

PC10372, Mathematics 2

Workshop Sheet 2 Solutions

1. (i) $\frac{\partial f}{\partial x} = 2xy - y^2, \frac{\partial f}{\partial y} = x^2 - 2xy$
(ii) $\frac{\partial f}{\partial x} = e^x \cos y, \frac{\partial f}{\partial y} = -e^x \sin y$
(iii) $\frac{\partial f}{\partial x} = -2axe^{-a(x^2+y^2)}, \frac{\partial f}{\partial y} = -2aye^{-a(x^2+y^2)}$
(iv) $\frac{\partial f}{\partial x} = 2xy \cos(x^2y), \frac{\partial f}{\partial y} = x^2 \cos(x^2y)$
(v) $\frac{\partial f}{\partial x} = \frac{1}{x+y^2}, \frac{\partial f}{\partial y} = \frac{2y}{x+y^2}$
(vi) $\frac{\partial f}{\partial x} = \frac{-y}{x^2}, \frac{\partial f}{\partial y} = \frac{1}{x}$
2. (i) $\frac{\partial^2 f}{\partial x^2} = 2y, \frac{\partial^2 f}{\partial y^2} = -2x$
(ii) $\frac{\partial^2 f}{\partial x^2} = e^x \cos y, \frac{\partial^2 f}{\partial y^2} = -e^x \cos y$
(iii) $\frac{\partial^2 f}{\partial x^2} = (4a^2x^2 - 2a)e^{-a(x^2+y^2)}, \frac{\partial^2 f}{\partial y^2} = (4a^2y^2 - 2a)e^{-a(x^2+y^2)}$
3. (i) $\frac{\partial f}{\partial x \partial y} = 2x - 2y, \frac{\partial f}{\partial y \partial x} = 2x - 2y$
(i) $\frac{\partial f}{\partial x \partial y} = -e^x \sin y, \frac{\partial f}{\partial y \partial x} = -e^x \sin y$
(i) $\frac{\partial f}{\partial x \partial y} = 4a^2xye^{-a(x^2+y^2)}, \frac{\partial f}{\partial y \partial x} = 4a^2xye^{-a(x^2+y^2)}$

Comment: Notice that $\frac{\partial f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x}$ in all three cases. This is a general result. The value of a mixed derivative is independent of the order in which the derivatives are taken.