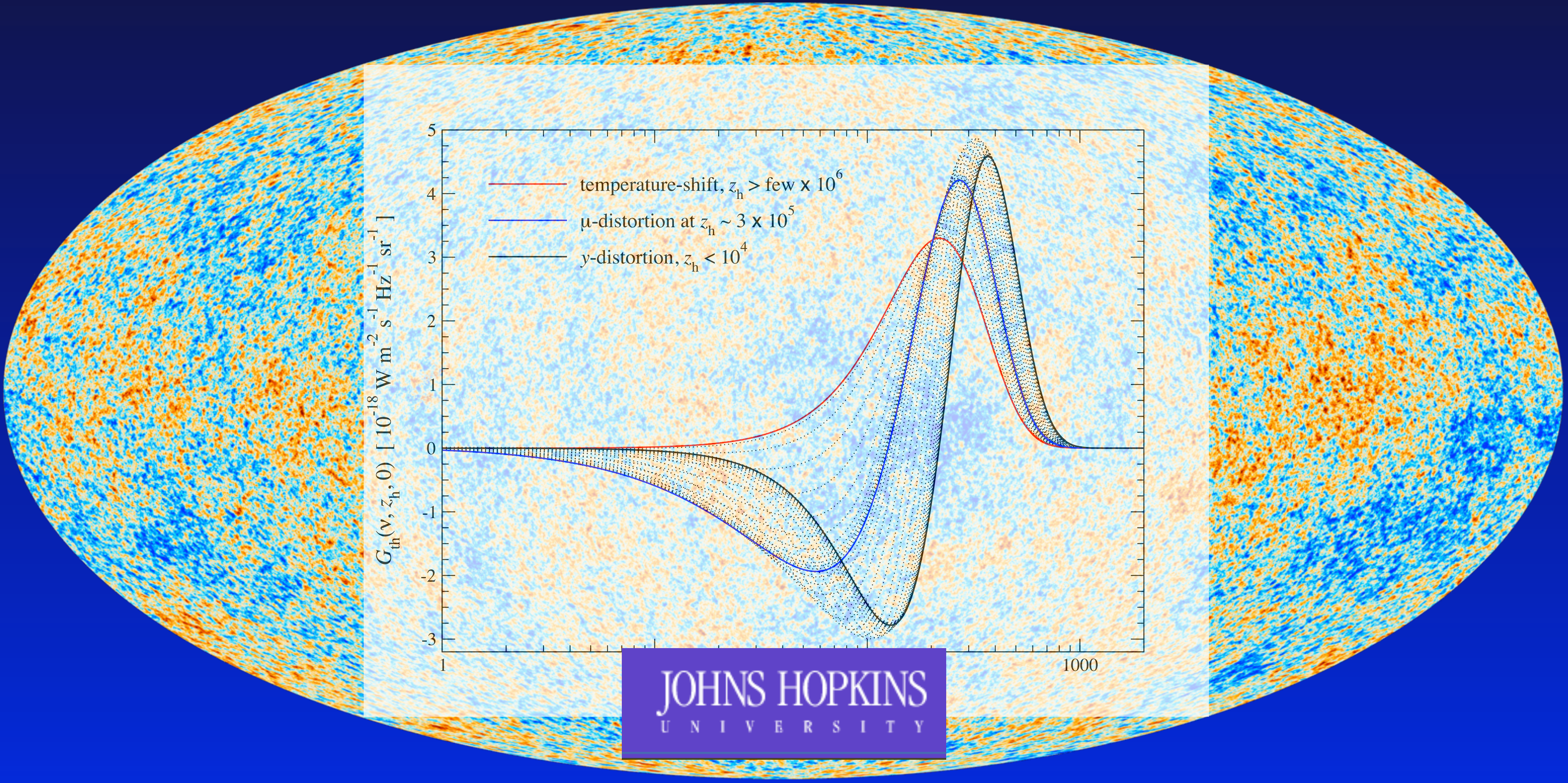
CMB Spectral Distortions as New Probe of Early-Universe Physics



JOHNS HOPKINS
UNIVERSITY

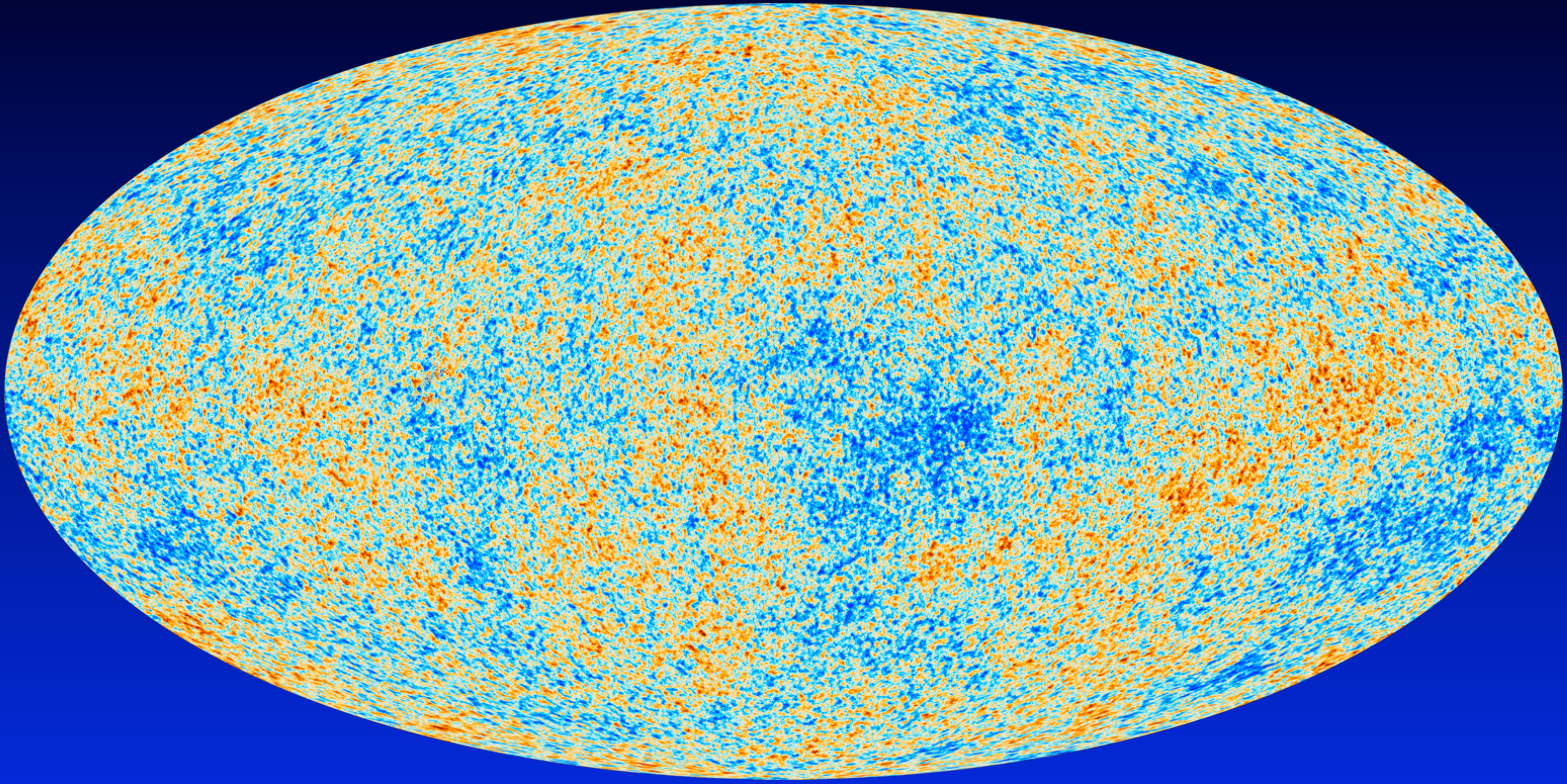
Jens Chluba

Lecture I: *Theory of CMB Spectral Distortions*

Les Houches, August 1st, 2013

CITA ICAT
Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique

Cosmic Microwave Background Anisotropies



Planck all sky map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Cosmic Microwave Background Anisotropies

Planck all sky map

- CMB has a blackbody spectrum in every direction
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Here we are Interested in the CMB Monopole Signal!!!

COBE/FIRAS

$$T_0 = (2.726 \pm 0.001)\,\mathrm{K}$$

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen, 2003, ApJ, 594, 67
Fixsen, 2009, ApJ, 707, 916

- CMB monopole is 10000 - 100000 times larger than fluctuations!

Main Questions for this Lecture

- What do we know about CMB spectral distortions?
- How are CMB spectral distortions created?
- How do distortions evolve / thermalize?
- Which physical processes are important?
- Definition of different types of distortions
- Simple approximations for spectral distortions

References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, SvA, 18, 413

Yakov Zeldovich

Rashid Sunyaev

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  - Illarionov & Sunyaev, 1975, SvA, 18, 413
- Additional milestones
  - Danese & de Zotti, 1982, A&A, 107, 39
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis

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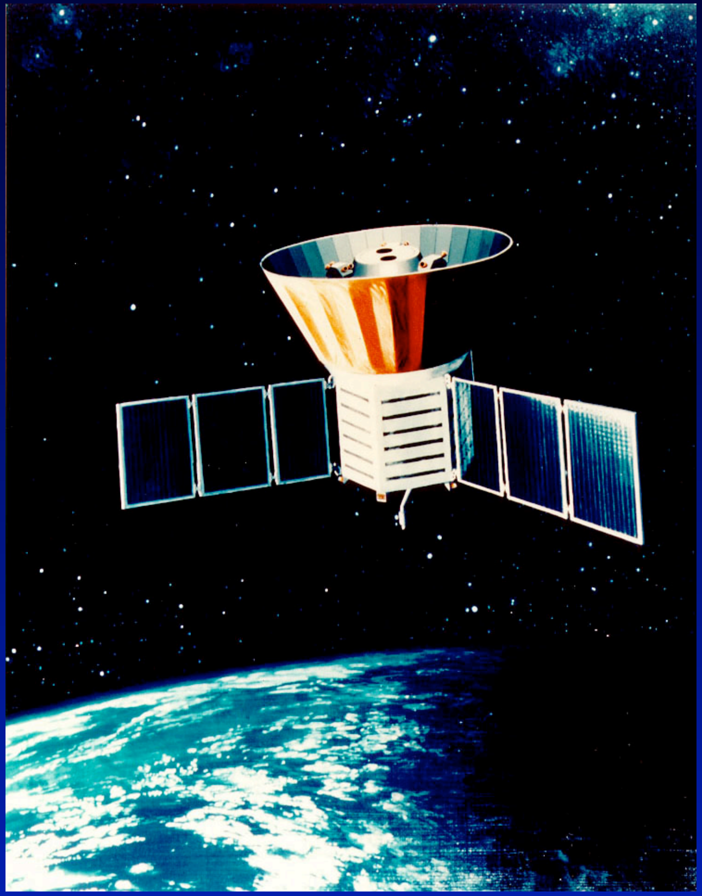
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis
- More recent work
  - JC & Sunyaev, 2012, MNRAS, 419, 1294
  - Khatri & Sunyaev, 2012, JCAP, 9, 16
  - JC, MNRAS, 2013, in print (ArXiv:1304.6120)

Current Spectral Distortion Constraints

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|y| \leq 1.5\times10^{-5}$$

$$|\mu| \leq 9\times10^{-5}$$

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
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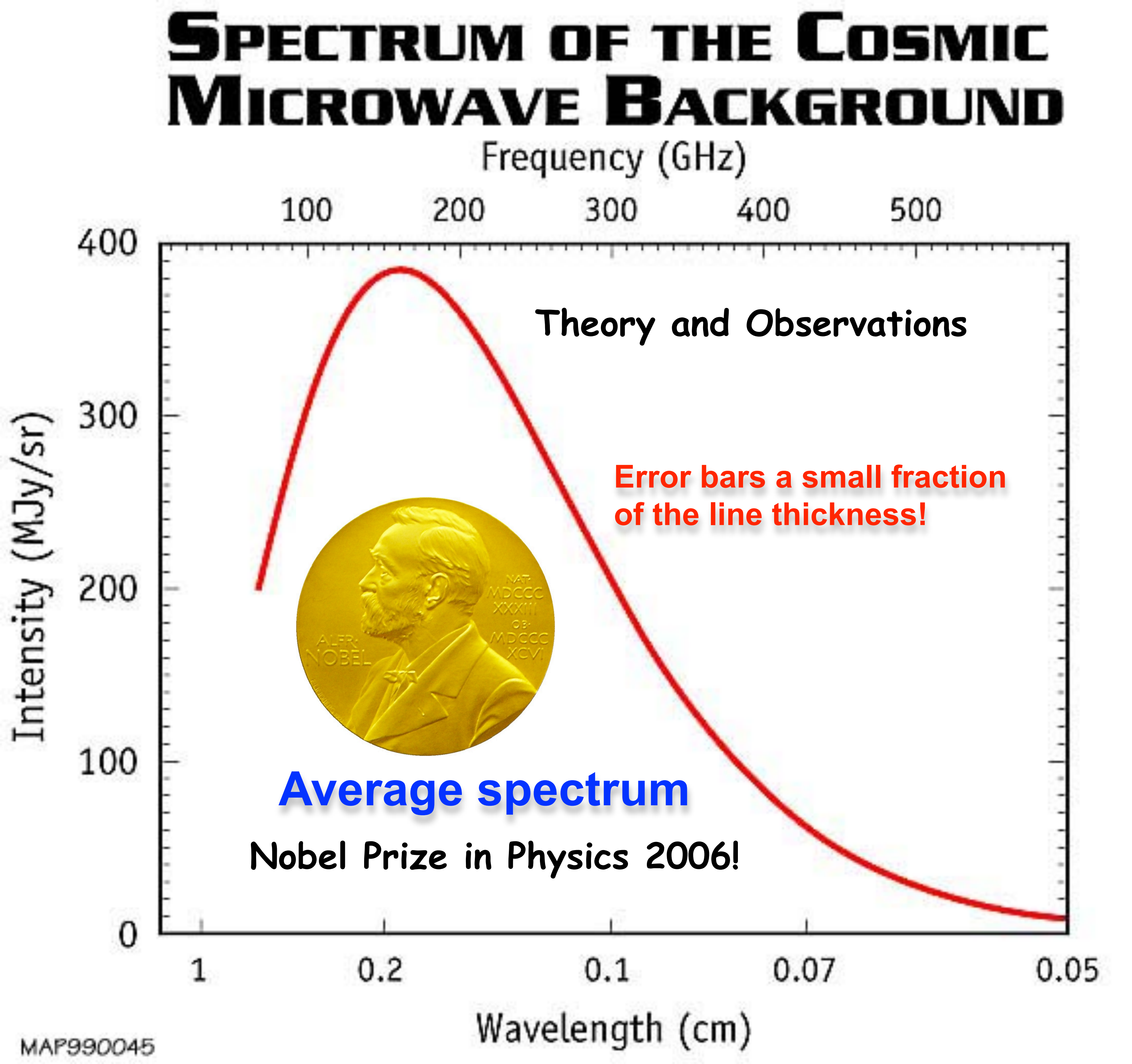


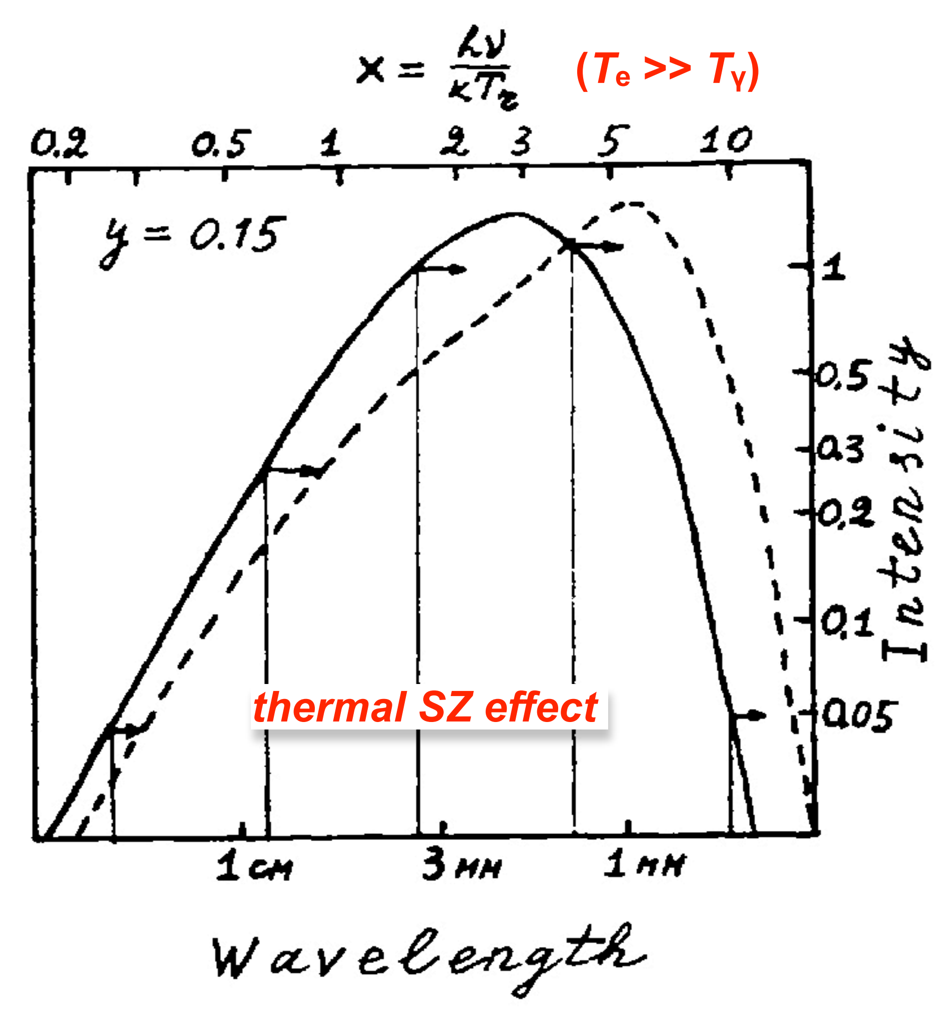
Error bars a small fraction of the line thickness!

Average spectrum

Nobel Prize in Physics 2006!

Small Sneak Preview....

Compton y-distortion



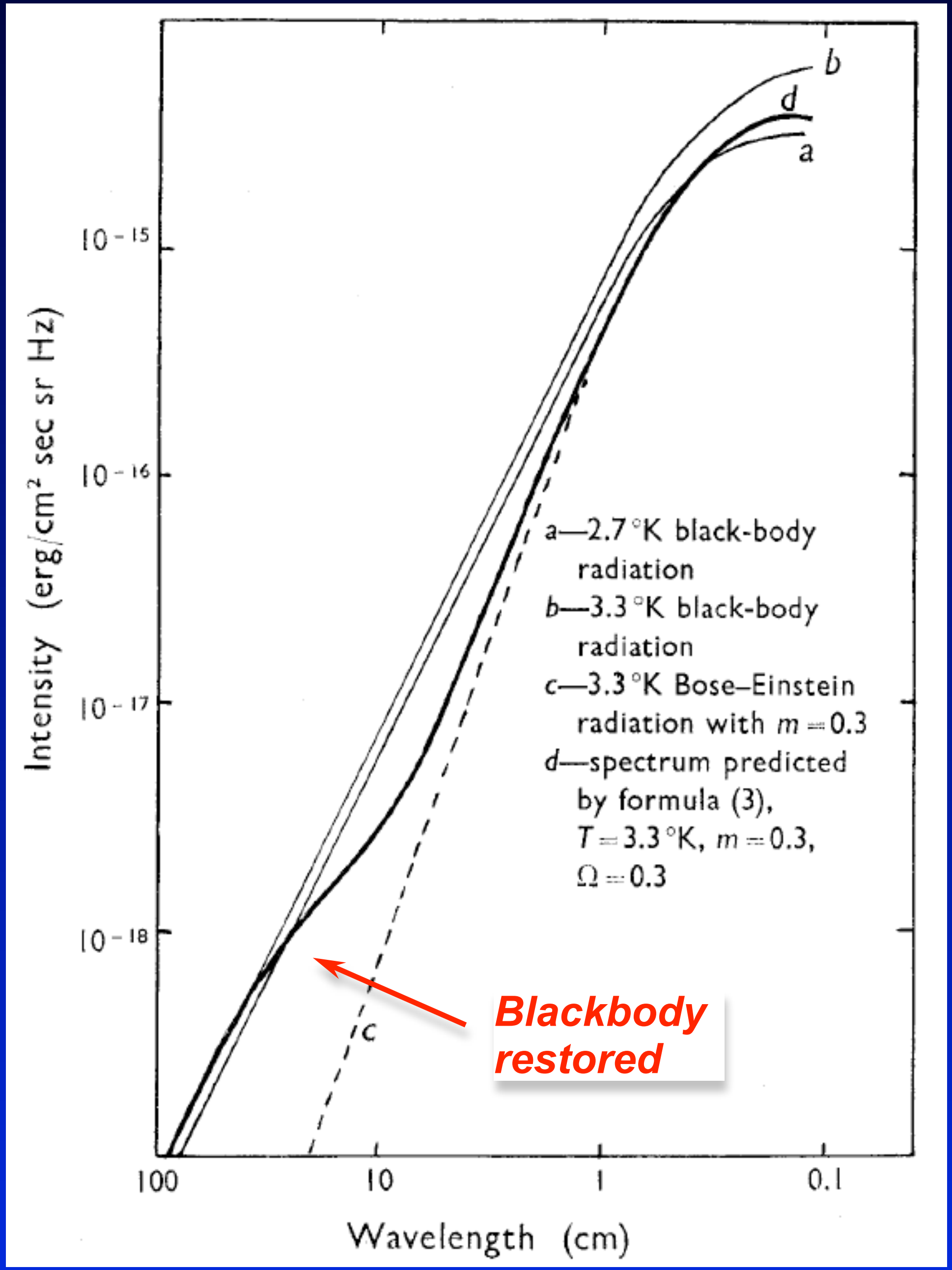
($T_e >> T_\gamma$)

thermal SZ effect

Sunyaev & Zeldovich, 1980, ARAA, 18, 537

- also known from thSZ effect
- up-scattering of CMB photon
- important at late times (z<50000)
- scattering inefficient

Chemical potential μ-distortion

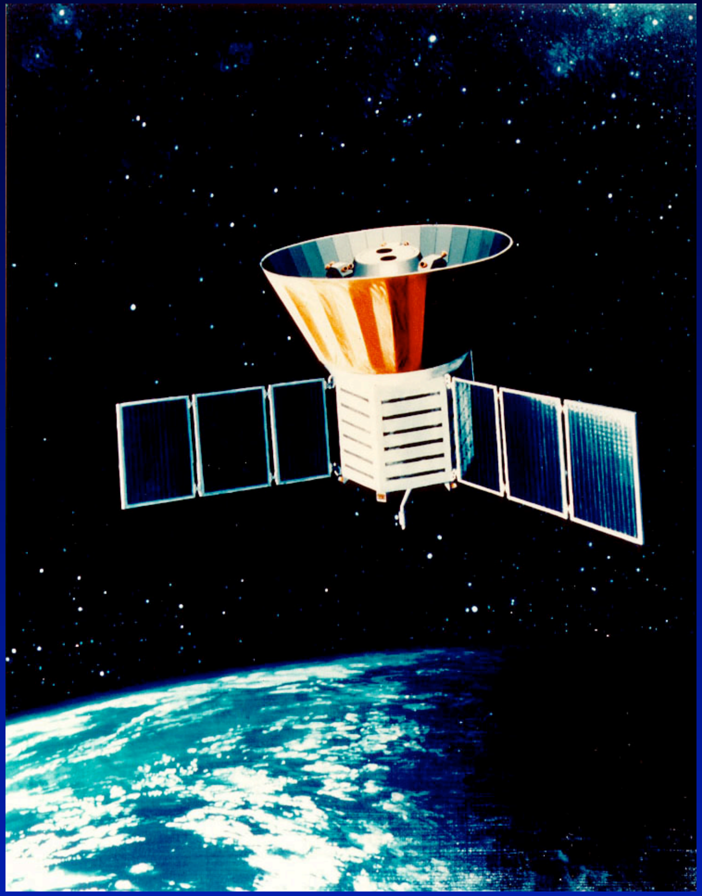


Blackbody restored

Sunyaev & Zeldovich, 1970, ApSS, 2, 66

- important at very times (z>50000)
- scattering very efficient

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



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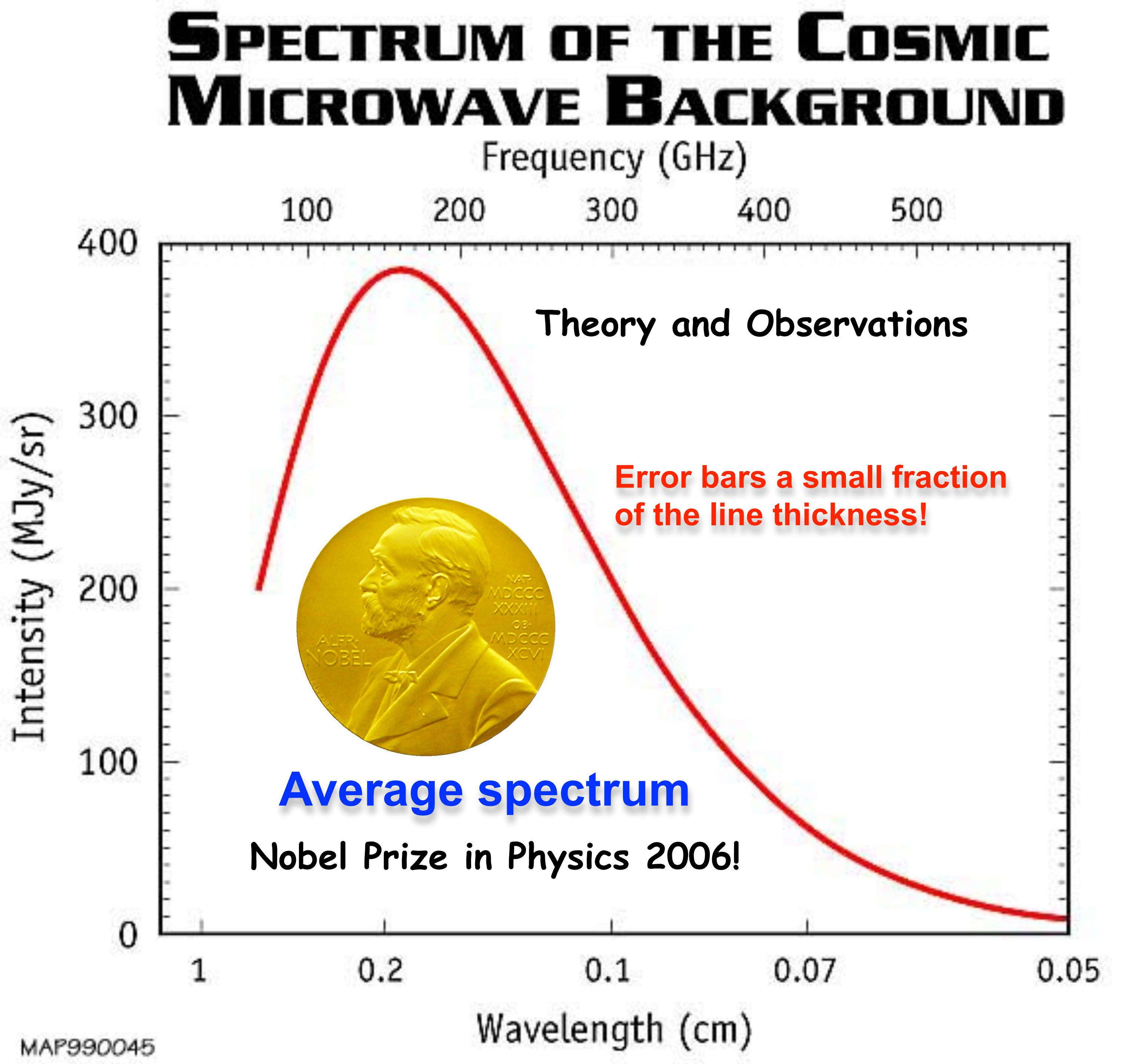

Theory and Observations

Error bars a small fraction of the line thickness!

Average spectrum

Nobel Prize in Physics 2006!

Only very small distortions of CMB spectrum are still allowed!

ARCADE

(Absolute Radiometer for Cosmology, Astrophysics and Diffuse Emission)

- Balloon experiment
- Several flights (2001, 2003, 2005, 2006)
- Frequencies ν = {3, (5), 8, 10, 30, 90} GHz

Kogut et al. 2006, New Astronomy Rev., 50, 925
Kogut et al., 2011, ApJ, 734, 9
Fixsen et al., 2011, ApJ, 734, 11
Seiffert et al., 2011, ApJ, 734, 8

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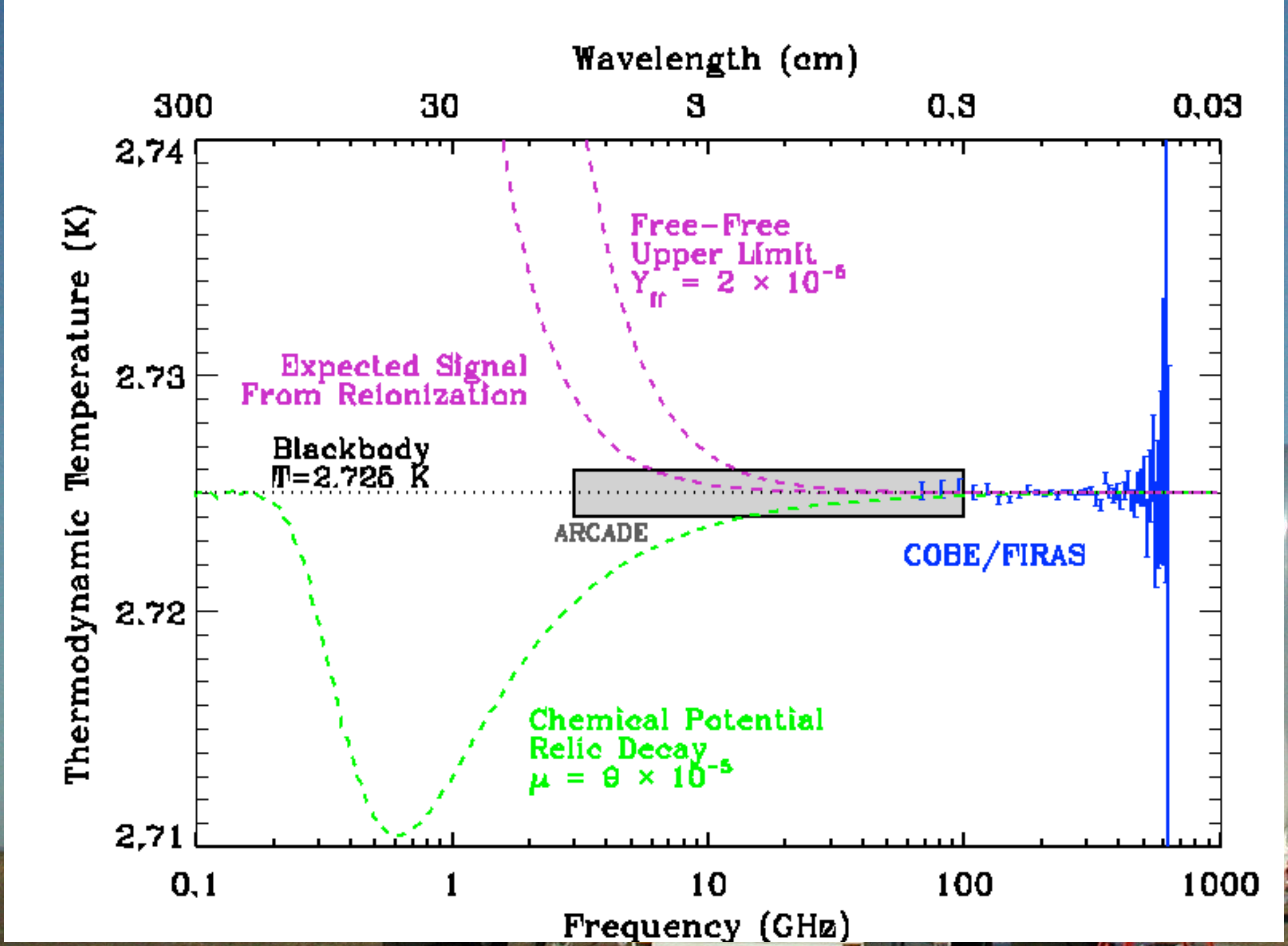

- Distortion constraints:

$$|\mu| < 6 \times 10^{-4}$$

$$|Y_{\rm ff}| < 10^{-4}$$

- No limit on y-parameter

Kogut et al. 2006, New Astronomy Rev., 50, 925
Kogut et al., 2011, ApJ, 734, 9
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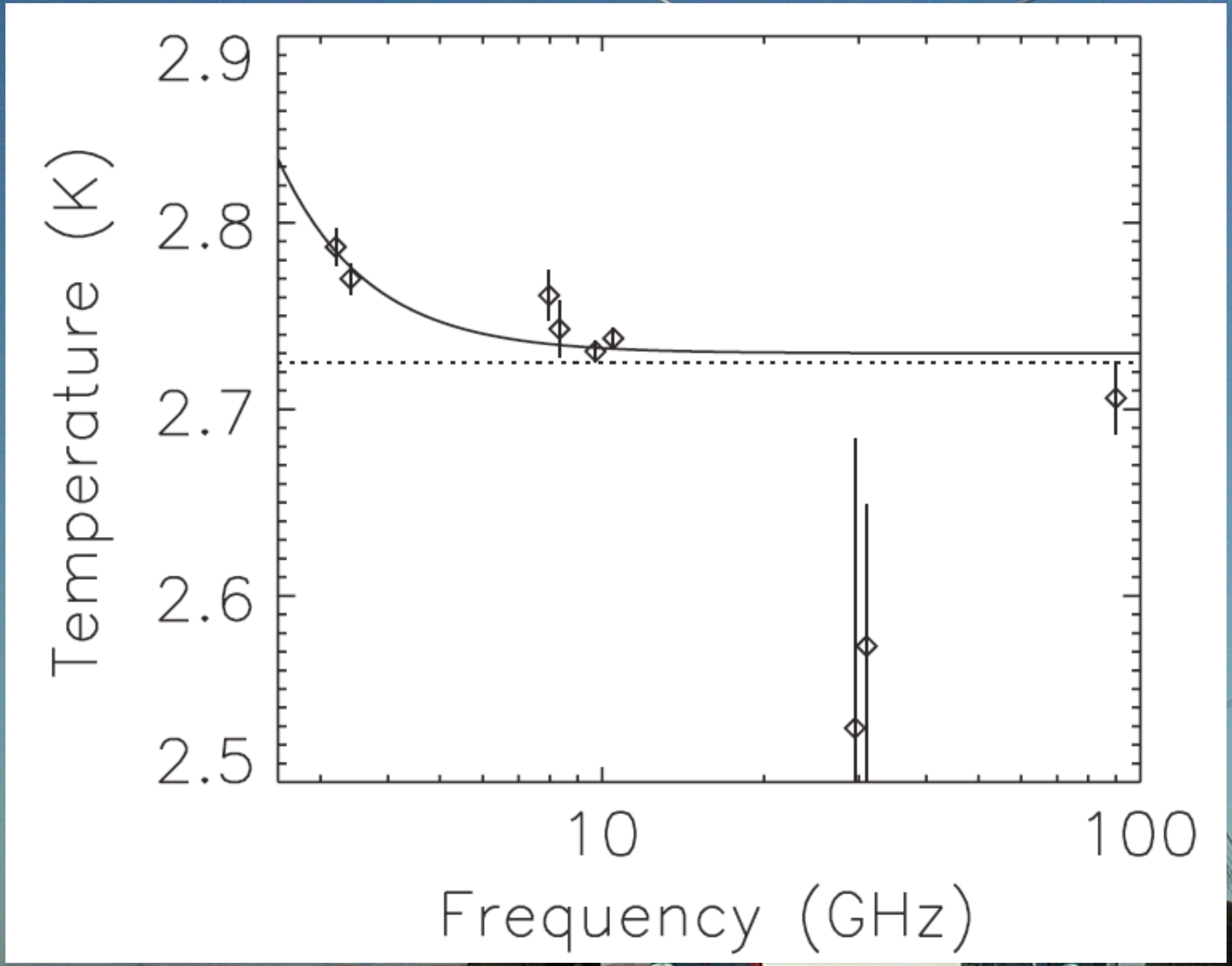

- Found low frequency excess
- Spectrum:

$$T(\nu) = (24.1 \pm 2.1)\mathrm{K}(\nu/\nu_0)^{-2.599\pm0.036}$$

$$\nu_0 = 310\,\mathrm{MHz}$$

- Origin of excess unclear
- New population of radio sources?
- Systematic effect?
- In tension with TRIS results? (next slide)

Kogut et al. 2006, New Astronomy Rev., 50, 925
Kogut et al., 2011, ApJ, 734, 9
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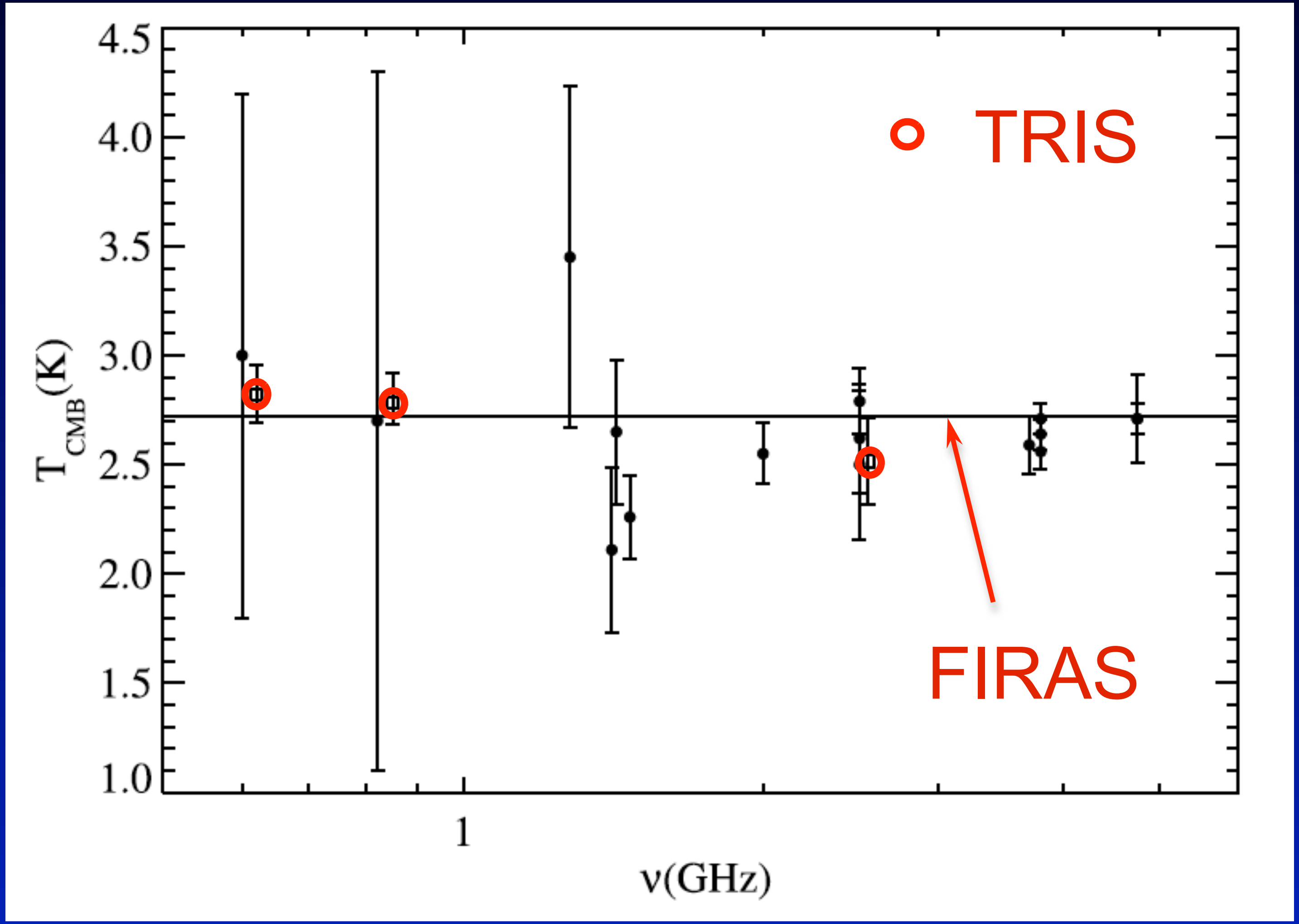
TRIS and Other Low Frequency Measurements



- Ground-based radio antenna
- Grand Sasso Lab, Italy
- Frequencies ν = {0.6, 0.82, 2.5} GHz

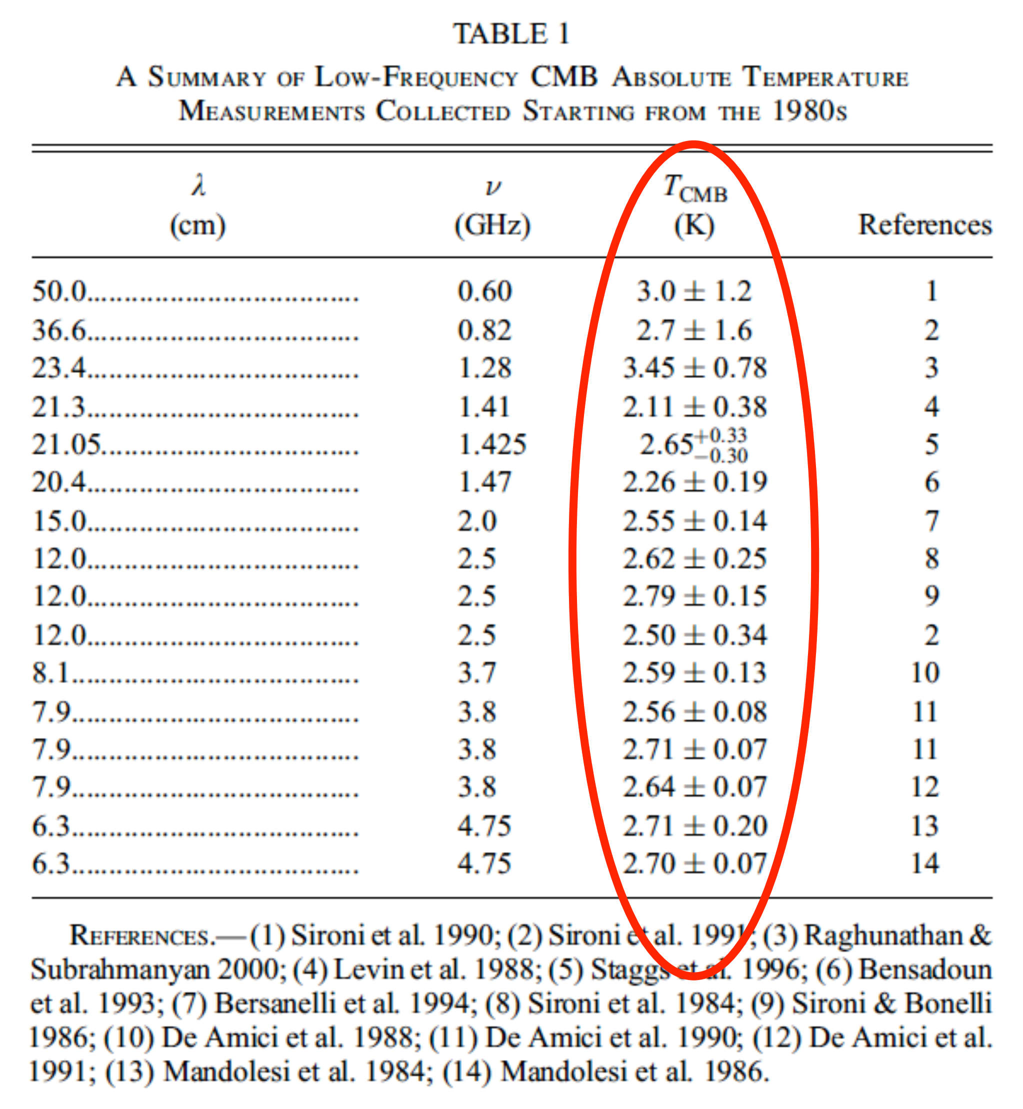


TABLE 1

A SUMMARY OF LOW-FREQUENCY CMB ABSOLUTE TEMPERATURE MEASUREMENTS COLLECTED STARTING FROM THE 1980s

λ (cm)	ν (GHz)	$T_{\rm CMB}$ (K)	References
50.0	0.60	3.0 ± 1.2	1
36.6	0.82	2.7 ± 1.6	2
23.4	1.28	3.45 ± 0.78	3
21.3	1.41	2.11 ± 0.38	4
21.05	1.425	$2.65^{+0.33}_{-0.30}$	5
20.4	1.47	2.26 ± 0.19	6
15.0	2.0	2.55 ± 0.14	7
12.0	2.5	2.62 ± 0.25	8
12.0	2.5	2.79 ± 0.15	9
12.0	2.5	2.50 ± 0.34	2
8.1	3.7	2.59 ± 0.13	10
7.9	3.8	2.56 ± 0.08	11
7.9	3.8	2.71 ± 0.07	11
7.9	3.8	2.64 ± 0.07	12
6.3	4.75	2.71 ± 0.20	13
6.3	4.75	2.70 ± 0.07	14

REFERENCES.—(1) Sironi et al. 1990; (2) Sironi et al. 1991; (3) Raghunathan & Subrahmanyan 2000; (4) Levin et al. 1988; (5) Staggs et al. 1996; (6) Bensadoun et al. 1993; (7) Bersanelli et al. 1994; (8) Sironi et al. 1984; (9) Sironi & Bonelli 1986; (10) De Amici et al. 1988; (11) De Amici et al. 1990; (12) De Amici et al. 1991; (13) Mandolesi et al. 1984; (14) Mandolesi et al. 1986.

