PC10372, Mathematics 2
Workshop Sheet 6

1) In spherical polar coordinates the Laplacian is

$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2}{\partial\theta^2}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

Use this result to show that the Coulomb potential

$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

satisfies Laplace's equation

$$\nabla^2 V = 0$$

for $r \neq 0$.

2) Electric charge is spread over the xy plane with density

$$\rho(x,y) = C(x^2+y^2)$$

where C is a constant.
Calculate the total charge enclosed:
(i) within the rectangle $0 < x < a$, $0 < y < b$
(ii) with a cirlce of radius R centred on the origin.

3) Find the mass of a rod of length L and cross sectional areas A whose density varies with position x along the rod as

$$\rho(x) = \frac{B}{x^2+L^2}$$

where B is a constant.

Hints: You need to work out the mass of a thin slice of rod from x to $x + dx$. You might also find the substitution $x = L\tan y$ useful.

4) For the rod in the previous queation, how far along the rod would you have to place your finger if you wanted it to balance?