PC10372, Mathematics 2
Example Sheet 8

1) Evaluate the following double integrals:

$$\text{a) } \int_0^1 dy \int_1^2 dx\, xy^2$$

$$\text{b) } \int_0^{\pi/2} dx \int_0^{\pi/2} dy\, \cos(x+y)$$

$$\text{c) } \int_0^1 dy \int_0^1 dx\, \frac{x}{y}$$

2) Find the volume enclosed by the surface $z = 2x - y + 3$ and the surface $z = 0$ and which lies in the region $0 \le x \le 1, 0 \le y \le 2$.

3) Evaluate the following double integrals. In each case sketch the region of integration and check your result by changing the order of integration

$$\text{a) } \int_0^1 dy \int_0^y dx$$

$$\text{b) } \int_0^1 dy \int_0^{\sqrt{1-y^2}} dx$$

4) a) Evaluate the integral
$$\int_R xyz\, dx\, dy\, dz$$
over the cuboid $0 \le x \le 2,\ 0 \le y \le 4,\ 2 \le z \le 4$.

b) Use spherical polar coordinates to evaluate
$$\int_R z^4\, dx\, dy\, dz$$
where R is the sphere $x^2 + y^2 + z^2 \le 4$.