- Distortion constraints:

$$|\mu| < 6 \times 10^{-5}$$ (30% improvement over FIRAS)

$$-6.3 \times 10^{-6} < Y_{\rm ff} < 1.3 \times 10^{-5}$$

- No limit on y-parameter (too low ν)

Zannoni et al. 2008, ApJ, 688, 12
Gervasi et al., 2008, ApJ, 688, 24
Tartari et al., 2008, ApJ, 688, 32

Why bother? No distortion detected so far!??

Physical mechanisms that lead to spectral distortions

- *Cooling by adiabatically expanding ordinary matter*: $T_\gamma \sim (1+z) \leftrightarrow T_m \sim (1+z)^2$
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
  - continuous *cooling* of photons until redshift *z* ~ 150 via Compton scattering
  - due to huge heat capacity of photon field distortion very small ($\Delta\rho/\rho \sim 10^{-10}$-$10^{-9}$)
- Heating by *decaying* or *annihilating* relic particles
  - How is energy transferred to the medium?
  - lifetimes, decay channels, neutrino fraction, (at low redshifts: environments), ...
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012)
  - rather fast, quasi-instantaneous but also extended energy release
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; Jedamzik et al. 2000)
- *Cosmological recombination*

„high“ redshifts

pre-recombination epoch

„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization (Heating of medium by X-Rays, Cosmic Rays, etc)

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Much more tomorrow!

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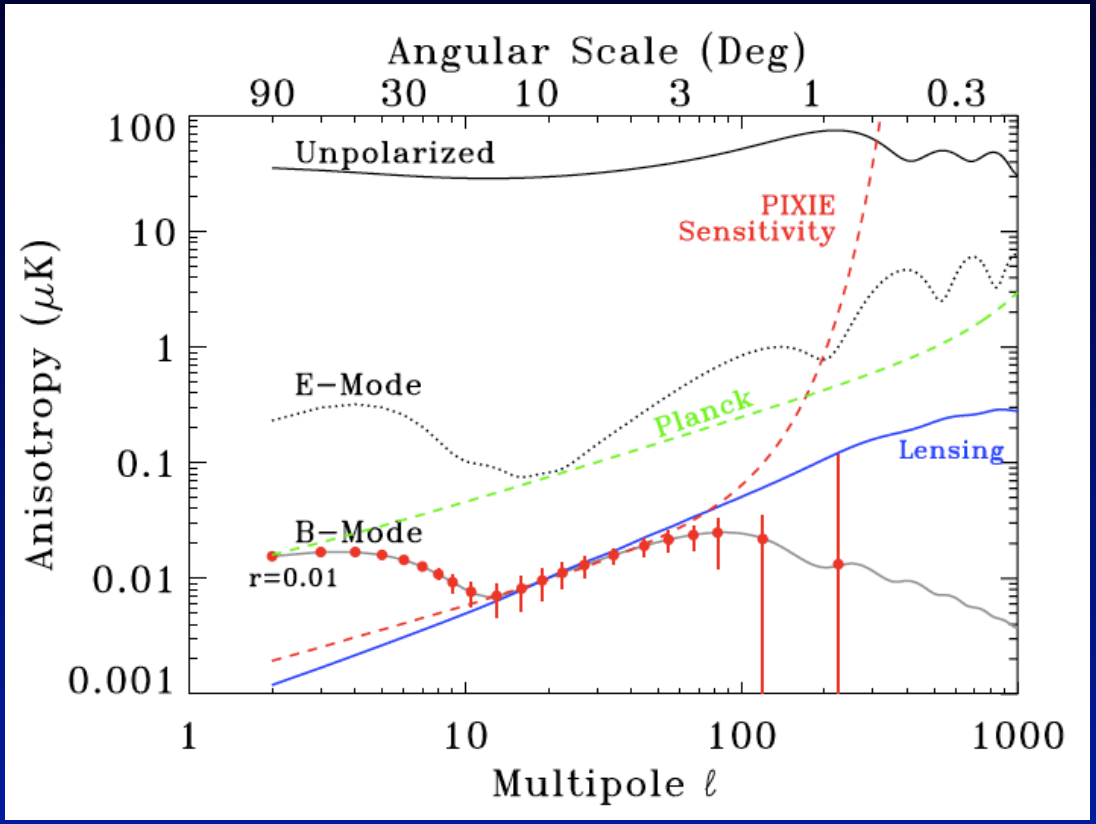
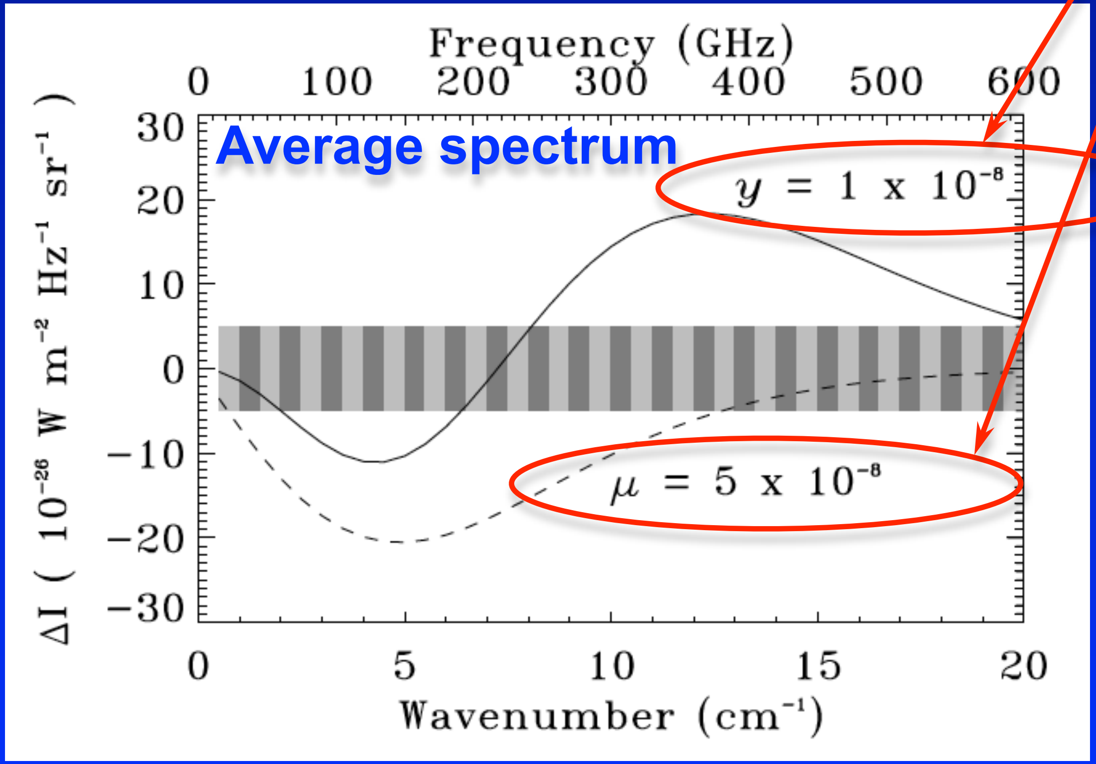
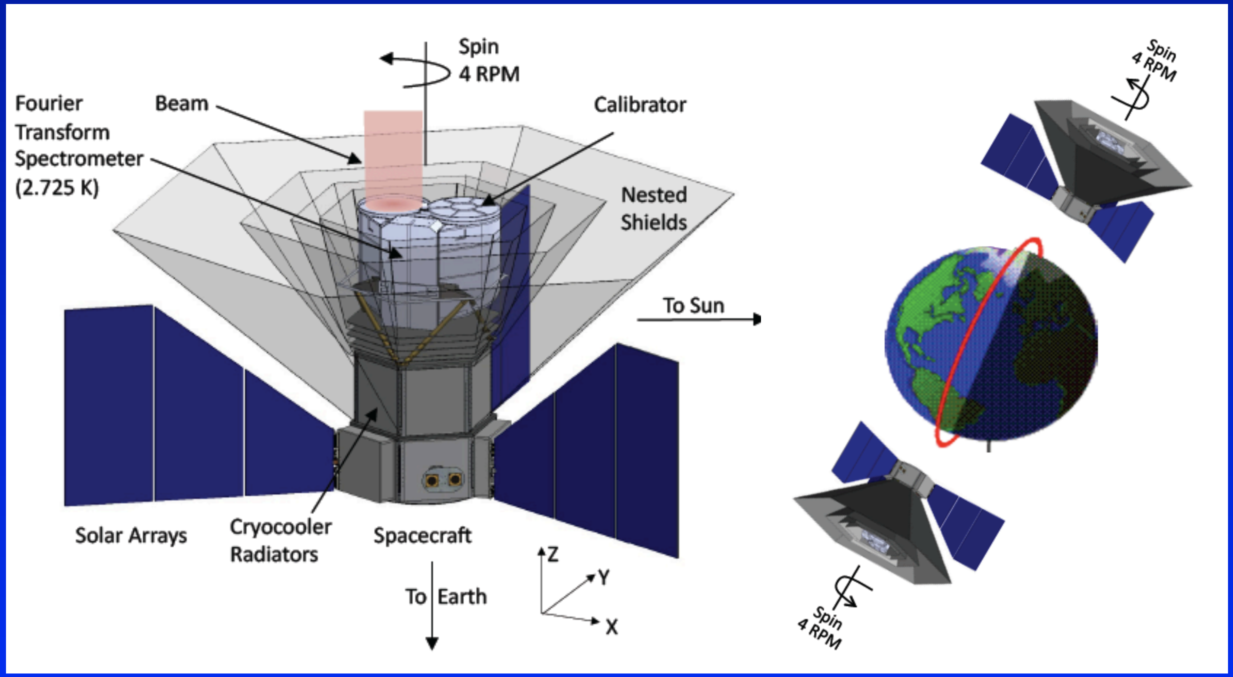
PIXIE: Primordial Inflation Explorer

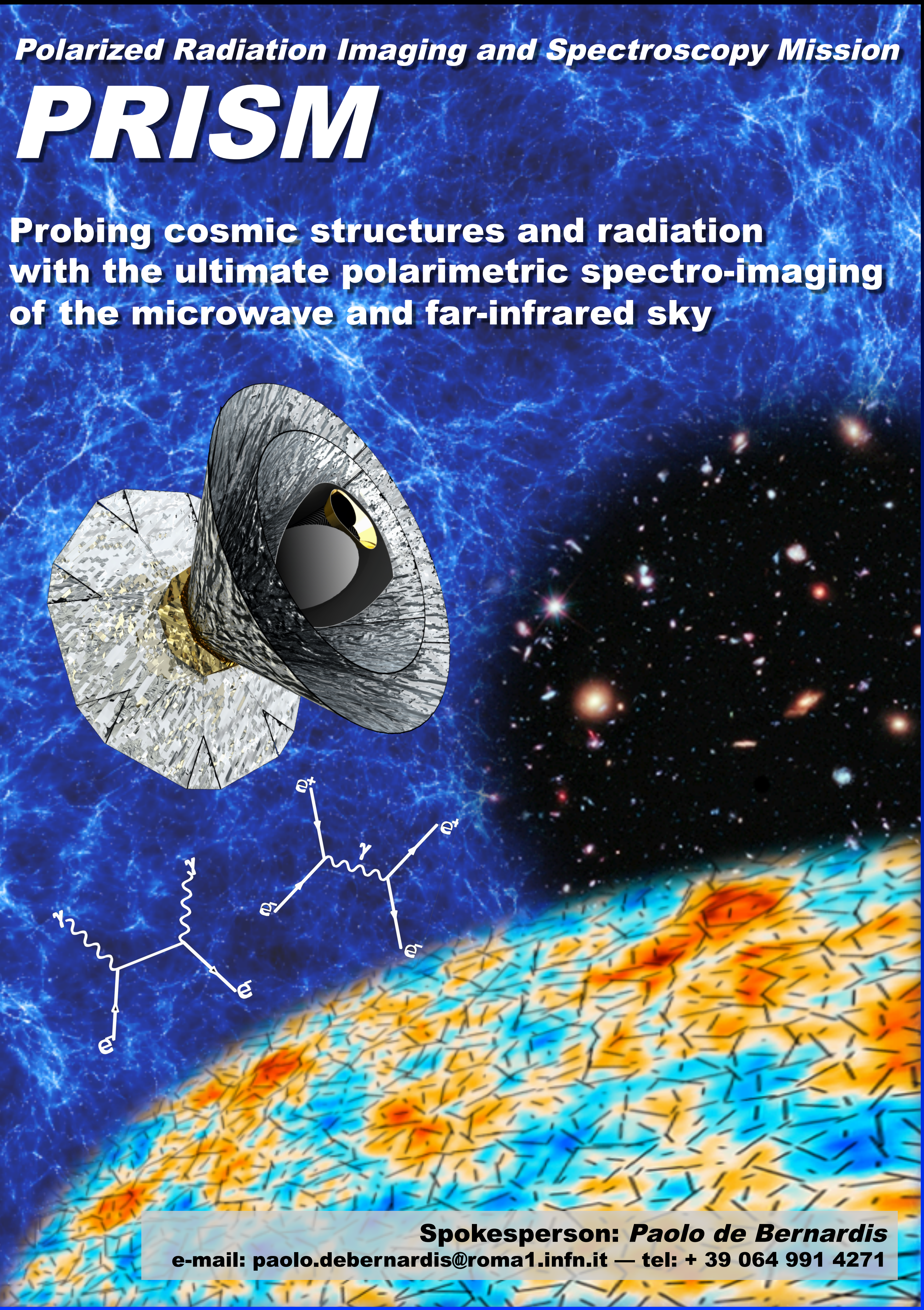


- 400 spectral channel in the frequency range 30 GHz and 6THz (Δv ~ 15GHz)
- about 1000 (!!!) times more sensitive than COBE/FIRAS
- B-mode polarization from inflation ($r \approx 10^{-3}$)
- improved limits on μ and y
- was proposed 2011 as NASA EX mission (i.e. cost ~ 200 M$)





Kogut et al, JCAP, 2011, arXiv:1105.2044



Polarized Radiation Imaging and Spectroscopy Mission

PRISM

Probing cosmic structures and radiation with the ultimate polarimetric spectro-imaging of the microwave and far-infrared sky

Spokesperson: ***Paolo de Bernardis***
e-mail: paolo.debernardis@roma1.infn.it — tel: + 39 064 991 4271

Instruments:

- L-class ESA mission
- White paper, May 24th, 2013
- Imager:
  - polarization sensitive
  - 3.5m telescope [arcmin resolution at highest frequencies]
  - 30GHz-6THz [30 broad (Δv/v~25%) and 300 narrow (Δv/v~2.5%) bands]
- Spectrometer:
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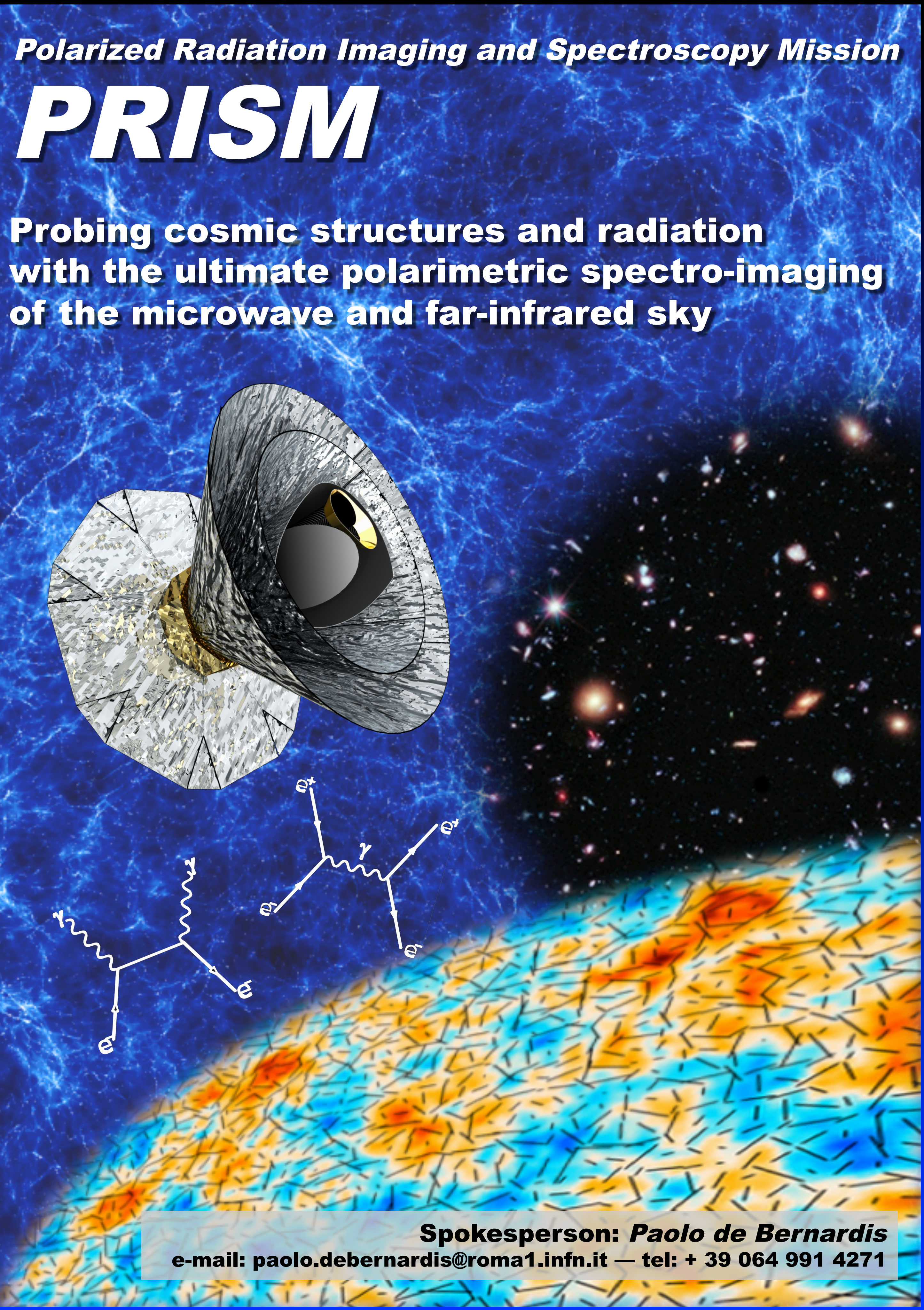
Sign up at:
http://www.prism-mission.org/



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Some of the science goals:

- B-mode polarization from inflation ($r \approx 5\text{x}10^{-4}$)
- count all SZ clusters $>10^{14}$ M_{sun}
- CIB/large scale structure
- Galactic science
- *CMB spectral distortions*

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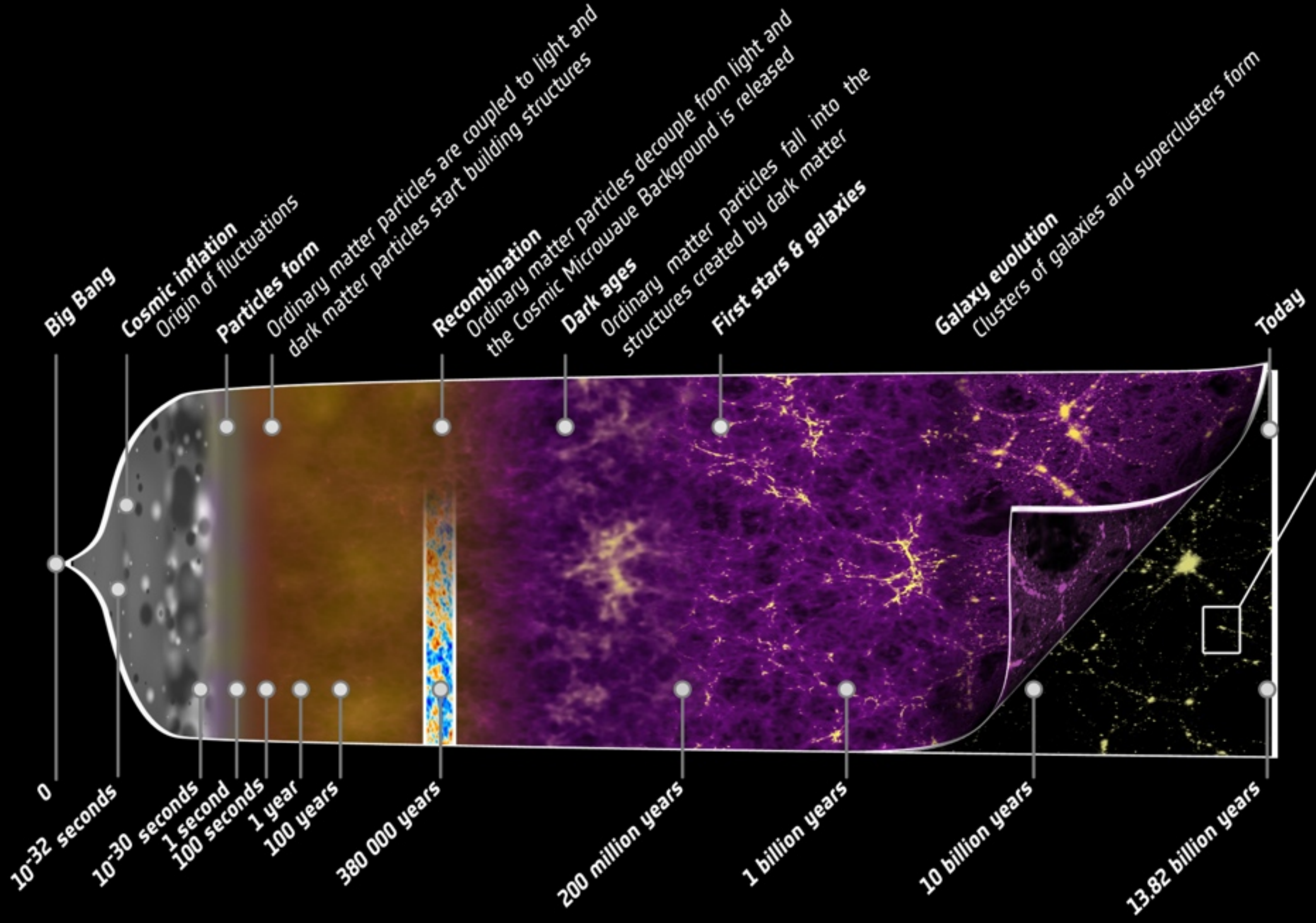
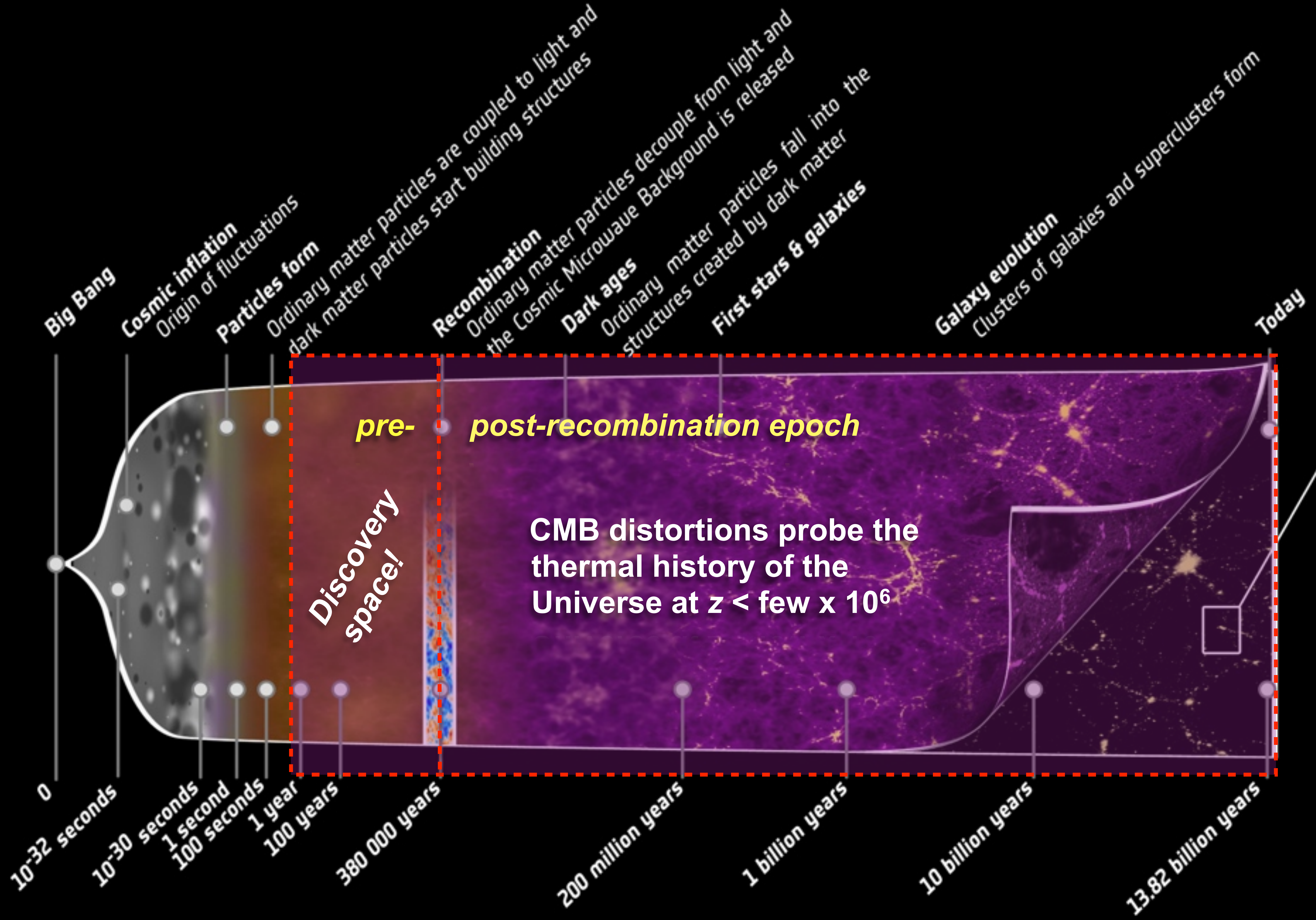

Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years



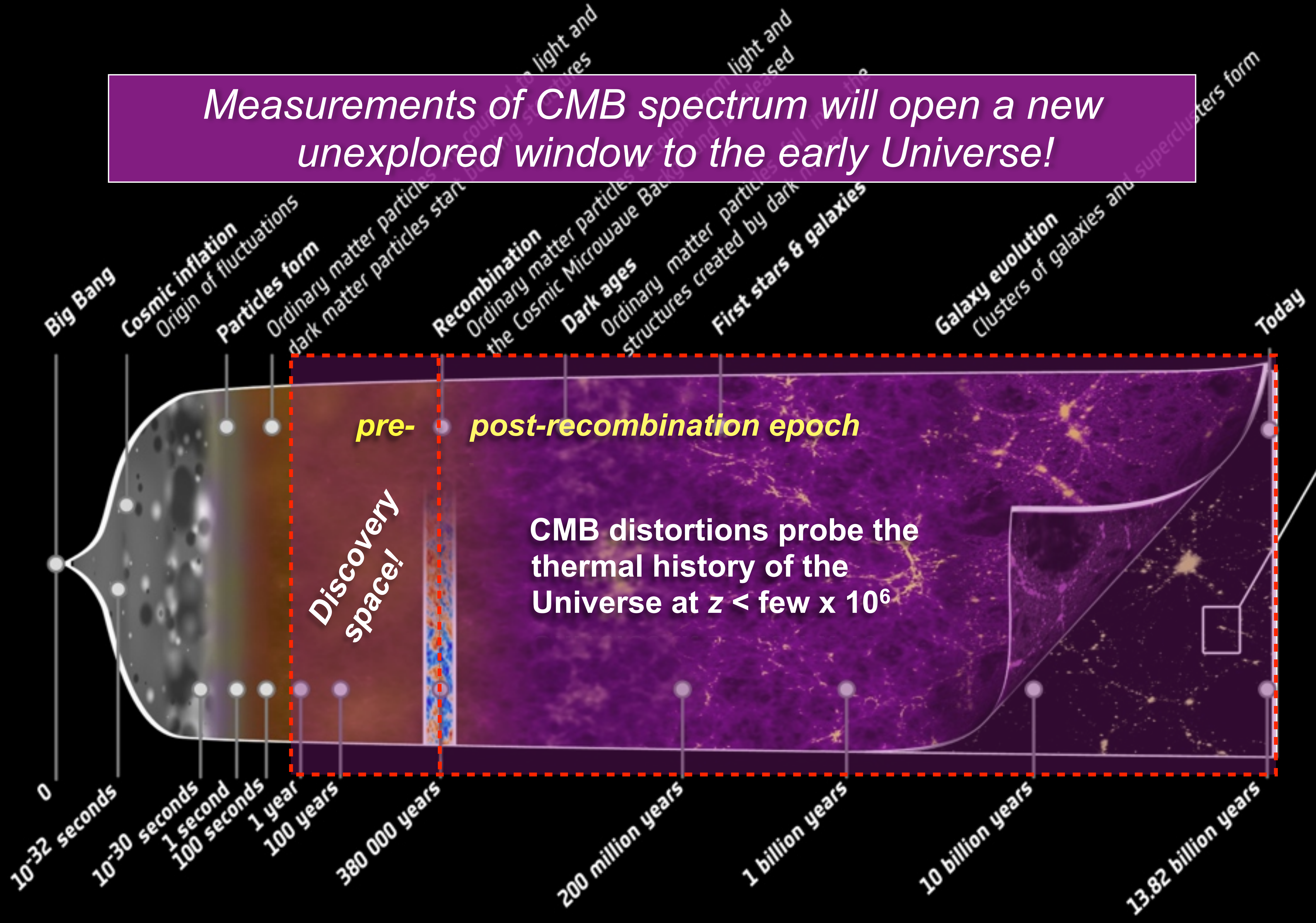
pre-

post-recombination epoch

Discovery space!

CMB distortions probe the thermal history of the Universe at $z <$ few x 10^6

Measurements of CMB spectrum will open a new unexplored window to the early Universe!



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Why should one expect some spectral distortion?

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Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411$ cm^{-3} $(1+z)^3 \sim 2{\times}10^9\, N_b$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1{\times}10^{-7}\, m_e c^2$ cm^{-3} $(1+z)^4 \sim \rho_b$ x $(1+z)$ / 925 ~ 0.26 eV cm^{-3} $(1+z)^4$

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Perturbing full equilibrium by

- Energy injection (interaction *matter* ←→ *photons*)
- Production of (energetic) photons and/or particles (i.e. change of entropy)

→ CMB spectrum deviates from a pure blackbody

→ thermalization process (partially) erases distortions

(Compton scattering, double Compton and Bremsstrahlung in the expanding Universe)

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Measurements of CMB spectrum place very tight constraints on the thermal history of our Universe!

Some simple statements about distortions

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- Start with blackbody: T_γ, $N_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^3$, and $\rho_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^4$

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$$\Delta\rho_\gamma = (4\pi/c) \int h\nu \, \Delta N_\nu \, {\rm d}\nu$$

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- Effective temperatures:

$$N_\gamma \equiv N_\gamma^{\mathrm{bb}}(T_N^*)$$

$$\rho_\gamma \equiv \rho_\gamma^{\mathrm{bb}}(T_\rho^*)$$

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$$\begin{aligned} N_\gamma &\equiv N_\gamma^{\rm bb}(T_N^*) \\ \rho_\gamma &\equiv \rho_\gamma^{\rm bb}(T_\rho^*) \end{aligned} \quad \Longrightarrow \quad \begin{aligned} T_N^* &= \left(\frac{h^3 c^3 N_\gamma}{16\pi k^3 \zeta(3)}\right)^{1/3} \approx T_\gamma \left(1 + \frac{1}{3}\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}\right) > T_\gamma \\ T_\rho^* &= \left(\frac{15 h^3 c^3 \rho_\gamma}{8\pi^5 k^4}\right)^{1/4} \approx T_\gamma \left(1 + \frac{1}{4}\frac{\Delta \rho_\gamma}{\rho_\gamma^{\rm bb}}\right) > T_\gamma. \end{aligned}$$

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Question: Is there enough time to restore full equilibrium?

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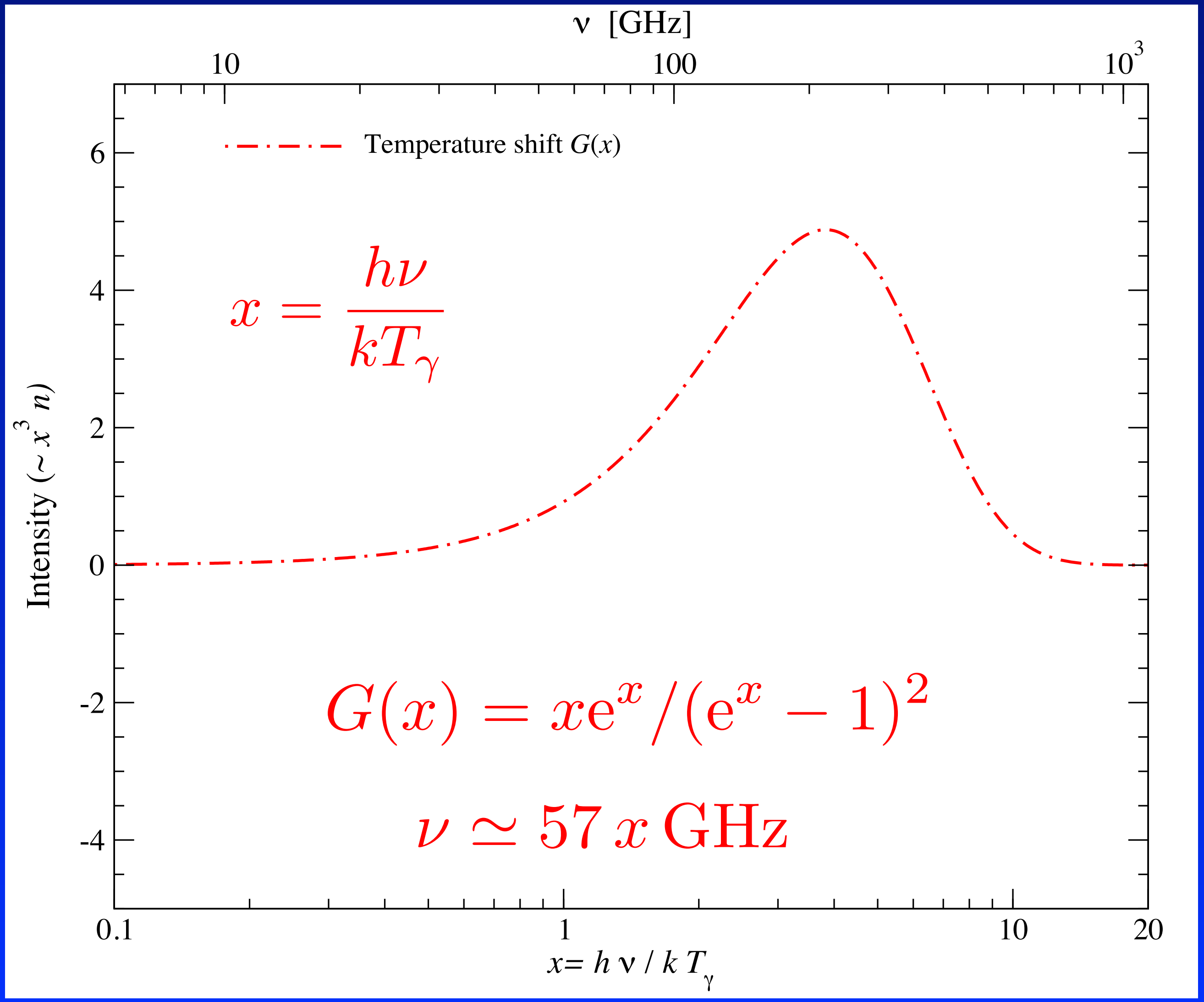

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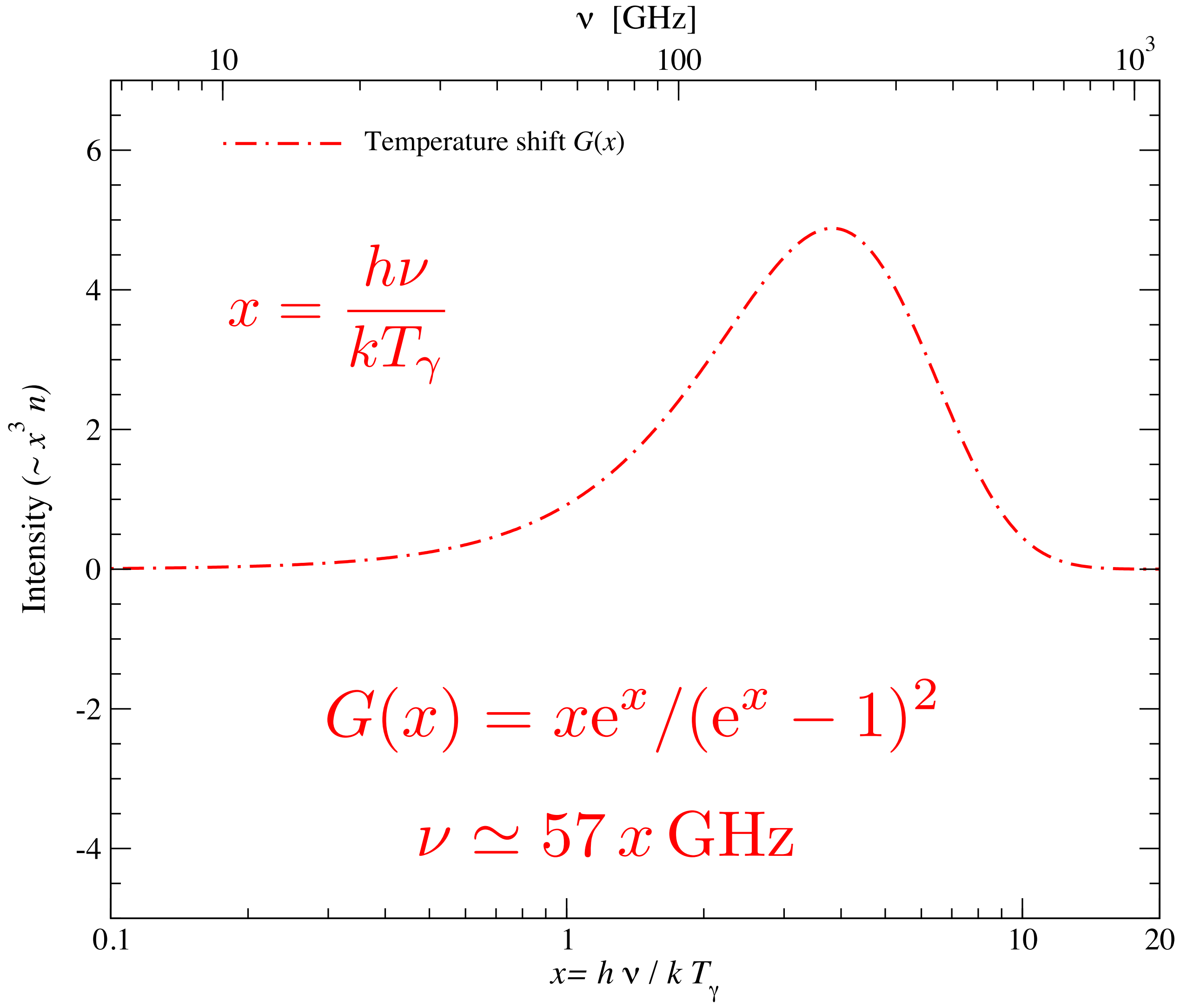

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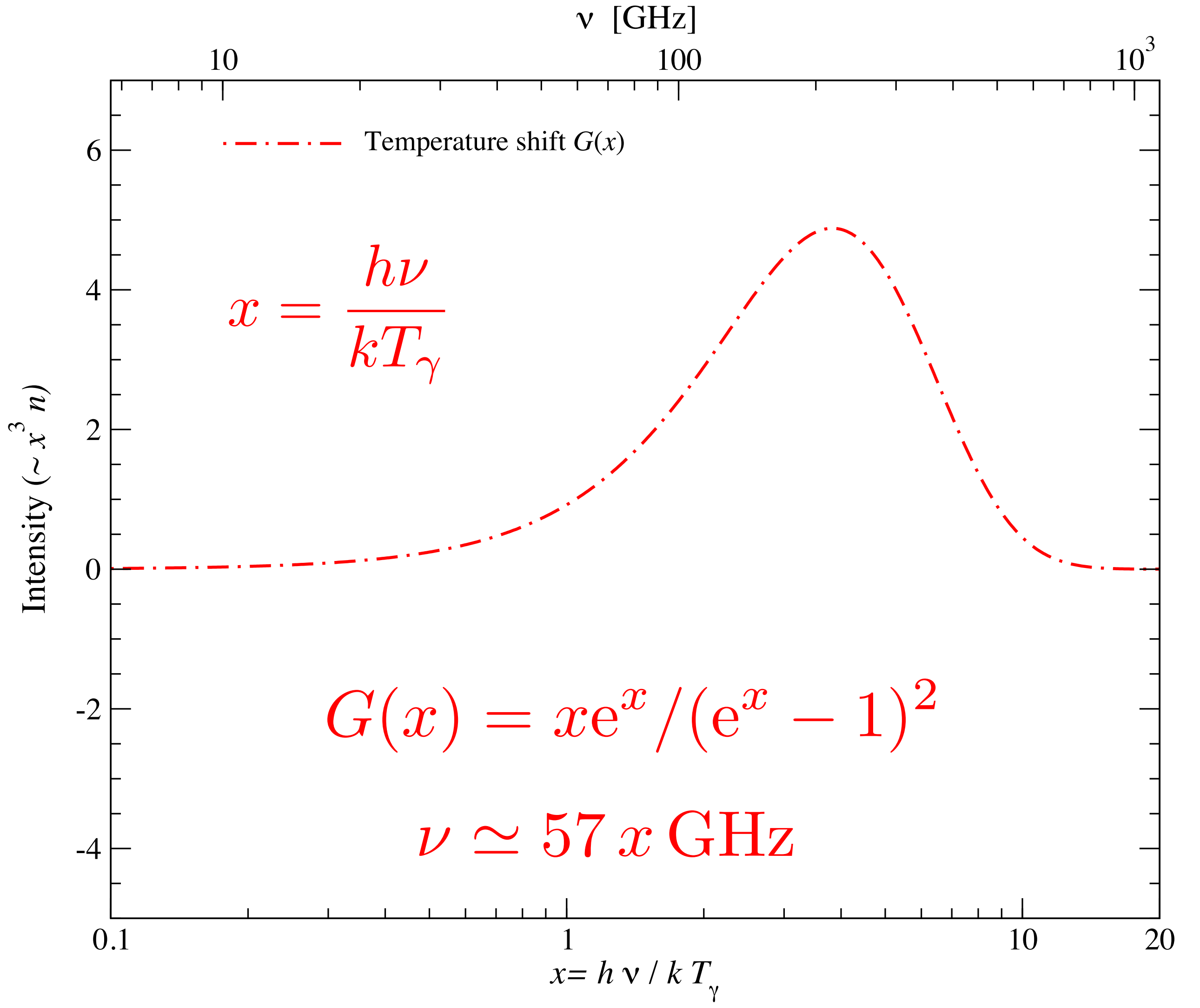

