Waveguide

Infinitely long rectangular membrane, width b, fixed at edges.
Displacement $\phi(x, y, t)$ satisfies 2-D wave equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

Subject to $\phi(x, 0, t) = \phi(x, b, t) = 0$
Separate using $\phi(x, y, t) = X(x)Y(y)T(t)$ giving

$$\frac{d^2X}{dx^2} = -k_x^2 X \qquad \frac{d^2Y}{dy^2} = -k_y^2 Y$$

$$\frac{d^2T}{dt^2} = -\omega^2 T \qquad \omega^2 = c^2\left(k_x^2 + k_y^2\right)$$

Boundary conditions

$$
\begin{aligned}
Y(0) = Y(b) = \phi(x, 0, t) &= 0 \\
Y(y) = \sin\frac{n\pi y}{b} &\rightarrow k_y = \frac{n\pi}{b} \text{ for } n = 1, 2, 3, \ldots
\end{aligned}
$$

For X use the real part of $X(x) = e^{ik_x x}$
For $T(t) = Ae^{i\omega t} + Be^{-i\omega t}$
Separable solutions:

$$
\begin{aligned}
\phi(x, y, t) &= e^{ik_x x}\sin\frac{n\pi y}{b}\left(Ae^{i\omega t} + Be^{-i\omega t}\right) \\
&= \sin\frac{n\pi y}{b}\left(Ae^{i(k_x x + \omega_n t)} + Be^{-i(k_x x - \omega_n t)}\right)
\end{aligned}
$$

where $\omega_n(k_x) = c\sqrt{k_x^2 + \frac{n^2\pi^2}{b^2}}$ - travelling waves with dispersive phase velocity $v_p = c\sqrt{1 + n^2\pi^2/k_x^2 b^2}$ and so group velocity $v_g = ck_x/(k_x^2 + n^2\pi^2/b^2)^{0.5}$

At $k_x = 0$, frequency is non-zero, $\omega_n(0) = \omega_{nc} = cn\pi/b$.
Travelling waves for $\omega_n > \omega_{nc}$.
But for $\omega_n < \omega_{nc}$, $k_x^2 < 0$, k_x imaginary - wave can not propagate.