Stars and Galaxies

Coursework Sheet 8 – Feedback

1. When in Keplerian rotation the velocity is given by

$$v = \sqrt{\frac{GM}{r}} \qquad \text{(1 mark)}$$

Therefore the radius is given by

$$r = \frac{GM}{v^2} \qquad \text{(1 mark)}$$

$$= \frac{7\times10^{-11}\times10^{9}\times2\times10^{30}}{\left(10^{4}\times10^{3}\right)^{2}}$$

$$= 1\times10^{15}\ \mathrm{m}$$

$$= 0.05\,\mathrm{pc} \qquad \text{(1 mark)}$$

The timescale for variation must be smaller than the light travel time across the emitting region, i.e.

$$l \le ct$$

$$t \ge \frac{l}{c} = \frac{1\times10^{15}}{3\times10^{8}} = 3\times10^{6}\,s = 40\ \text{days} \qquad \text{(1 mark)}$$

This is similar to the shortest observed variability timescale in AGN and therefore it is consistent with a picture in which most of the light is emitted from the same region as that where the broad emission lines originate, i.e. an accretion disc around the super-massive black hole.

2. Luminosity of material falling onto a compact object is given by

$$L = \frac{1}{2}\dot{m}c^2$$

$$\dot{m} = \frac{2L}{c^2} = \frac{2\times10^{42}}{\left(3\times10^{8}\right)^{2}} = 2\times10^{25}\ \mathrm{kg\,s^{-1}} \qquad \text{(2 marks)}$$

$$\dot{m} = 2\times10^{25}\times\frac{3\times10^{7}}{2\times10^{30}} = 320\ \text{solar masses per year} \qquad \text{(1 mark)}$$

(Alternatively:

$$L = \frac{GM\dot{m}}{R} \qquad \text{(1 mark)}$$

Take the radius of the black hole to be the Schwartzschild radius

$$R = \frac{2GM}{c^2}$$

$$= \frac{2 \times 7 \times 10^{-11} \times 10^{9} \times 2 \times 10^{30}}{\left(3 \times 10^{8}\right)^2} \qquad \text{(1 mark)}$$

$$= 3 \times 10^{12}\, m$$

The mass infall rate required is given by

$$\dot{m} = \frac{LR}{GM} = \frac{10^{42} \times 3 \times 10^{12}}{7 \times 10^{-11} \times 10^{9} \times 2 \times 10^{30}} = 2 \times 10^{25} \text{ kg s}^{-1}$$

$$= 2 \times 10^{25} \times \frac{3 \times 10^{7}}{2 \times 10^{30}} = 320 \text{ solar masses per year} \qquad \text{(1 mark)}$$

The actual infall rate would have to be higher than this because this assumes a perfect conversion of the gravitational potential energy into the luminosity of the radiation emitted. In reality it is only about 10% and so the required mass infall rate required is more like 3000 solar masses per year onto the black hole. (1 mark)

3. From the plot of absolute magnitude versus period we find that a period of 15 days corresponds to an absolute magnitude of -3.1 in the visual assuming it is a Type I Cepheid.

(1 mark)

Ignoring extinction, i.e. A_V=0, then

$$m_V - M_V = 5\log d - 5$$

$$\log d = \frac{m_V - M_V + 5}{5} = \frac{17.6 - (-3.1) + 5}{5} = \frac{25.7}{5} = 5.1$$

$$d = 1.2 \times 10^{5} \text{ pc} = 120 \text{ kpc} \qquad \text{(1 mark)}$$