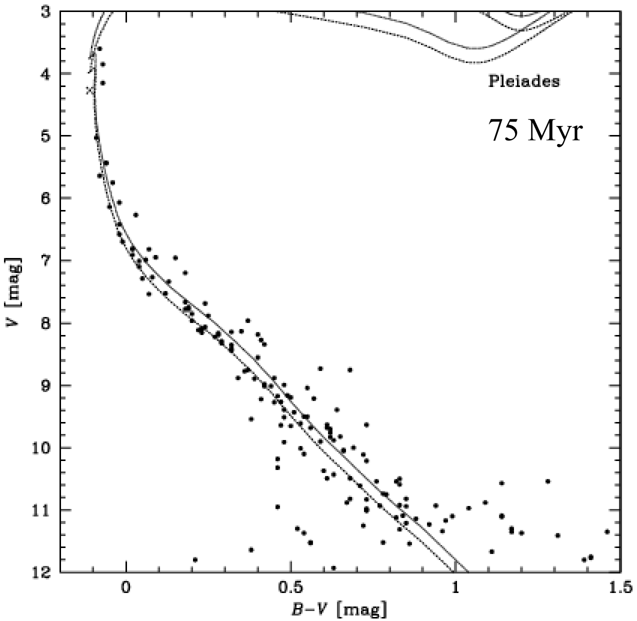
http://stev.oapd.inaf.it/cgi-bin/cmd

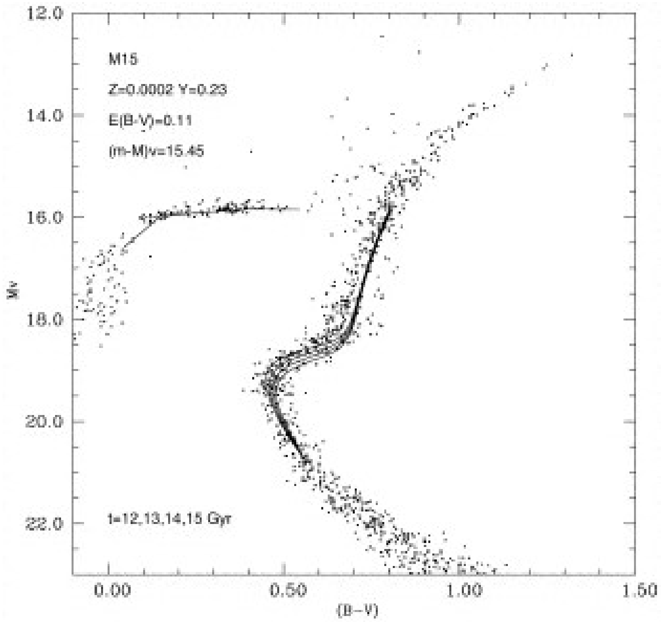
Stellar Evolution

- As nuclear fusion proceeds the composition in the core changes
- Adjust μ in equations of stellar structure
- Major changes when particular nuclear reactions stop (no more fuel) or ignite (increasing T and ρ)
- Can compute evolutionary track for a star with a given mass and initial composition

Isochrone fitting

- Isochrone is a locus for stars of different masses but same age and composition

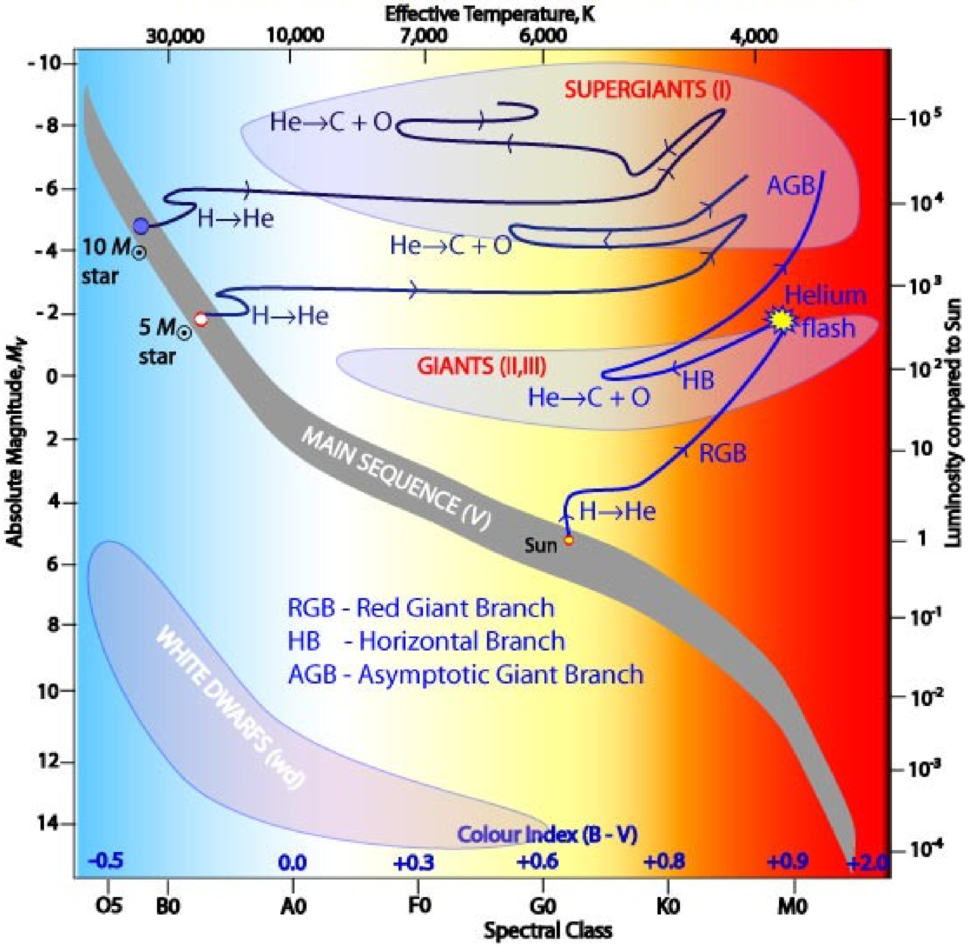




Summary

- The internal structure of a main sequence star can be calculated from first principles
- The evolution of this structure can be tracked through most stages of evolution
- Predictions compare well with observations in most cases
- Convection, magnetic fields, mass-loss, binary star evolution not well understood

Evolutionary Tracks off the Main Sequence



www.atnf.csiro.au/outreach/education/senior/astrophysics/stellarevolution_postmain.html