Cookery...

How to boil an egg: Assume a spherical egg..
An egg is a sphere of radius $a = 2$cm, almost entirely made of water.

heat capacity	$c =$	4184	J/kg/K
heat conductivity	$\kappa =$	0.609	W/m/K
density	$\rho =$	1000	kg/m^3
$\rightarrow$ diffusivity	D=$\kappa/(c\rho) =$	1.46×10^{-7}	m^2/s

Back of the envelope estimate: only two dimensional quantities, D and a:
$t \sim a^2/D = 2700\mathrm{s} = 45\mathrm{min}$

Calculate time using heat flow equation

Spherical symmetry:

$$\nabla^2 f \quad = \quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) = \frac{1}{D}\frac{\partial f}{\partial t}$$

Boundary and initial conditions:

1. f regular at $r = 0$.
2. $f(a, t) = 0$ at $r = a$.
   Place egg in boiling water, so define $T = 100^\circ\mathrm{C} \equiv 0\mathrm{B}$ (Blumenthals).
3. Initial condition: $f(r, 0) = T_0$

To soft boil need centre to reach $T_f = 45^\circ\mathrm{C} = -55\mathrm{B}$.
Initially at room temperature, so $T_0 = 15^\circ\mathrm{C} = -85\mathrm{B}$.

Separate variables...

$f(r,t) = R(r)T(t)$. Substitute and divide by RT

$$\frac{1}{Rr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = \frac{1}{DT}\frac{dT}{dt} = -k^2$$

where k is a constant.

T equation

$$\begin{aligned} \frac{dT}{dt} &= -Dk^2 T \\ \rightarrow T &= Ae^{-\gamma t} \end{aligned}$$

$\gamma = Dk^2$

Radial equation

$$
\begin{aligned}
\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) &= -k^2 R \\
\text{Define } R(r) &= \frac{u(r)}{r} \\
\frac{dR}{dr} &= \frac{1}{r}\frac{du}{dr} - \frac{1}{r^2}u \\
\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) &= \frac{d}{dr}\left(r\frac{du}{dr} - u\right) = r\frac{d^2u}{dr^2} \\
\text{So ODE becomes } \frac{d^2u}{dr^2} &= -k^2 u
\end{aligned}
$$

Radial solution

$$
\begin{aligned}
u &= A\cos kr + B\sin kr \\
\rightarrow R &= A\frac{\cos kr}{r} + B\frac{\sin kr}{r}
\end{aligned}
$$

Boundary conditions:

1. regular at $r = 0 \rightarrow A = 0$
2. $R(a) = 0 \rightarrow \sin ka = 0, \rightarrow k = n\pi/a,\ \ n = 1, 2, 3, ...$

So separable solutions are

$$
f_n(r,t) = \frac{B_n}{r}\sin\left(\frac{n\pi r}{a}\right)e^{-\gamma_n t}
$$

and the general solution

$$
f(r,t) = \sum_{n=1}^{\infty}\frac{B_n}{r}\sin\left(\frac{n\pi r}{a}\right)e^{-\gamma_n t}
$$

where $\gamma_n = D\left(\frac{n\pi}{a}\right)^2$.

Initial Condition

$$f(r,0) = \sum_{n=1}^{\infty} \frac{B_n}{r} \sin\left(\frac{n\pi r}{a}\right) = T_0$$

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi r}{a}\right) = T_0 r$$

Fourier sine series with

$$B_n = \frac{2}{a} \int_0^a T_0 r \sin\left(\frac{n\pi r}{a}\right) dr$$

Integrate by parts:

$$B_n = -\frac{2T_0 a}{n\pi} \cos n\pi = \frac{2T_0 a}{n\pi} (-1)^{n+1}$$

So finally, the temperature inside the egg is given by

$$f(r,t) = 2T_0 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin \frac{n\pi r}{a}}{\left(\frac{n\pi r}{a}\right)} e^{-\gamma_n t}$$

Look at time term

$$\gamma_1 = \frac{D\pi^2}{a^2} = 0.0036, \; t_1 = 1/\gamma_1 = 277\mathrm{s}$$
$$\gamma_2 = \frac{4D\pi^2}{a^2} = 0.0144, \; t_2 = 1/\gamma_2 = 6.9\mathrm{s}$$

So for long times, i.e. $t >> t_2$, so t of a few minutes, only the first term is significant.
Time, t_{soft} for centre ($r = 0$) to reach T_f is given by

$$\begin{aligned} T_f &= 2T_0 e^{-\left(D\pi^2/a^2\right)t_{\mathrm{soft}}} \\ \rightarrow t_{\mathrm{soft}} &\simeq \frac{a^2}{D\pi^2} \ln \frac{2T_0}{T_f} \\ &\simeq 277 \ln \frac{2 \times 85}{55} \\ &\simeq 5.2 \text{ minutes} \end{aligned}$$

Delia Smith: 6.5 minutes (initially boiling water, but then leave).