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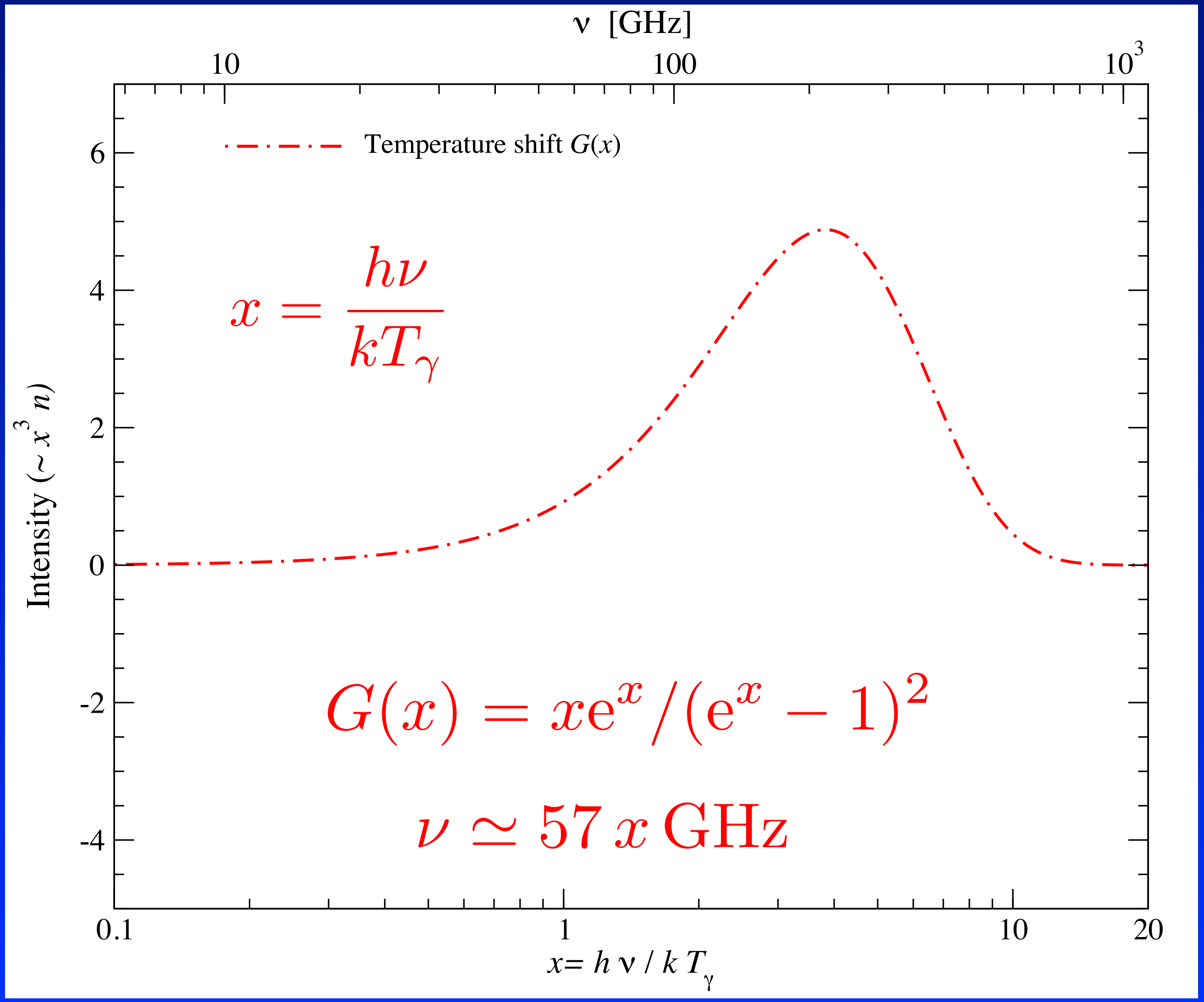

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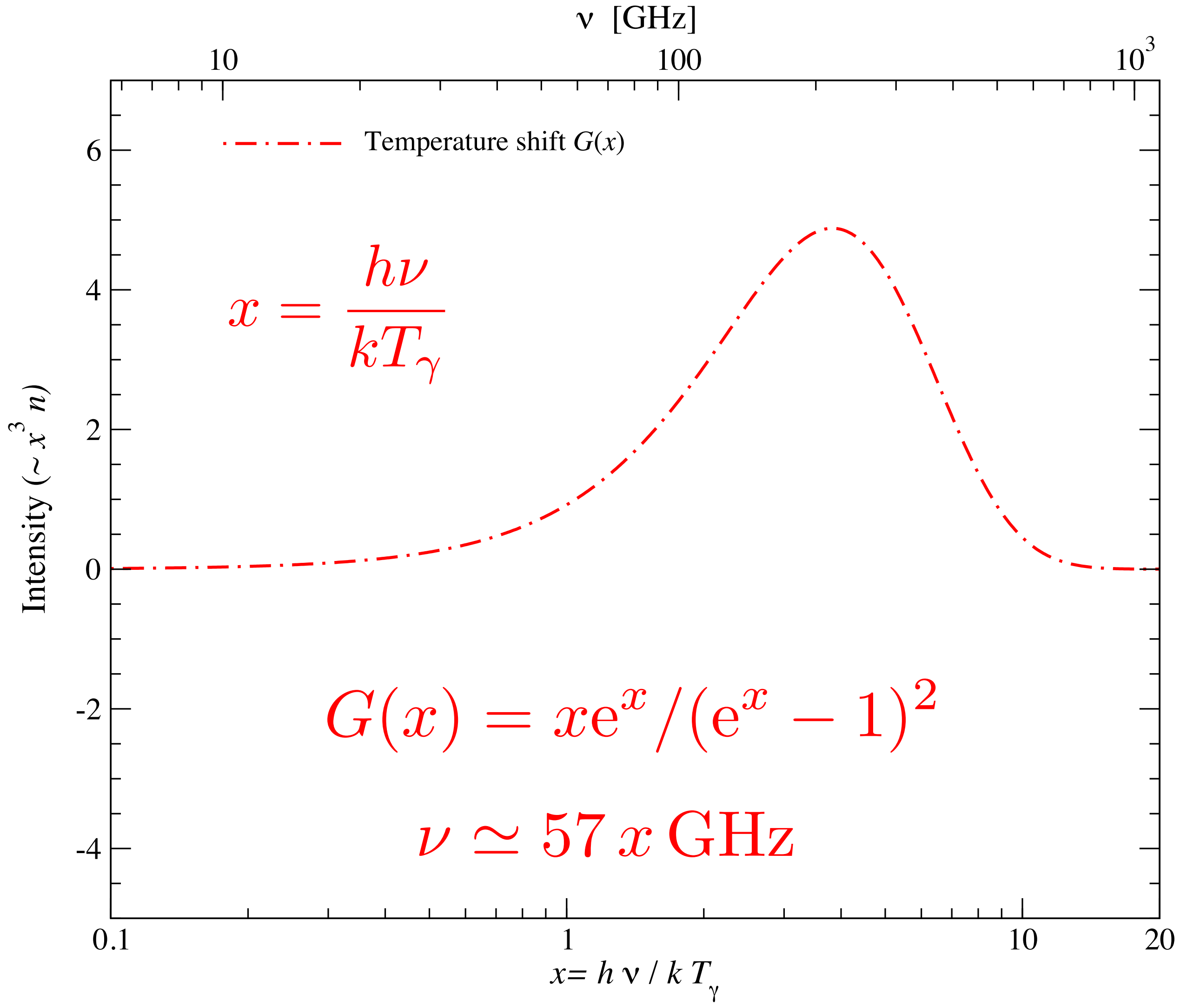

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(
- Only lowest order in Δ*T*/*T*
  → *superposition of blackbodies*

)

How does the thermalization process work?

Some important conditions

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- Hubble expansion
  - → adiabatic cooling of photons [$T_\gamma \sim (1+z)$] and ordinary matter [$T_\mathrm{m} \sim (1+z)^2$]
  - → redshifting of photons

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

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redistribution of photon over frequency

adjusting photon number

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- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$

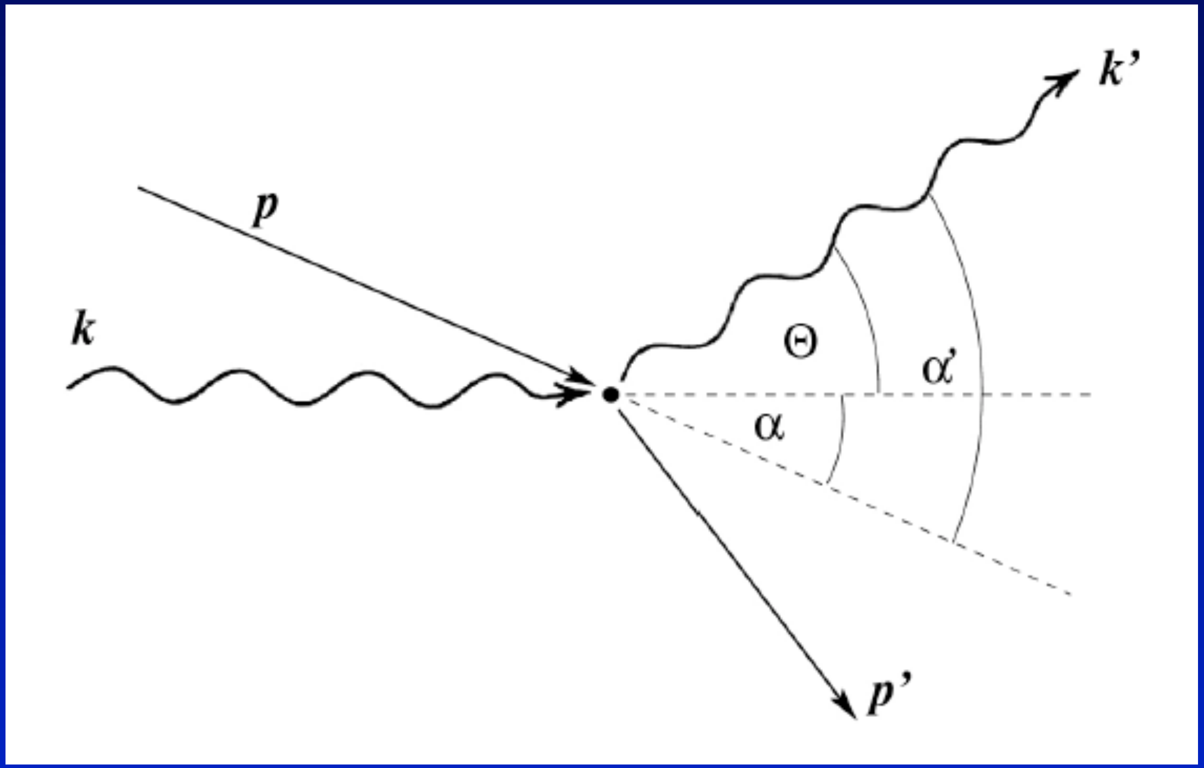
- Full equilibrium: $\mathcal{C}[n] \equiv 0$ ⇒ blackbody spectrum conserved

- Energy release: $\mathcal{C}[n] \neq 0$ ⇒ *thermalization process starts*

Compton scattering

Redistribution of photons by Compton scattering

- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$



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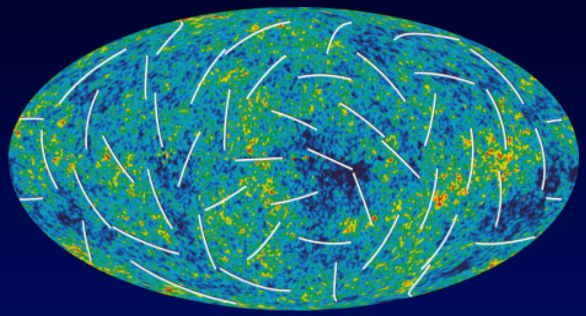
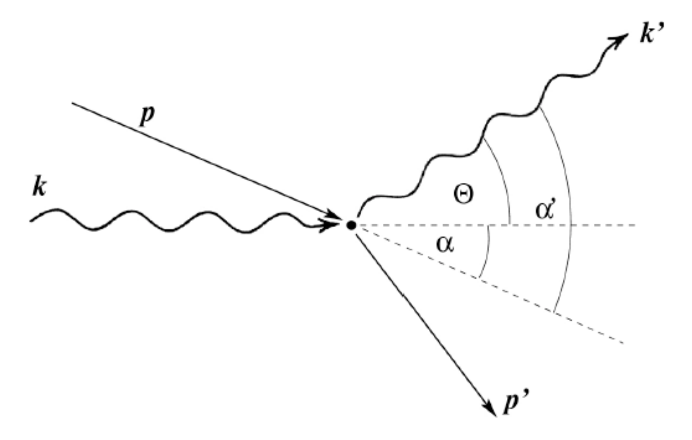


- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$

→ *no energy exchange ⇒ Thomson limit*

⇒ important for anisotropies

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_{\mathrm{T}}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$



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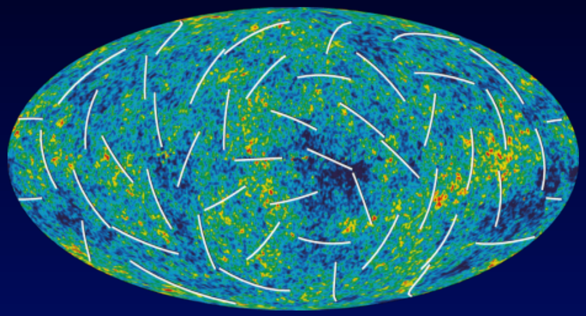

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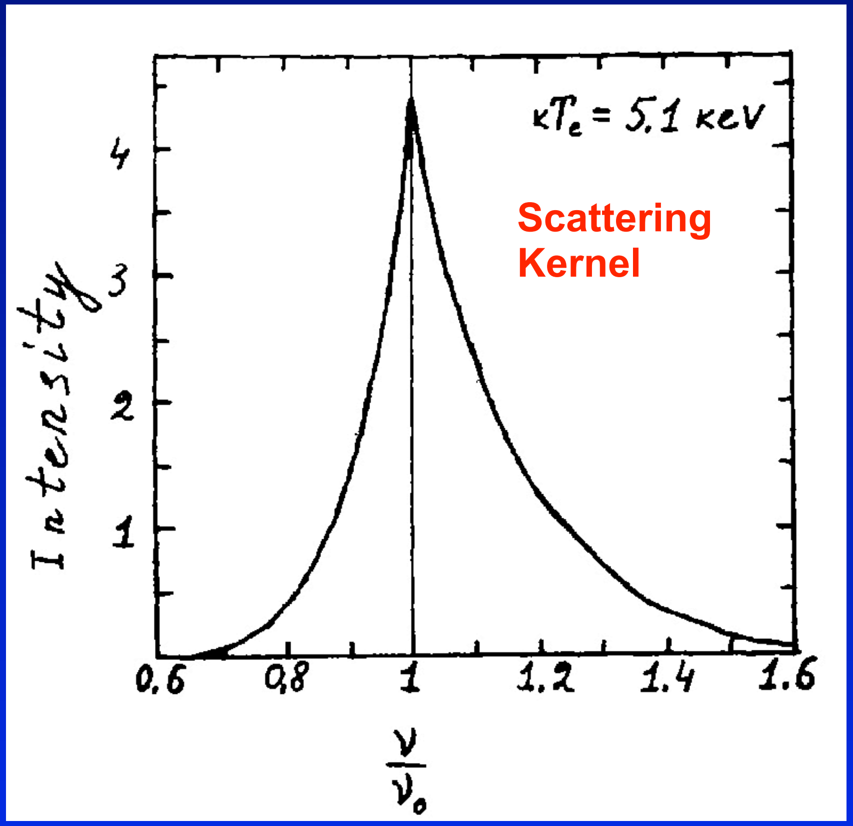
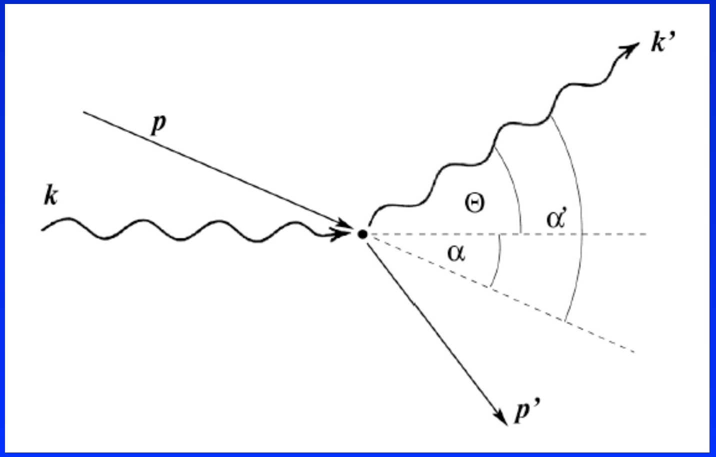
→ *energy exchange included*

- up-scattering due to the ***Doppler*** effect for $h\nu < 4kT_{\mathrm{e}}$
- down-scattering because of ***recoil*** (and stimulated recoil) for $h\nu > 4kT_{\mathrm{e}}$
- ***Doppler*** broadening $\frac{\Delta\nu}{\nu} \simeq \sqrt{\frac{2kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}}$



Sunyaev & Zeldovich, 1980, ARAA, 18, 537



Compton Collision Term / Kompaneets Equation

$$\left.\frac{\mathrm{d}n_\nu}{\mathrm{d}\tau}\right|_{\mathrm{C}} = \int \left[\left(\frac{\nu'}{\nu}\right)^2 P(T_{\mathrm{e}}, \nu' \to \nu)\, n_{\nu'}(1+n_\nu) - P(T_{\mathrm{e}}, \nu \to \nu')\, n_\nu(1+n_{\nu'}) \right] \mathrm{d}\nu'$$

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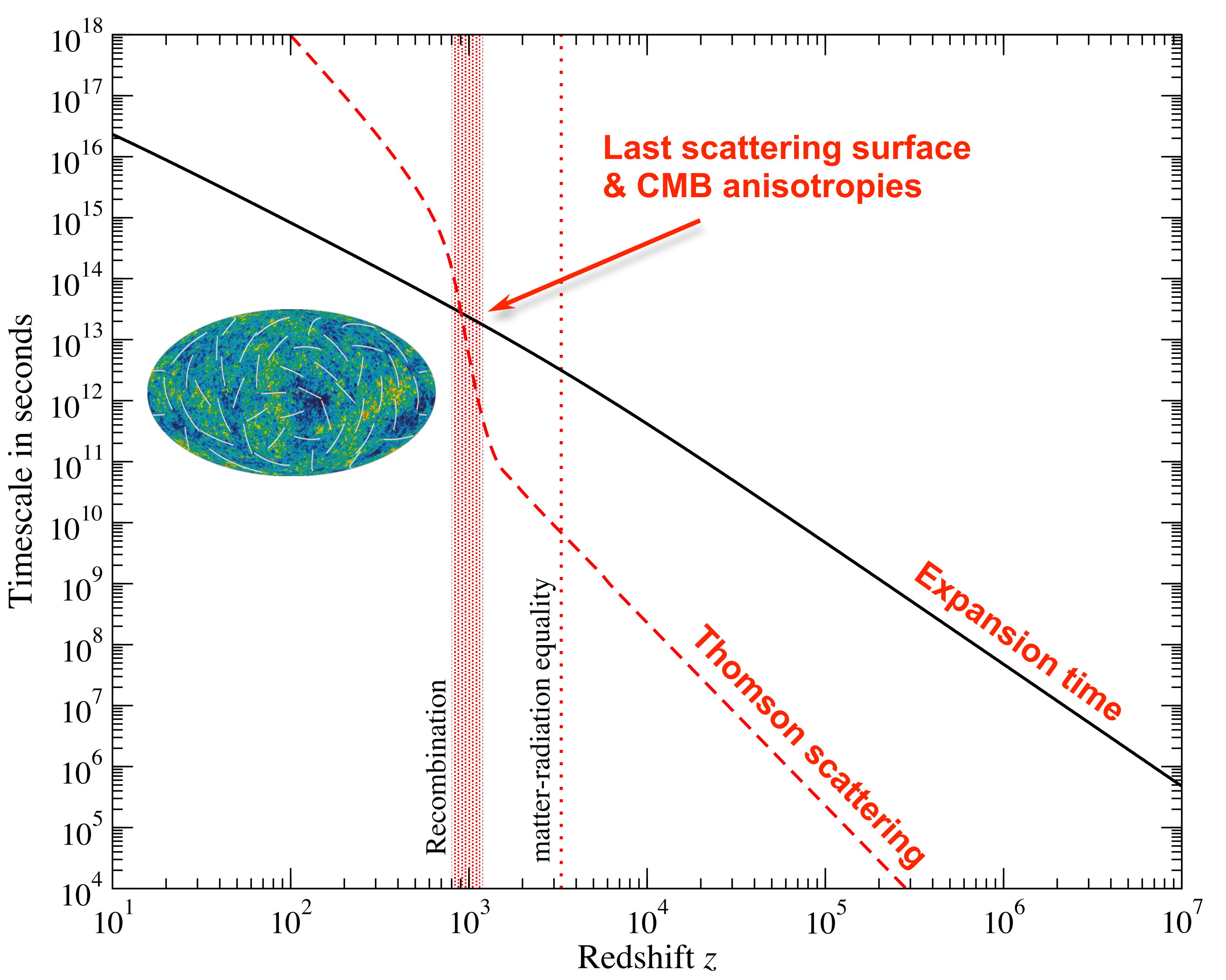
Thomson optical depth

Important Timescales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,{\rm sec}$



Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,{\rm sec}$$
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Thomson optical depth

Scattering Kernel

Stimulated scattering

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Doppler broadening & boosting **recoil and stimulated recoil**

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Kompaneets equation (A.S. Kompaneetz, 1956, JETP, 47, p. 1939)

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Kompaneets equation (A.S. Kompaneetz, 1956, JETP, 47, p. 1939)

- *Initially developed to describe repeated scattering of thermal neutrons*

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$$N = N_\mathrm{e} + N_\mathrm{H} + N_\mathrm{He}$$

total number of gas particles

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While Compton cooling is fast:

$$T_\mathrm{e} \simeq T_\mathrm{m} \simeq T_\mathrm{eq}$$

Important Timescales for Compton Process

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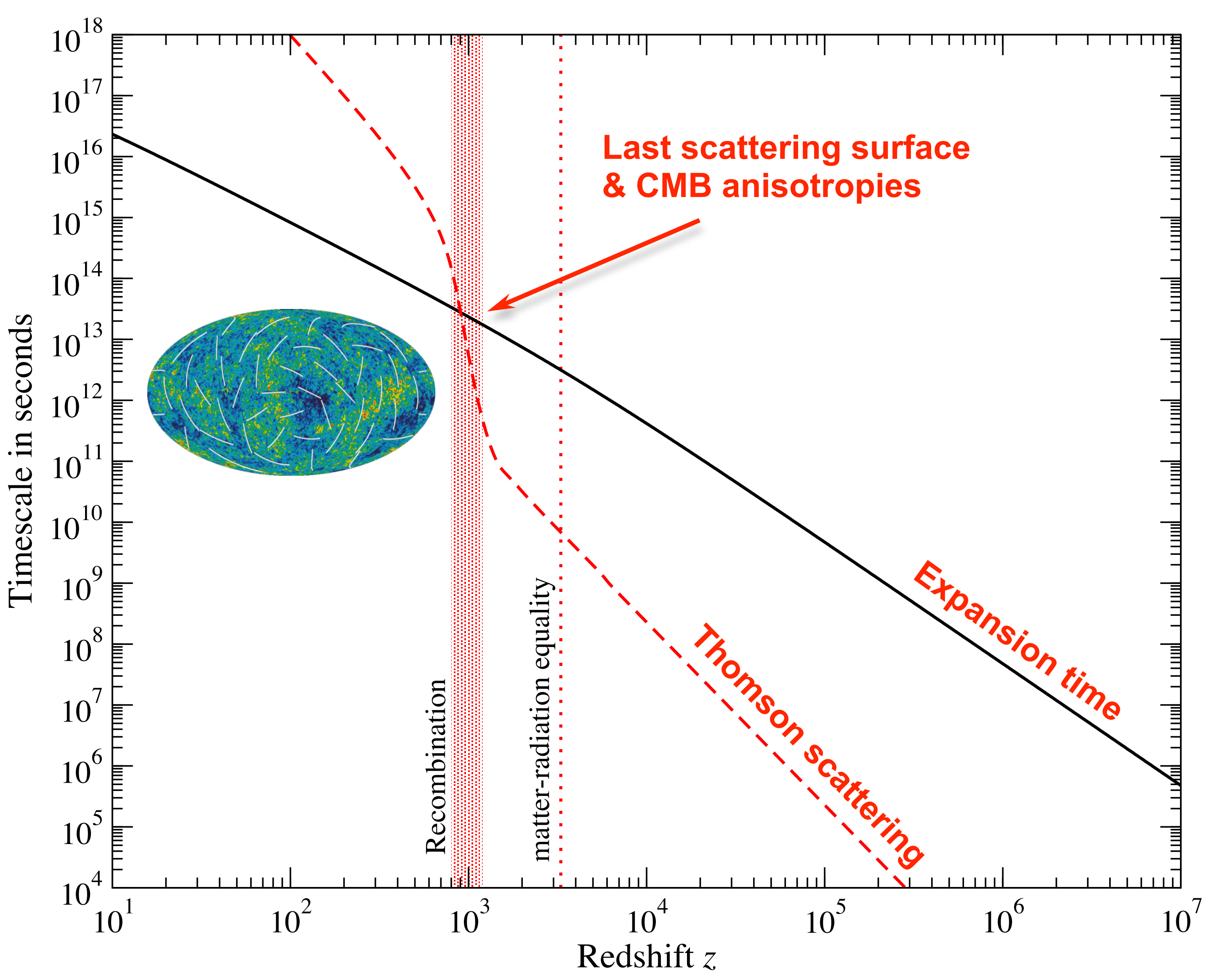

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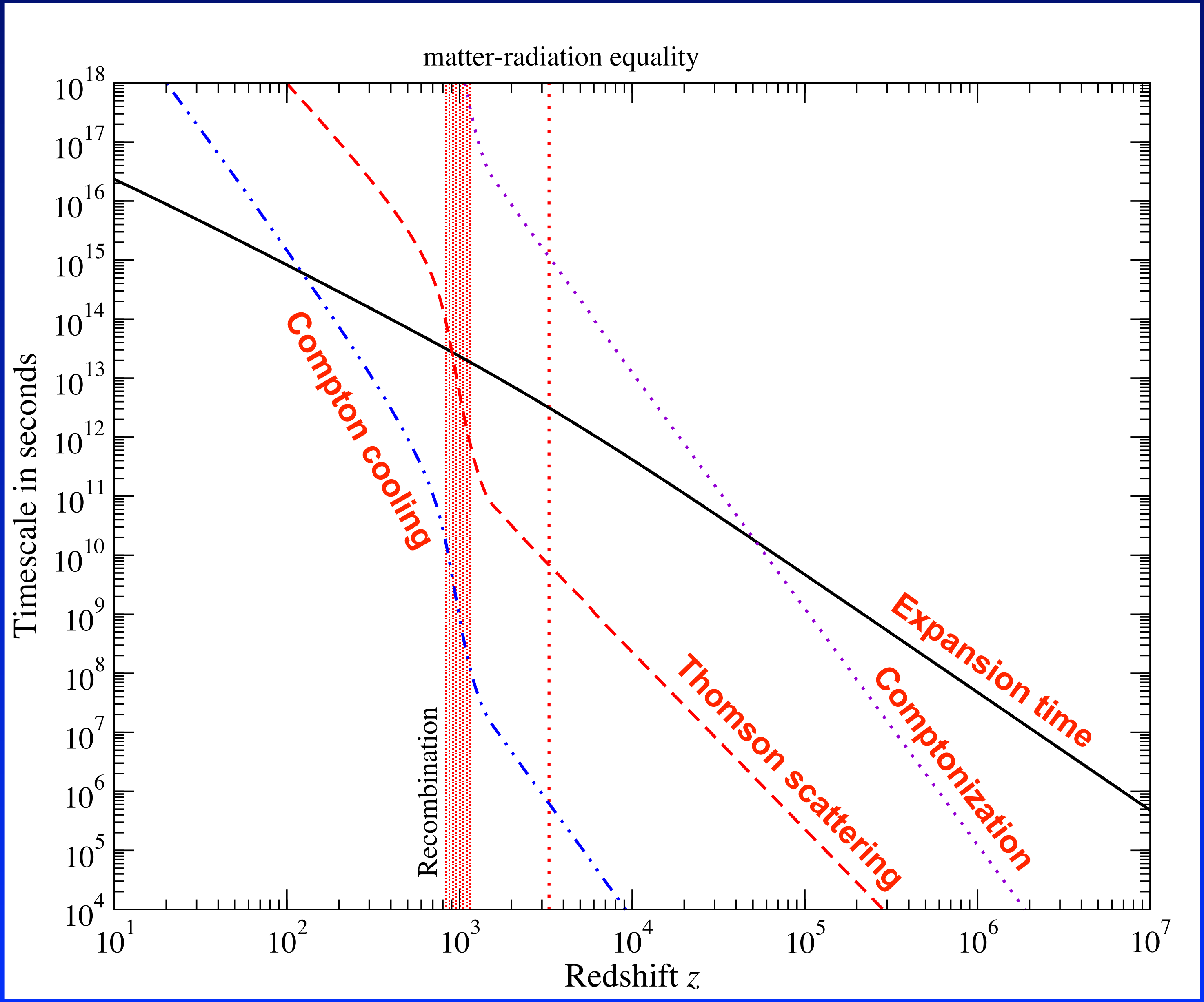

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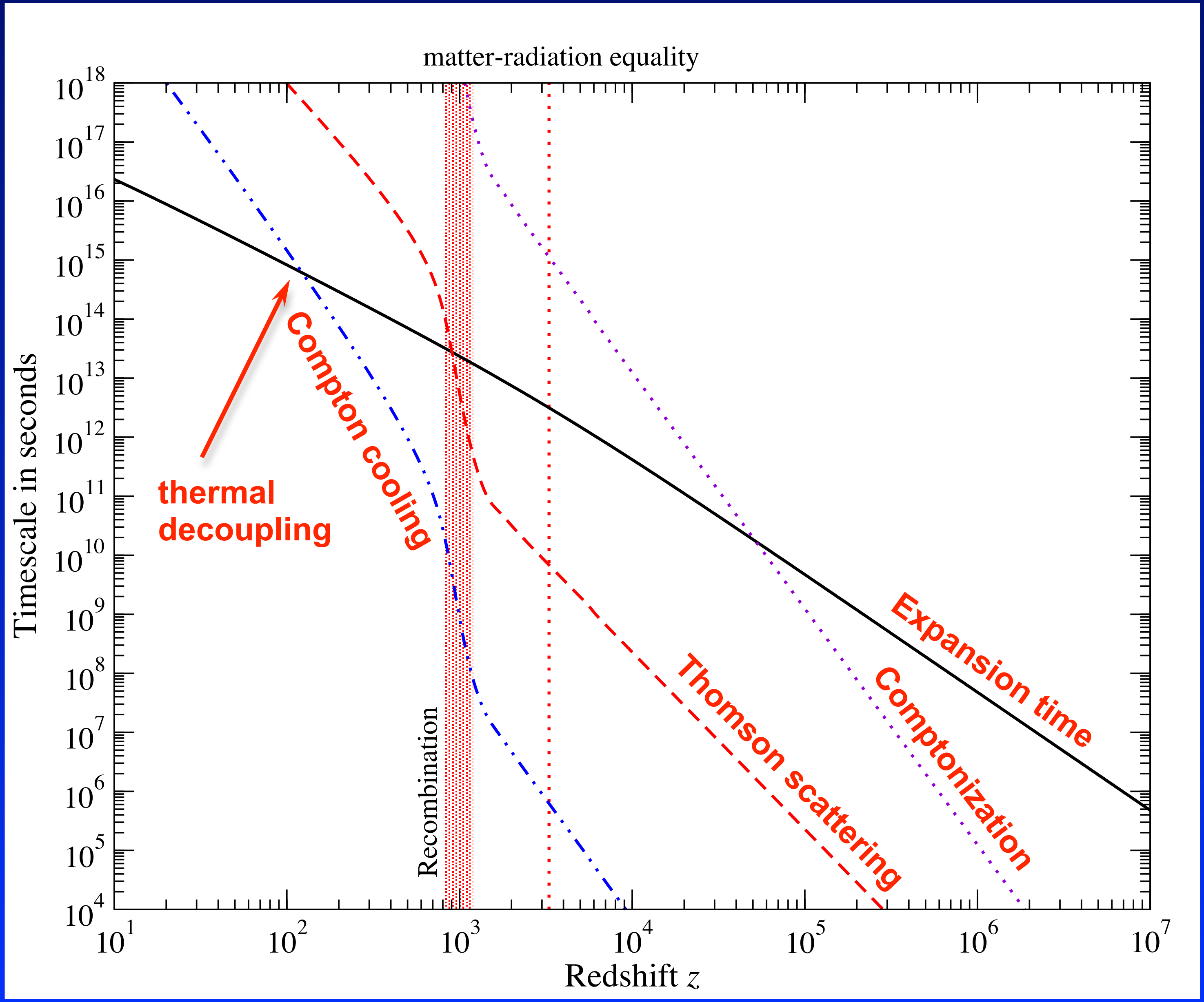

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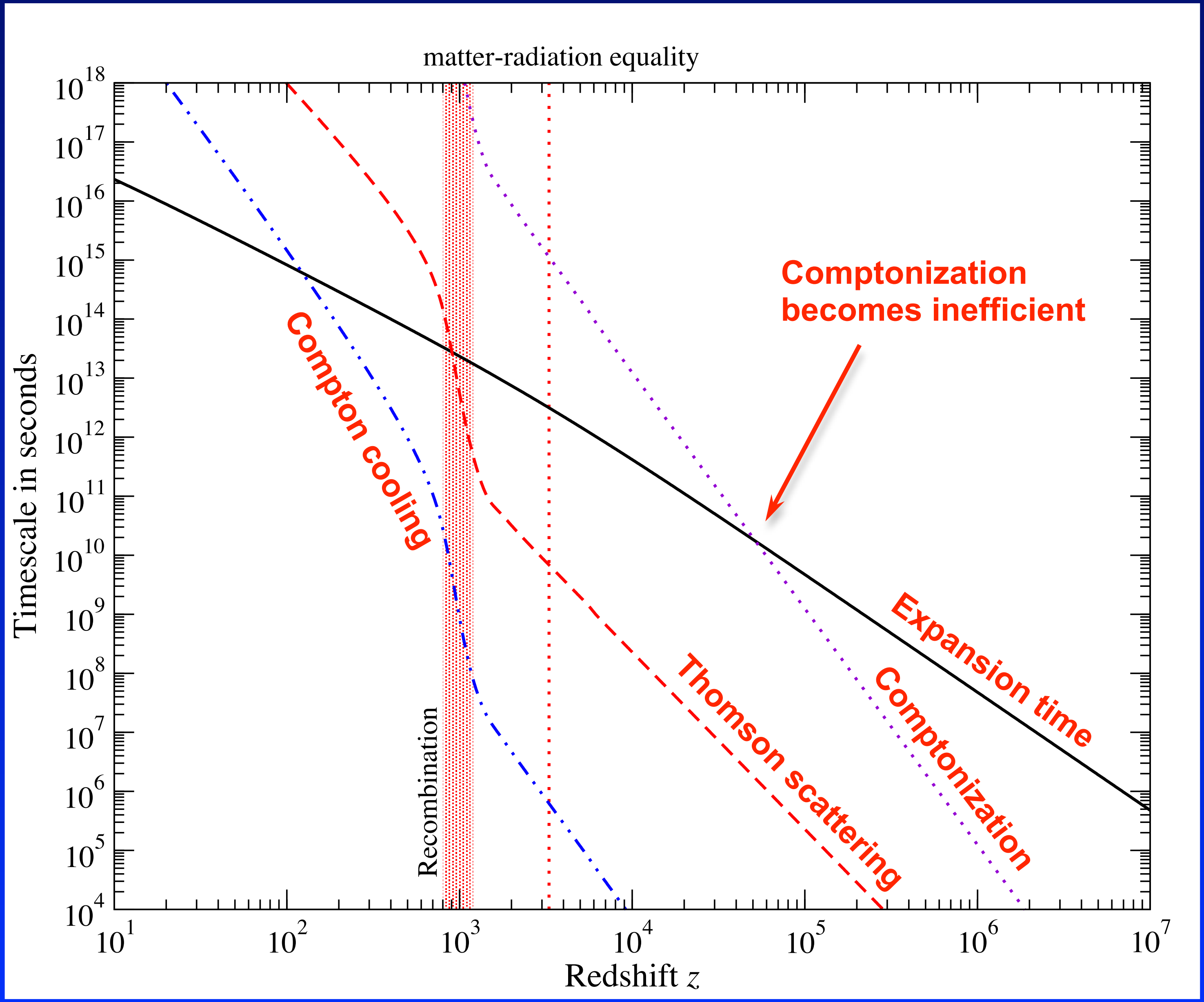

- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*
- *Comptonization becomes inefficient at $z_{\rm K} \simeq 50000$*

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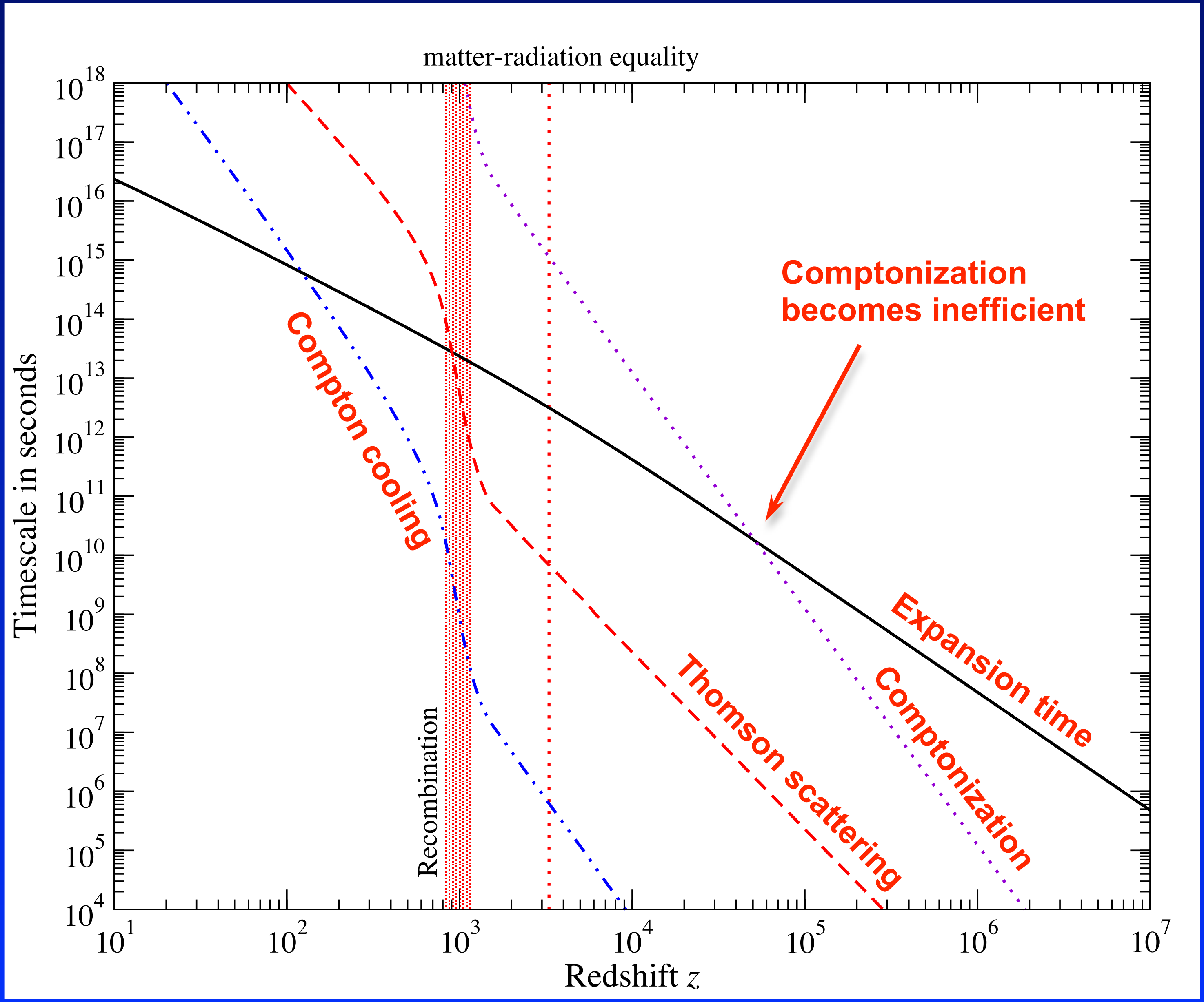

- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*
- *Comptonization becomes inefficient at* $z_{\rm K} \simeq 50000$

⇒ character of distortion should change at $z_{\rm K}$ *!*

Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,\rm sec$$

$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,\rm sec$$

Matter dominated

What are y- and μ-distortions?

