PC1372, Mathematics 2
Example Sheet 2 Solutions

1) (a) i) $f_x = 3, f_y = 7$, ii) $f_x = 3x^2 - 3y,\ f_y = -3x - 8y$
iii) $f_x = 4x^3y^2 - 2xy^4,\ f_y = 2x^4y - 4x^2y^3$, iv) $f_x = -1/(yx^2),\ f_y = -1/(xy^2)$
v) $f_x = -x/(x^2+y^2)^{3/2}$, $f_y = -y/(x^2+y^2)^{3/2}$
vi)

$$\begin{aligned} f_x = \frac{df}{dr}\frac{\partial r}{\partial x} &= -\alpha e^{-\alpha r} x(x^2+y^2)^{-1/2} \\ &= -\alpha e^{-\alpha r}\frac{x}{r} \\ f_y &= -\alpha e^{-\alpha r}\frac{y}{r} \end{aligned}$$

b) For (a) ii) $f_{xx} = 6x, f_{yy} = -8, f_{xy} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = -3, f_{yx} = -3$
For (a) iv)

$$f_{xx} = \frac{2}{x^3y}, f_{yy} = \frac{2}{xy^3}, f_{xy} = \frac{1}{x^2y^2}, f_{yx} = \frac{1}{x^2y^2} = f_{xy}$$

2) (a) Write down the total differential of the function

$$\begin{aligned} g(x,y) &= 2xy - x^3 - 8y^3 \\ dg &= \frac{\partial g(x,y)}{\partial x} + \frac{\partial g(x,y)}{\partial y} \\ &= g_x\,dx + g_y\,dy \\ &= \left(2y - 3x^2\right)\,dx + \left(2x - 24y^2\right)\,dy \end{aligned}$$

(b)

$$x(t) = t \qquad \frac{dx}{dt} = 1$$

$$y(t) = \frac{1}{2} - t \qquad \frac{dy}{dt} = -1$$

$$\begin{aligned} \frac{dg}{dt} &= \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \\ &= \left(2y - 3x^2\right)\frac{dx}{dt} + \left(2x - 24y^2\right)\frac{dy}{dt} \\ &= \left(1 - 2t - 3t^2\right)(1) + \left(2t - 24(\frac{1}{2} - t)^2\right)(-1) \\ &= \left(1 - 2t - 3t^2\right) - \left(2t - 6 + 24t - 24t^2\right) \\ &= 7 - 28t + 21t^2 \end{aligned}$$

3) i)

$$\begin{aligned}
\frac{dg}{dx} &= \frac{\partial g}{\partial x} + \frac{\partial g}{\partial y}\frac{dy}{dx} \\
\frac{dy}{dx} &= 2x \\
\frac{\partial g}{\partial x} = 2x + 2y &\qquad \frac{\partial g}{\partial y} = 2x \\
\text{therefore } \frac{dg}{dx} &= (2x + 2y) + 2x\,2x \\
&= 4x^2 + 2x + 2y = 4x^2 + 2x + 2\,x^2 = 6x^2 + 2x
\end{aligned}$$

ii) First substitute $y = x^2$,

$$\begin{aligned}
g(x, y) &= x^2 + 2xy \\
g(x) &= x^2 + 2xx^2 \\
&= x^2 + 2x^3 \\
\text{Now differentiate} & \\
\frac{dg}{dx} &= 2x + 6x^2
\end{aligned}$$

- the same as using the chain rule.