PC10372, Mathematics 2
Workshop Sheet 8

Apart from the first question, this week's questions are fairly hard and should really test your ability to use cylindrical and spherical polar coordinates.

1) Show, using spherical polar coordinates, that the volume of a sphere is equal to $4\pi R^3/3$ and that its surface areas is $4\pi R^2$. Use the elements $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$ and $dA = R^2 \sin\theta \, d\theta \, d\phi$.

2) a) Write the triple integral in spherical polar coordinates for the volume inside the cone $z^2 = x^2 + y^2$ and the planes $z = 1$ and $z = 2$. Evaluate this integral. Hint: the substitution $u = \cos\theta$ may be useful.

b) Repeat a) using cylindrical polar coordinates (where the volume element is $dV = r \, dr \, d\theta \, dz$).

3) Find the location of the centre of mass (centroid) of a uniform solid cone of height h and base radius h. Hint: Imagine putting the tip of the cone at the origin and its axis along the z-axis then use cylindrical polar coordinates.

4) Find the moment of interia about the z-axis of the ellipsoid which lies inside

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Assume that the ellipsoid has uniform density ρ.

Hint: You change variables to $x' = x/a$, $y' = y/b$, $z' = z/c$, but be careful about the volume element. It is not $dx' \, dy' \, dz'$.