Compton y-distortion / thermal SZ effect

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}\,n(1+n)\right]$$

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Compton y-parameter

Compton y-distortion / thermal SZ effect

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Compton y-parameter *spectrum of y-distortion*

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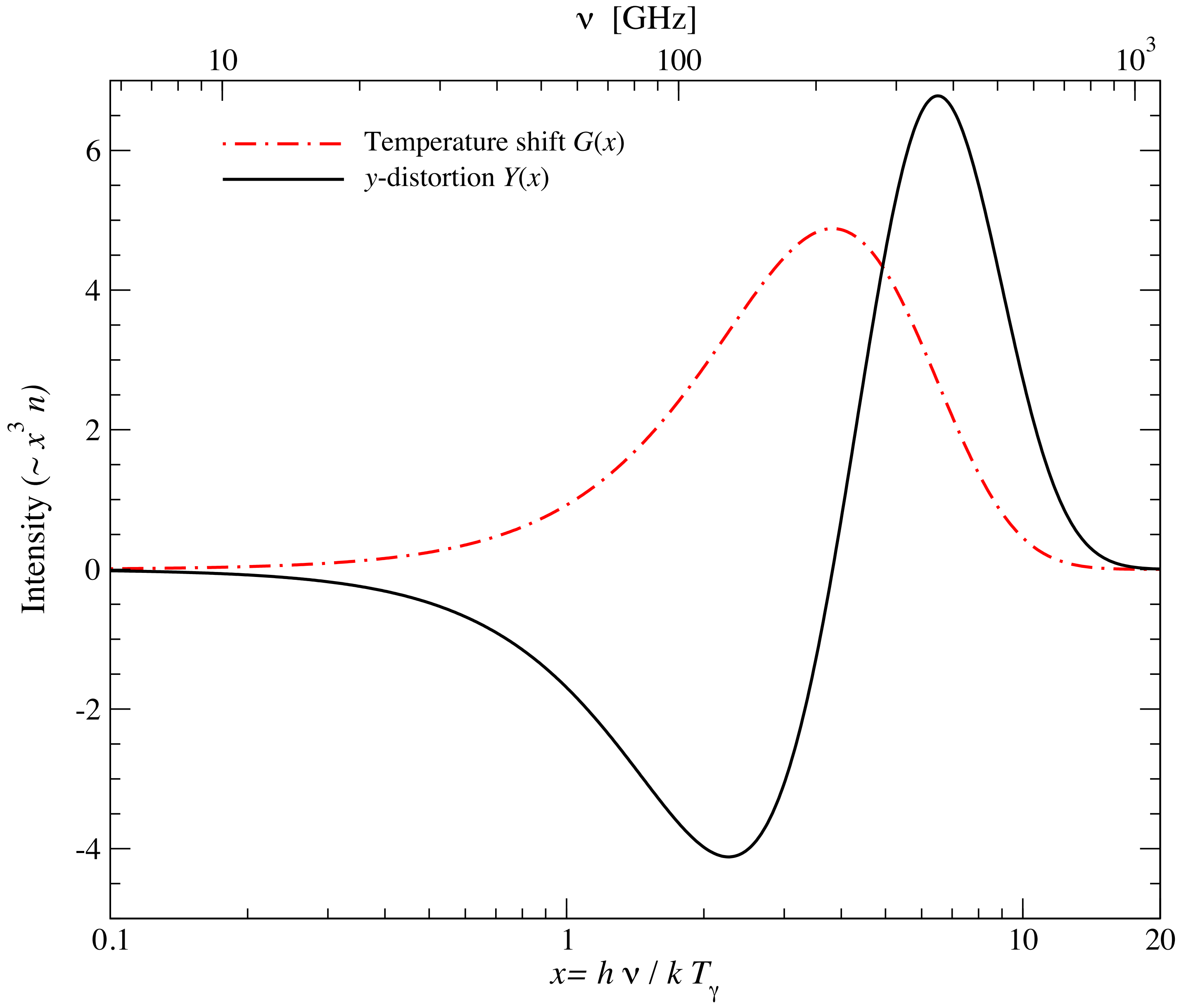
- *if* $T_{\mathrm{e}} > T_\gamma \implies$ *up-scattering of photons / cooling of electrons*
- *for* $T_{\mathrm{e}} \gg T_\gamma \implies$ *thermal Sunyaev-Zeldovich effect (up-scattering)*

Temperature shift ↔ y-distortion



$$Y(x) = \frac{x e^x}{(e^x - 1)^2}\left[x\frac{e^x + 1}{e^x - 1} - 4\right]$$

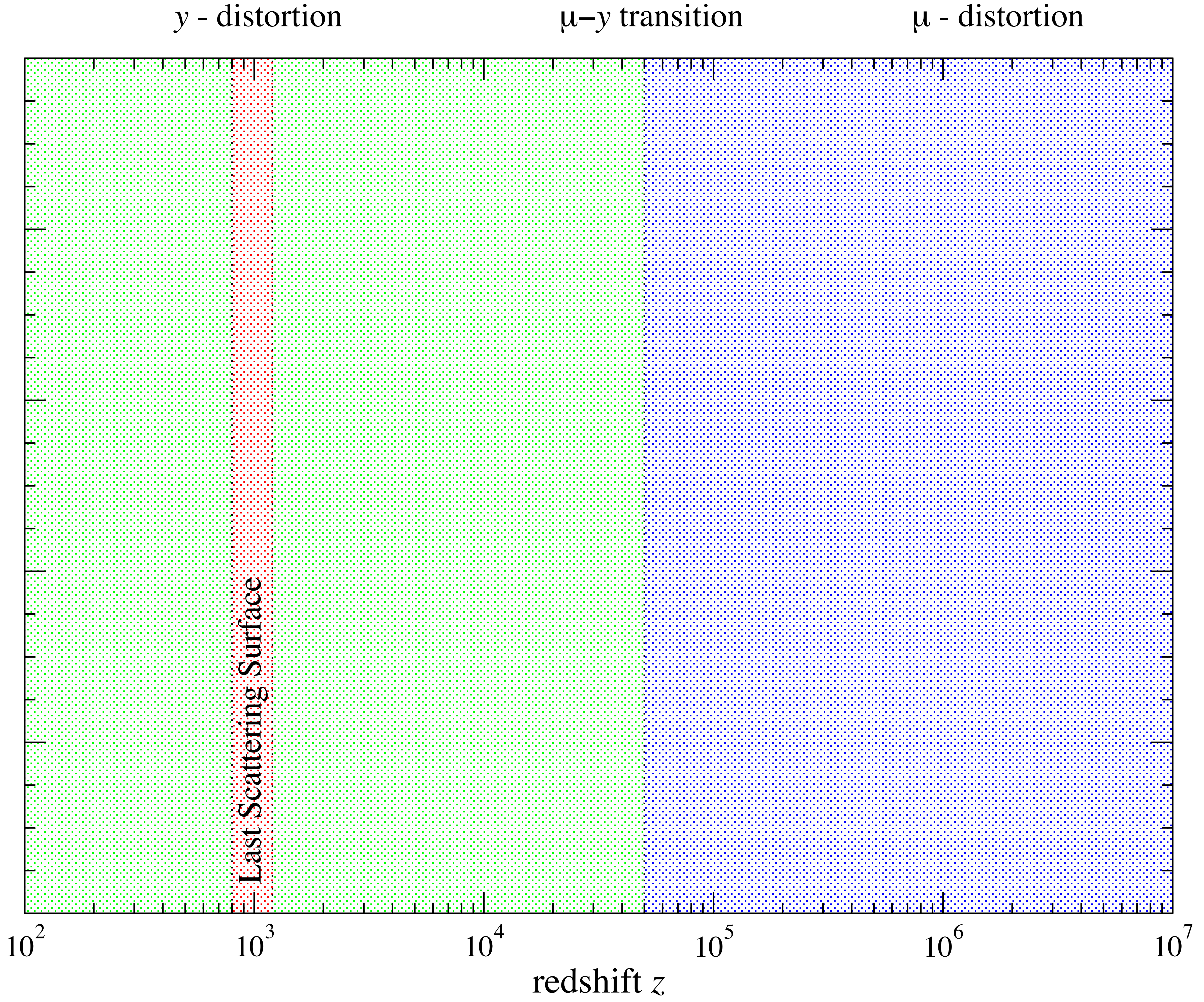
- thermal SZ spectrum (non-relativistic)
- *important for* $y \ll 1$
- null at ν ~ 217 GHz (x~3.83)
- photon number conserved

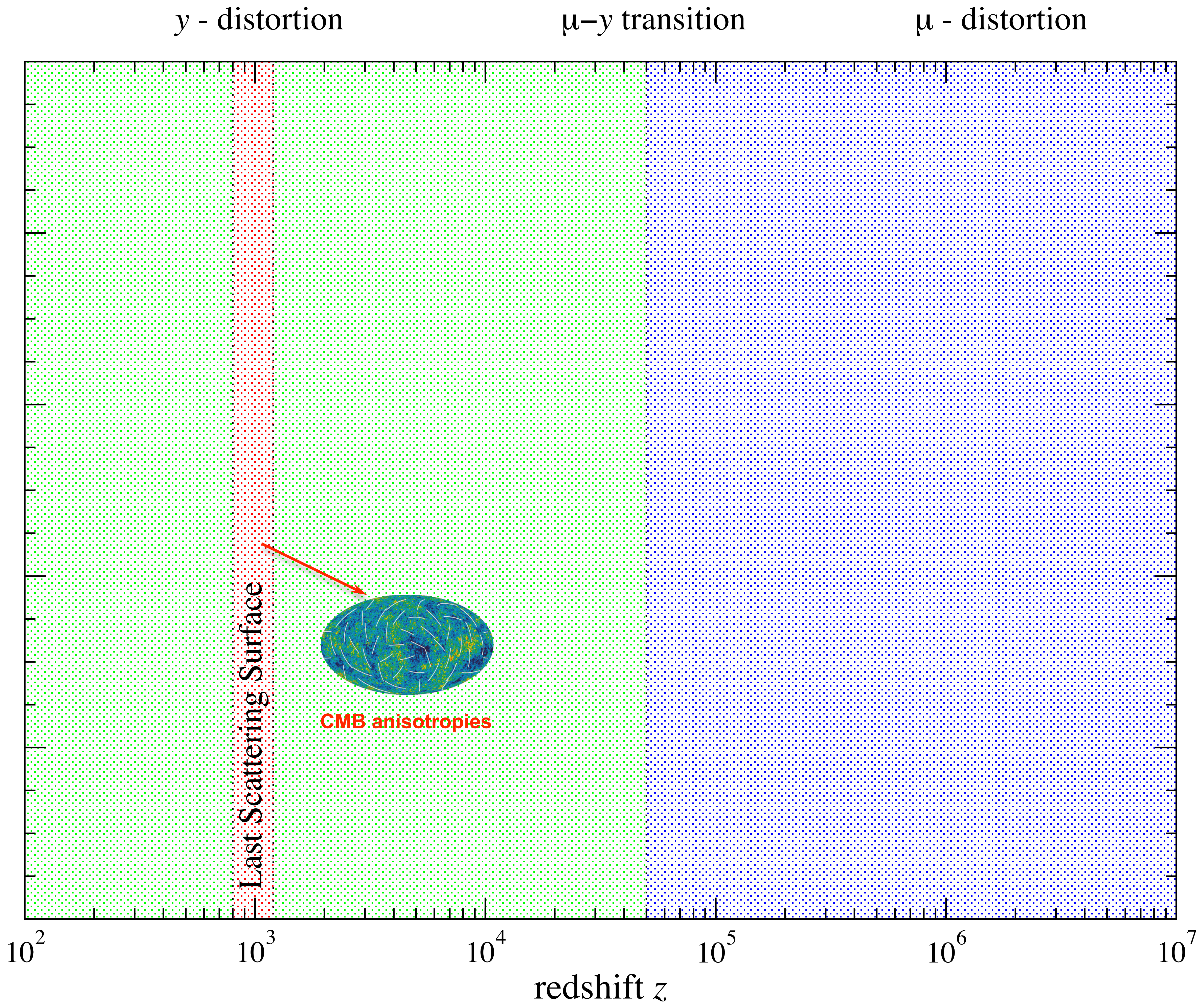
$$\int x^2 Y(x)\,dx = 0$$

- energy exchange

$$\int x^3 Y(x)\,dx = \frac{4\pi^4}{15} \leftrightarrow 4\rho_\gamma$$

- *direction of energy flow* depends on difference between T_e and T_γ


y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z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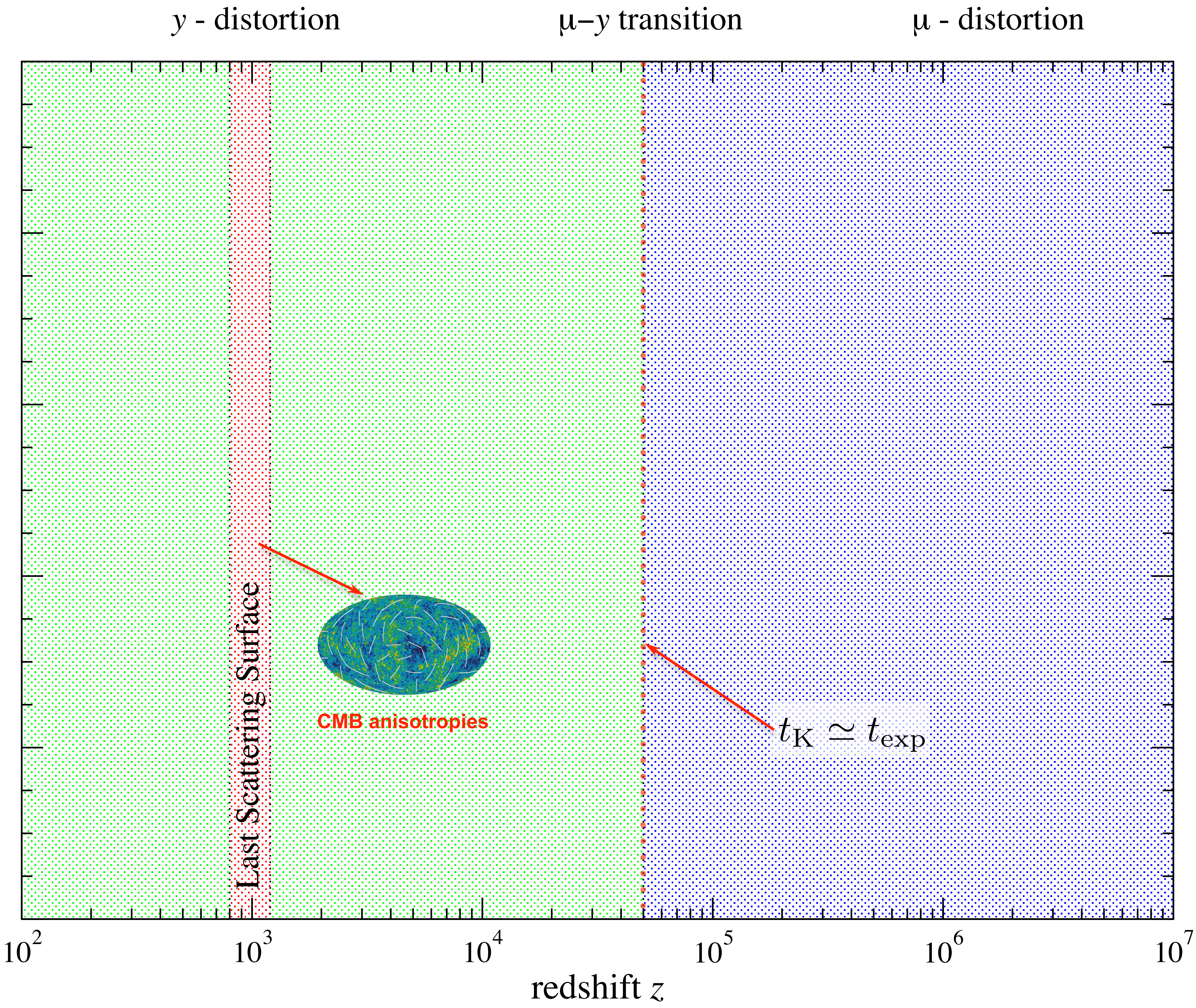
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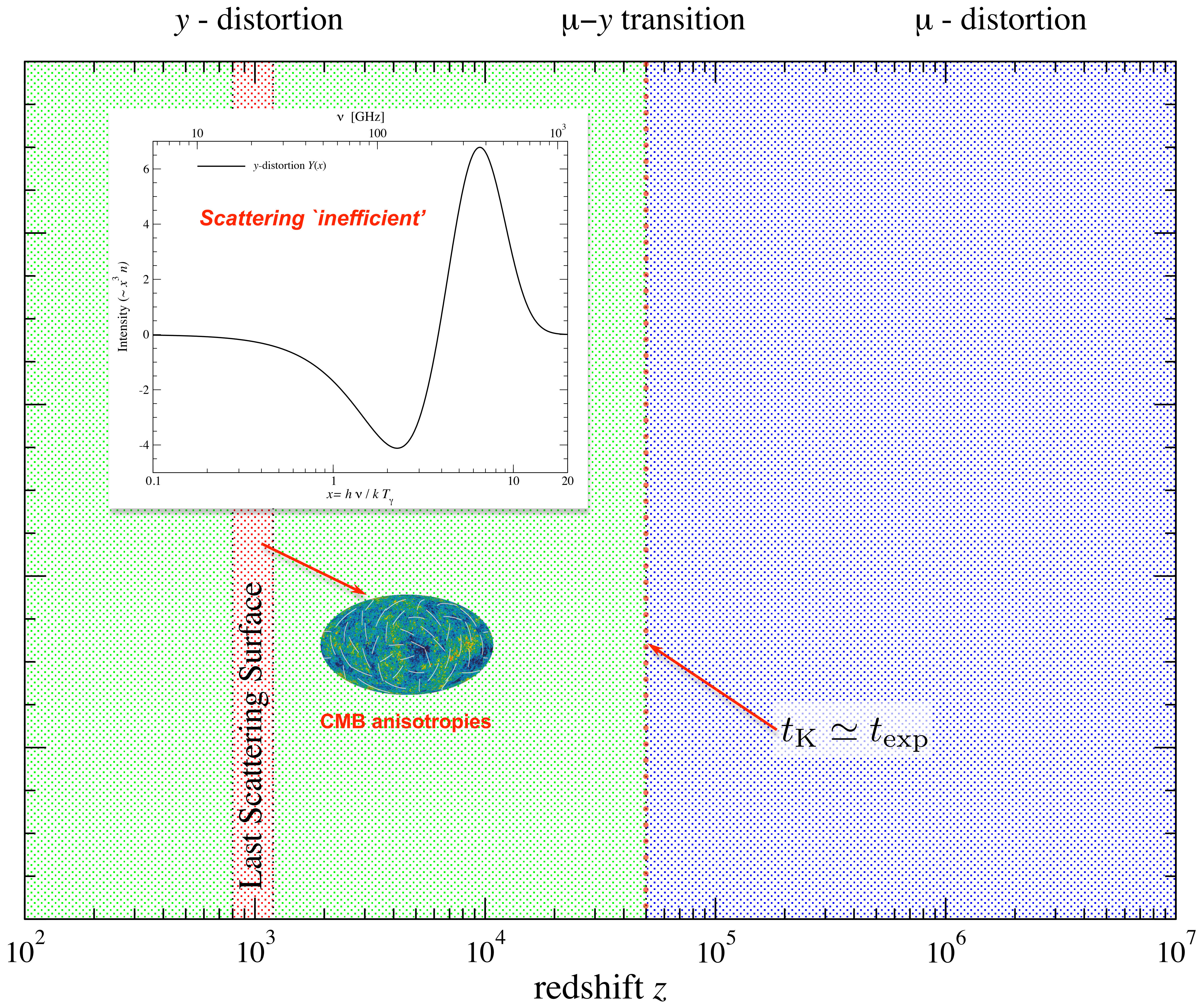
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Intensity (~ x^3 n)
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Chemical Potential / μ-parameter

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- *Limit of “many” scatterings*

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constant

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$T_\gamma = T_\mathrm{e} \equiv T_{\mathrm{eq}} \quad \text{and} \quad \mu_0 = \text{const}$ ($\equiv 0$ for blackbody)

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Something is missing? How do you fix T_e and μ_0?

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

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$$\approx 1.368$$

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$$\Longrightarrow \quad M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

μ-distortion spectrum
(photon number conserved)

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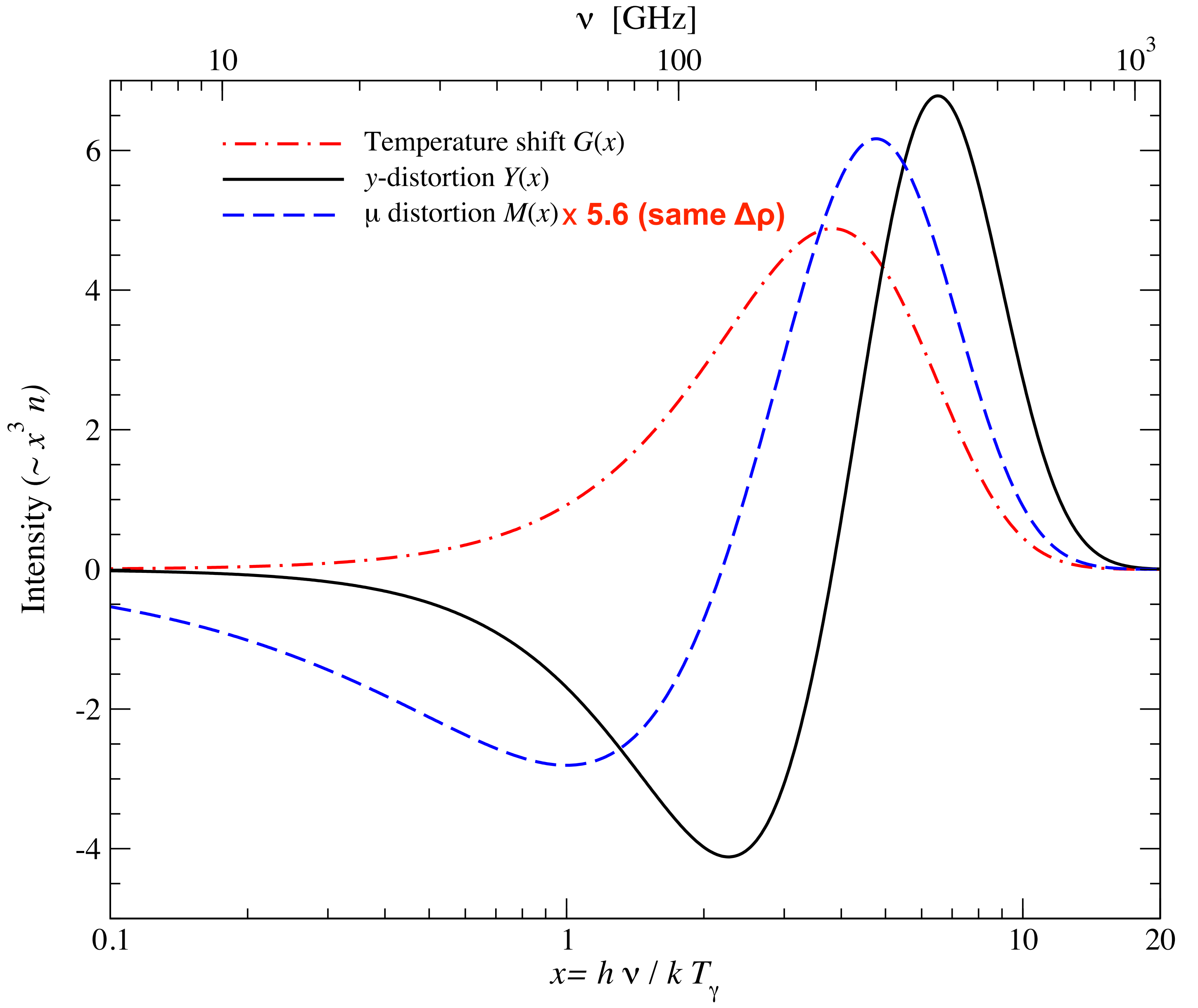
μ-distortion spectrum
(photon number conserved)

- $\mu_0 > 0$ ⇒ *too few photons / too much energy*
- $\mu_0 < 0$ ⇒ *too many photons / too little energy*

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

Temperature shift ↔ y-distortion ↔ μ-distortion



- *important for in the limit of many scatterings*
- null at *ν* ~ 125 GHz (*x*~2.19)
- photon number conserved

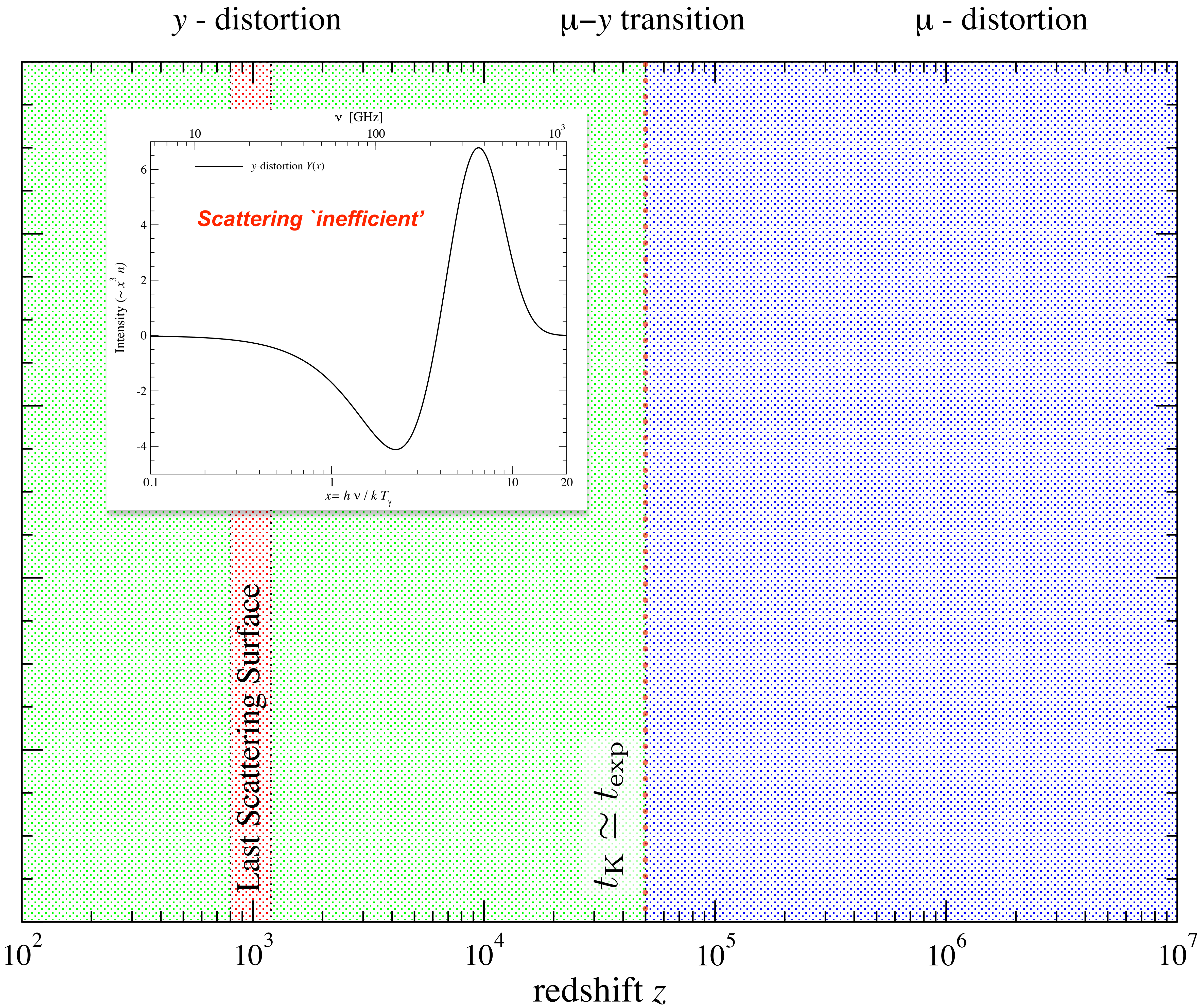
$$\int x^2 M(x) \mathrm{d}x = 0$$

- energy exchange

$$\int x^3 M(x) \mathrm{d}x \approx \frac{\pi^4/15}{1.401} \leftrightarrow \frac{\rho_\gamma}{1.401}$$

- *can only be created at very early times*

$$M(x) = G(x) \left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x} \right]$$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10^3
10^4
10^5
10^6
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ν [GHz]
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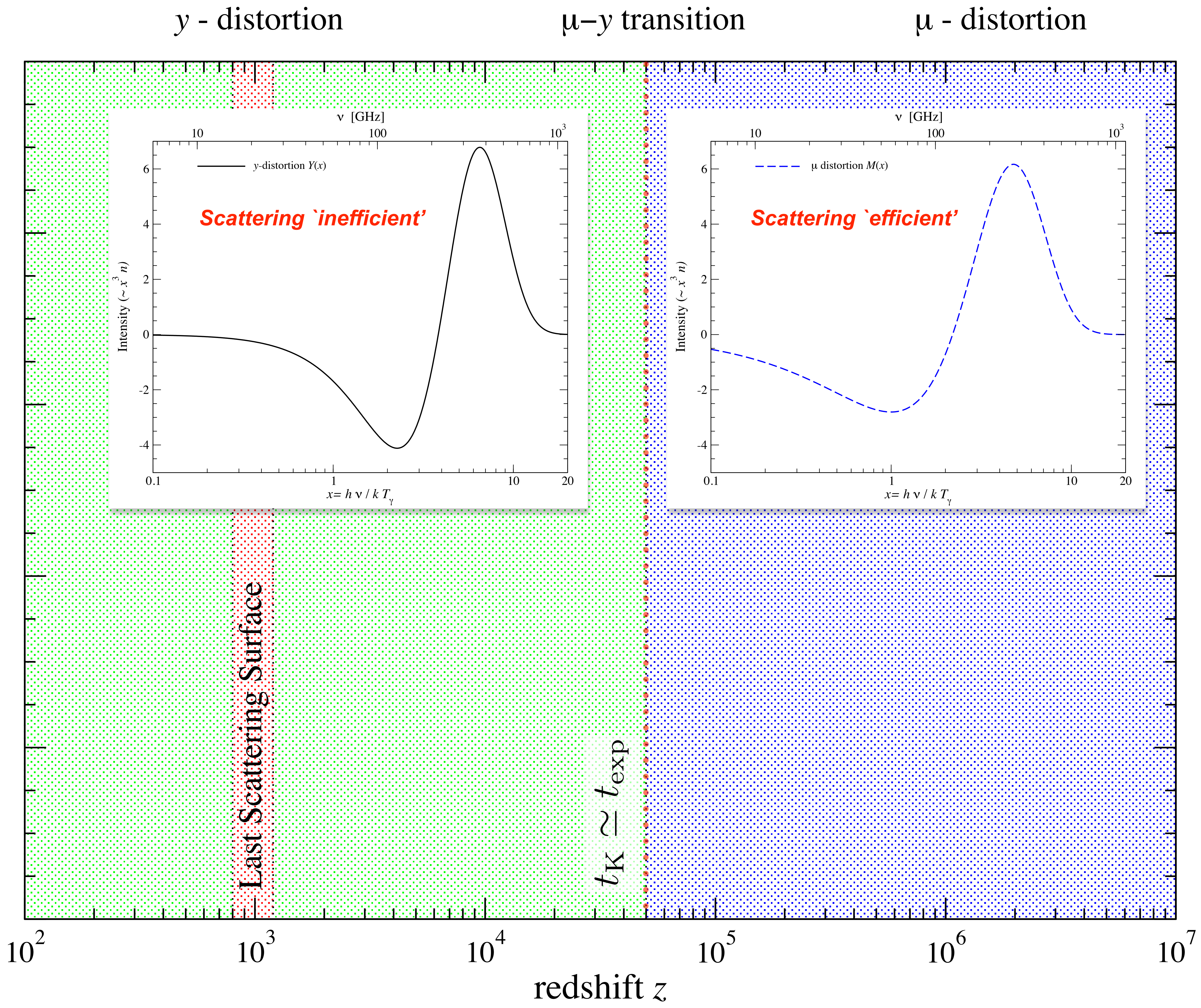
y-distortion $Y(x)$
Scattering `inefficient'
Intensity ($\sim x^3\,n$)
6
4
2
0
-2
-4
0.1
1
10
20
$x = h\,\nu\,/\,k\,T_\gamma$



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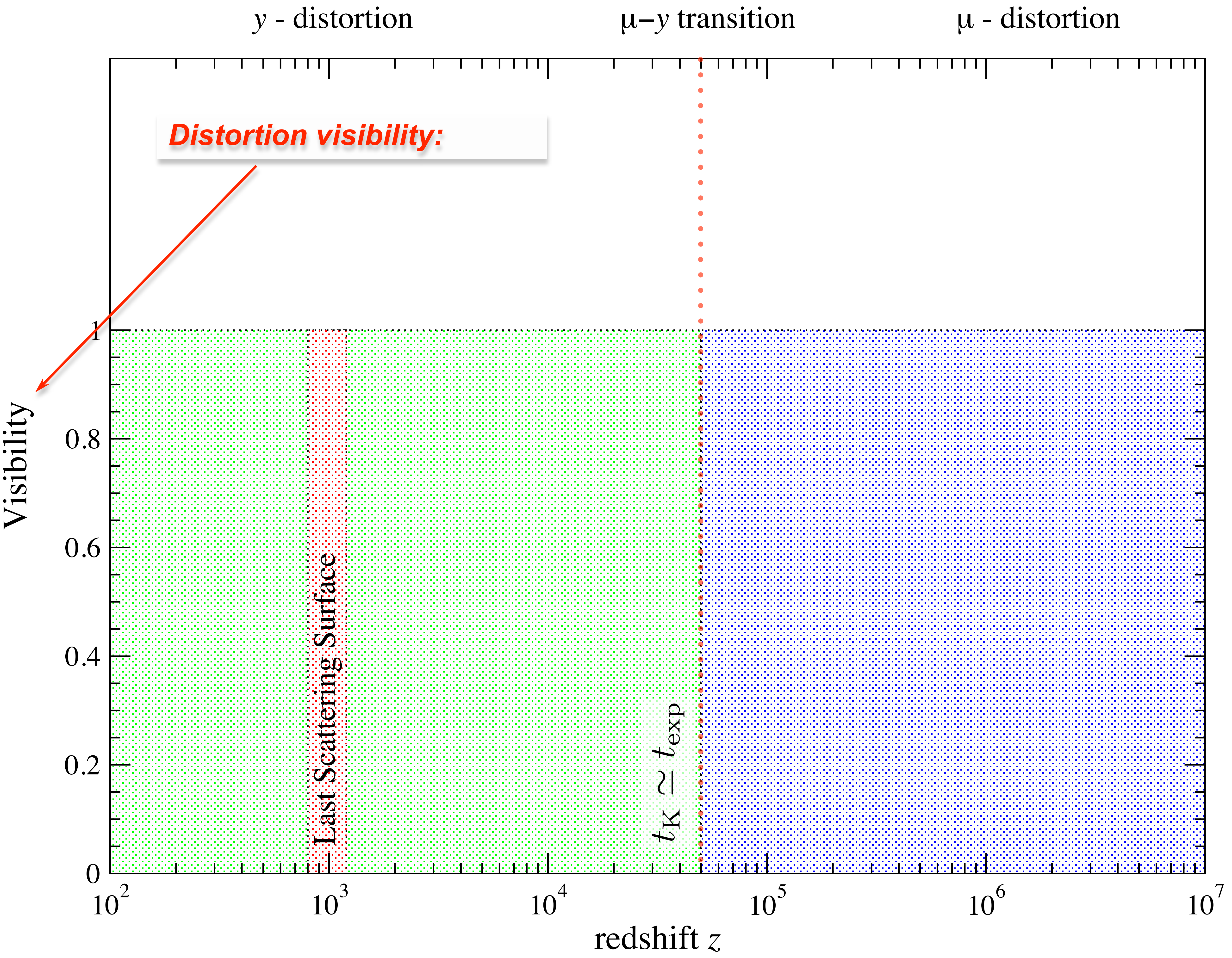
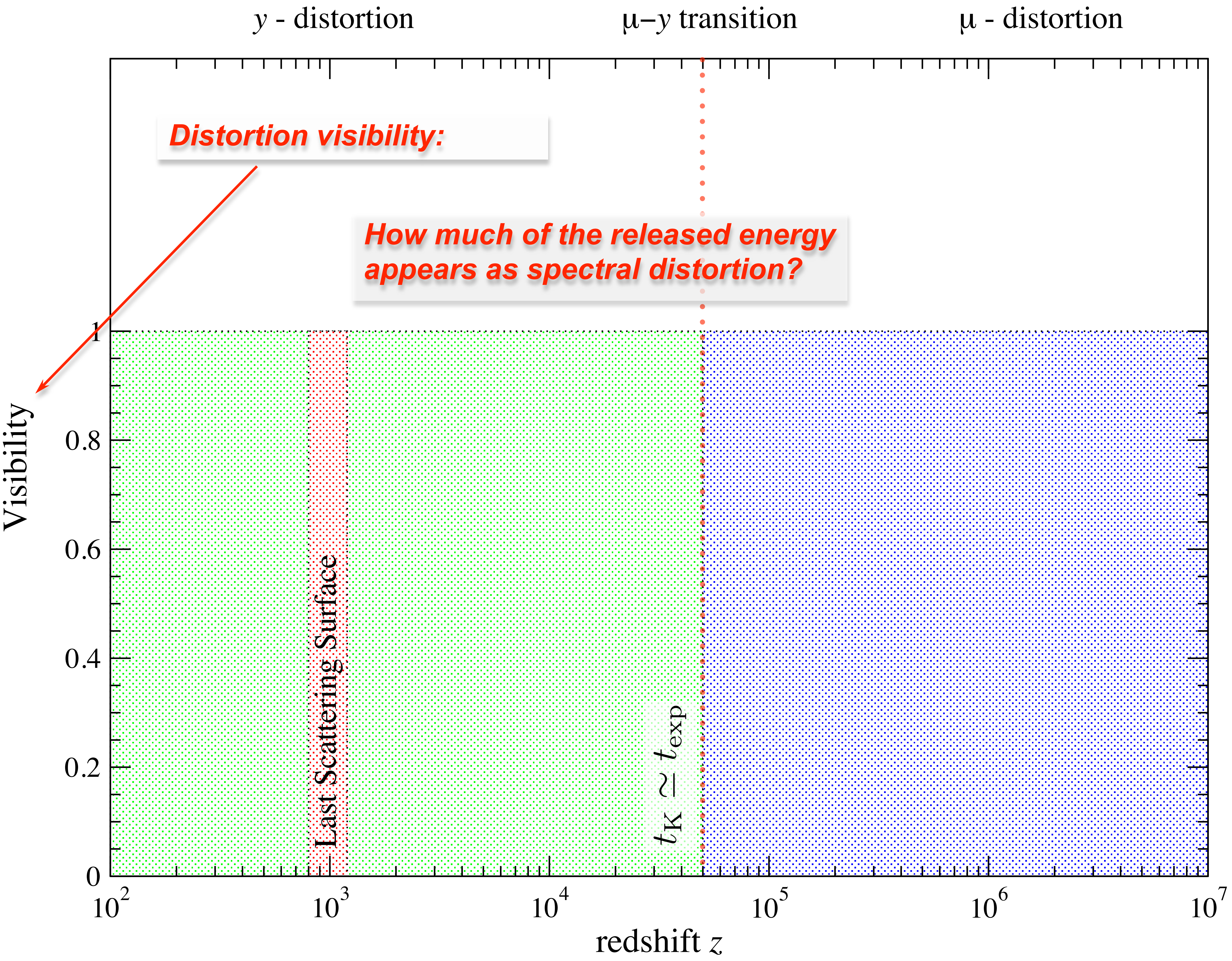
10^7
redshift z

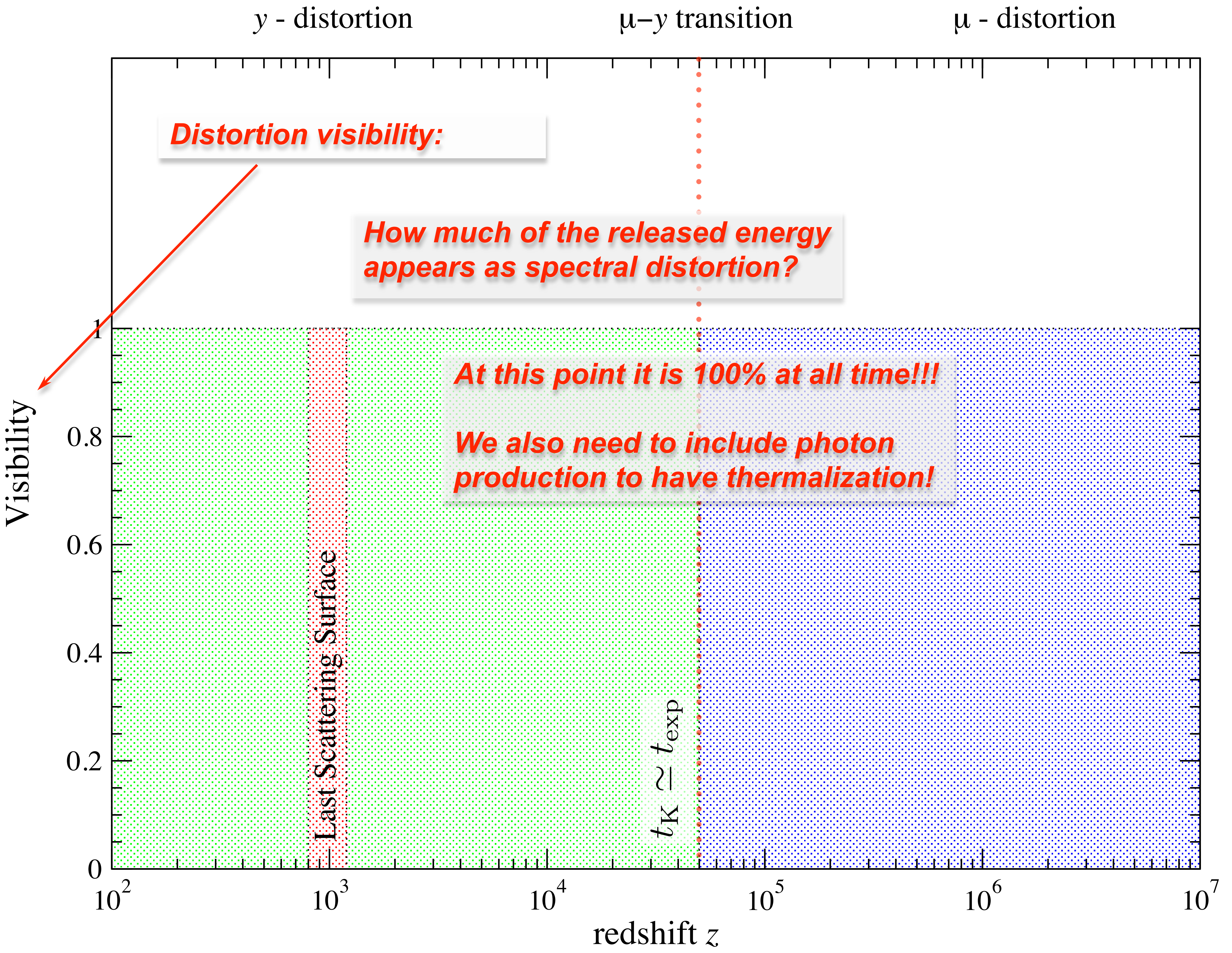


Distortion visibility:



Distortion visibility:

How much of the released energy appears as spectral distortion?



Distortion visibility:

How much of the released energy appears as spectral distortion?

At this point it is 100% at all time!!!

We also need to include photon production to have thermalization!

