Electromagnetism

For electric field $\underline{E}$ and charge density ρ, Maxwell says

$$\underline{\nabla} \cdot \underline{E} \quad = \quad \frac{\rho}{\epsilon_0}$$

Integrate both sides over volume V

$$\int_V \underline{\nabla} \cdot \underline{E}\, dV \quad = \quad \frac{1}{\epsilon_0} \int_V \rho\, dV$$

$$\int_S \underline{E} \cdot \underline{\mathbf{d}A} = \frac{Q}{\epsilon_0}$$

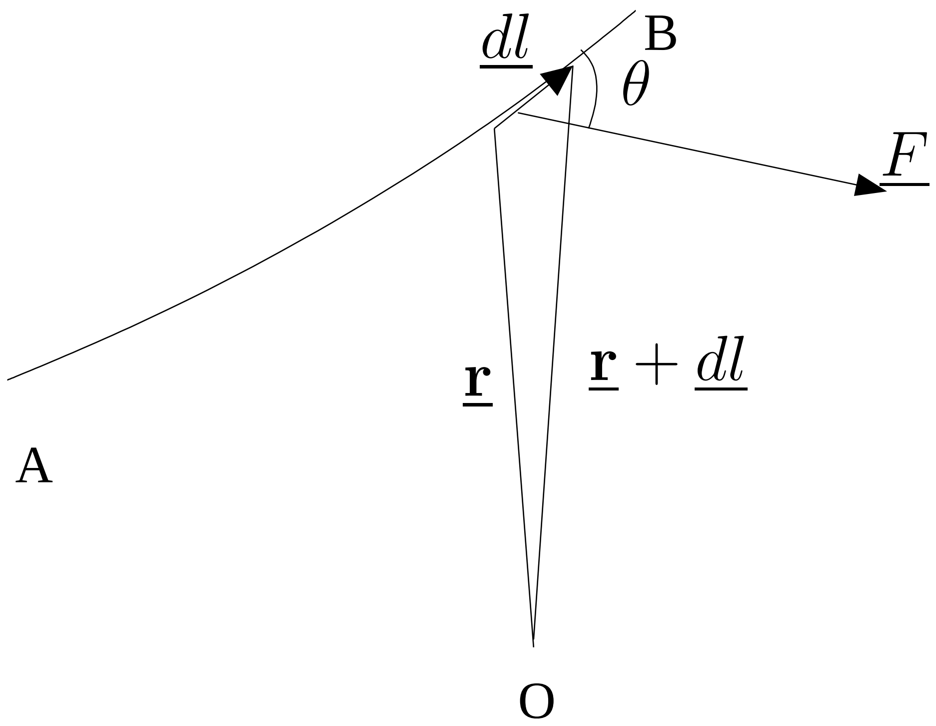
where Q is the total charge in volume V - Gauss' Law - equivalent to Maxwell equation.

Line Integrals

Integrate a function along some defined path.

Example: Calculate the work done by a force acing on a particle moving on a particular path.

$\underline{dl}$ is the element of path - direction of gives the direction of path magnitude gives the length of element- Analogous to $\underline{\mathbf{d}A}$.



dW is the work done by force $\underline{F}$ on a particle moving from $\underline{\mathbf{r}}$ to $\underline{\mathbf{r}} + \underline{dl}$ - force$\times$ distance along direction of force - so

$$\begin{aligned} dW &= |\underline{F}| \cos\theta \, |\underline{dl}| \\ dW &= \underline{F} \cdot \underline{dl} \end{aligned}$$

So the total amount of work done in moving from A to B is

$$W = \int_A^B \underline{F} \cdot \underline{dl}$$

In general the work depends on path taken from A to B.
If it does **not** , the force is said to be conservative.
Note that $\underline{F}$ is a function of x, y and z and so varies along the path.
$\int_A^B \underline{F} \cdot \underline{dl}$ is a *line integral*.
where $\underline{dl} = dx\underline{\mathbf{i}} + dy\underline{\mathbf{j}} + dz\underline{\mathbf{k}}$.
Example