PC10372, Mathematics 2
Example Sheet 1

Revision

1) Find the stationary points of the function $y = x^2 e^{-x}$. Hence sketch a graph of the function y.

2) Convert $-2 + 2i$ into exponential form, i.e. the form $r\, e^{i\theta}$. Using this form find *all* the solutions of the equation $z^4 = -2 + 2i$.

3) Use L'Hopital's rule to evaluate

$$\lim_{x \to \pi} \frac{\cos^2(x/2)}{e^x - e^\pi}$$

Differential Equations

4) Solve the following *separable* ordinary differential equations:

i) $\frac{dy}{dt} = \cos t - e^{-3t}$, if $y = 3$ when $t = 0$.

ii) $y\left(1 - x^2\right)^2 \frac{dy}{dx} = x\left(1 + y^2\right)$

iii) $\frac{dy}{dx} = x + xy$ if $y = 0$ when $x = 0$

5) The motion of a space probe launched from the surface of a planet of mass M and radius R is described by the equation

$$v\frac{dv}{dr} = -MG\frac{1}{r^2}$$

where r is the distance of the probe from the centre of the planet and $v = v(r)$ is the probe's velocity at distance r. (Note that the LHS of this equation is the acceleration and the RHS is the inverse-square law for the gravitational force.) If the probe has velocity u when it leaves the planet's surface, solve the differential equation to find $v(r)$.

6) Solve the following first order ODEs using an integrating factor.

i) $\frac{dy}{dt} - 3y = e^{-2t}$ where $y = 1$ at $t = 0$

ii) $\frac{dy}{dt} + 4y = t - 3$

iii) $\sec x \frac{dy}{dx} + y = 1$

7) Solve the *homogeneous* equation

$$\frac{dy}{dx} = \frac{y - \sqrt{x^2 + y^2}}{x}$$

8) Use a substitution to linearise the following Bernoulli equation, and hence find the general solution

$$\frac{dy}{dx} + y = y^3$$

9) The motion of a projectile falling under gravity with air resistance which is proportional to the speed is given by

$$\frac{dv}{dz} = -g - kv$$

where k is a constant.

a) Solve this i) as a separable equation and ii) using an integrating factor. Show that the answers are the same.

b) Find the particular solution satisfying $v = u$ at $t = 0$.