Photon production processes

Collision Term for Photon Production Processes

- *emission/absorption term*

$$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx \frac{K(x)}{x^3}\left[1 - n \times (\mathrm{e}^{x_\mathrm{e}} - 1)\right] \qquad \textit{with} \qquad x_\mathrm{e} = \frac{h\nu}{kT_\mathrm{e}}$$

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- *emission/absorption most efficient at low frequencies!*

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$

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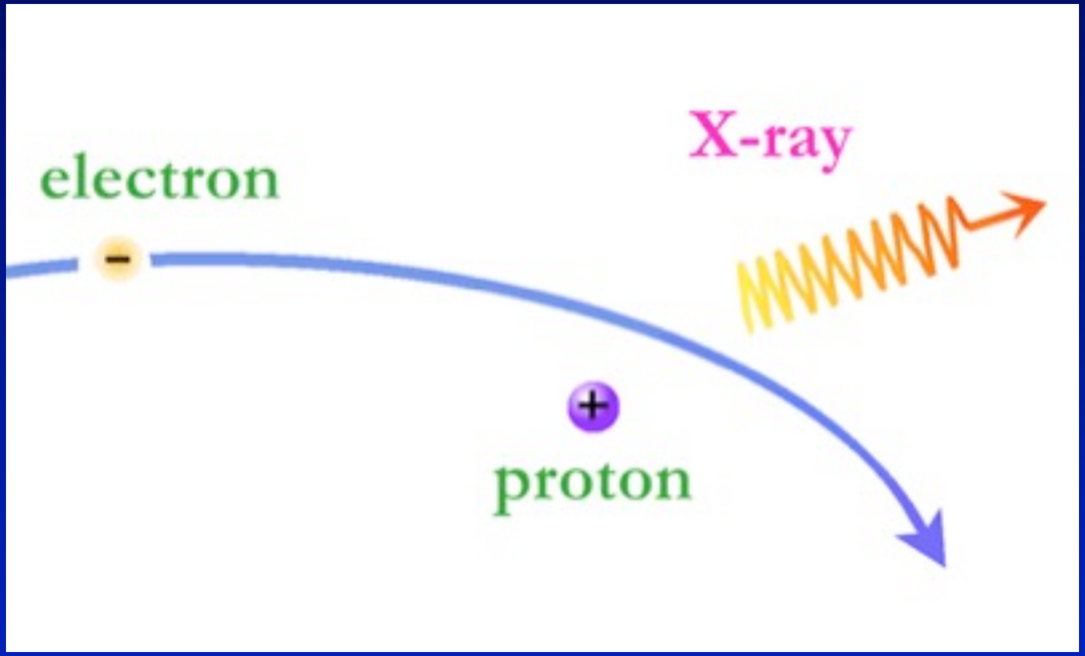

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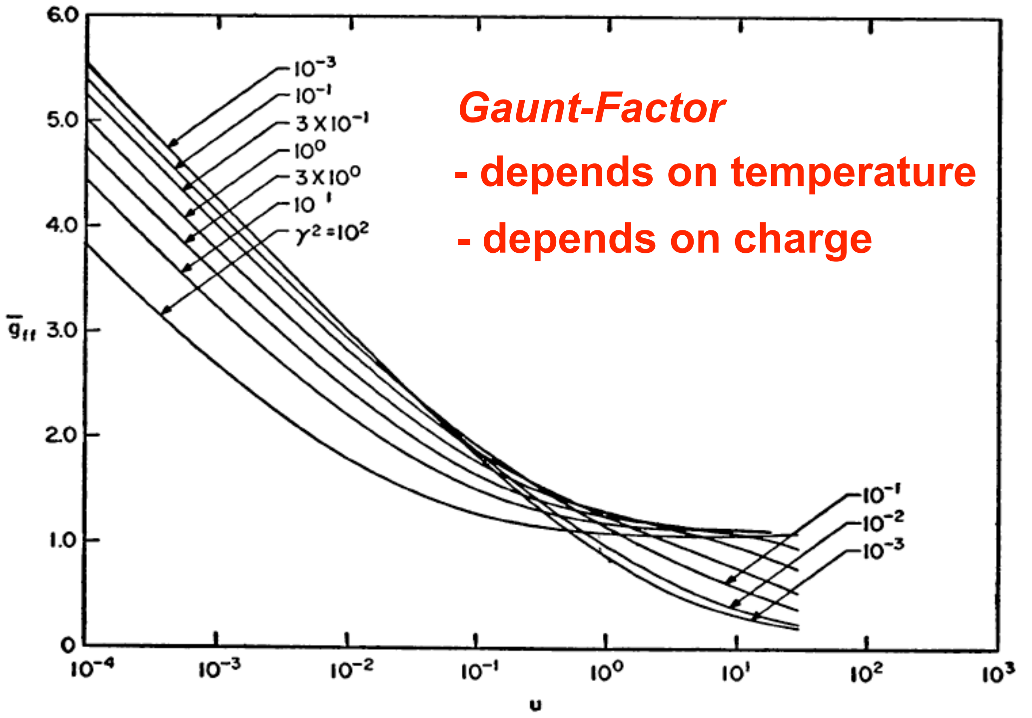

Gaunt-Factor

- depends on temperature

- depends on charge

FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2Ry/kT$.

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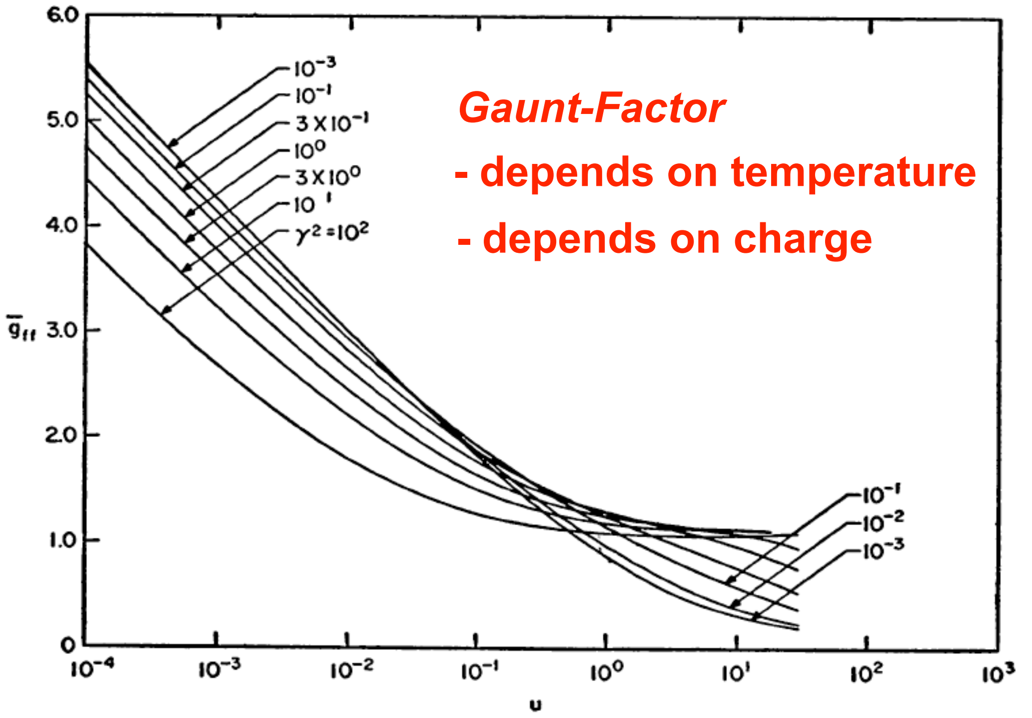

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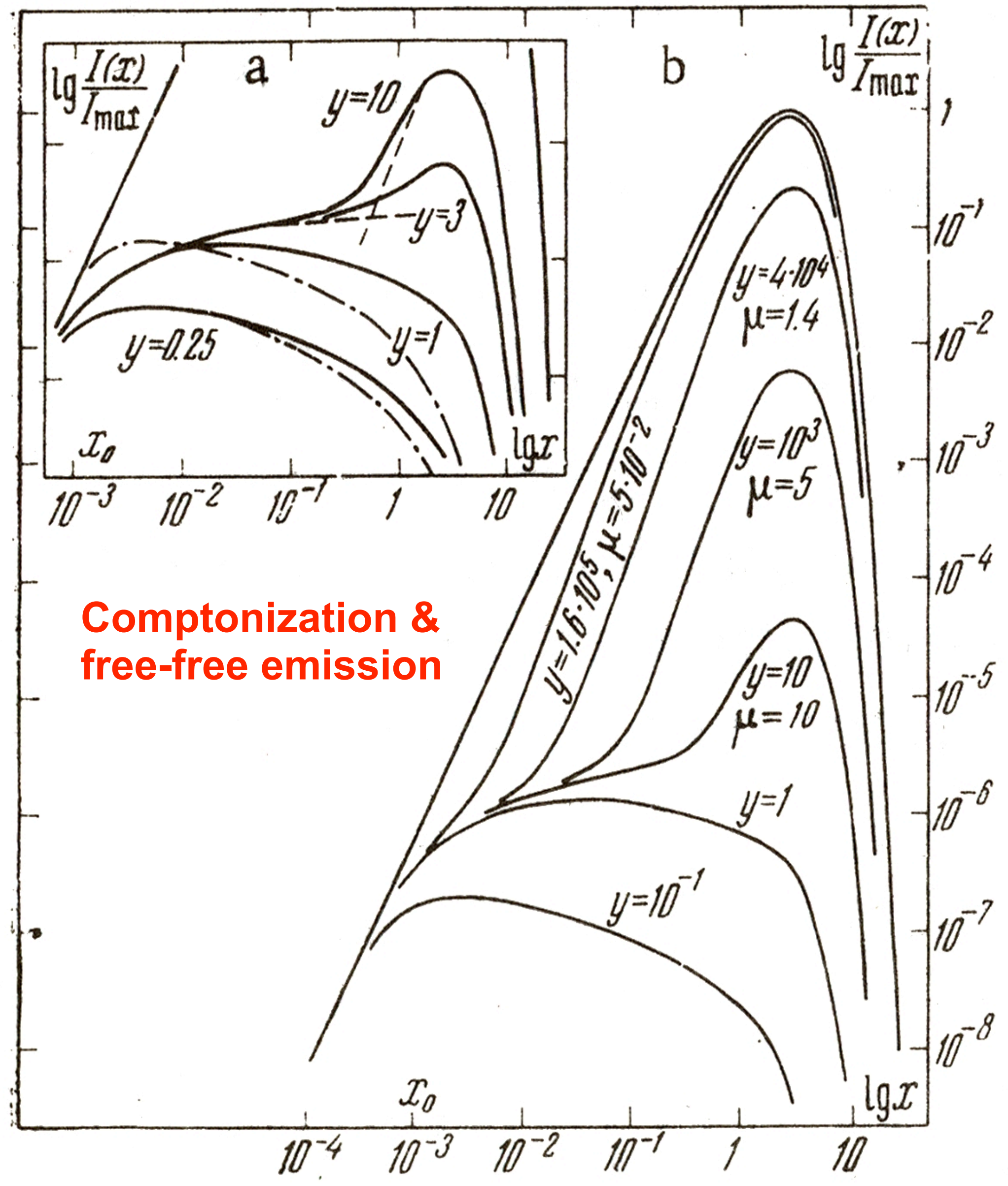

Comptonization & free-free emission

Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

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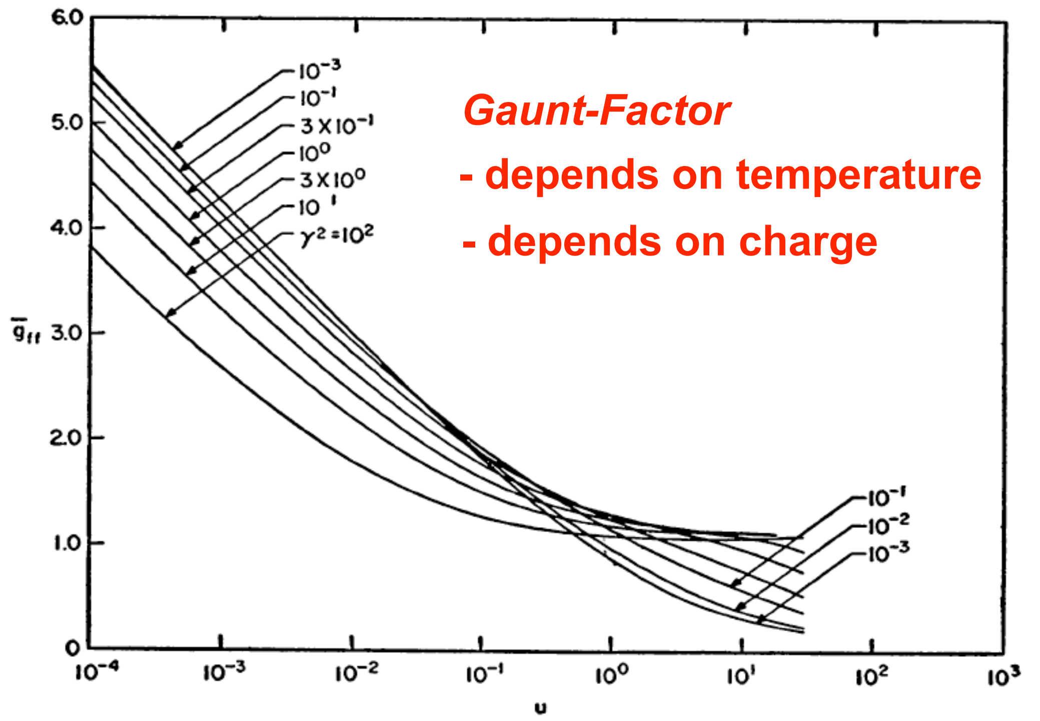

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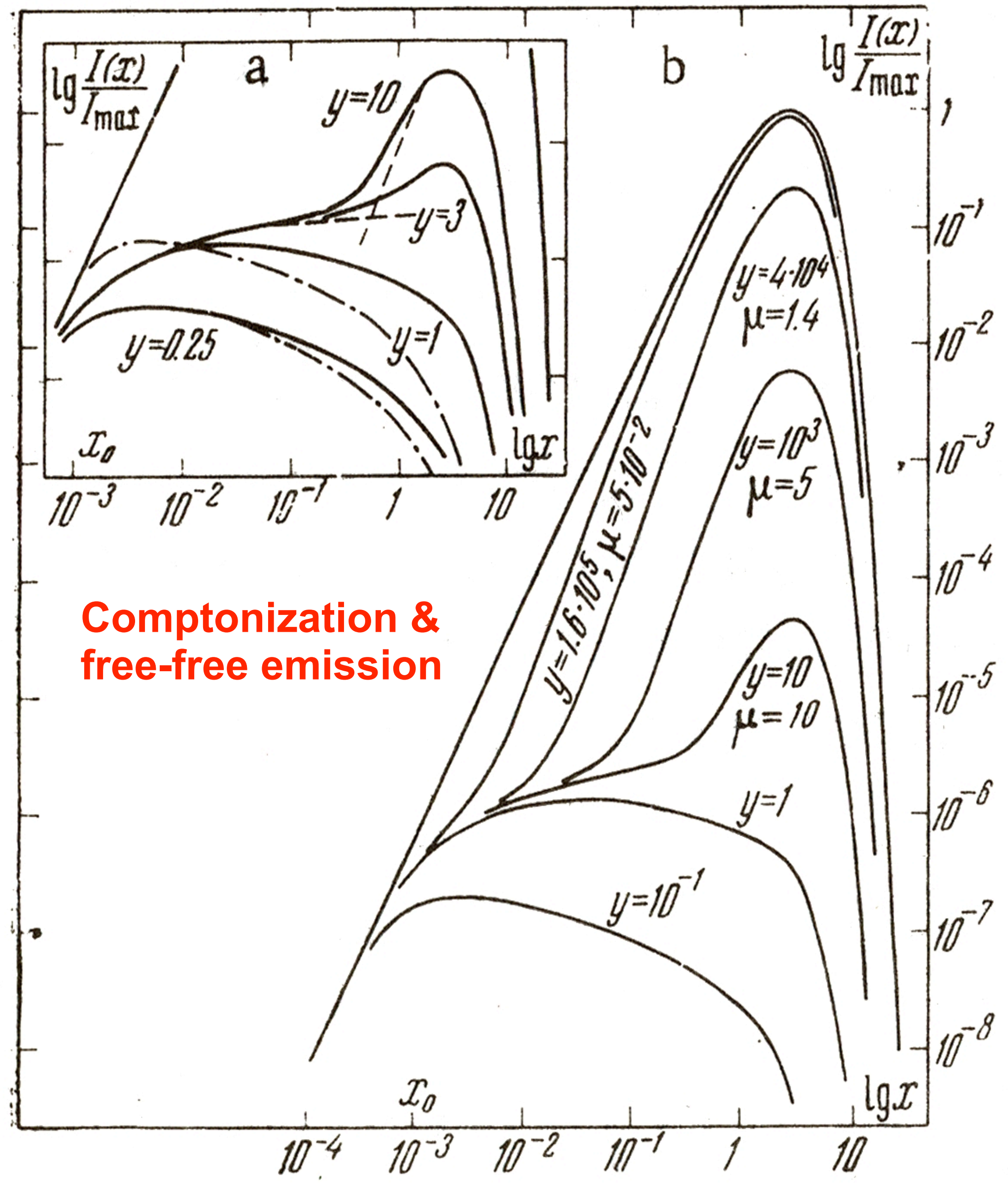

Comptonization & free-free emission

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Thermalization inefficient already at z ≲ 10^7 with Bremsstrahlung alone!

Double Compton Emission

- Reaction: $e+\gamma \leftrightarrow e'+\gamma\,'+\gamma_2$

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$$K_{\rm DC} \propto N_{\rm e} N_\gamma \left(\frac{kT_\gamma}{m_{\rm e} c^2}\right)^{-1} \propto (1+z)^5 \iff K_{\rm BR} \propto N_{\rm e} N_{\rm b} \left(\frac{kT_{\rm e}}{m_{\rm e} c^2}\right)^{-7/2} \propto (1+z)^{5/2}$$

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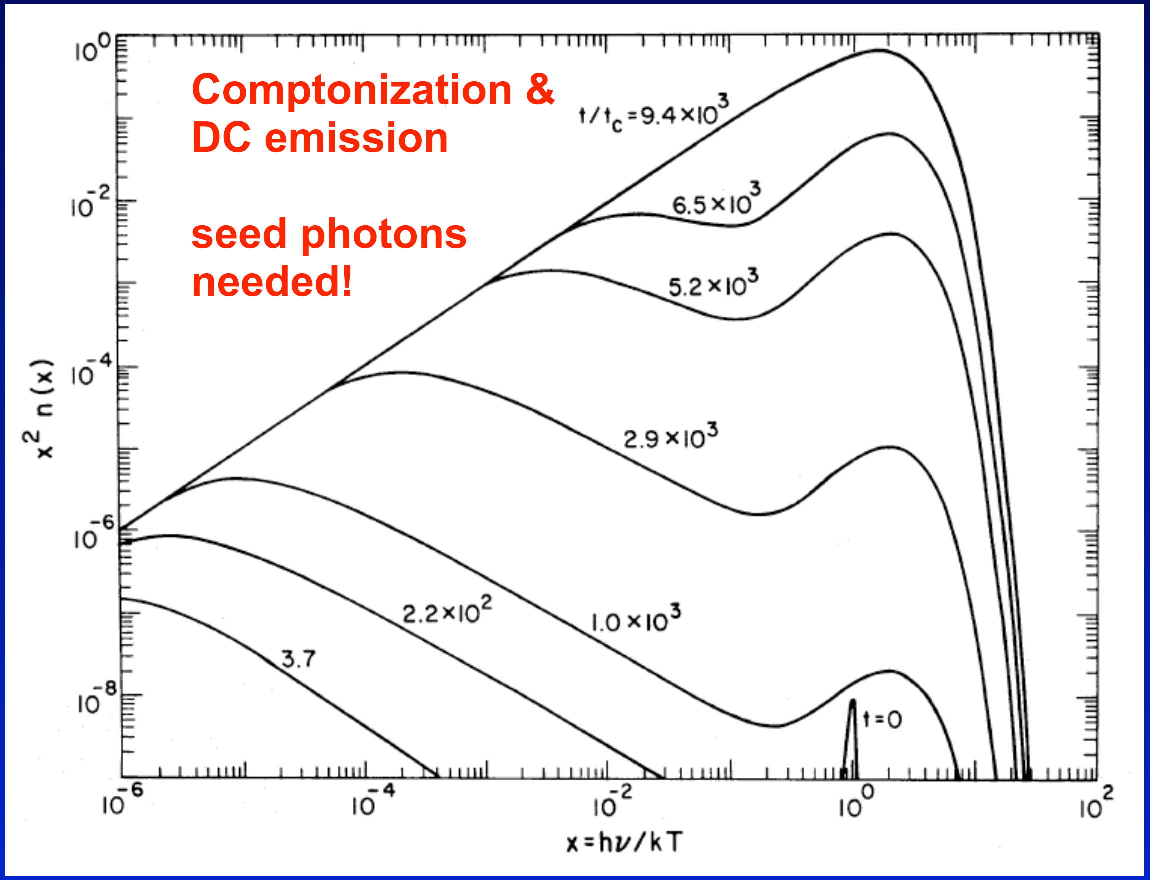

Comptonization & DC emission

seed photons needed!

Lightman 1981

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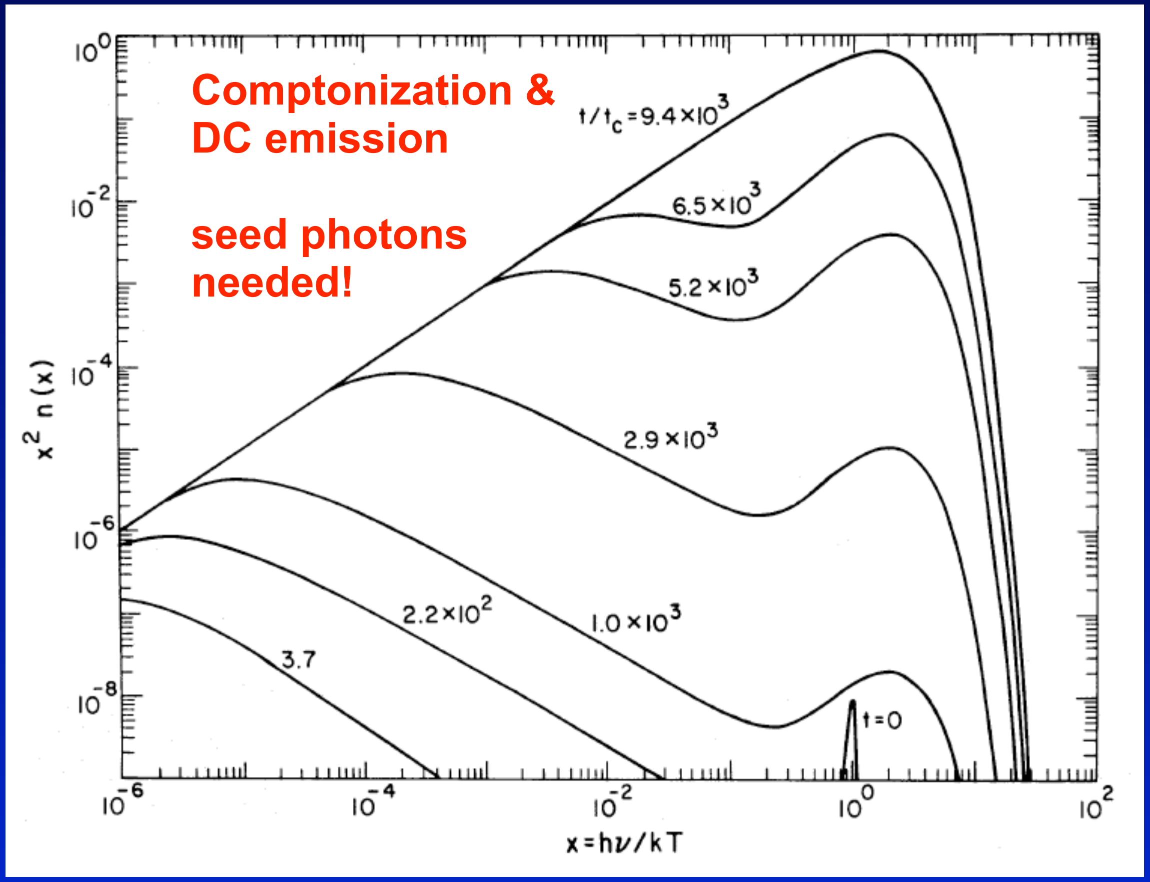

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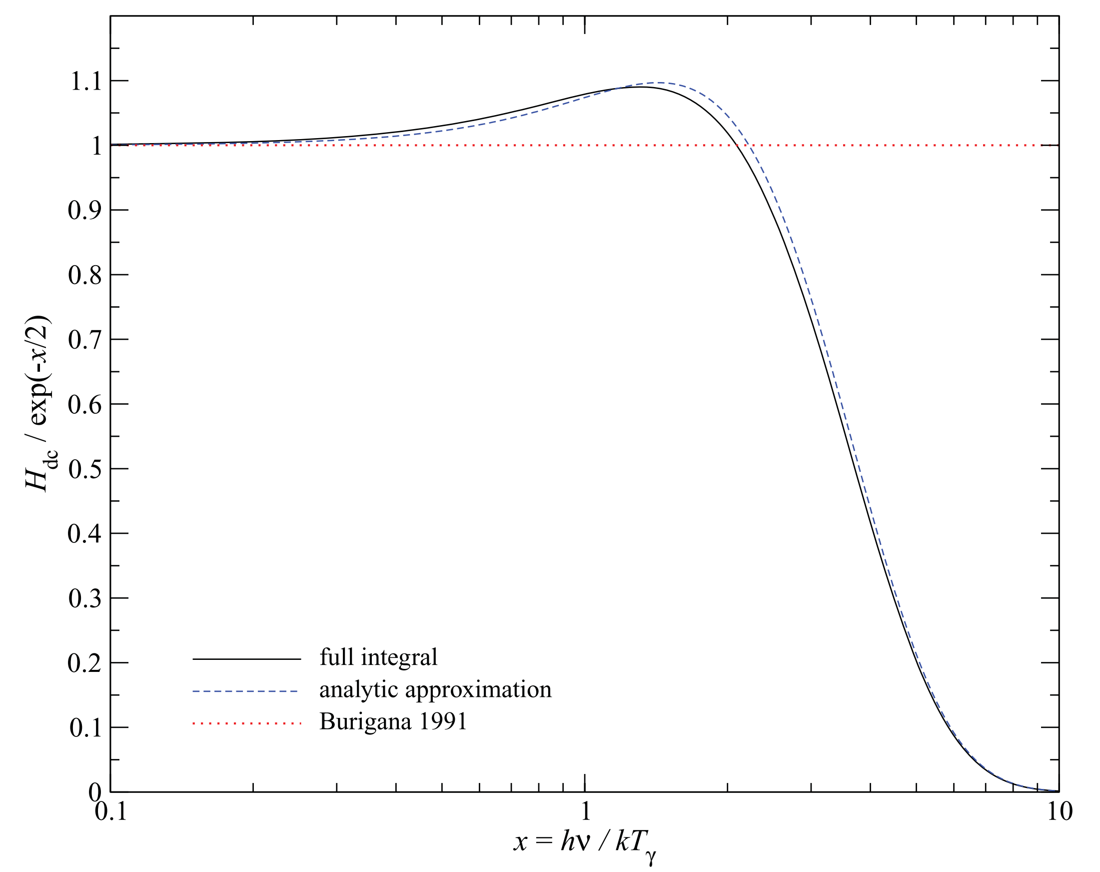

JC, 2005 (PhD Thesis); JC, Sazonov & Sunyaev, 2007; JC & Sunyaev, 2012

→ was only included later **(Danese & De Zotti, 1982)**

→ DC Gaunt-factor and temperature corrections included by latest computations, but the effect is small

Example: *Energy release by decaying relict particle*

redshift

difference between electron and photon temperature

- initial condition: *full equilibrium*
- total energy release:

  $\Delta\rho/\rho \sim 1.3\text{x}10^{-6}$
- most of energy release around:

  $z_X \sim 2\text{x}10^6$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5\text{x}10^5$
- late ($z<10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

today x=2 x 10^{-2} means $\nu \sim$1GHz

Computation carried out with *CosmoTherm*
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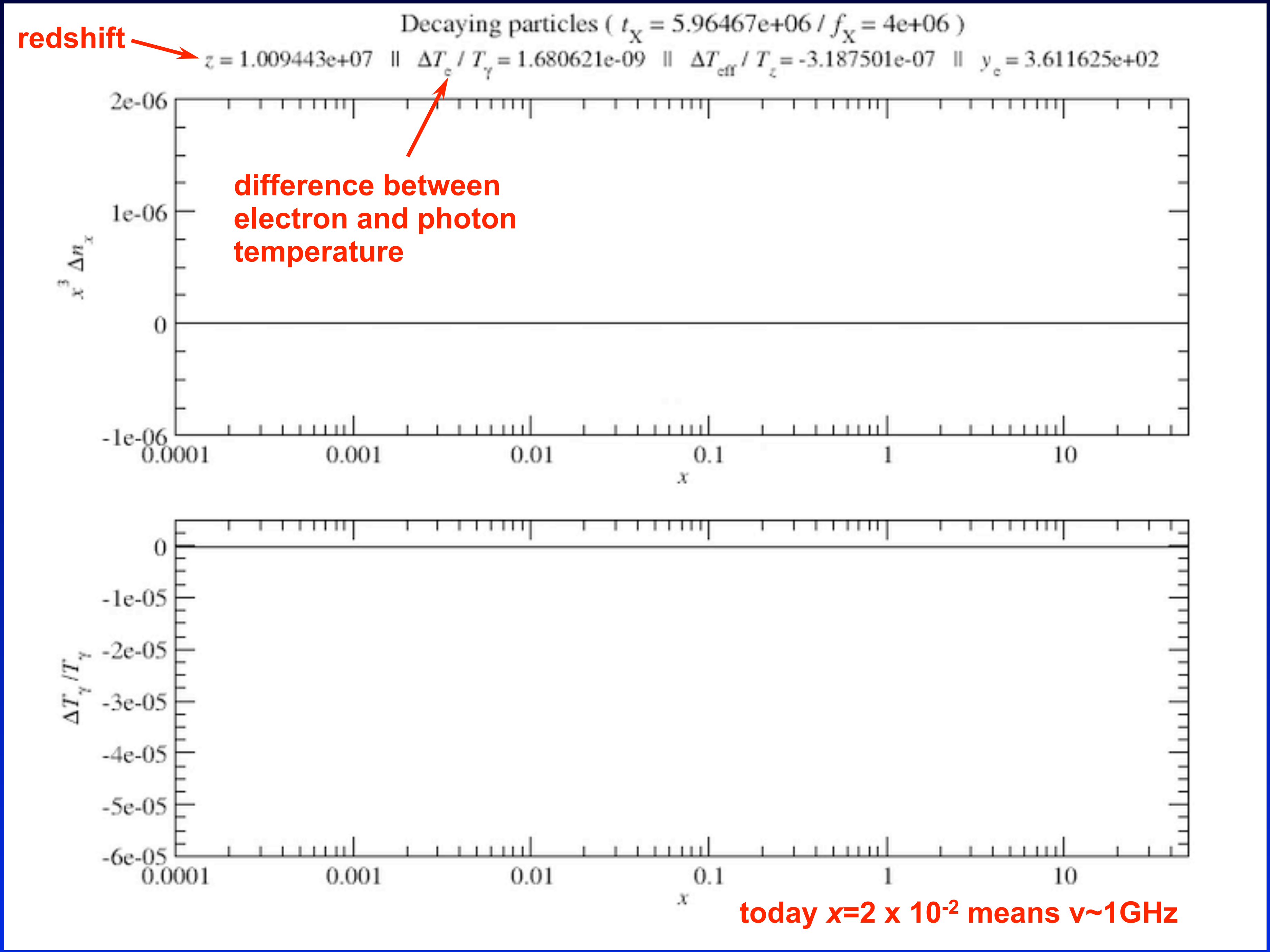

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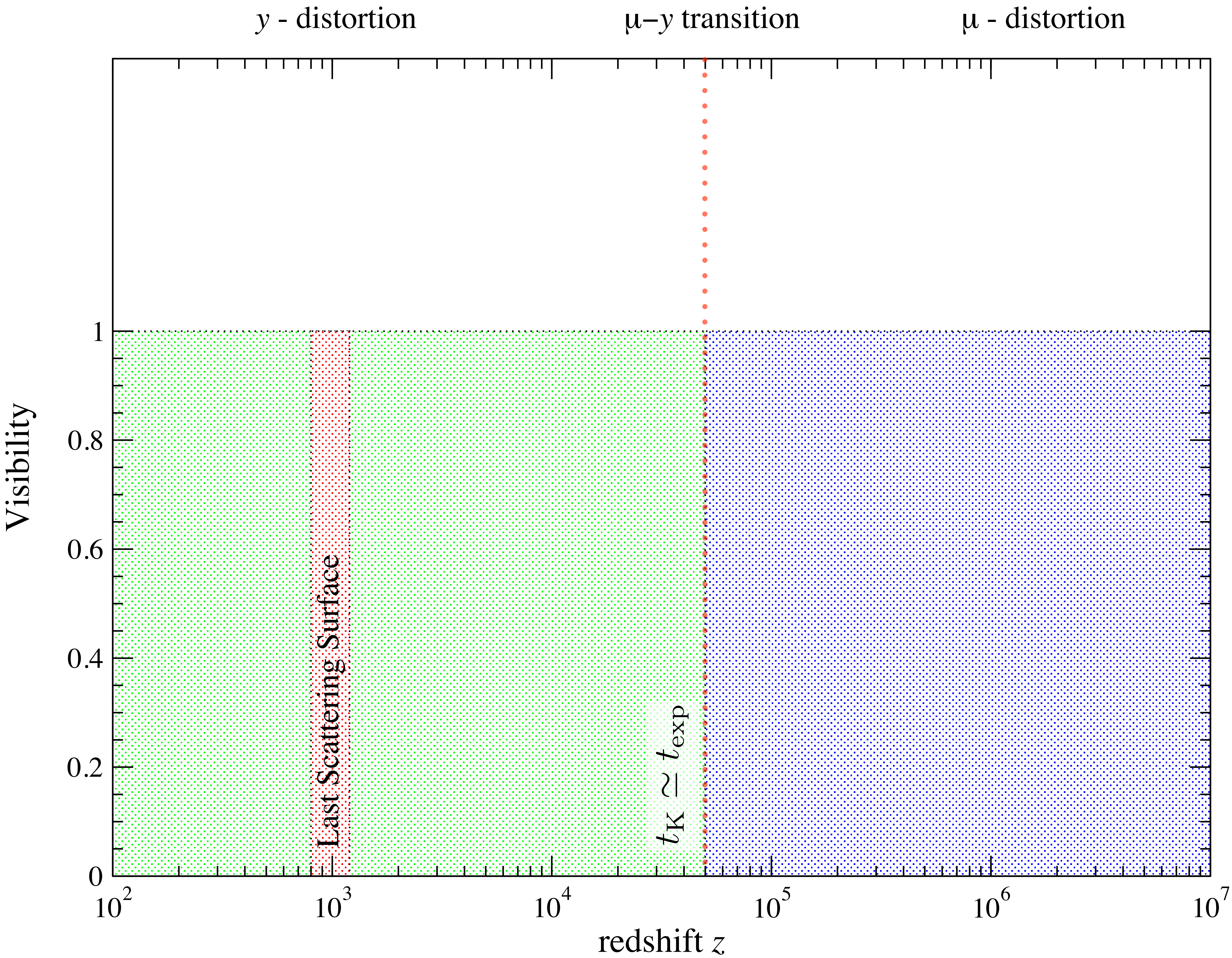
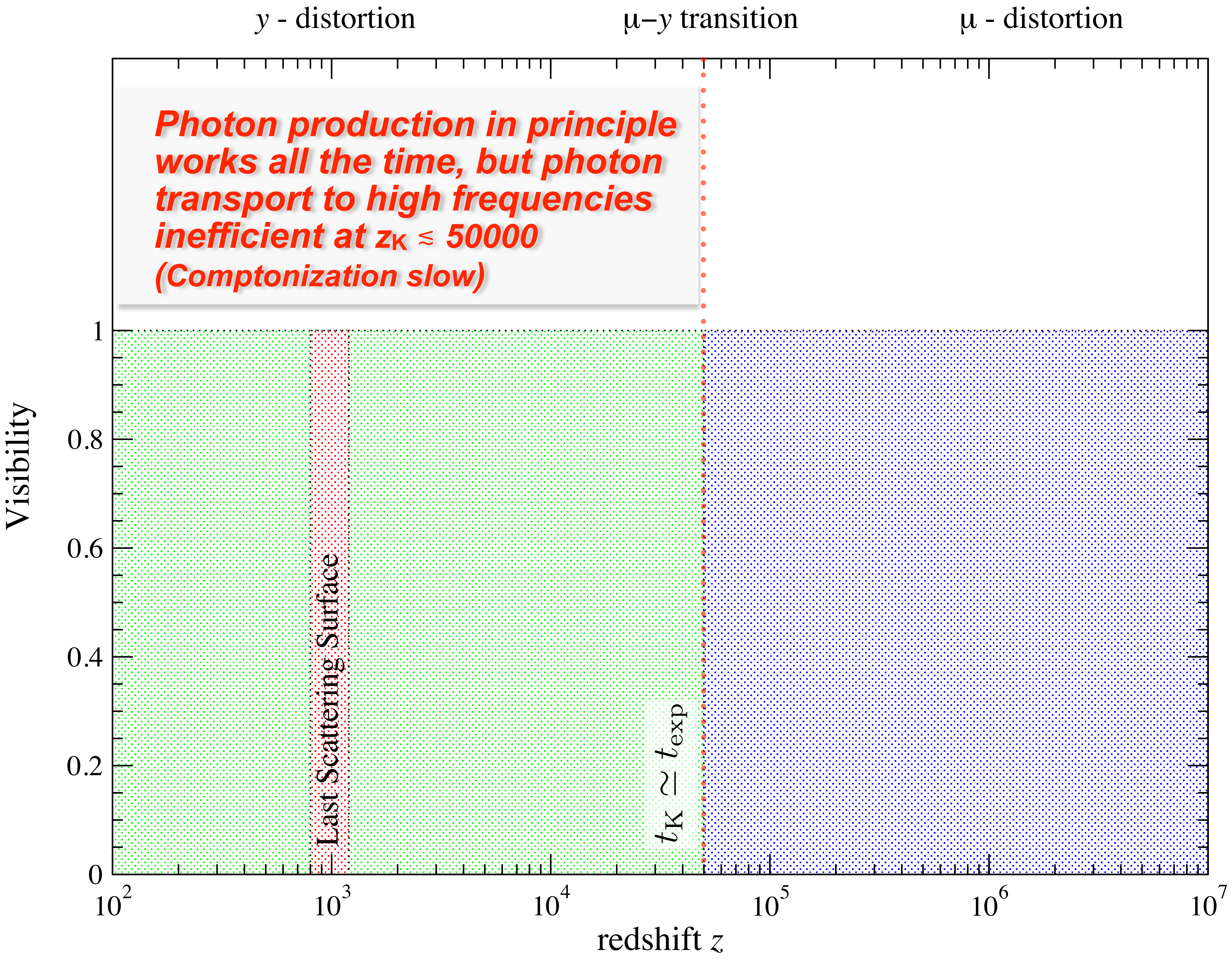
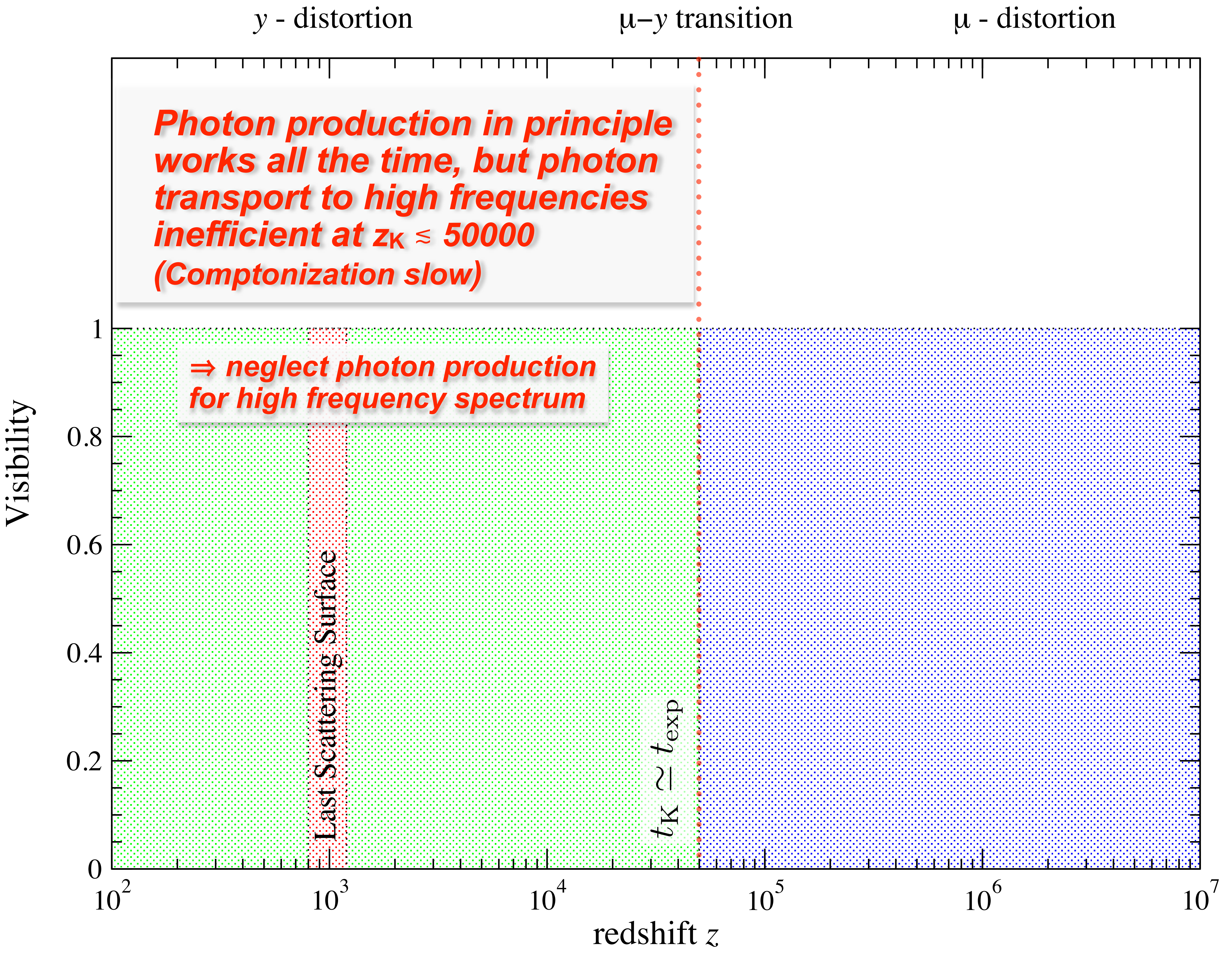
Let’s try to understand the evolution of distortions with photon production analytically!


y - distortion
μ–y transition
μ - distortion
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)



