

## PC10372, Mathematics 2

### Example Sheet 3

1) Calculate the gradient of the following scalar fields and sketch the vector field of the gradient: i)  $\phi = x + y$ , ii)  $\phi = xy$ , iii)  $\phi = x^2 - y^2$ .

2) If  $r = \sqrt{x^2 + y^2 + z^2}$ , calculate the

$$i) \frac{\partial r}{\partial x}, \quad ii) \nabla r, \quad iii) \nabla \left(\frac{1}{r}\right)$$

3) The height,  $h$ , of the land above sea level (in metres) on an island is described by the function

$$h^2 = h_0^2 - \frac{x^2}{4} - y^2$$

$h_0 = 100\text{m}$  and  $h > 0$ .

- Sketch a map of the island, showing contours of the height of the land at 20m intervals.
- Calculate the gradient of  $h$  at a point  $(x, y)$ . If you start at the point  $(-50\text{m}, 30\text{m})$  and walk in the direction  $2\mathbf{i} + \mathbf{j}$  are you travelling uphill or downhill?
- A party lands at the point  $(-120\text{m}, -80\text{m})$  and climbs to the summit of the island by the steepest route. Show that their route is given by  $y = Nx^4$  and determine the constant  $N$ .