Multichannel interference mitigation techniques and their effect on image formation techniques

Amir Leshem and Alle-Jan van der Veen

Delft University of Technology, Dept. Electrical Eng./DIMES

The Netherlands

`allejan@cas.et.tudelft.nl`

Thanks

- A-J. Boonstra, A. Kokkeler, G. Schoonderbeek (ASTRON).
- R. Calders, J. Coupard (Delft University).

References

[1] A. Leshem and A.J. van der Veen. Radio-astronomical imaging in the presence of strong radio- intrerference. *IEEE trans. on IT*, August 2000, pp. 1730-1747. A special issue on Information theoretic imaging.

[2] A. Leshem, A.J. van der Veen, and A.J. Boonstra. Multichannel interference mitigation techniques in radio astronomy. *ApJS*, October 2000 (in press).

Overview of the talk

Examples of RFI.

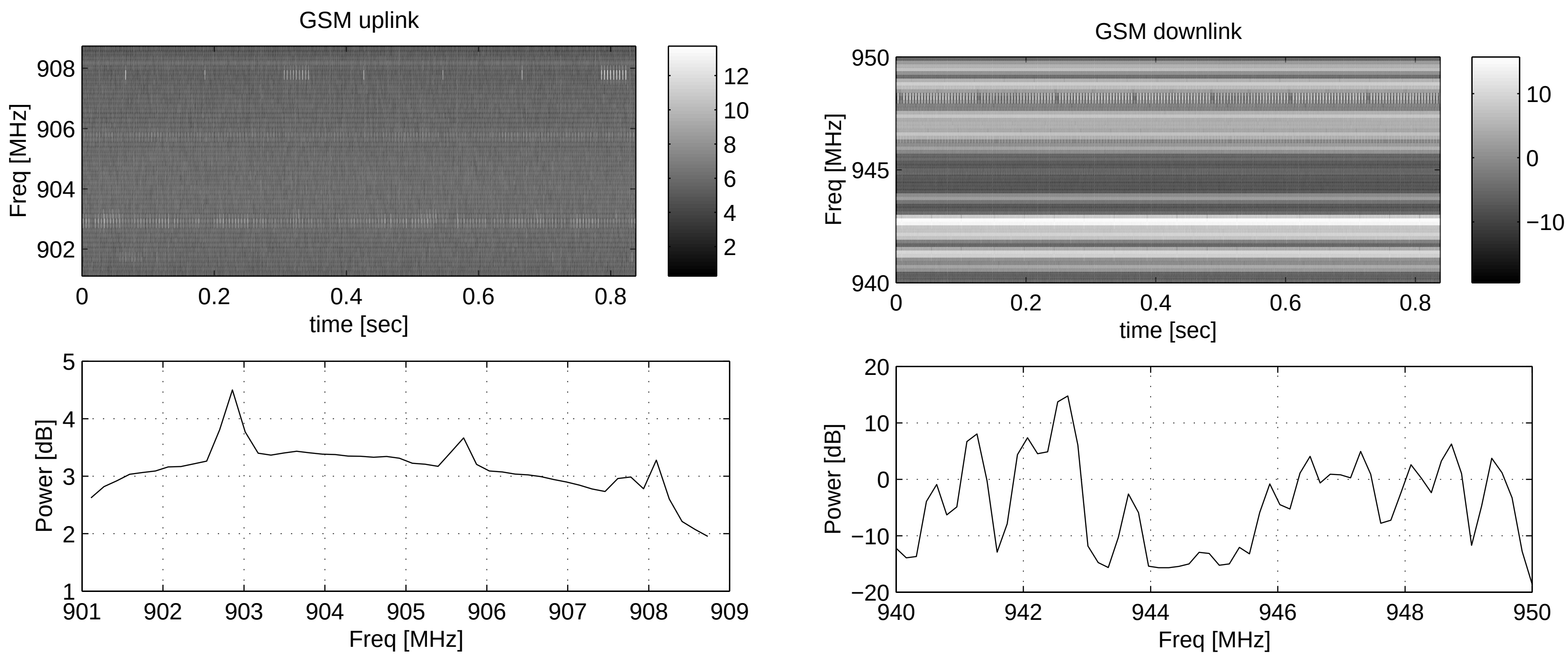
Multichannel interference mitigation techniques.

Reformulation of radio astronomical imaging.

Effect of interference suppression on image formation.

