PHYS 20171 MATHEMATICS OF WAVES AND FIELDS

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The aim of this course is to develop some of techniques needed to solve linear partial differential equations (PDE's). These equations appear in many areas of physics and describe waves and fields which can vary in one or more space dimensions and in time. They include:

- Laplace's equation

$$\nabla^2\phi(\mathbf{r}) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

  The field $\phi(\mathbf{r})$ could be, for example, the electrostatic potential in a region of space without electric charge, or the steady-state distribution of temperature inside some body.

- The ordinary (or nondispersive) wave equation

$$\nabla^2\phi(\mathbf{r}) = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

  This describes waves with a constant speed c. These could be sound waves, if $\phi(\mathbf{r}, t)$ is the displacement of a vibrating string or membrane or medium. They could also be electromagnetic waves, such as light or radio, if $\phi(\mathbf{r}, t)$ is one of the components of the electric field.

- The Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) = \mathrm{i}\hbar\frac{\partial\psi}{\partial t}$$

  This is the central equation of quantum mechanics. It describes the quantum mechanical wave for a particle of mass m moving in a potential $V(\mathbf{r})$. The wave function $\psi(\mathbf{r}, t)$ gives the probability amplitude for finding the particle at the point $\mathbf{r}$ at time t.

- The heat-flow (or diffusion) equation

$$\nabla^2\phi(\mathbf{r}, t) = \frac{1}{D}\frac{\partial\phi}{\partial t}$$

  This describes the flow of heat inside a body with no internal source of heat; the field $\phi(\mathbf{r}, t)$ is the (time-dependent) temperature distribution inside the body.

Recommended books

The recommended books for this course is:

- **M. L. Boas,** *Mathematical methods in the physical sciences*, **3rd edn., (Wiley, 2006).** Material in: Chapter 7, Sections 5, 10 and 11 of Chapter 8, Chapter 12, and Chapter 13. This book will provide the necessary mathematical backup for your physics courses over the next two or three years.
- **K. F. Riley, M. P. Hobson and S. J. Bence,** *Mathematical Methods for Physics and Engineering* **(Cambridge, 1997)** Chapters 10, 11, 13.1, 14, 15, 16 and 17.

Another useful book, on applications of these ideas to PDE's, is: G. Stephenson, *Partial differential equations for scientists and engineers* (Imperial College, 1996).

Webpage

`http://www.jb.man.ac.uk/~gaf/lecture/phys20171.html`

and *Blackboard* page.

Course outline

References to book chapters or sections, as follows:

B3 Boas, 3rd edition

B2 Boas, 2nd edition

R+ Riley, Hobson and Bence

Examples of partial differential equations in physics

B3 and **B2** 13.1; **R+** 16.1

0. **Ordinary differential equations** ($\sim$ 1 lecture)

   B3 and **B2** 8.5, 2.9, 2.11, 2.12; **R+** 13.1

   First-order, linear

   Second-order, linear

   Complex exponentials

1. **Wave problems in one dimension** ($\sim$ 2 lectures)

   B3 and **B2** 13.2 (separation of variables), 13.4; **R+** 16.1.1, 17.1, 17.2

   Separation of variables

   Normal modes of a string: eigenfunctions and eigenvalues

   General motion of a string

2. **Fourier series** ($\sim$ 4 lectures)

   B3 and **B2** 7.1–7.11; **R+** 10

   Orthogonality and completeness of sines and cosines

   Fourier coefficients

   Complex exponential form of Fourier series

   Initial conditions on PDE's

3. **Other PDE's** ($\sim$ 2 lectures)

   B3 and **B2** 13.2, 13.3; **R+** 16.1.2, 16.1.3, 17.2

   Laplace's equation

   The heat-flow equation

4. **Integral transforms** ($\sim$ 3 lectures)

   B3 7.12, 8.10, 8.11; **B2** 15.4, 15.5, 15.7; **R+** 11.1

   Fourier transform

   Convolutions

   Wave packets and dispersion

5. **Series solution of ODE's** ($\sim$ 4 lectures)

   B3 and **B2** 1.6C, 1.10, 1.12, 12.2, 12.6, 12.7–12.9, 12.11, 12.12;
   R+ 3.3, 3.6, 14.2, 14.3, 14.6, 14.7

   Taylor series

   Legendre polynomials and related functions

   Bessel functions

   Orthogonal sets of eigenfunctions

   Legendre series

6. **Problems in two and three dimensions** ($\sim$ 6 lectures)

   B3 and **B2** 13.5–13.7; **R+** 17.3

   Normal modes of a square membrane; degeneracy

   Wave guide

   Normal modes of a circular and spherical systems

   Heat flow and Laplace's equation in circular and spherical systems