Outline

- Fourier Transforms
- The Dirac δ Function

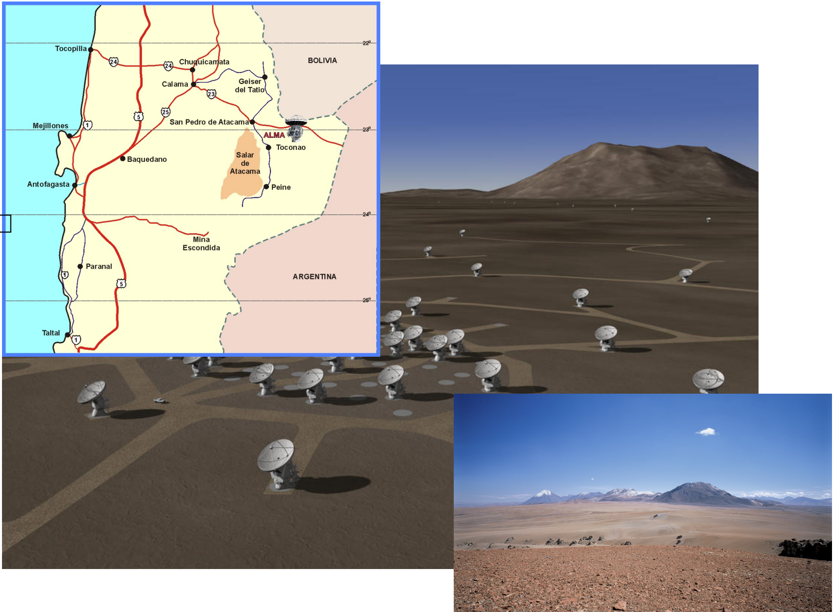
Fourier Transform

Fourier Transform of a non-perodic function, $f(x)$. If $g(k)$ is the FT of $f(x)$, then

$$
\begin{aligned}
g(k) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx \\
f(x) &= \int_{-\infty}^{\infty} g(k)e^{ikx}\,dk
\end{aligned}
$$

and $f(x)$ is the inverse transform of $g(k)$.
x and k have inverse units.
If x is a wavelength, k is a wavenumber.
If x is time, k is frequency.

ALMA: Atacama Large Millimetre/Submillimetre Array



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Interferometers

What you measure with two elements of an interferometer is the complex visibility

$$V = \int_{-\infty}^{\infty} I(\underline{\sigma}) e^{i2\pi \underline{b}\cdot\underline{\sigma}} \, d\sigma$$

where $I(\underline{\sigma})$ is the distribution of the brightness of the sources on the sky (as described by the direction on the sky by $\underline{\sigma}$). $\underline{b}$ is vector baseline between the two elements.
V is just the (inverse) Fourier transform of the sky brightness distribution I. So to determine the structure of the source take the Fourier transform V measurements.

Fourier Transform

Transform of a non-perodic function, $f(x)$. If $g(k)$ is the FT of $f(x)$, then

$$\begin{aligned} g(k) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx \\ f(x) &= \int_{-\infty}^{\infty} g(k)e^{ikx}\,dk \end{aligned}$$

and $f(x)$ is the inverse transform of $g(k)$.

Example:

FT of square pulse is a sinc ($= (\sin x)/x$) function.

Example:

Shift theorem for Fourier transforms.

Dirac δ-function

$$\begin{aligned} \delta(x) &= 0 \qquad \text{for } x \neq 0 \\ \int_{-\infty}^{\infty} \delta(x)\, dx &= 1 \end{aligned}$$

By considering the Fourier transform of a Dirac δ function and then the inverse of the result

$$\delta(x - x_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-x_0)}\, dk$$