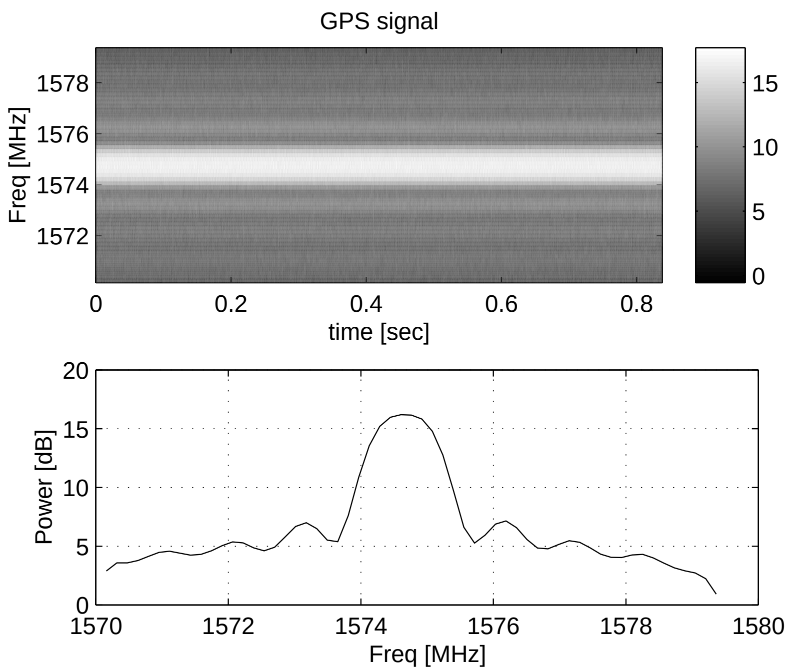
Data driven image formation.

