# VCHI

### Scientific motivation

Magnetic fields are dynamically important in the very-early and verylate stage of stellar evolution. In particular, they are almost certainly dynamically important in the formation process of massive stars, and possibly in the shaping of proto-planetary nebulae. Masers are commonly emitted in the circumstellar material of nascent or ageing stars. They are not only excellent probes of the overall dynamical properties of the regions where they are emitted but also of their magnetic fields through the Zeeman effect. Zeeman measurements can allow the recovery of both the strength and the direction of the local magnetic fields as demonstrated by many observational studies in the recent past (Etoka & Diamond 2010, Leal-Ferreira et al. 2012, Vlemmings et al 2017). Though the theory of propagation of polarised-maser emission already exists along with 1D implementations (Tobin, Kemball & Gray 2019), to date to our knowledge no truly 3D implementation has been tackled. Here we present preliminary results of the 3-D polarisation model we are currently developing. As a first step towards a more general (multi-level) case, this first model considers the basic (3level) J = 1-0 Zeeman pattern.

• The model presented here uses a modified version of the 1-level 3D maser model originally developed for the investigation of rotation as a possible explanation for {e.g. methanol} flaring events (Gray, Mason & Etoka 2018).

• The non-linear Orthomin algorithm, based on the implementation of Chen and Cai (2001), is used for the computation of the populations {fractional inversions} of the  $\sigma^{+/-} \& \pi$  transitions.

• The domain is generated by Delaunay triangulation of irregularly distributed points (which represent the nodes of the maser domain) following Zienkiewicz & Taylor (2000).

• The figure here below, displays the triangulated domain used for the work presented here:

## **3D Polarisation Model for Maser Emission**

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#### The model

♦ The current model is a 3-{upper-}levels (1 lower level) & solves for the propagation of the polarised emission following the methodology of Landi Del'Innocenti (1987).

#### The maser domain



 The figure here below presents the fractional inversions

achieved for the  $\sigma^+$ ,  $\pi$  and  $\sigma^-$  for selected nodes (the colour code is the same as used for the nodes in the figure displaying the "maser domain"):

#### Inversions

• The full radiative transfer equation,  $di/d\tau = \eta i$ (where  $\tau$  is the optical depth and  $\eta$  represents the Stokes matrix), leads to the following system of equations to be solved:

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \begin{pmatrix} i_n \\ q_n \\ u_n \\ v_n \end{pmatrix} = \begin{bmatrix} \eta_{\mathrm{I}} & \eta_{\mathrm{Q}} & \eta_{\mathrm{U}} & \eta_{\mathrm{V}} \\ \eta_{\mathrm{Q}} & \eta_{\mathrm{I}} & \rho_{\mathrm{V}} & -\rho_{\mathrm{U}} \\ \eta_{\mathrm{U}} & -\rho_{\mathrm{V}} & \eta_{\mathrm{I}} & \rho_{\mathrm{Q}} \\ \eta_{\mathrm{V}} & \rho_{\mathrm{U}} & -\rho_{\mathrm{Q}} & \eta_{\mathrm{I}} \end{bmatrix} \begin{pmatrix} i_n \\ q_n \\ u_n \\ v_n \end{pmatrix} =$$

$$\Delta^{\pm,0}(r) = \frac{1}{1+\overline{j}^{\pm,0}(r)}$$



#### Brightness maps & Spectra

 In the current model a uniform field is used, allowing the selection of a frame of reference for the formal solutions where Stokes Q or U is 0. For this implementation, we chose a frame of reference such that Stokes U=0.

• The figures here-below present typical {Stokes I, Q & V} brightness maps with the associated spectra:



Chen & Cai, 2001, Appl. Math Comput 124, 351 Etoka & Diamond, 2010, MNRAS 406, 2218 Landi Degl'Innocenti E., 1987, in: W. Kalkofen (ed.), Numerical Radiative Fransfer (Cambridge, CUP), p265 Vlemmings et al. 2017, A&A 603, 92 Tobin, Kemball & Gray 2019, 971, 189 Zienkiewicz & Taylor, 2000, The Finite Element Method: The basis. Butterworth-Heinemann, Oxford

Gray, Mason & Etoka, 2018, MNRAS, 477, 2628 Leal-Ferreira et al. 2012, 540 A41