Coordinate Systems

Cartesian

Coordinates: $x, y, z\ -\infty < x < \infty,\ -\infty < y < \infty,\ -\infty < z < \infty$
Volume element: $dV = dx\, dy\, dz$

$$\nabla f = \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} + \mathbf{k}\frac{\partial f}{\partial z}$$

Cylindrical Polar Coordinates

Coordinates: $x = r\cos\theta, y = r\sin\theta, z = z$ where $0 \le r < \infty,\ 0 \le \theta \le 2\pi,\ -\infty < z < \infty$
Volume element: $dV = r\, dr\, d\theta\, dz$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z}\mathbf{k}$$

Spherical Polar Coordinates

Coordinates: $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$ where $0 \le r < \infty,\ 0 \le \theta < \pi,\ 0 \le \phi \le 2\pi$
Volume element: $dV = r^2\sin\theta\, dr\, d\theta\, d\phi$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$