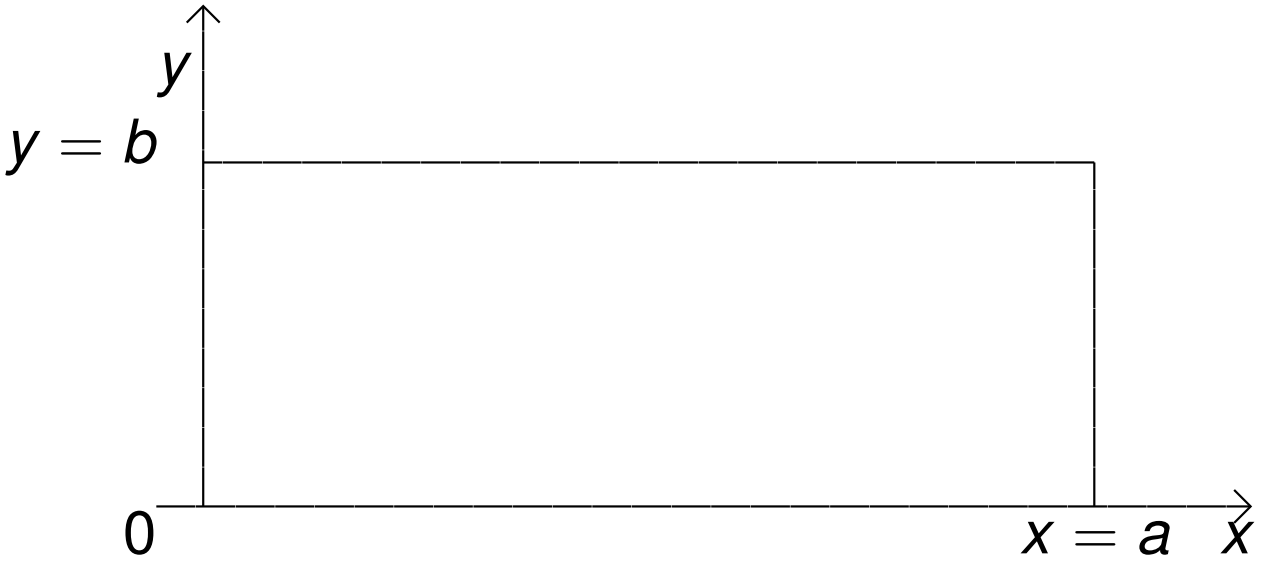
Problems in 2 and 3 dimensions

Rectangular membrane fixed to rectangular frame.



Displacement $\phi(x, y, t)$ satisfies 2-D wave equation

$$\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

Boundary conditions

$$\phi(0, y, t) = \phi(a, y, t) = \phi(x, 0, t) = \phi(x, b, t) \quad = \quad 0$$

Separate using $\phi(x, y, t) = X(x)Y(y)T(t)$.

Solutions

$$\begin{aligned}\phi_{n_x,n_y}(x,y,t) &= \sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{b} \\ &\quad \left(A_{n_x n_y}\cos\omega_{n_x n_y}t + B_{n_x n_y}\sin\omega_{n_x n_y}t\right)\end{aligned}$$

with frequencies

$$\begin{aligned}\omega_{n_x n_y} &= c\sqrt{k_x^2+k_y^2} \\ &= c\sqrt{\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{b^2}}\end{aligned}$$