PC10372, Mathematics 2
Example Sheet 9

Before you start make sure you understand how to evaluate the following scalar products.

For cylindrical polar coordinates:	For spherical polar coordinates:
$\underline{\hat{\mathbf{r}}} \cdot \underline{\mathbf{i}} = \cos\theta$	$\underline{\hat{\mathbf{r}}} \cdot \underline{\mathbf{i}} = \sin\theta\cos\phi$
$\underline{\hat{\mathbf{r}}} \cdot \underline{\mathbf{j}} = \sin\theta$	$\underline{\hat{\mathbf{r}}} \cdot \underline{\mathbf{j}} = \sin\theta\sin\phi$
	$\underline{\hat{\mathbf{r}}} \cdot \underline{\mathbf{k}} = \cos\theta$

1) Calculate the net flux of the vector field $\underline{\mathbf{v}} = x^2\underline{\mathbf{k}}$ out of the hemisphere defined by $x^2 + y^2 + z^2 = r^2$, for $z \geq 0$. Don't forget that the hemisphere has a base as well as a curved surface.

2) Calculate the flux of the vector field $\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$ out of a cube of side L. You can assume that the cube is in the region of space defined by $0 \leq x \leq L, 0 \leq y \leq L, 0 \leq z \leq L$ so that one corner of the cube is at the origin and the faces of the cube are parallel to the xy, yz and xz planes.

3) Calculate the flux of the vector field $\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$ out of a cylinder of height R and radius R. Place the cylinder so that it is aligned with the z-axis with the origin at the centre of the circular base.

4) Calculate the flux of the vector field $\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$ out of a sphere of radius R centred on the origin.

Note that the final results for parts 2, 3 and 4) do not in fact depend on the location of the volume you are considering.

If you have done things correctly, you should see that you results are consistent with

$$\text{Volume of Shape} \quad = \quad \frac{1}{3}\text{flux of vector r out of volume} = \frac{1}{3}\int_S \underline{\mathbf{r}} \cdot \underline{\mathbf{d}A}$$