RFI example: observed GSM mobile/base station transmissions

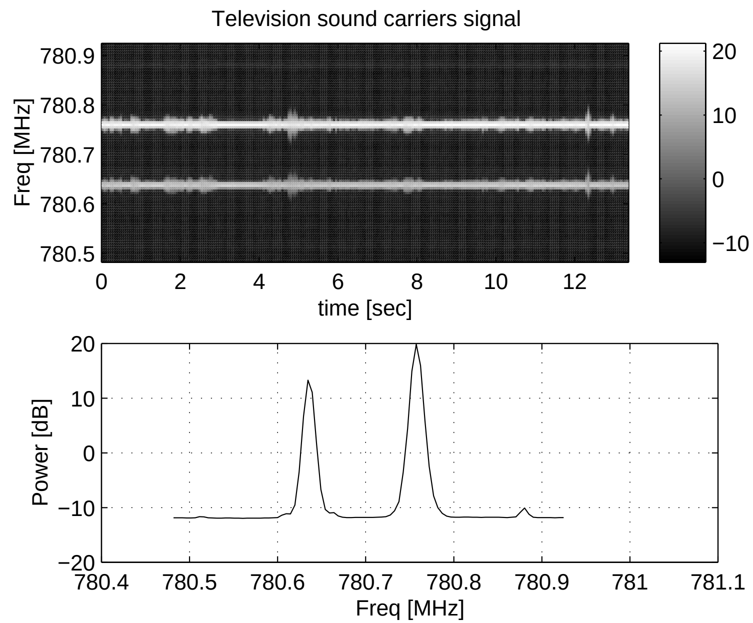


GSM mobile band **GSM base stations band**

RFI example: GPS transmissions/Television carriers

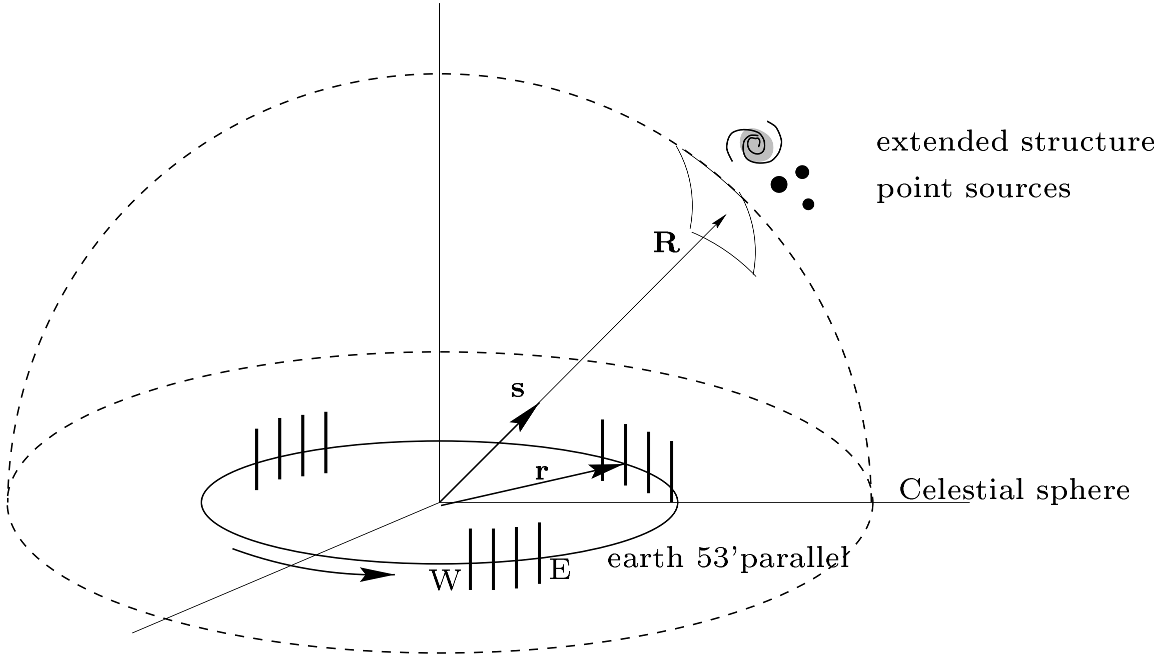


GPS signal band

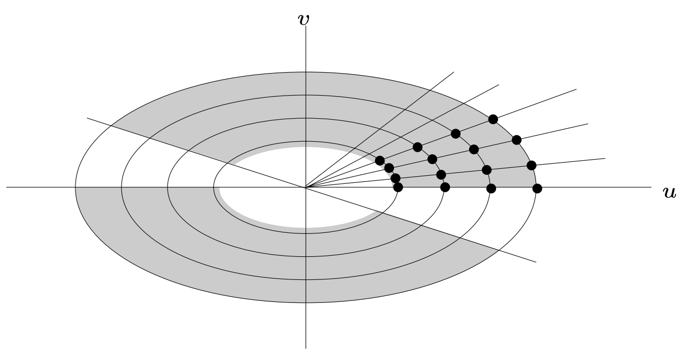


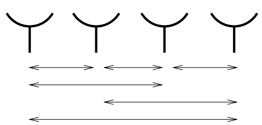
Television signal sound carriers

Astronomical imaging



We can measure only a discrete set of baselines $\mathbf{r}_i(t_k) - \mathbf{r}_j(t_k)$:





