PC10372, Mathematics 2
Workshop Sheet 2

This sheet is an introduction to *partial differentiation.*

The function $f(x, y)$ is a function of two variables x and y. The partial derivative $\frac{\partial f}{\partial x}$ of $f(x, y)$ is obtained by differentiating with respect to x *treating* y *as a constant.* The partial derivative gives the rate of change of $f(x, y)$ with respect to x at a fixed value of y. Similarly $\frac{\partial f}{\partial y}$ is obtained by differentiating with respect to y treating x as a constant, and gives the rate of change of f with respect to y at a fixed value of x.

1) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions $f(x, y)$:

(i) $x^2y - xy^2$

(ii) $e^x \cos y$

(iii) $e^{-a(x^2+y^2)}$

(iv) $\sin(x^2y)$

(v) $\log(x + y^2)$

(vi) y/x

The second derivative $\frac{\partial^2 f}{\partial x^2}$ is obtained by differentiating twice with respect to x while keeping y fixed, while $\frac{\partial^2 f}{\partial y^2}$ is obtained by differentiating twice with respect to y while keeping x fixed.

2) Calculate $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for the functions $f(x, y)$ in parts (i)-(iii) of question 1.

The mixed derivative $\frac{\partial^2 f}{\partial y \partial x}$ means differentiate first with respect to x (keeping y constant) and then with respect to y (keeping x fixed).

3) Calculate the mixed derivatives $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$ for the functions $f(x, y)$ in parts (i)-(iii) of question 1, and comment on your results.