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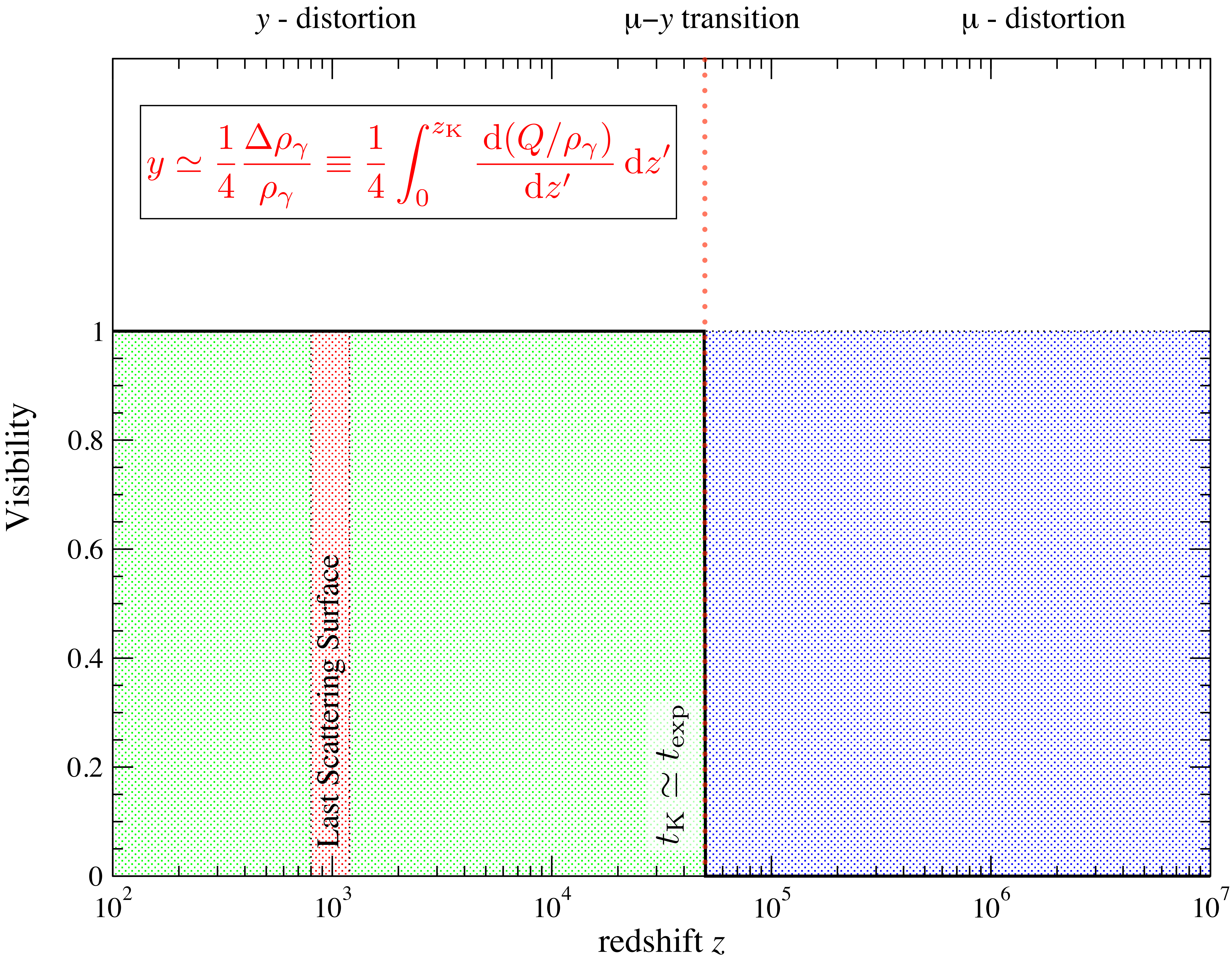
⇒ neglect photon production for high frequency spectrum



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Last Scattering Surface
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Visibility
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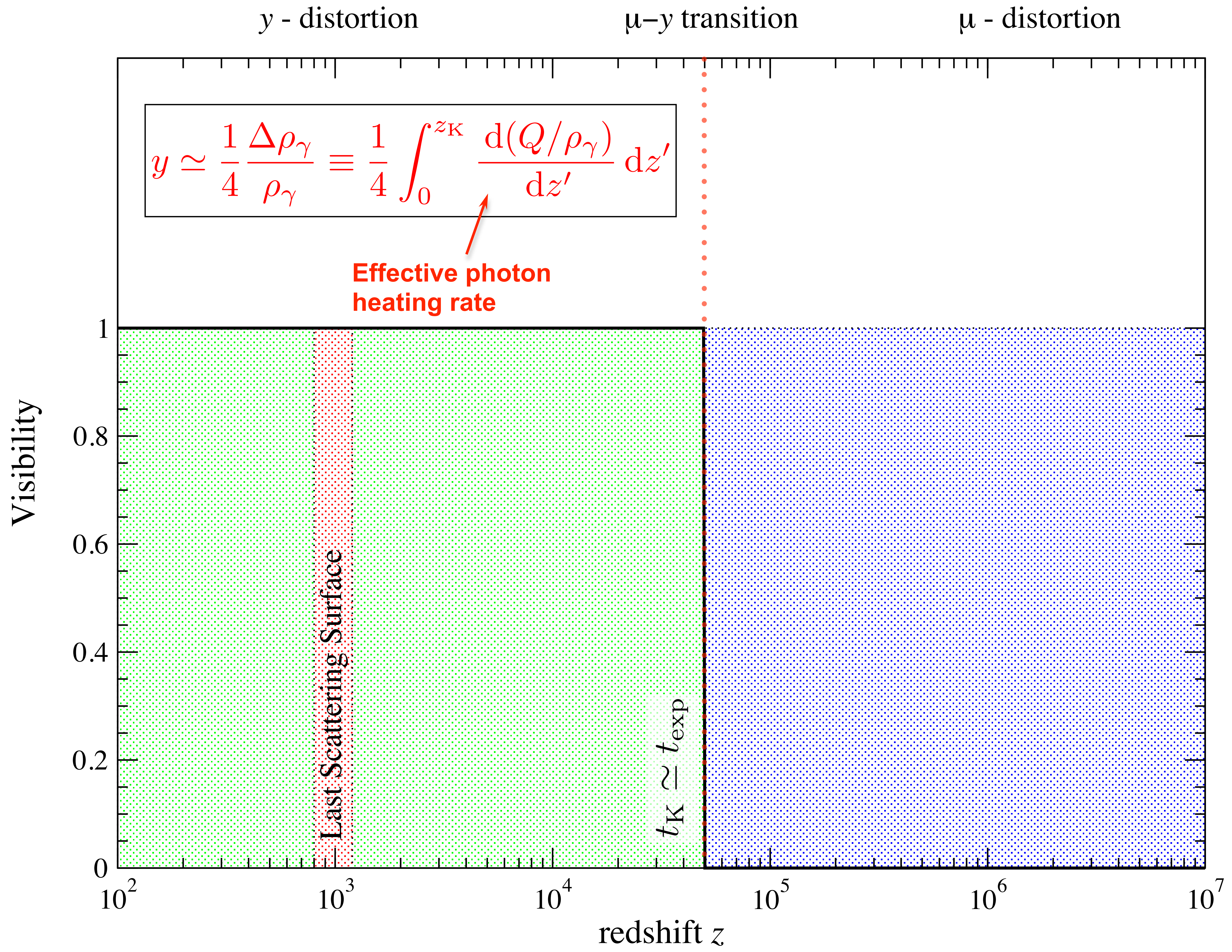
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Effective photon heating rate
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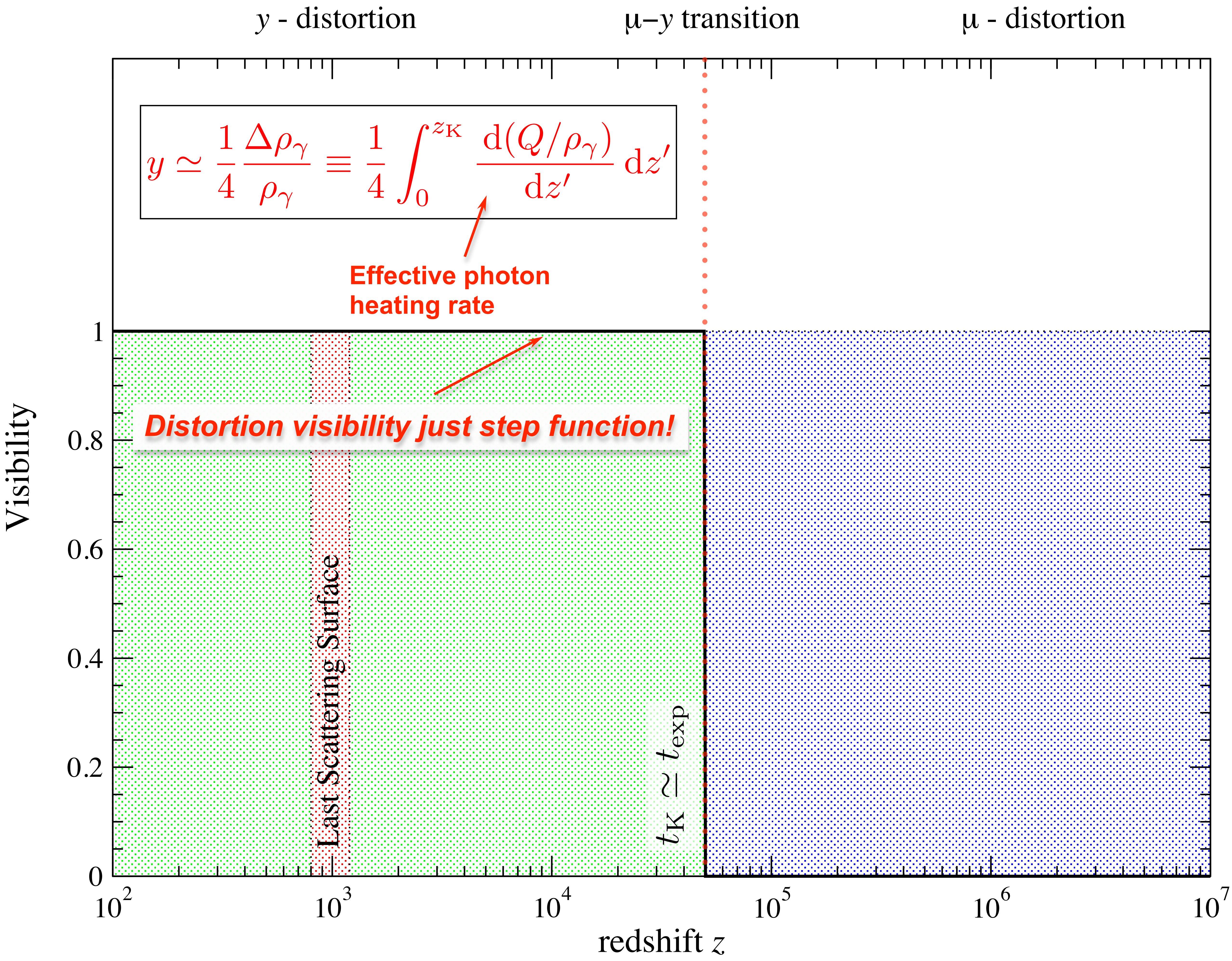

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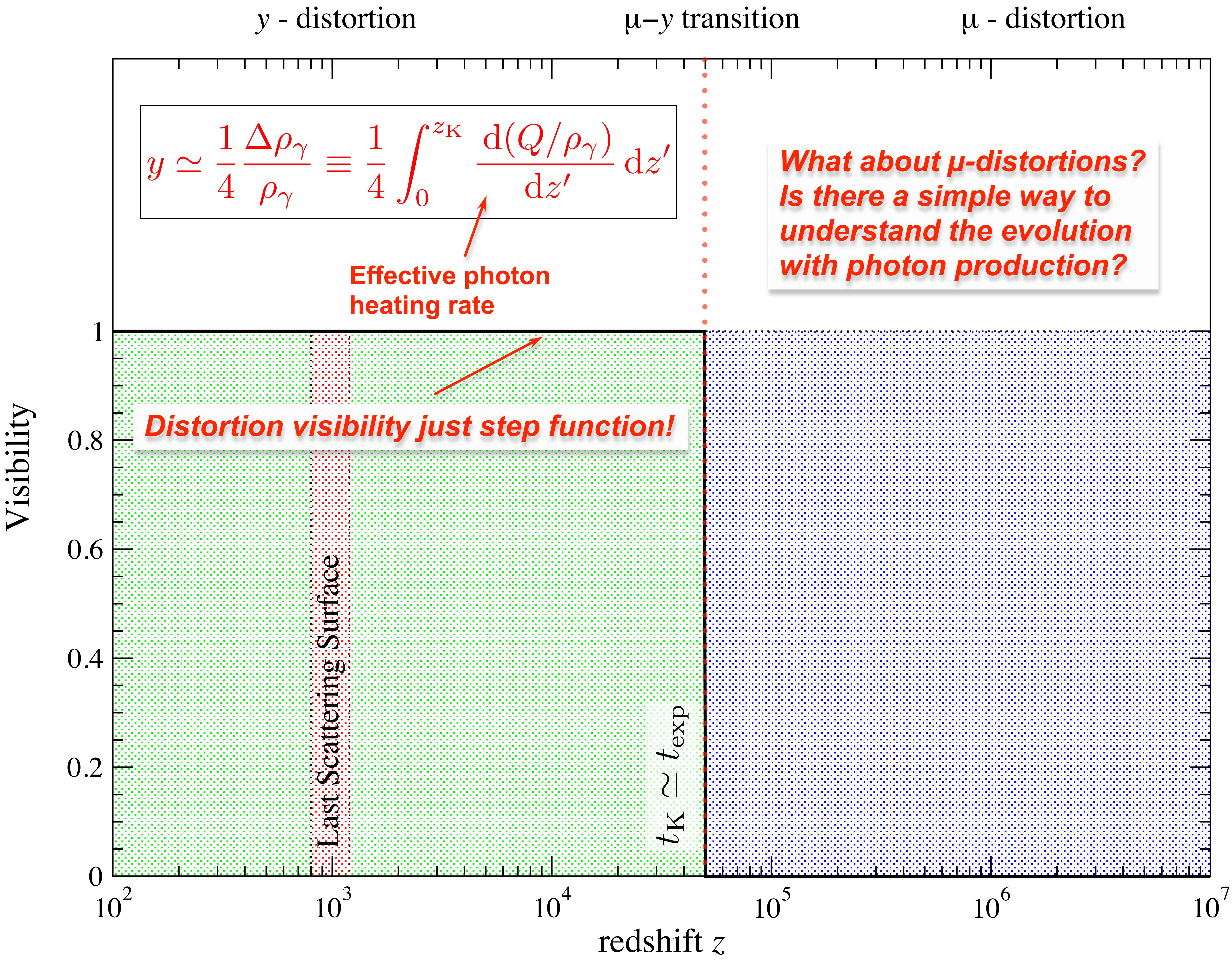

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Effective photon heating rate

What about μ-distortions? Is there a simple way to understand the evolution with photon production?

Distortion visibility just step function!

Analytic Approximation for μ-distortion

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- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm em/abs} \approx 0$

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(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

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chemical potential at high frequencies

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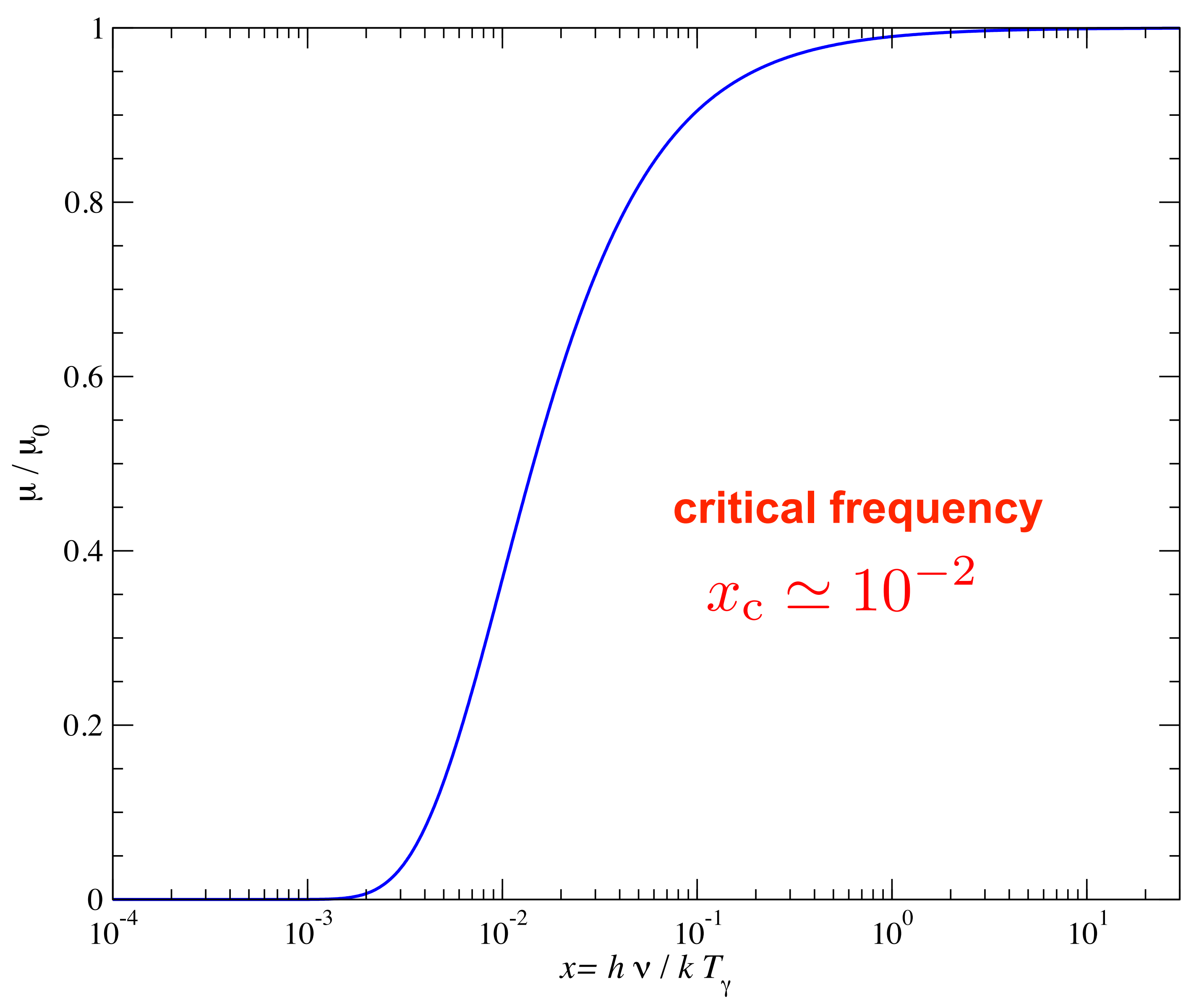

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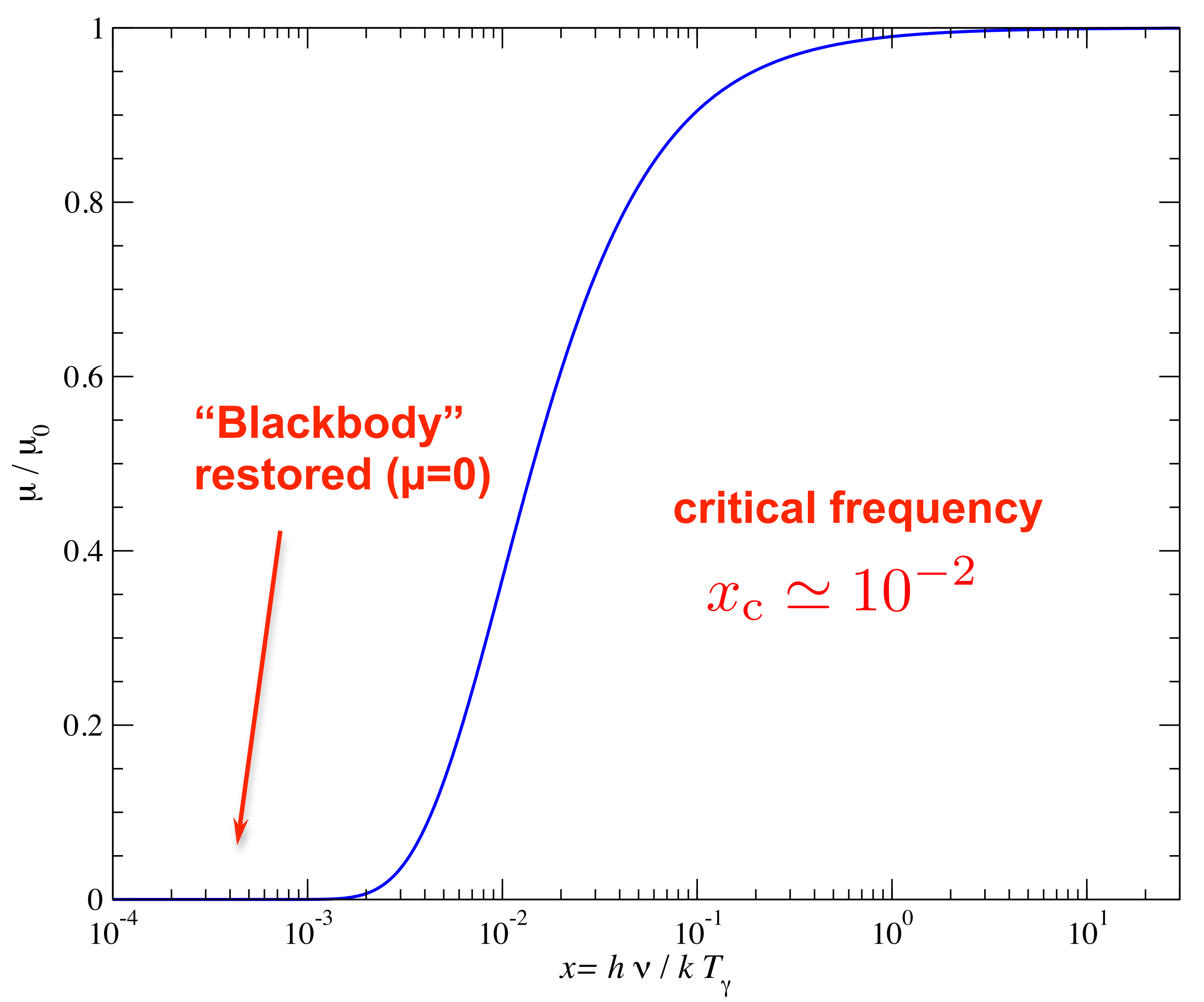

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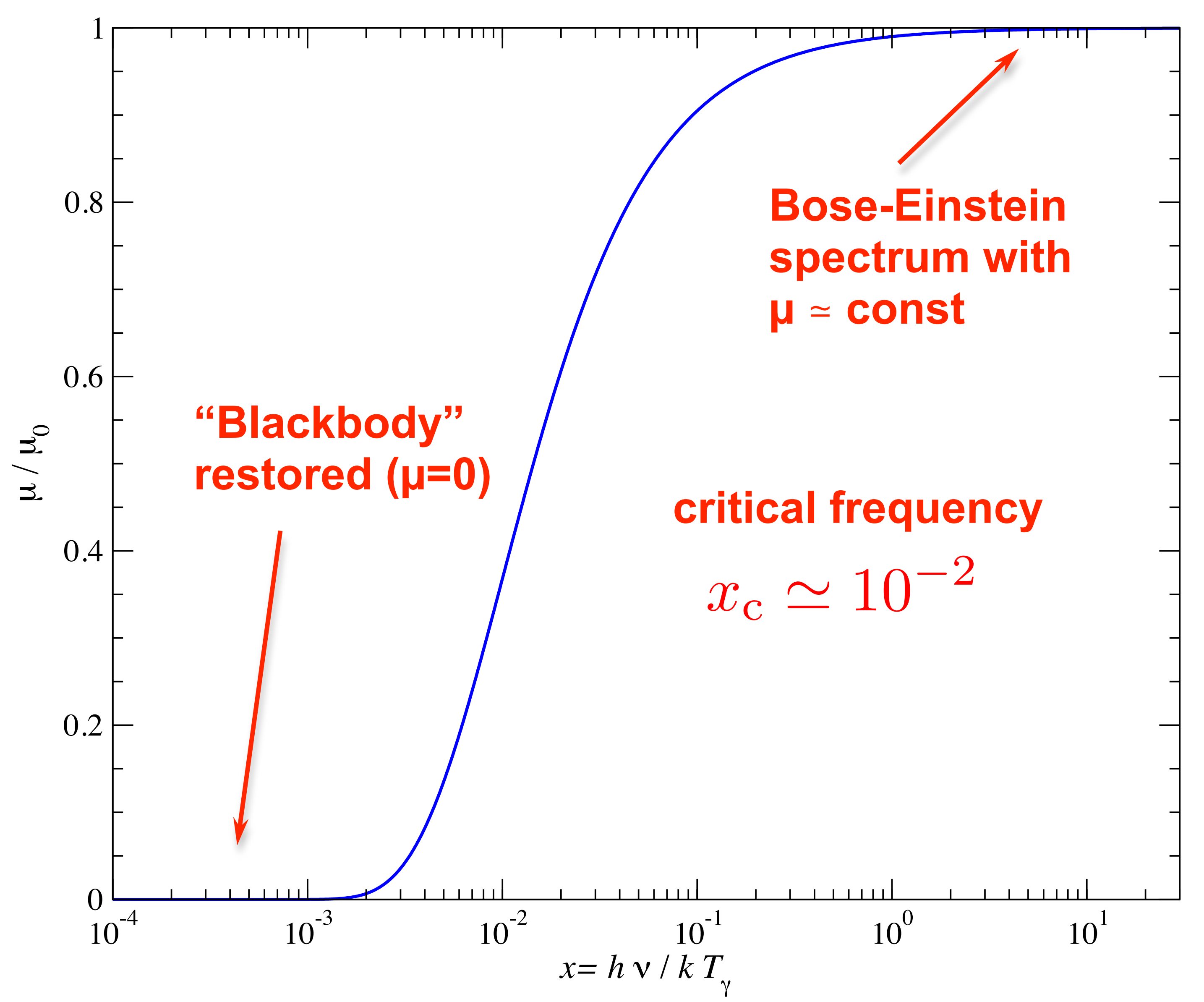

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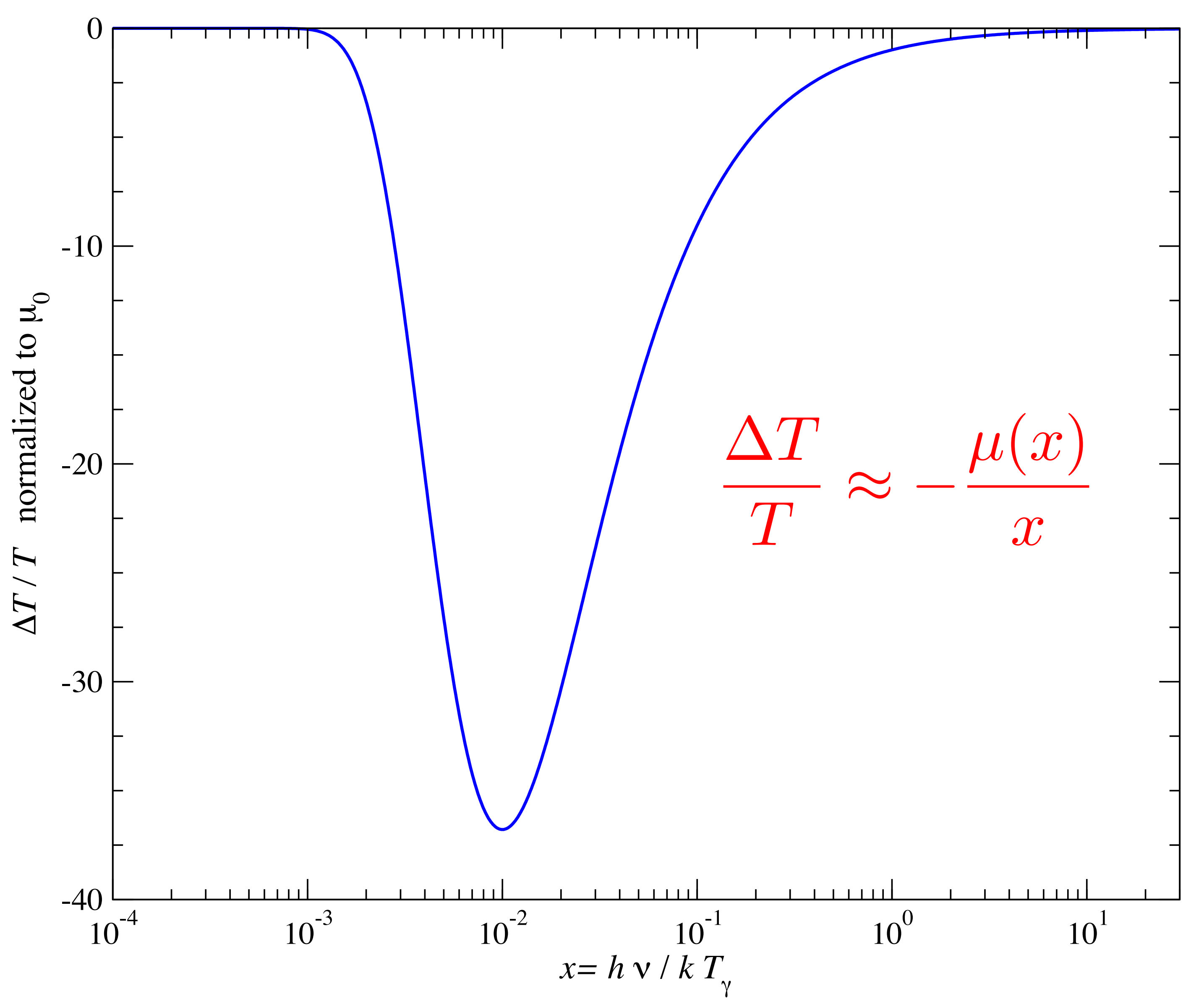

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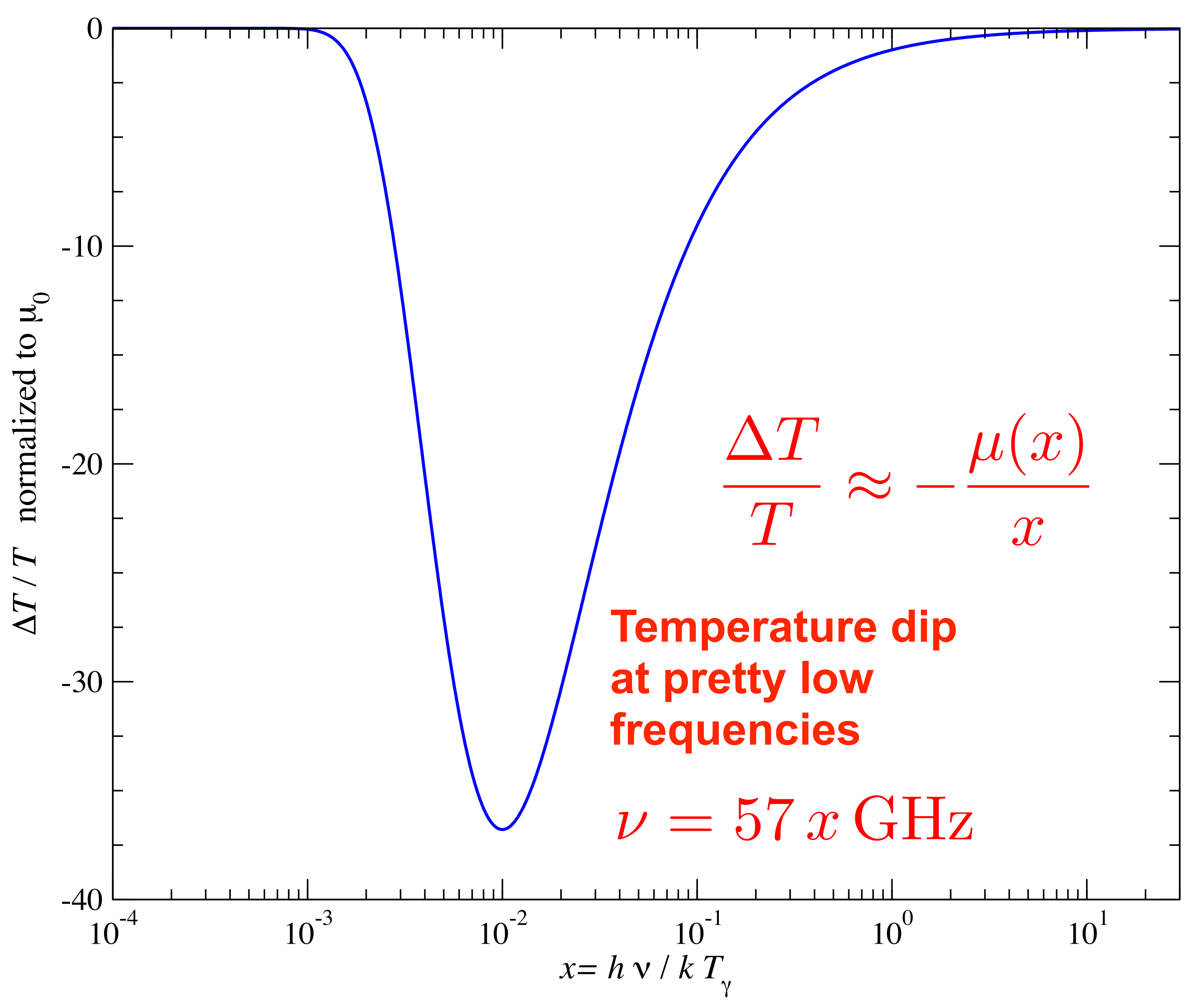

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Temperature dip at pretty low frequencies

$$\nu = 57\,x\,\mathrm{GHz}$$

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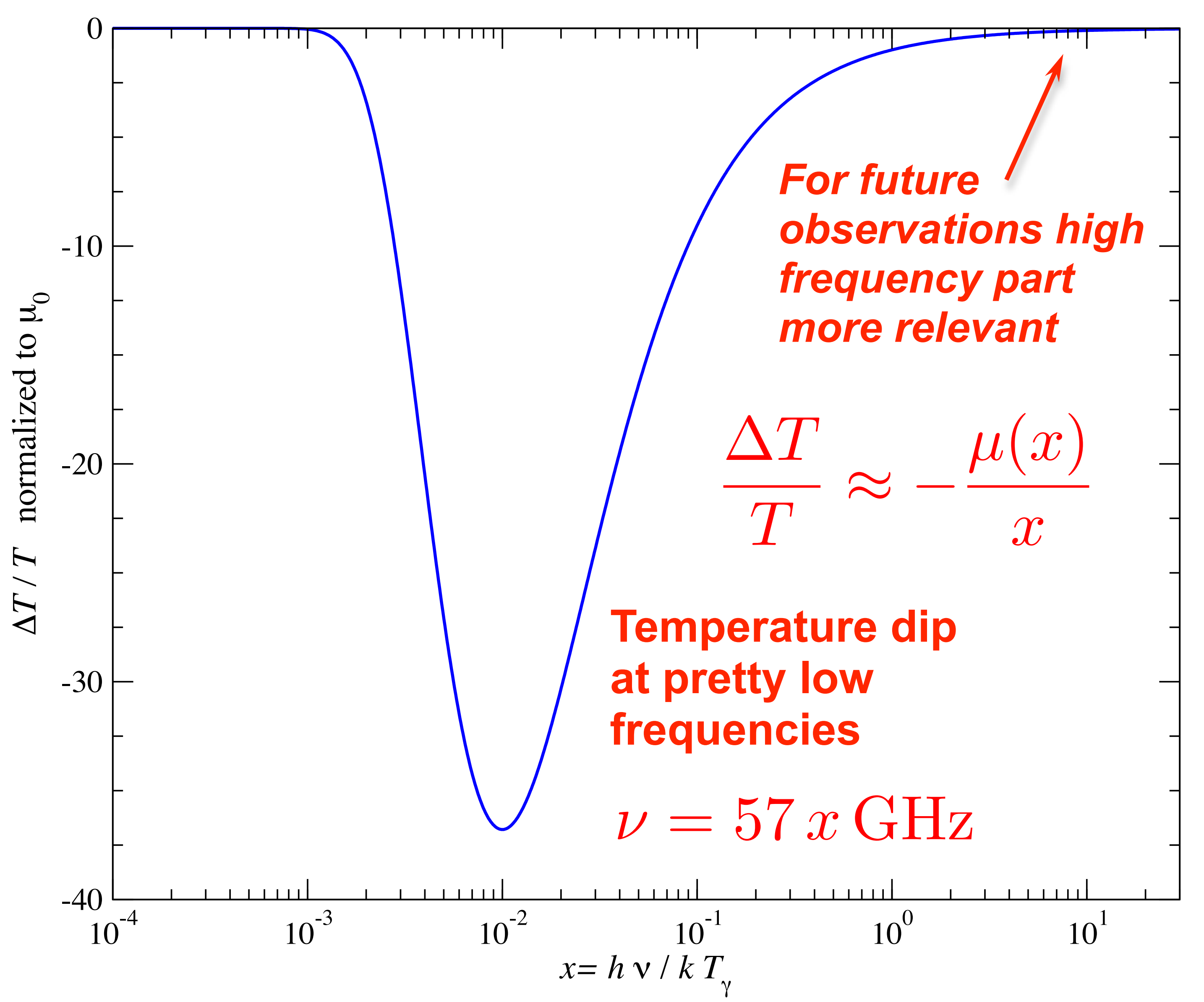

For future observations high frequency part more relevant

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Last step: How does $\mu_0(z)$ depend on z?

Analytic Approximation for μ-distortion

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- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

Analytic Approximation for μ-distortion

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- *low frequency limit & small distortion* $\Longrightarrow$ $\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

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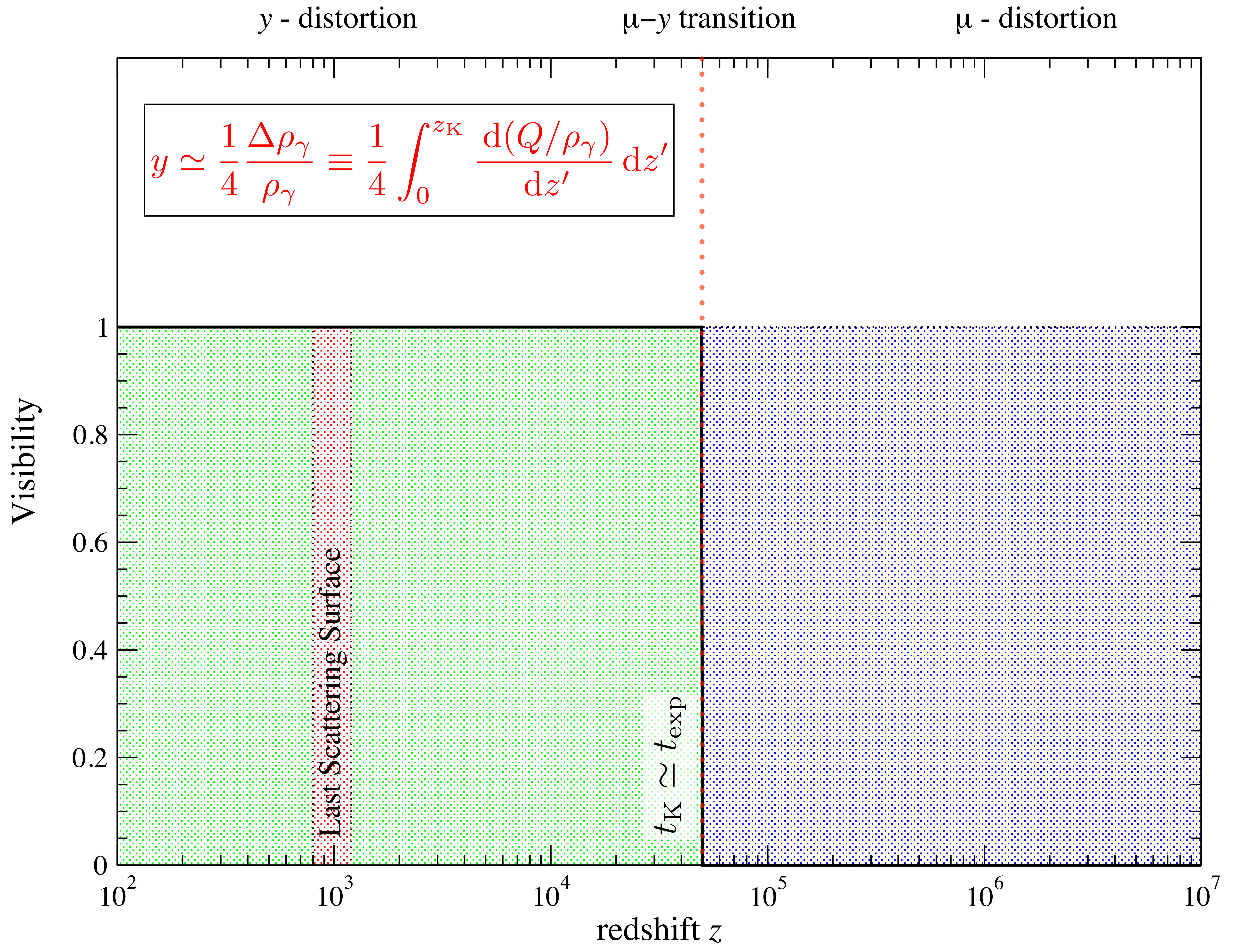
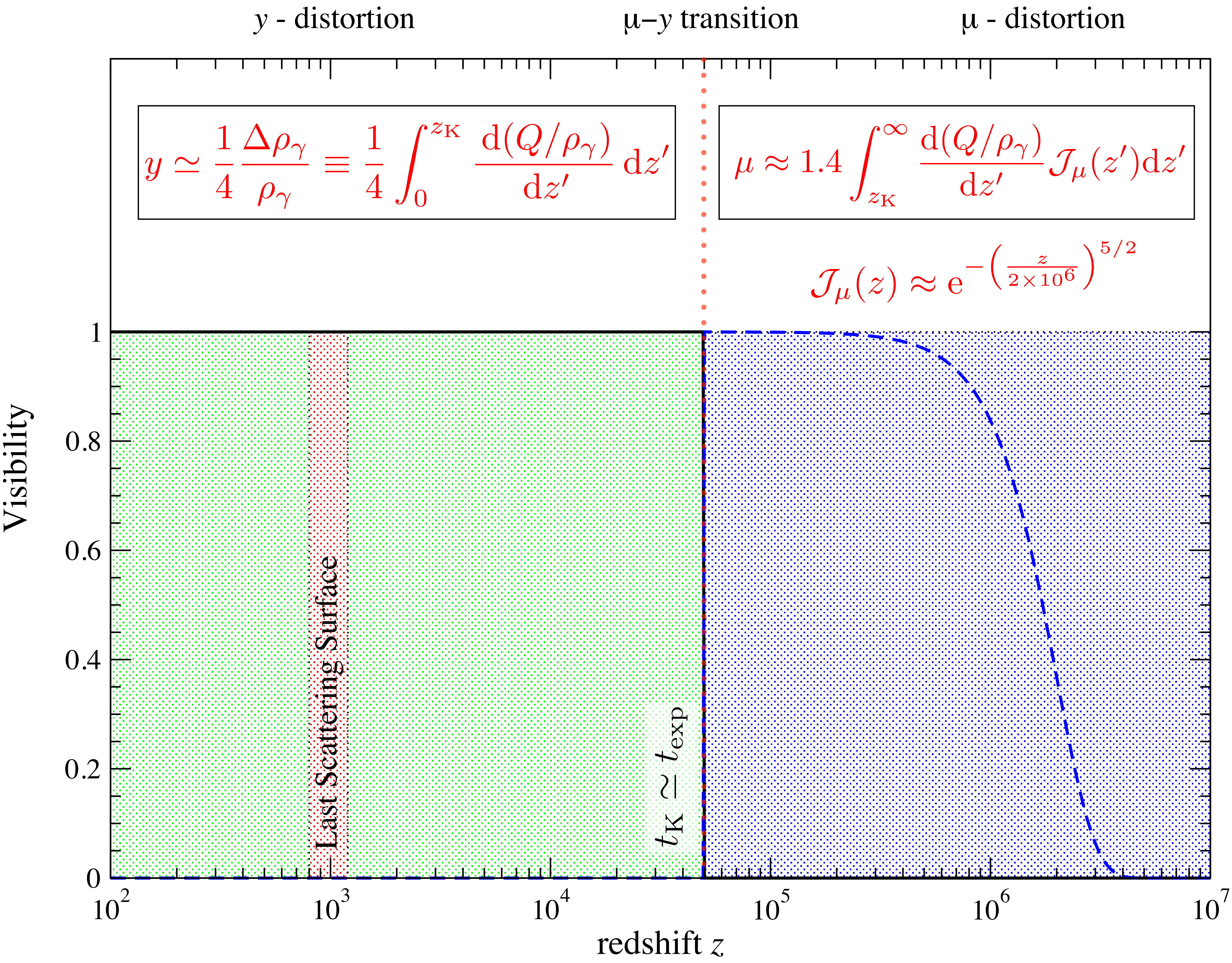
- *Transition between μ and y modeled as simple step function*


