PC10372, Mathematics 2
Workshop Sheet 4

The first question gives some practice in calculating the divergence and curl of vector fields. The second question asks you show generally that the divergence of a curl of any vector field and the curl of a gradient of any scalar field are always zero. To prove these results you just need to use the definitions of gradient, divergence and curl.

1) Calculate the divergence and the curl of each of the following vector fields:

$$\begin{aligned}(i)\ \mathbf{v}(x,y,z) &= x\mathbf{i} + 2y\mathbf{j} + 3\mathbf{k},\\ (ii)\ \mathbf{v}(x,y,z) &= y\mathbf{i} + x\mathbf{j} + x\ln y\mathbf{k},\\ (iii)\ \mathbf{v}(x,y,z) &= e^{-x}\sin(2y)\,(\mathbf{i}+\mathbf{j}+\mathbf{k})\end{aligned}$$

2) For each of the three vector fields in the previous questions show that the divergence of the curl is equal to zero, i.e. show that

$$\nabla\cdot(\nabla\times\mathbf{v}) = 0$$

Can you prove that this is a general result? Can you show that

$$\nabla\times(\nabla f) = 0$$

for any scalar field f.