PC10372, Mathematics 2
Workshop Sheet 3

The gradient, ∇f, of a scalar field $f(x, y, z)$ is a vector field defined by

$$\nabla f \;=\; \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

The rate of change of the function $f(x, y, z)$ in the direction $\mathbf{u}$, where $\mathbf{u}$ is a unit vector is $\nabla f \cdot \mathbf{u}$.

1) Calculate the gradient ∇f for each of the following functions:

1. $f(x, y, z) = x + y - 2z$
2. $f(x, y, z) = 5x^2 - xz$
3. $f(x, y, z) = e^{-2x} \sin \pi y$

2) For each of the functions in Question 1, compute the rate of change of the function in the direction $\mathbf{i} + \mathbf{j}$, evaluated at the origin.

3) At the origin, in which direction is the rate of change of the function greatest for each function in Question 1? What is the rate of change in that direction?

4) Find a unit vector which is normal to the surface $x^3y^2z = 1$ at the point $(1, 1, 1)$.

5) The vector field $\mathbf{v}(x, y)$ is given by $\mathbf{v} = e^{-x}\mathbf{i} + \mathbf{j}$. Derive an equation for the field lines of $\mathbf{v}$ and sketch them.