y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z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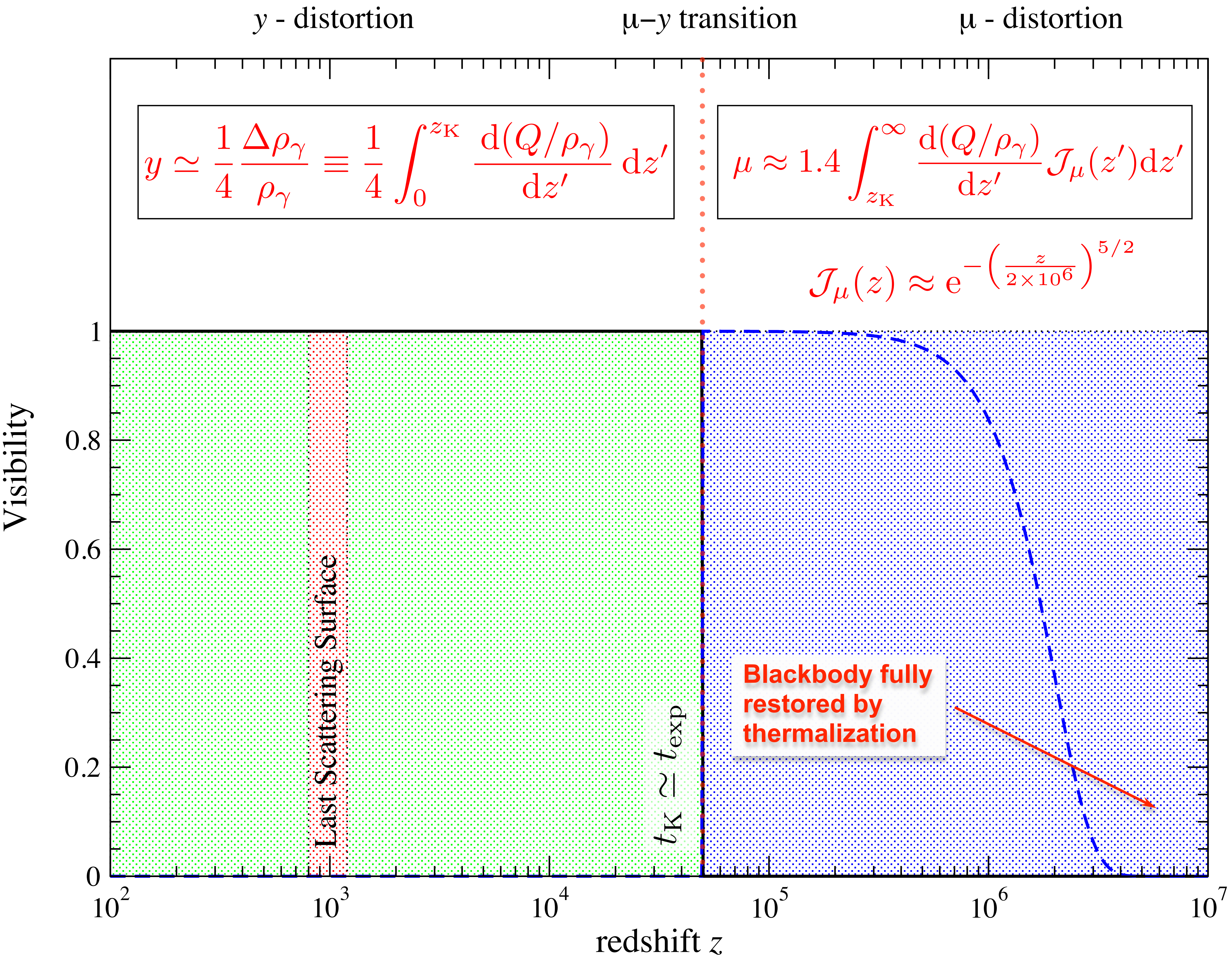
$\mathcal{J}_\mu(z) \approx \mathrm{e}^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Last Scattering Surface
$t_\mathrm{K} \simeq t_\mathrm{exp}$
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z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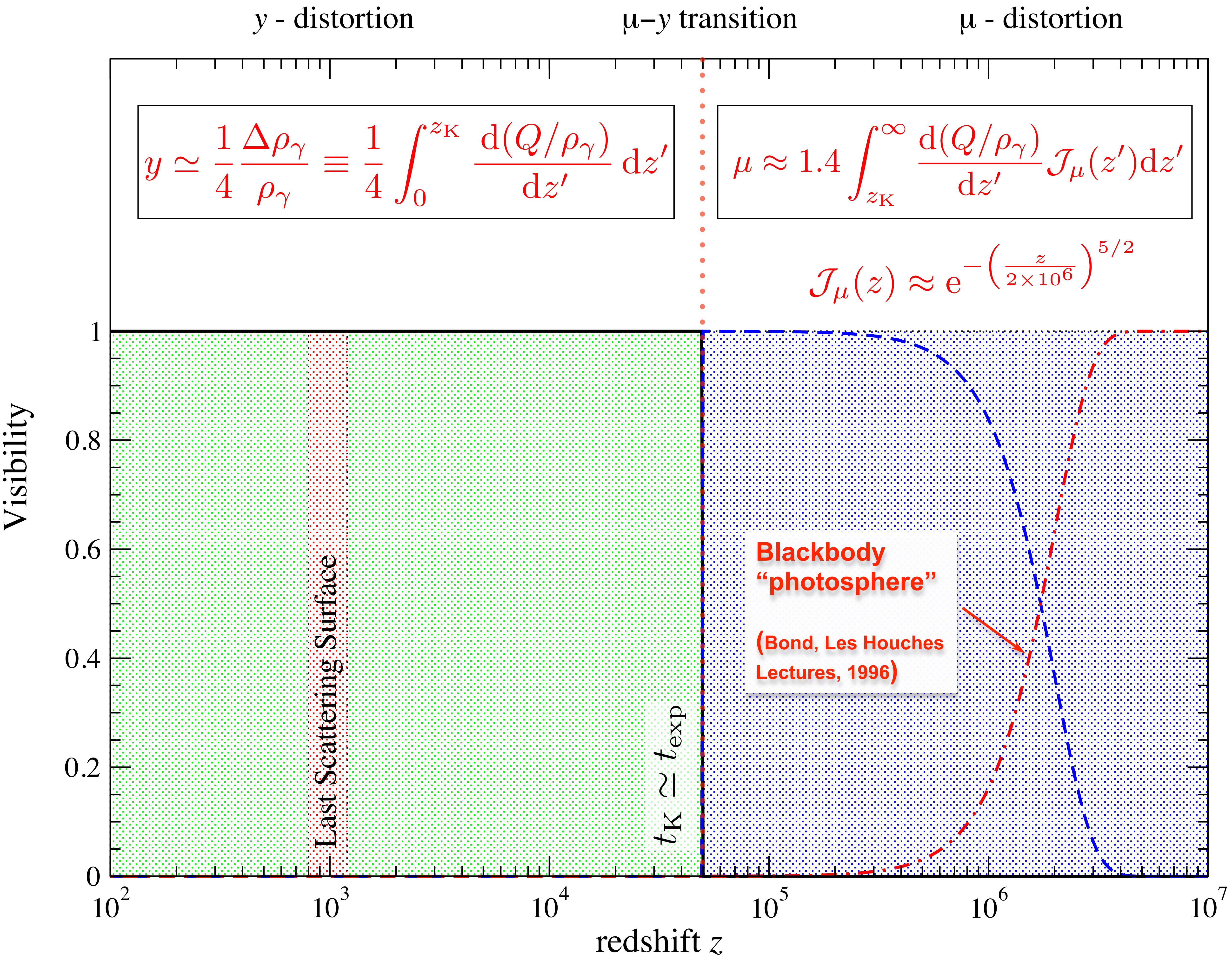
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Blackbody fully restored by thermalization
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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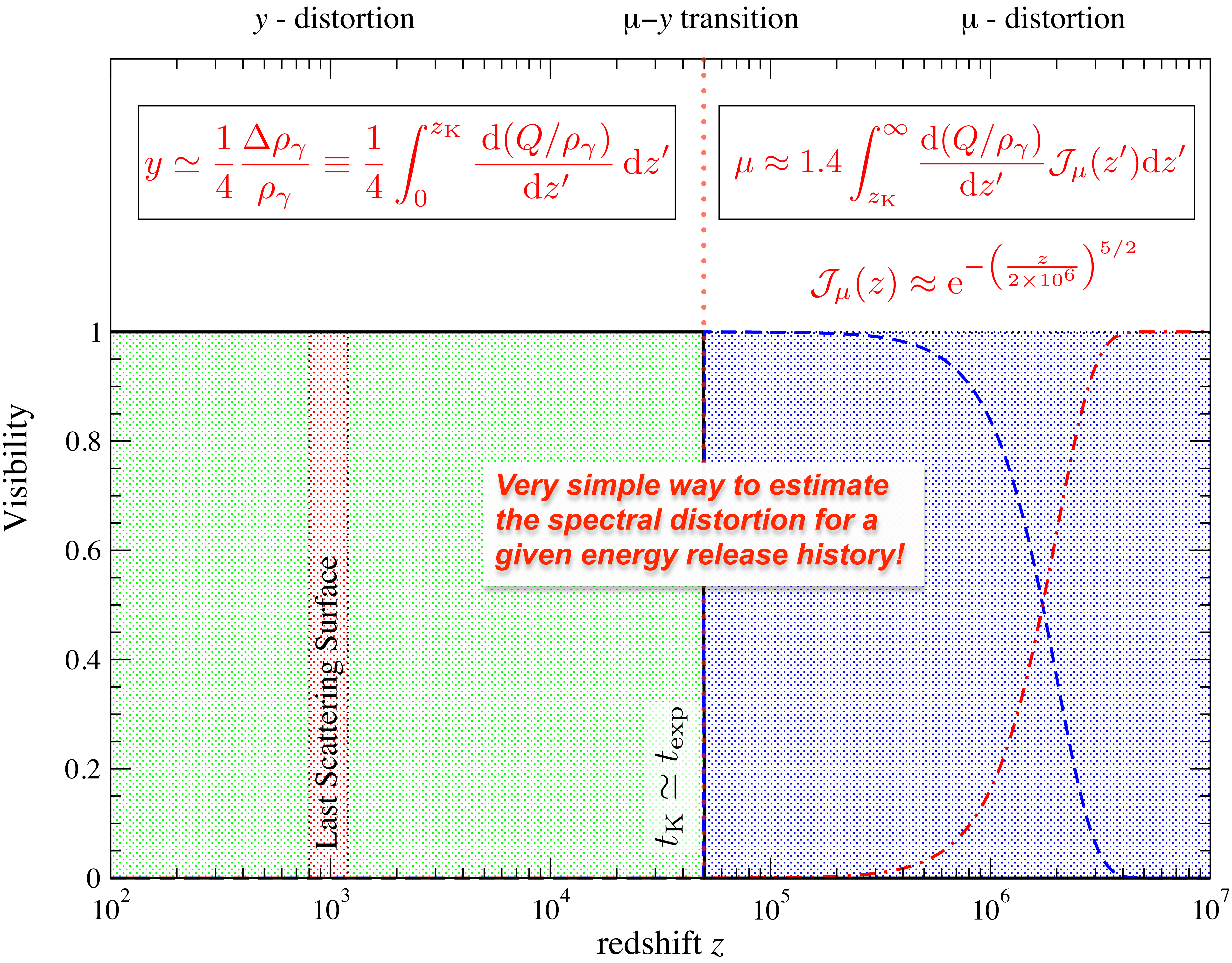
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_\mathrm{K} \simeq t_\mathrm{exp}$
Blackbody "photosphere"
(Bond, Les Houches Lectures, 1996)
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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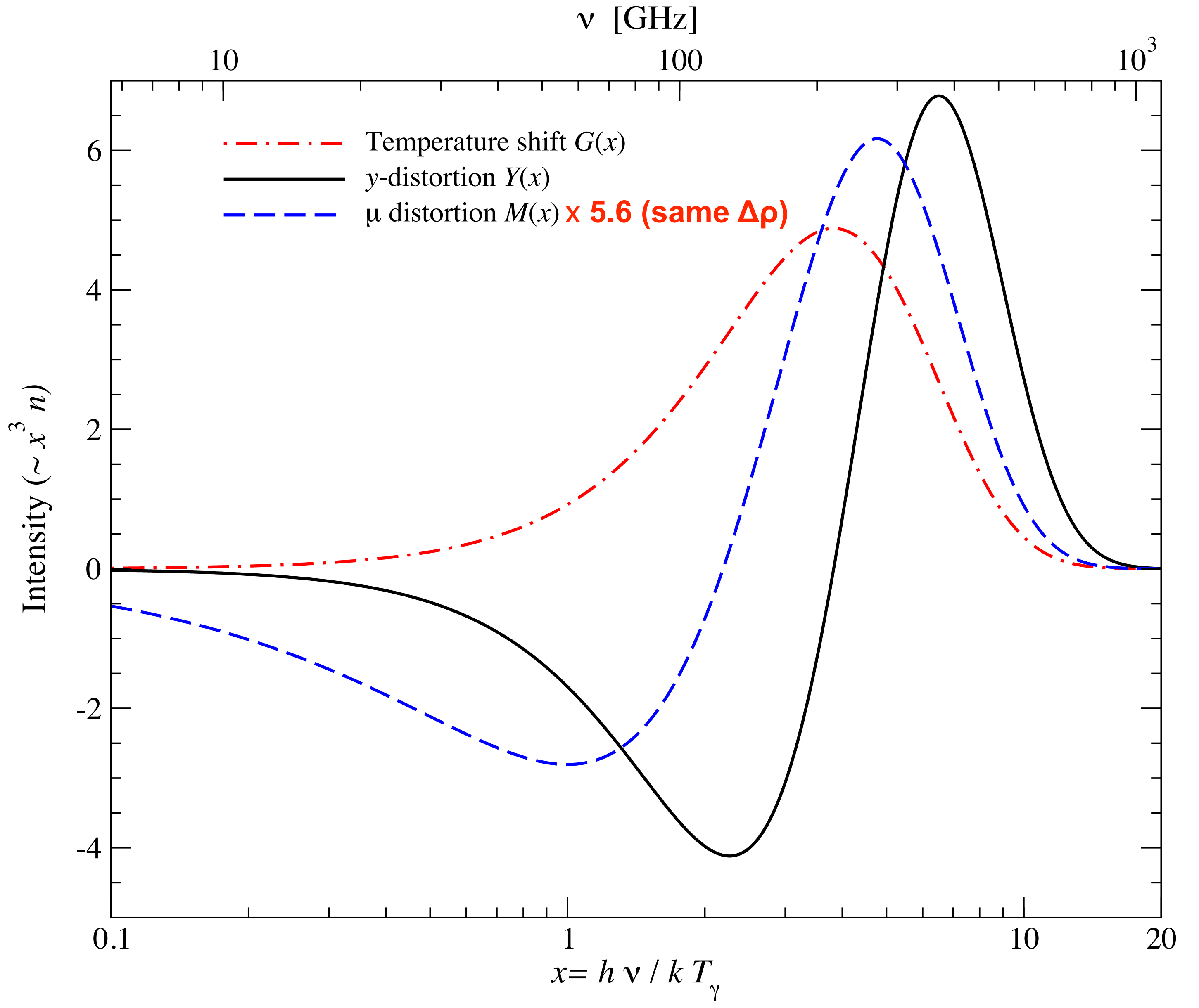
$$\mathcal{J}_\mu(z) \approx {\rm e}^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$$

Very simple way to estimate the spectral distortion for a given energy release history!

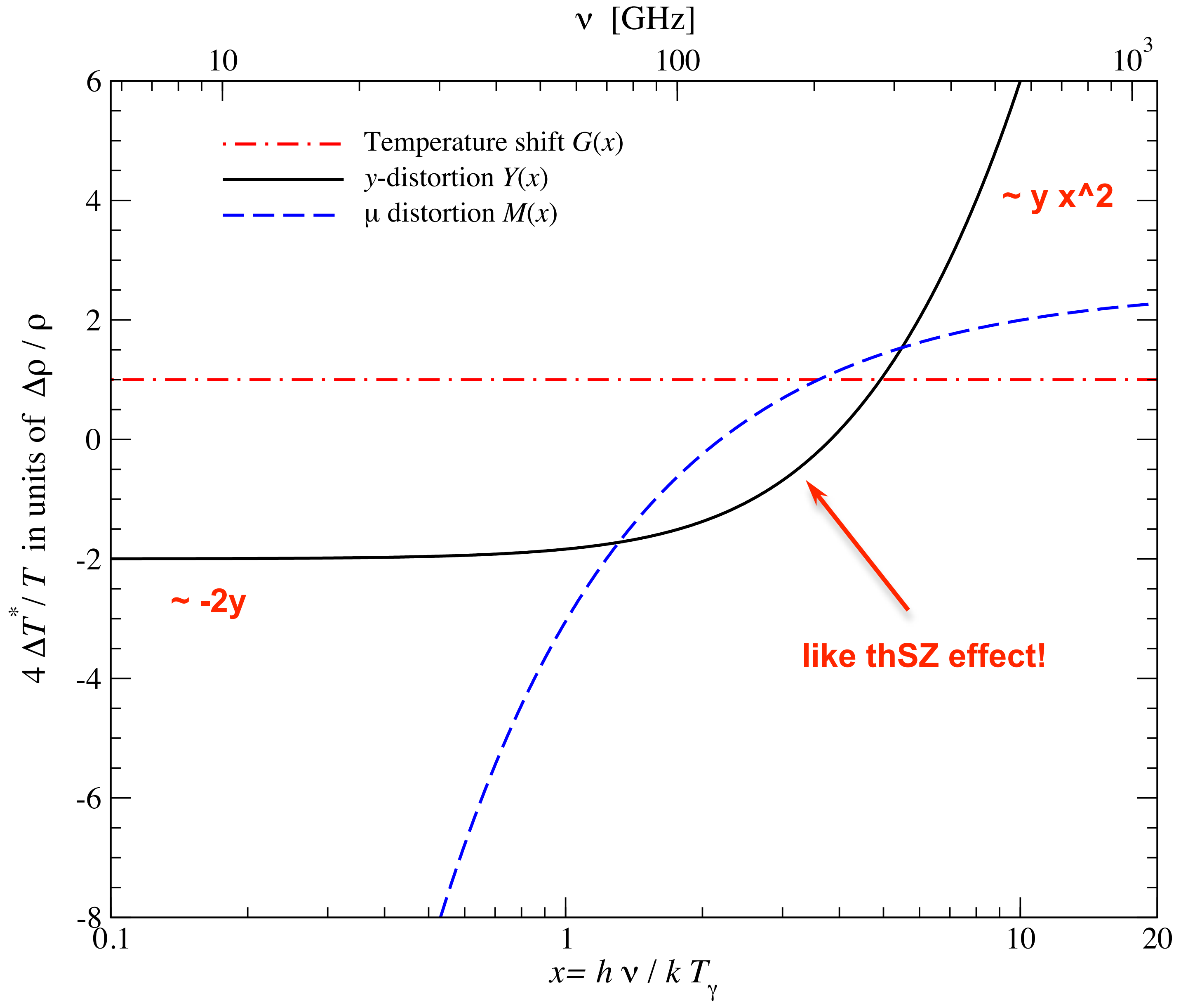
What about the μ-y transition regime?

Is the transition really as abrupt?

Temperature shift ↔ y-distortion ↔ μ-distortion



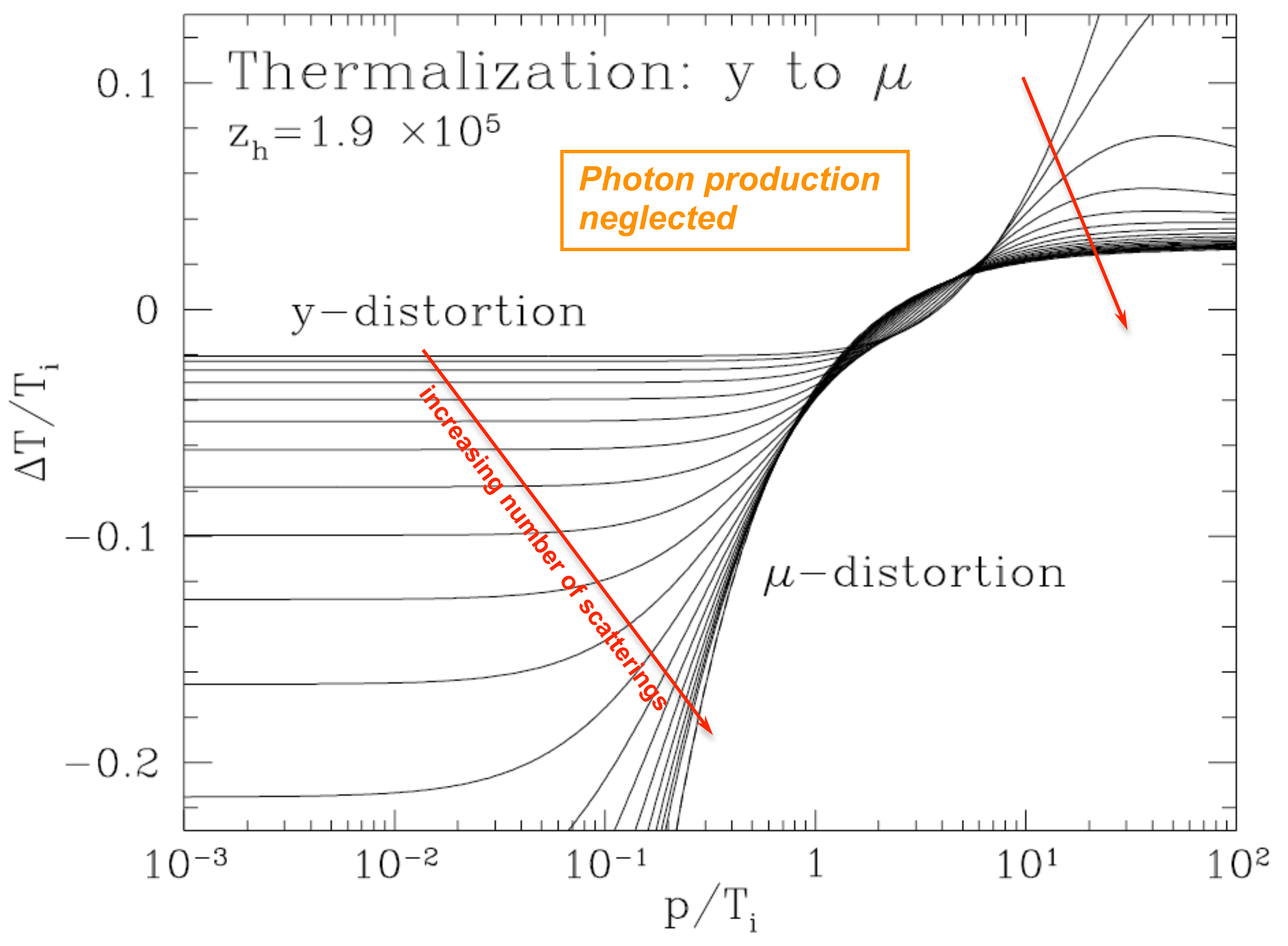
Temperature shift ↔ y-distortion ↔ μ-distortion



Same as before but as effective temperature

$$\frac{\Delta T^*}{T} \approx \frac{\Delta n(x)}{G(x)}$$

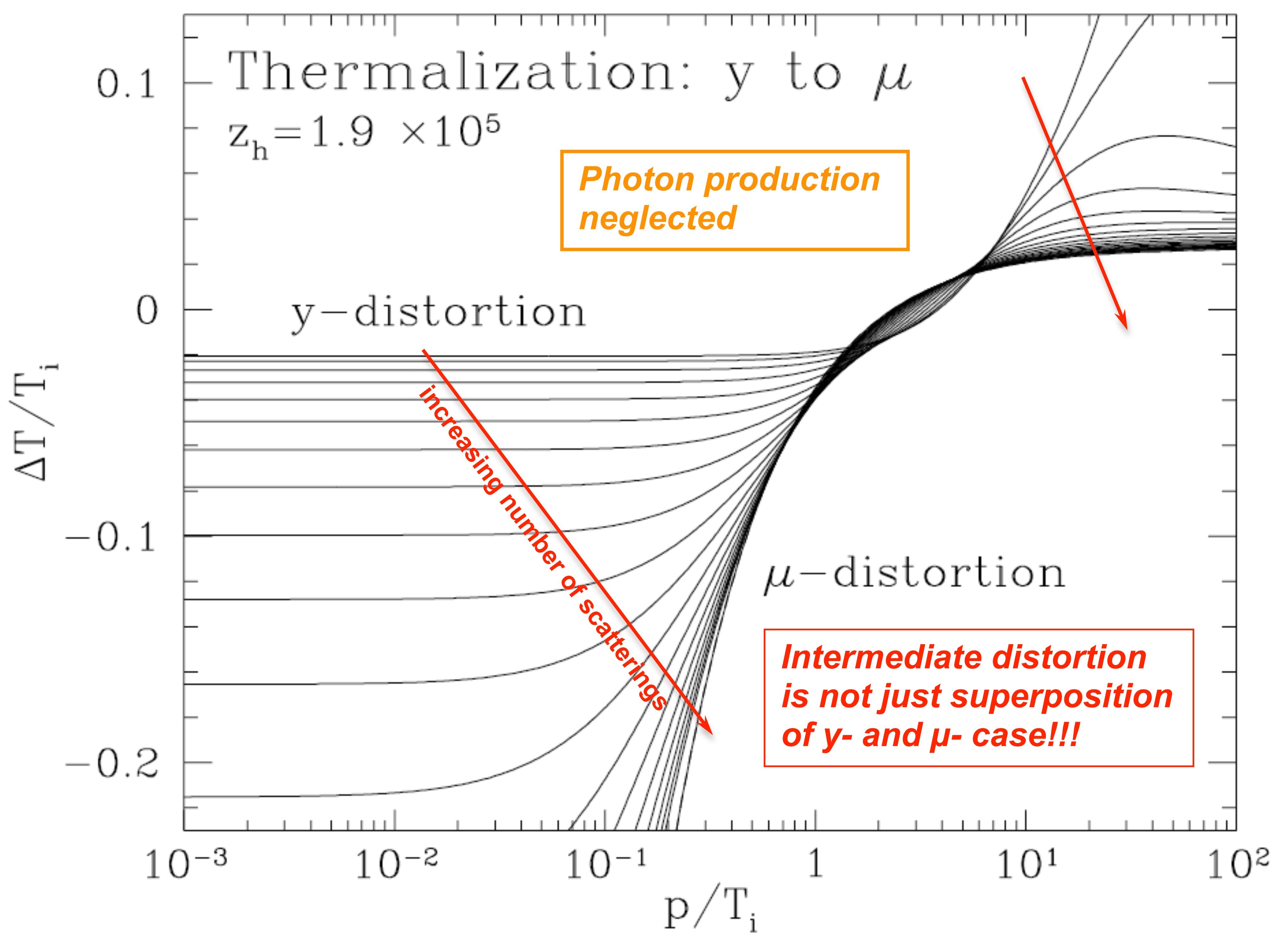
Transition from y-distortion → μ-distortion



Photon production neglected

Figure from Wayne Hu's PhD thesis, 1995

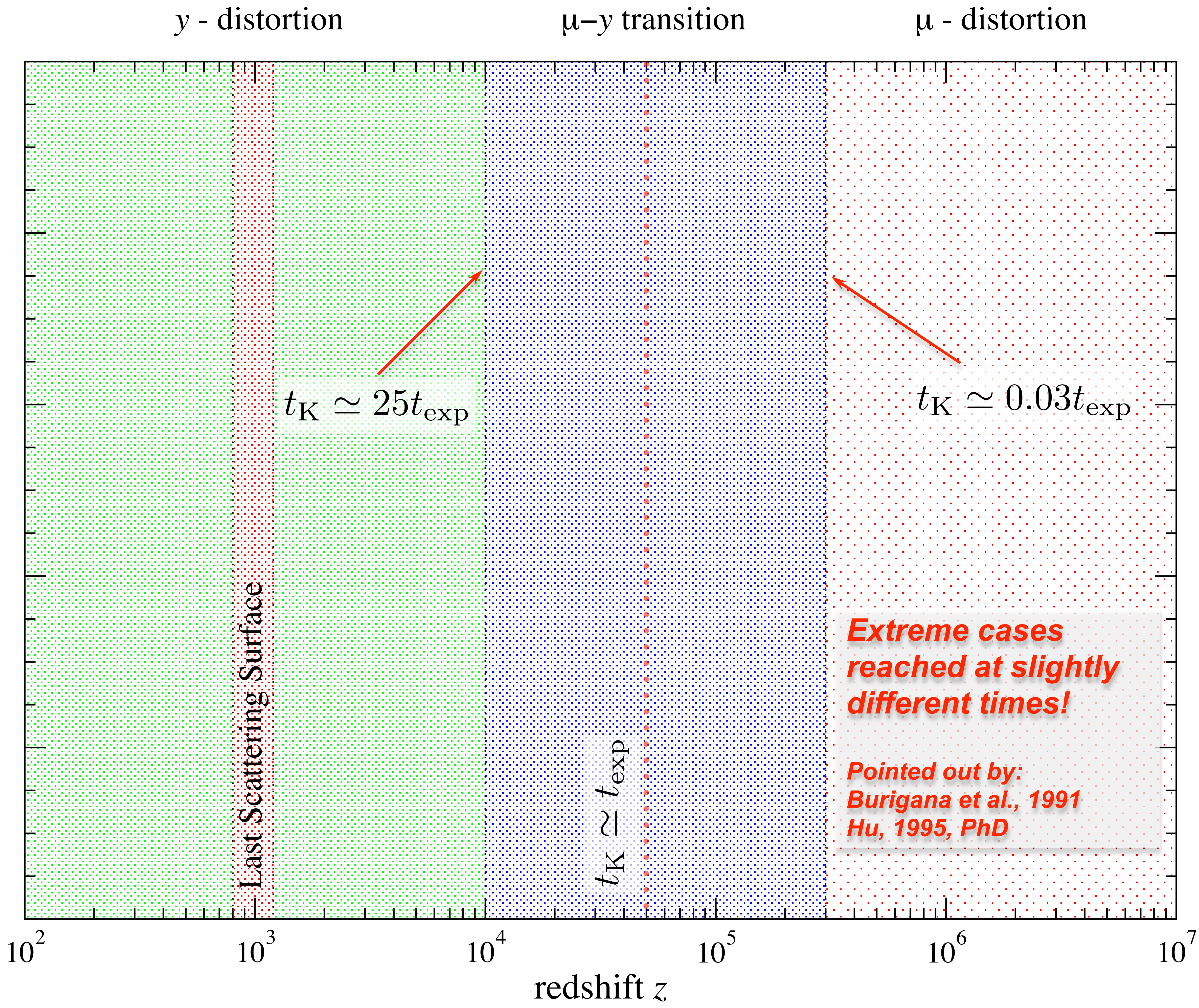
Transition from y-distortion → μ-distortion



Photon production neglected

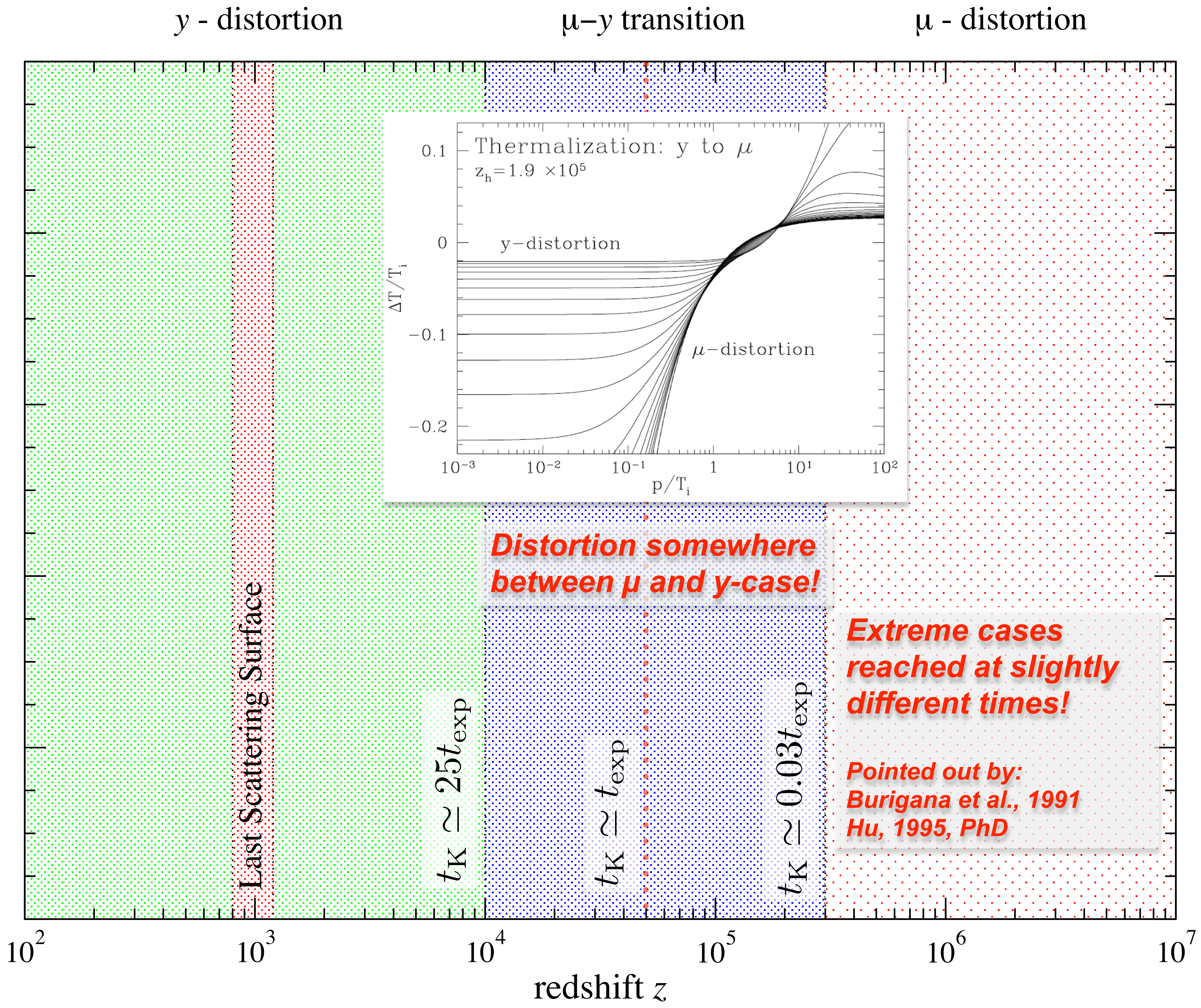
Intermediate distortion is not just superposition of y- and μ- case!!!

Figure from Wayne Hu's PhD thesis, 1995



Extreme cases reached at slightly different times!

Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD



Distortion somewhere between μ and y-case!

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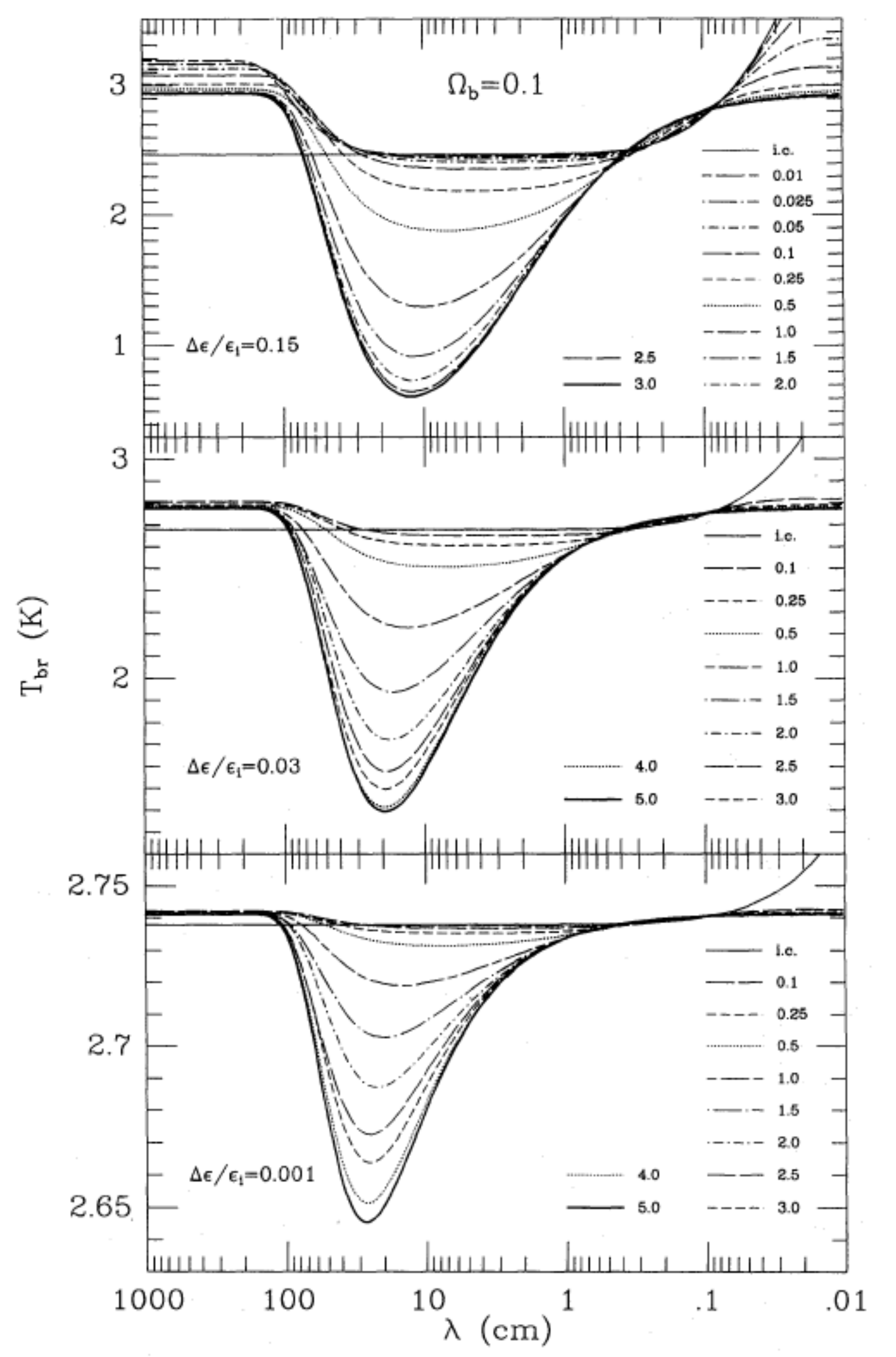
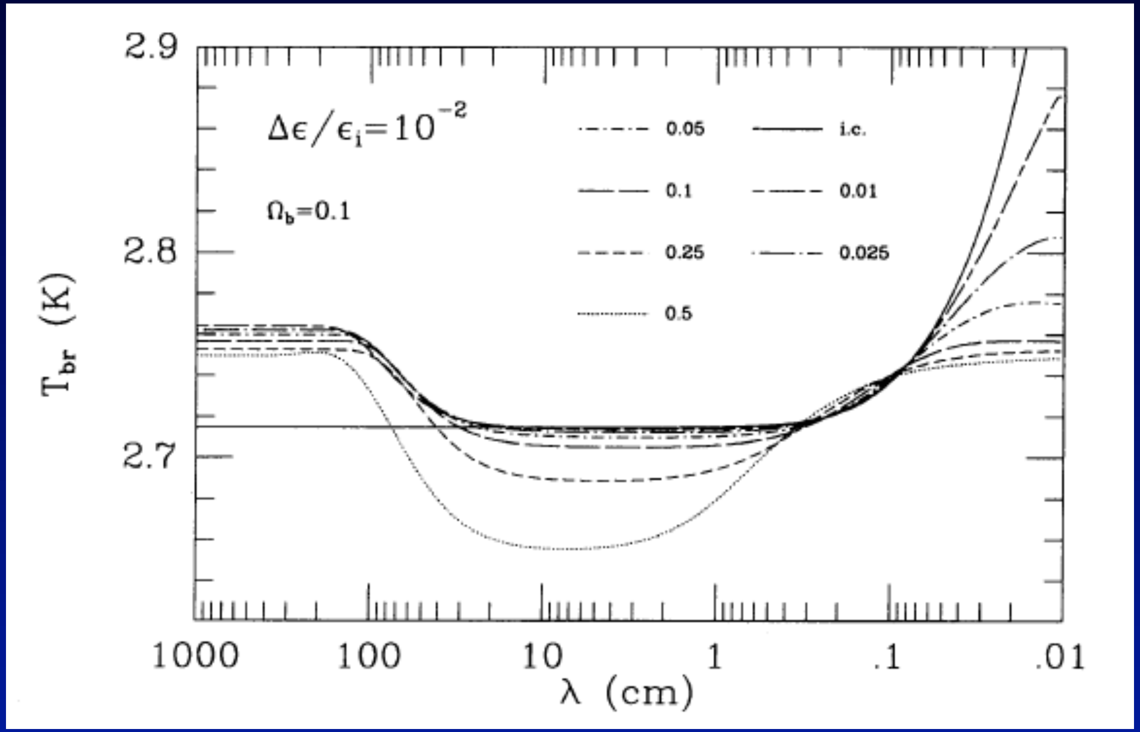
Thermalization from $y \rightarrow \mu$ *at low frequencies*





- amount of energy

  ↔ amplitude of distortion

  ↔ position of 'dip'

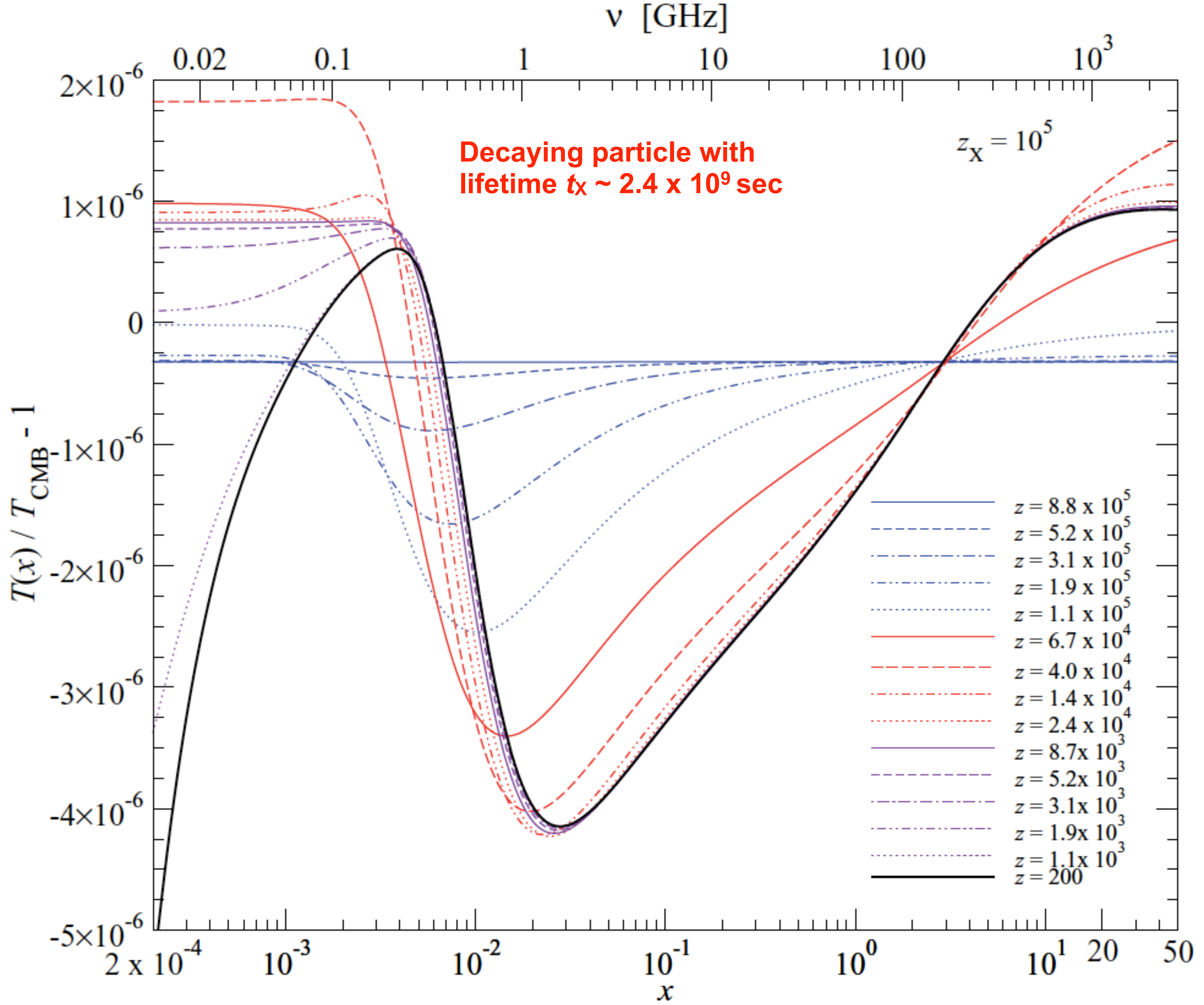
- Intermediate case ($3\text{x}10^5 \geq z \geq 10000$)

  ⇒ mixture between μ & y + *residual*

- details at very low frequencies change

Burigana, De Zotti & Danese, 1991, ApJ
Burigana, Danese & De Zotti, 1991, A&A

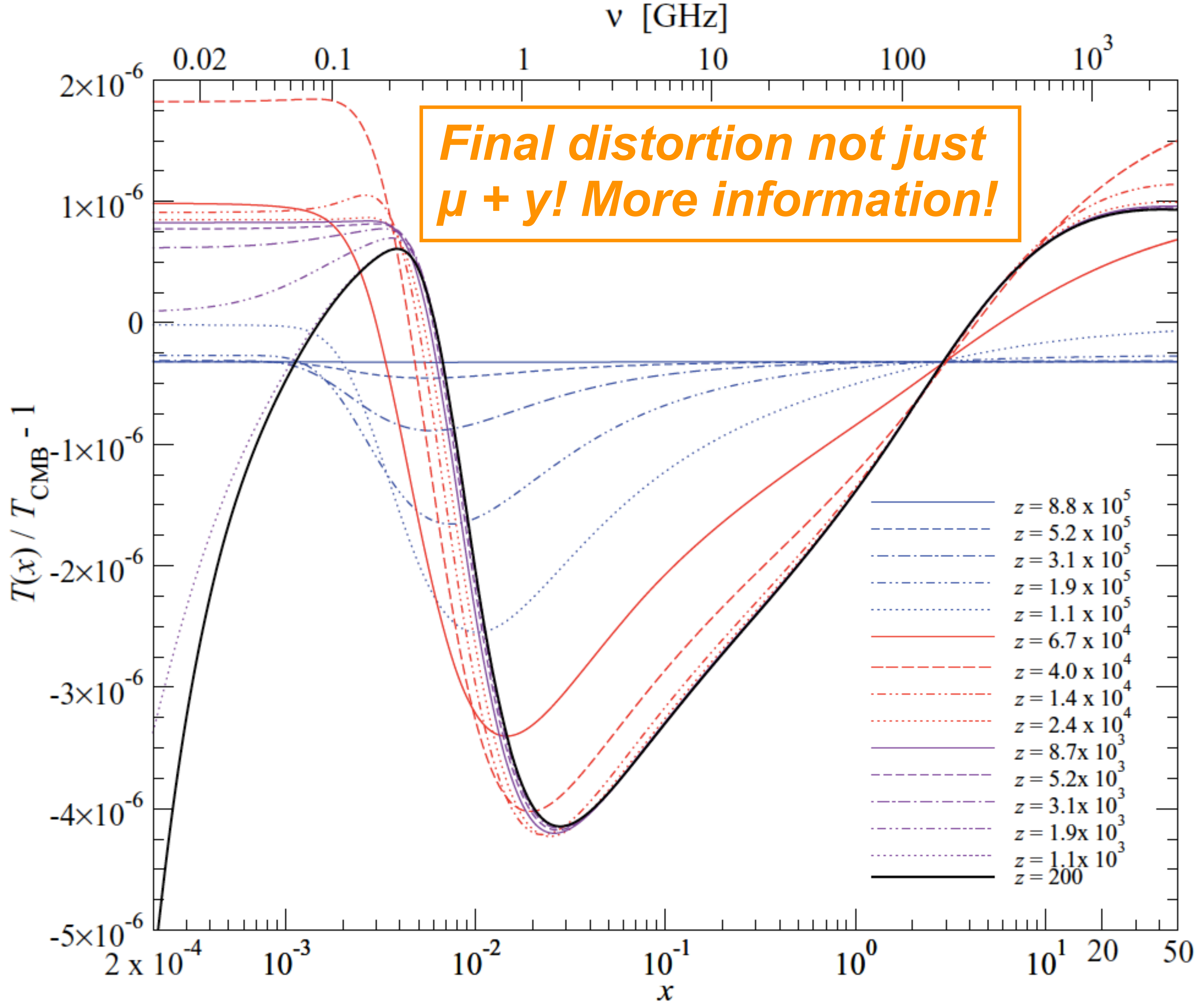
Distortion not just *μ* and *y*-distortion



Decaying particle with lifetime t_X ~ 2.4 x 10^9 sec

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2011)

Distortion not just μ and y-distortion



Final distortion not just μ + y! More information!

