PC10372, Mathematics 2
Example Sheet 5

1) Calculate the divergence of the following vector fields:

a) $x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$ b) $yz\underline{\mathbf{i}} + xz\underline{\mathbf{j}} + xy\underline{\mathbf{k}}$
c) $e^{x+y}\underline{\mathbf{i}} + e^{4z}\underline{\mathbf{j}} + e^{-3zx}\underline{\mathbf{k}}$ d) $4\underline{\mathbf{i}} - \underline{\mathbf{j}} + 3xz^2\underline{\mathbf{k}}$
e) $(-xy\underline{\mathbf{i}} + x^2\underline{\mathbf{j}})/(x^2+y^2)^{1/2}$ f) $x\ \sin y\underline{\mathbf{i}} + \cos y\underline{\mathbf{j}} + xy\underline{\mathbf{k}}$

2) Calculate the curl of the following vector fields:

a) $z^2\underline{\mathbf{i}} + x^2\underline{\mathbf{j}} - y^2\underline{\mathbf{k}}$ b) $4xz\underline{\mathbf{i}} + x^2\underline{\mathbf{k}}$
c) $e^{-y}\underline{\mathbf{i}} + e^{-z}\underline{\mathbf{j}} + e^{-x}\underline{\mathbf{k}}$ d) $(x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}})/(x^2+y^2+z^2)^{1/2}$

3) Calculate $\nabla(2xz)$ in both spherical polar and cylindrical polar coordinates.

4) Calculate ∇z in spherical polar coordinates and also in Cartesian coordinates. Hence show that

$$\underline{\mathbf{k}} \ = \ \hat{\underline{\mathbf{r}}} \cos\theta - \hat{\underline{\theta}} \sin\theta$$

Coordinate Systems

Cartesian

Coordinates: x, y, z $-\infty < x < \infty,\ -\infty < y < \infty,\ -\infty < z < \infty$
Volume element: $dV = dx\, dy\, dz$

$$\nabla f = \underline{\mathbf{i}}\frac{\partial f}{\partial x} + \underline{\mathbf{j}}\frac{\partial f}{\partial y} + \underline{\mathbf{k}}\frac{\partial f}{\partial z}$$

Cylindrical Polar Coordinates

Coordinates: $x = r\cos\theta, y = r\sin\theta, z = z$ where $0 \le r < \infty,\ 0 \le \theta \le 2\pi,\ -\infty < z < \infty$
Volume element: $dV = r\, dr\, d\theta\, dz$

$$\nabla f = \frac{\partial f}{\partial r}\underline{\hat{\mathbf{r}}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\underline{\hat{\boldsymbol{\theta}}} + \frac{\partial f}{\partial z}\underline{\mathbf{k}}$$

Spherical Polar Coordinates

Coordinates: $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$ where $0 \le r < \infty,\ 0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi$
Volume element: $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$\nabla f = \frac{\partial f}{\partial r}\underline{\hat{\mathbf{r}}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\underline{\hat{\boldsymbol{\theta}}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\underline{\hat{\boldsymbol{\phi}}}$$