Computation carried out with *CosmoTherm*
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Quasi-Exact Treatment: Thermalization Green’s Function

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$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'} {\rm d}z'$$

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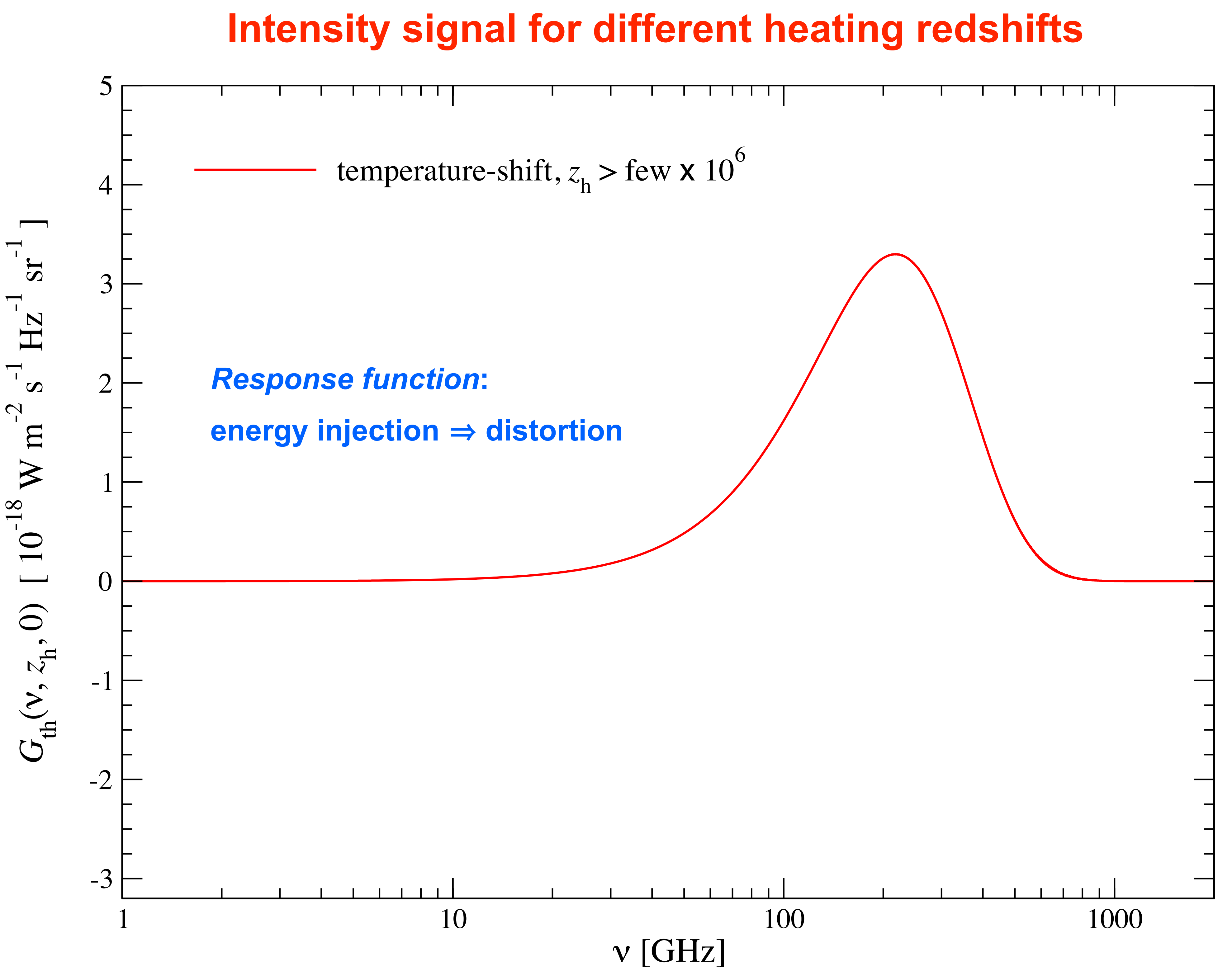
Thermalization Green's function

- *Fast and quasi-exact! No additional approximations!*

What does the spectrum look like after energy injection?



***Response function*:**

energy injection ⇒ distortion

JC & Sunyaev, 2012, ArXiv:1109.6552
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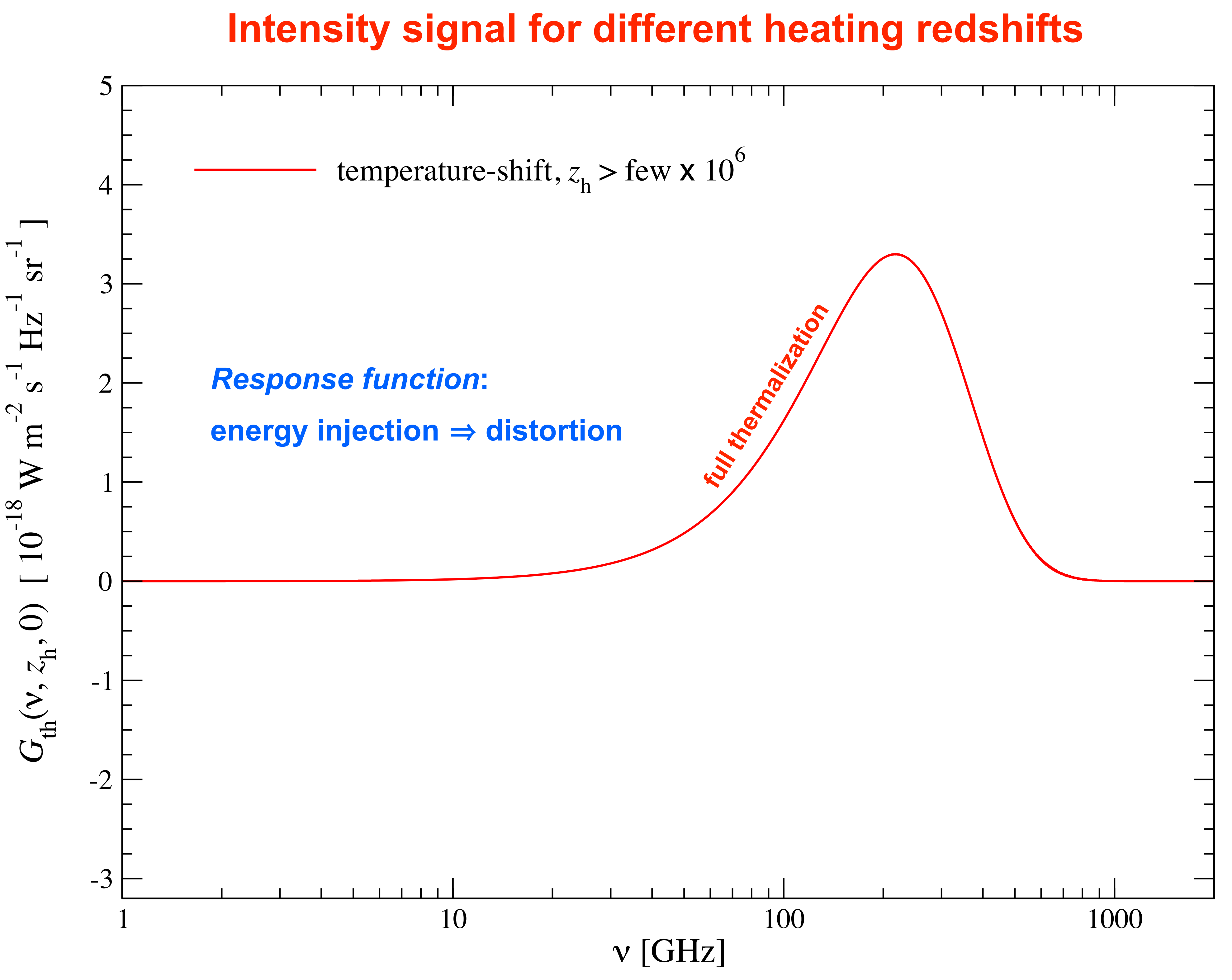

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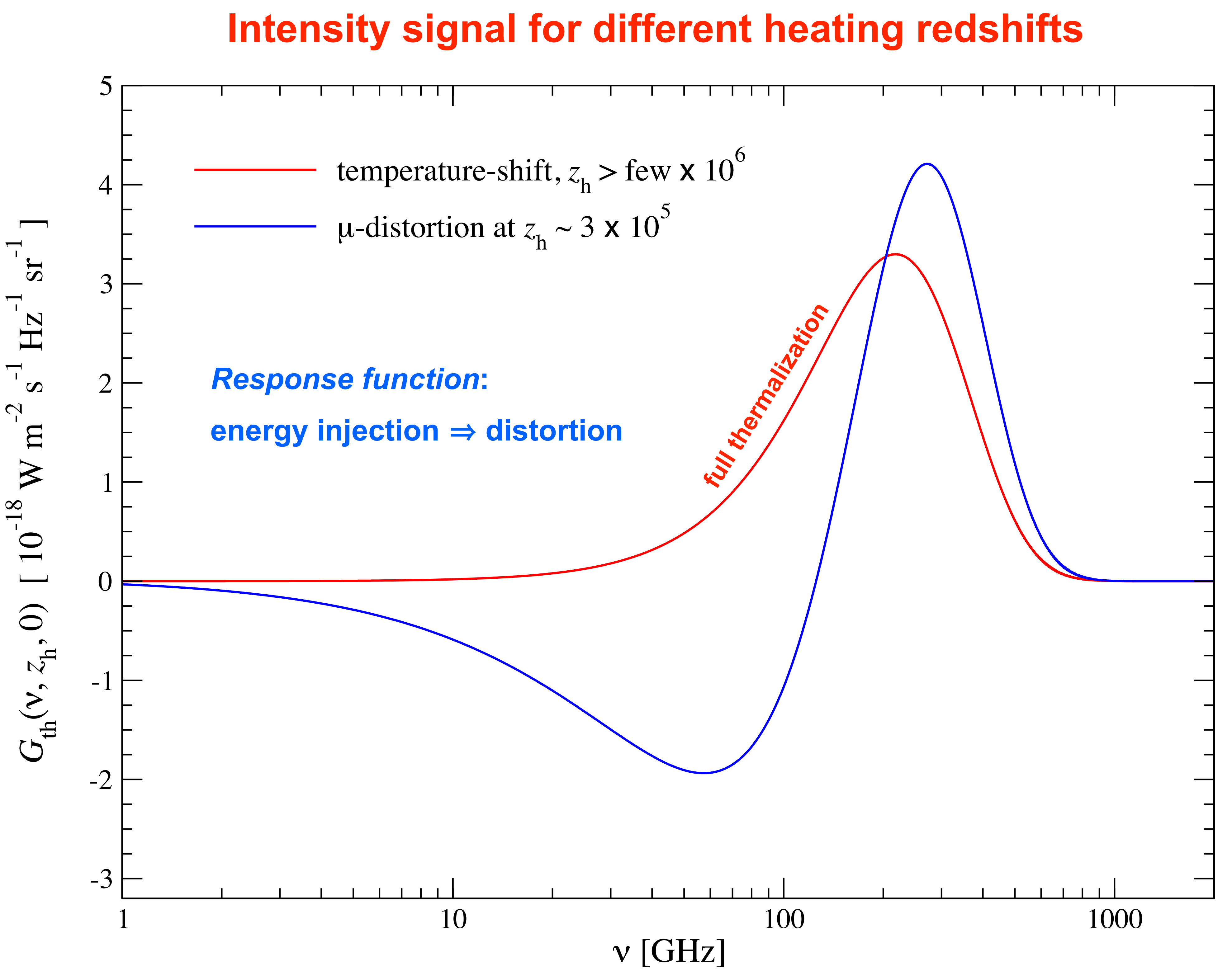

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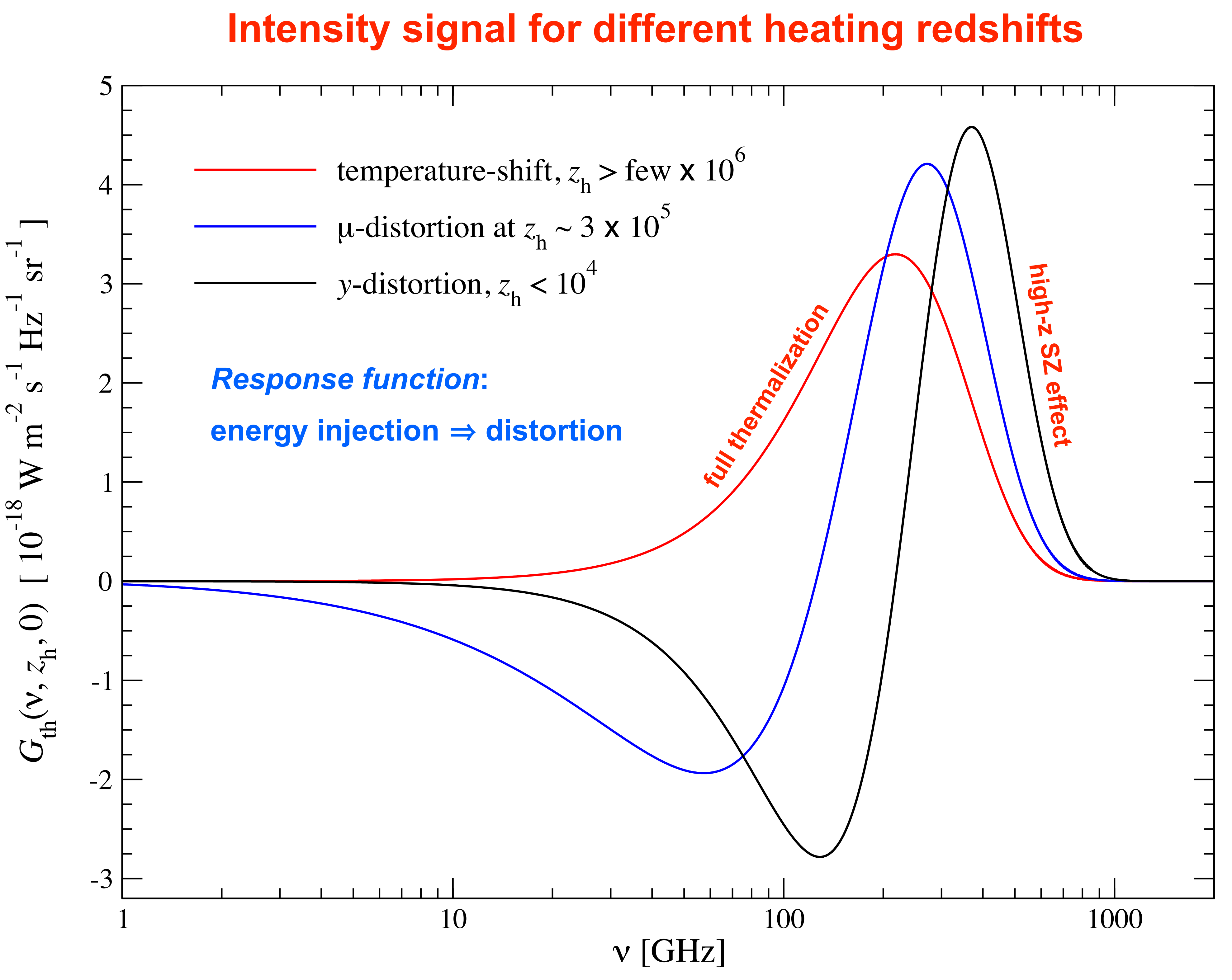

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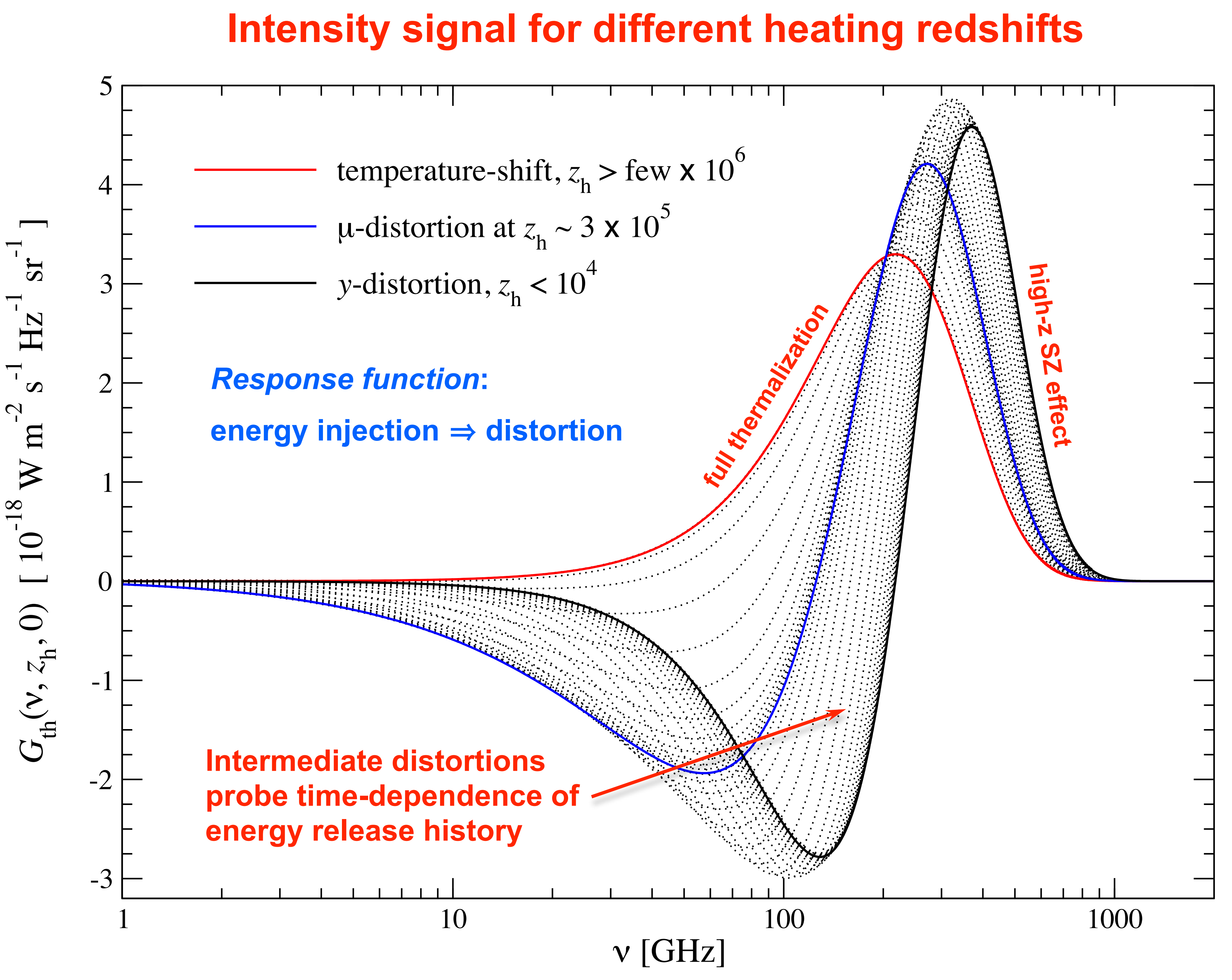

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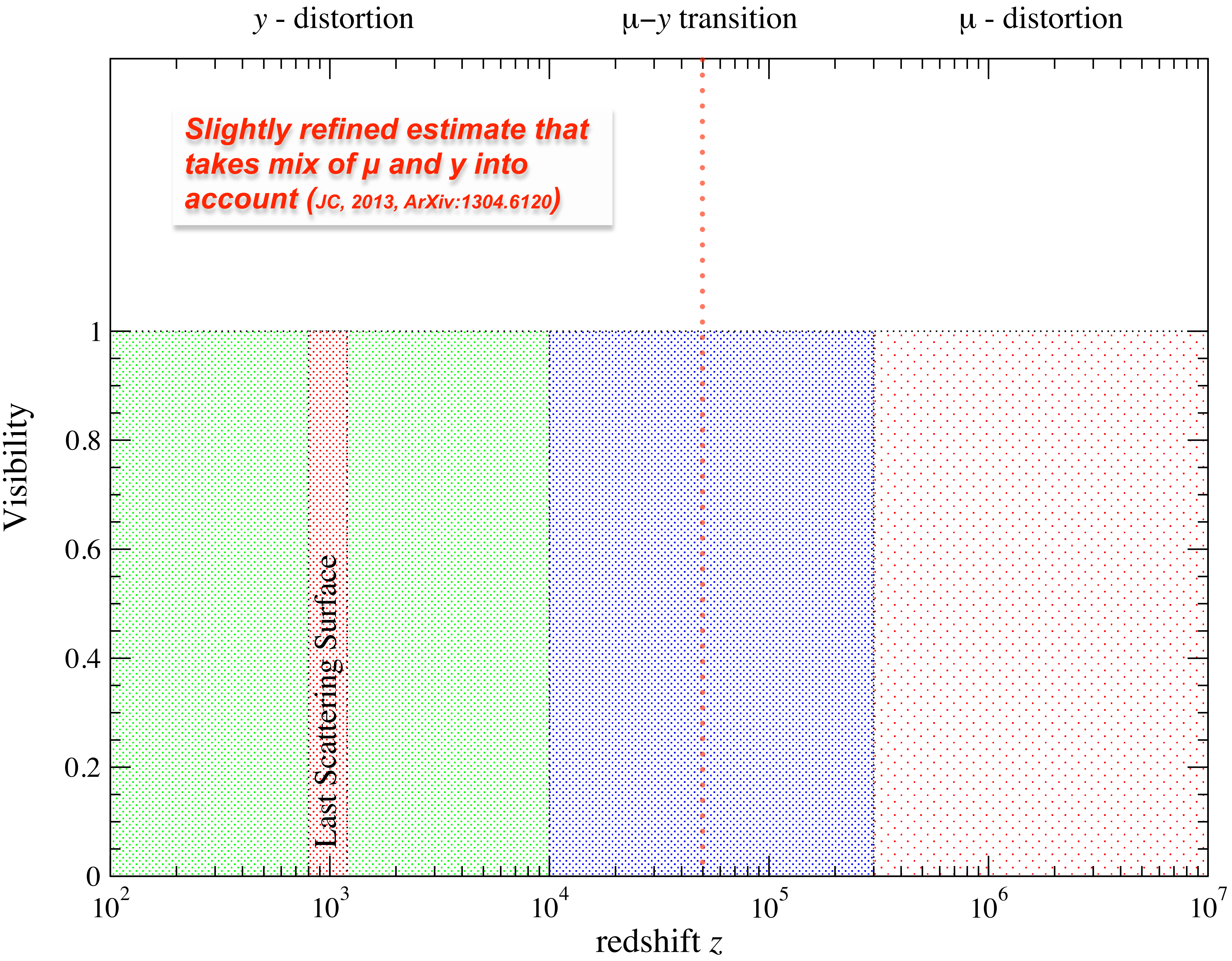
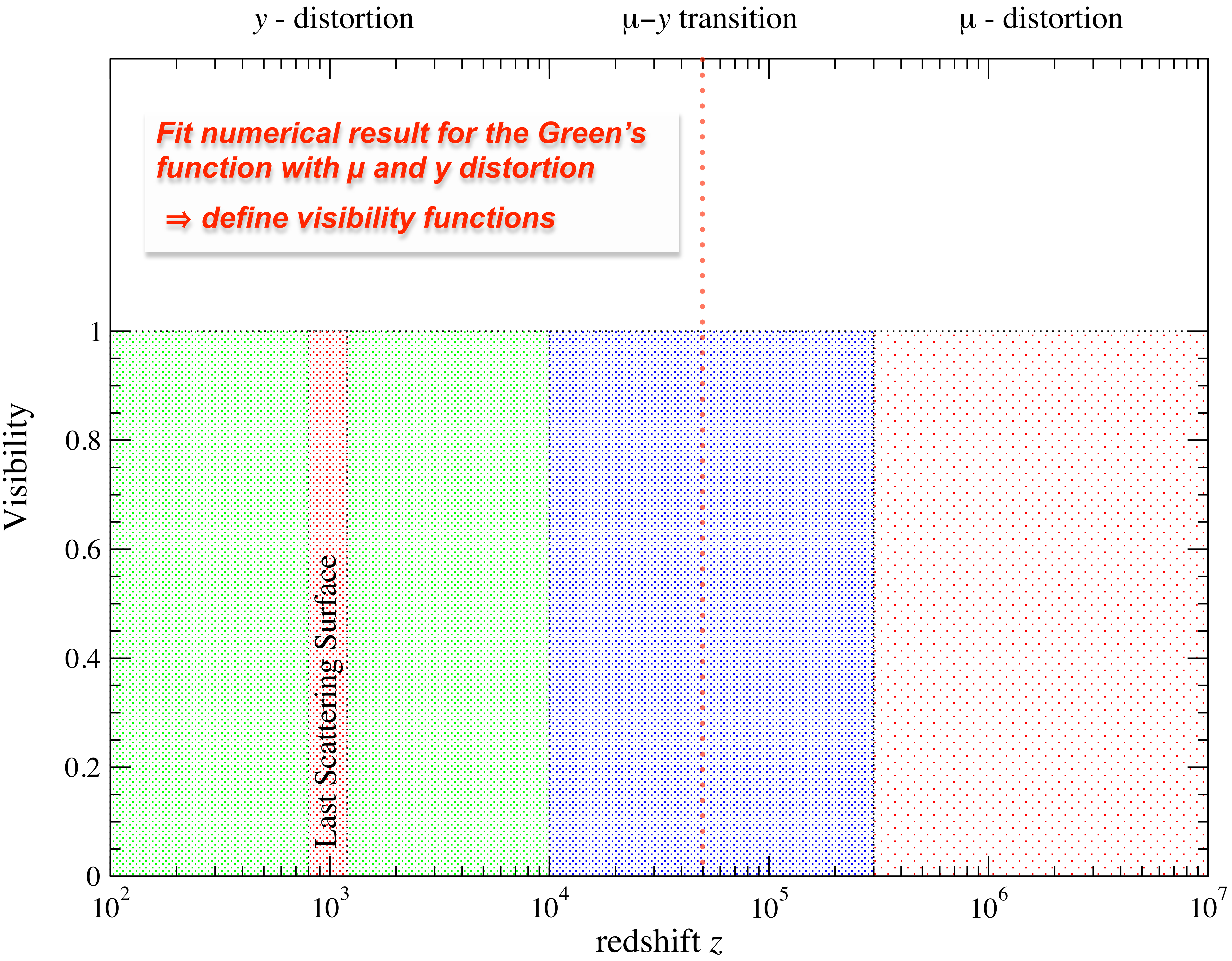
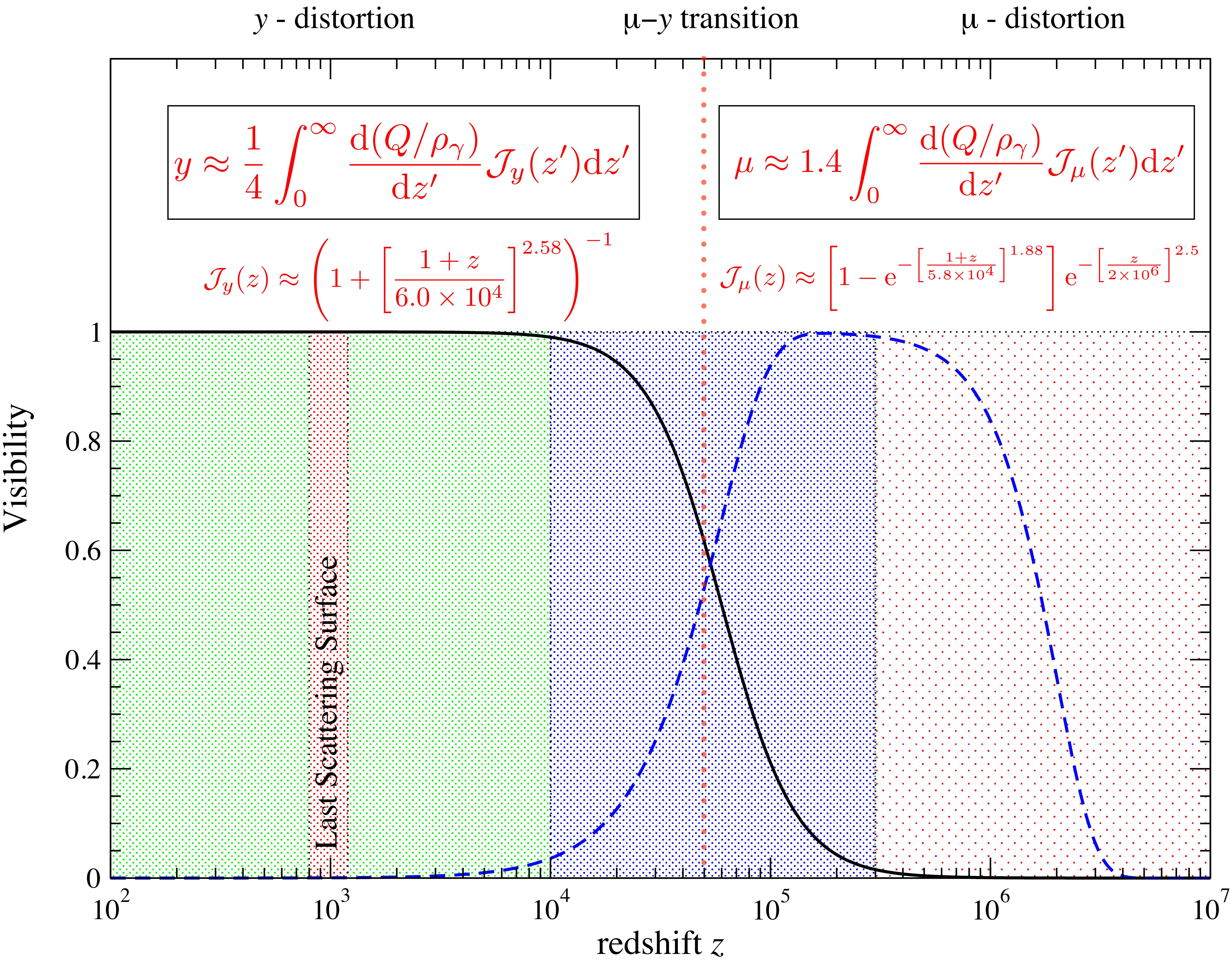


Slightly refined estimate that takes mix of μ and y into account (JC, 2013, ArXiv:1304.6120)



Fit numerical result for the Green's function with μ and y distortion

⇒ define visibility functions


y - distortion
μ–y transition
μ - distortion
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$\mu \approx 1.4\int_0^\infty \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\mathcal{J}_\mu(z')\mathrm{d}z'$
$\mathcal{J}_y(z) \approx \left(1+\left[\frac{1+z}{6.0\times10^4}\right]^{2.58}\right)^{-1}$
$\mathcal{J}_\mu(z) \approx \left[1-\mathrm{e}^{-\left[\frac{1+z}{5.8\times10^4}\right]^{1.88}}\right]\mathrm{e}^{-\left[\frac{z}{2\times10^6}\right]^{2.5}}$
Last Scattering Surface
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
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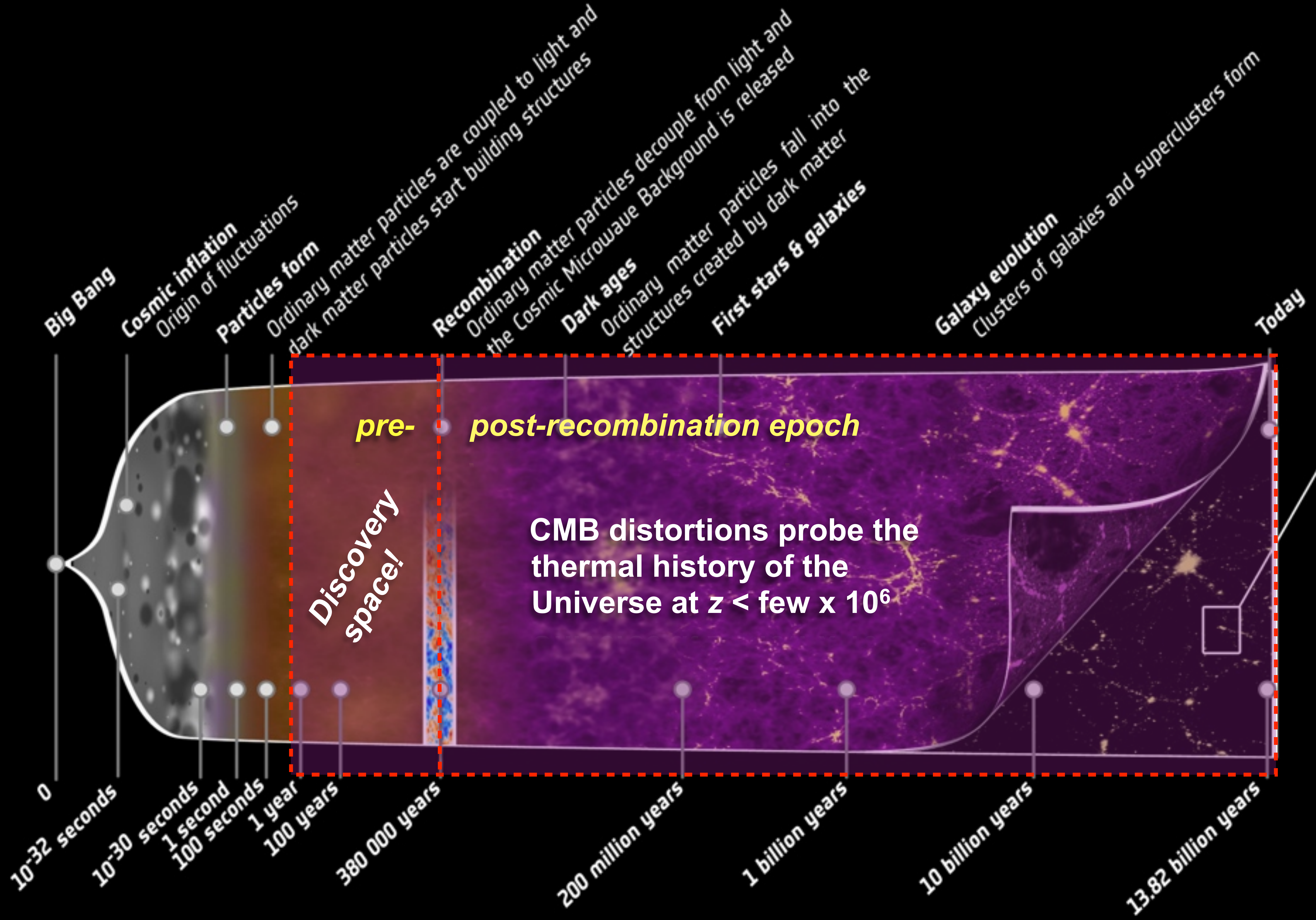
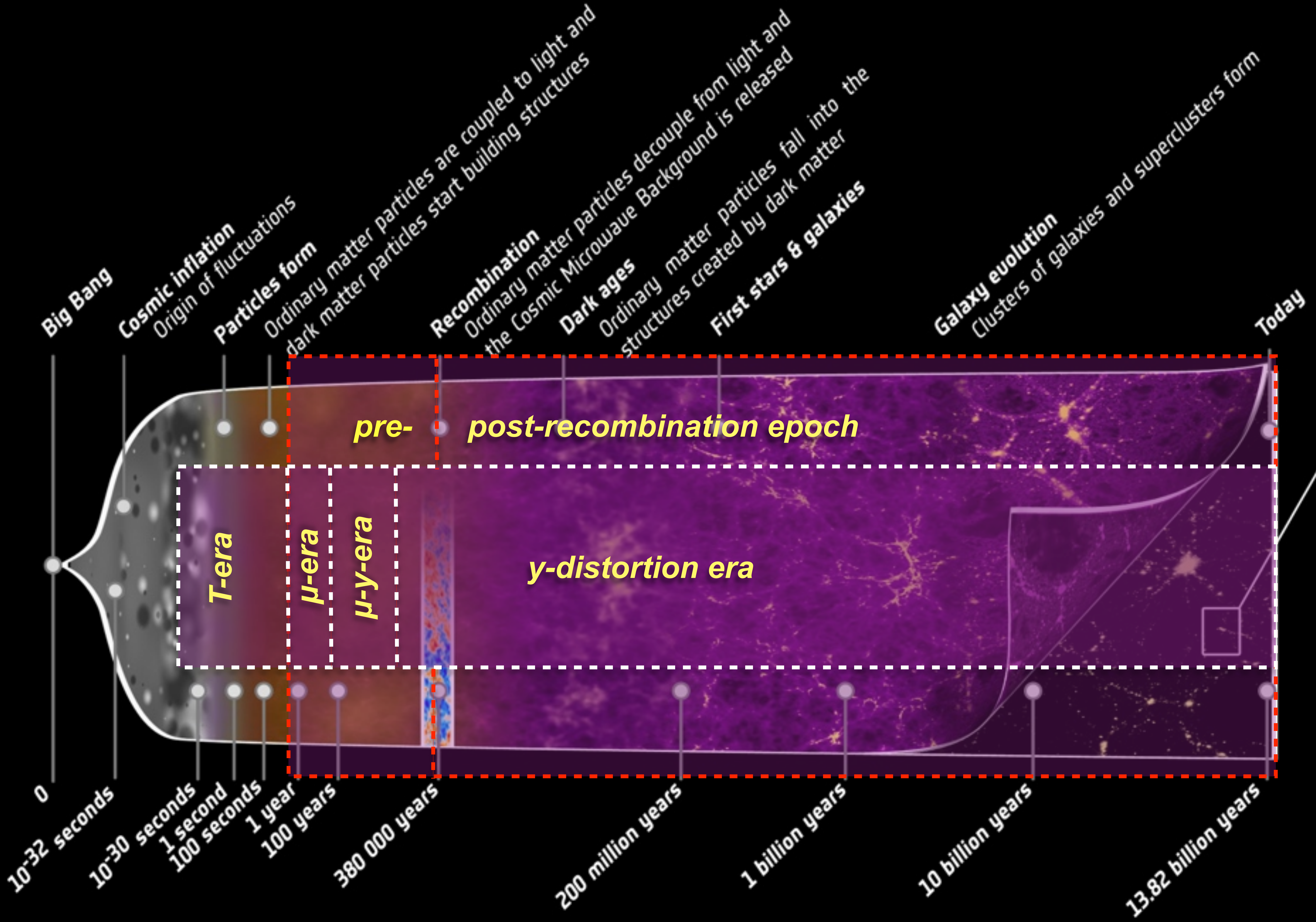


pre-

post-recombination epoch

Discovery space!

CMB distortions probe the thermal history of the Universe at $z <$ few x 10^6


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
y-distortion era
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years