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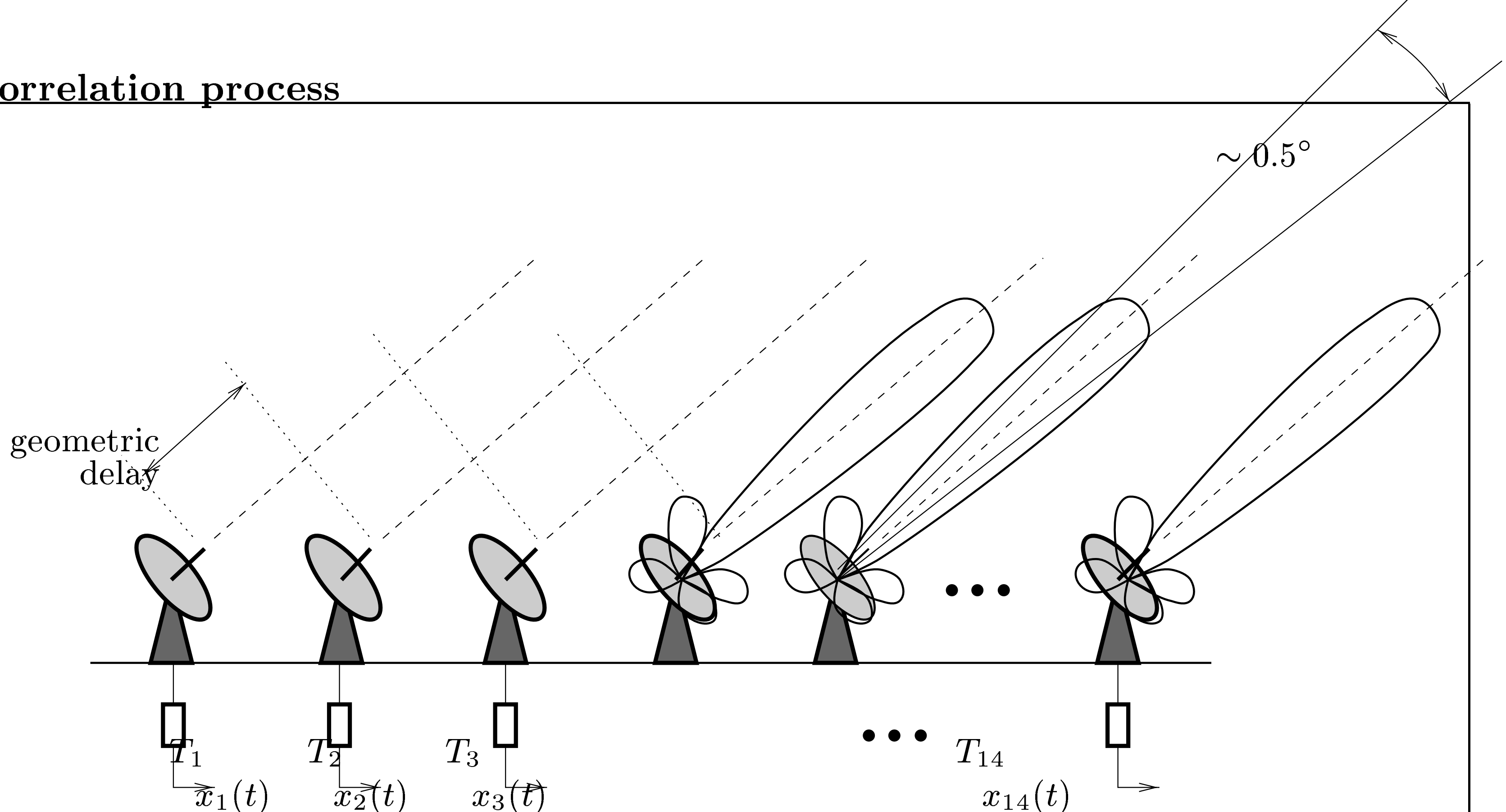
The correlation process

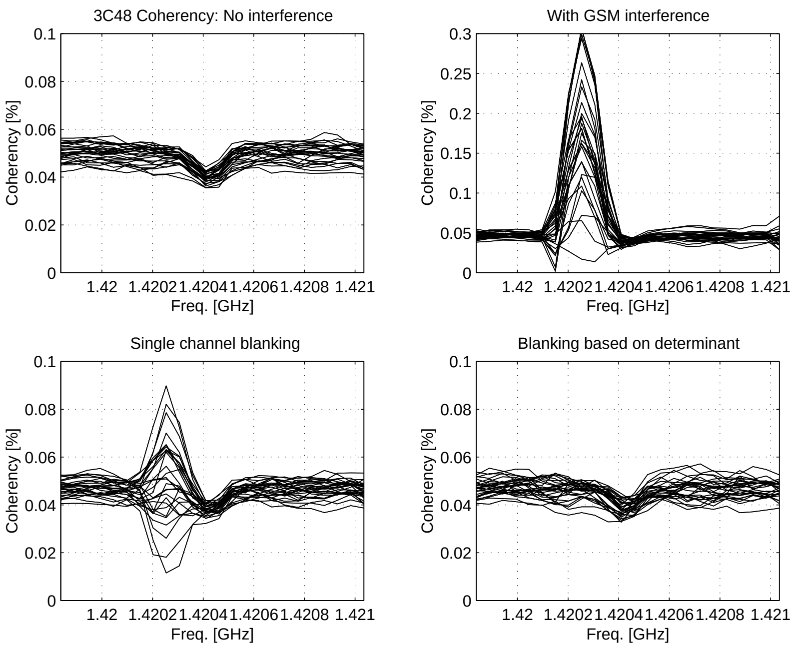


$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_{14}(t) \end{bmatrix}, \qquad \mathbf{R}(\tau) \;=\; \mathrm{E}\{\mathbf{x}(t)\mathbf{x}(t-\tau)^H\} \;\sim\; \frac{1}{N}\sum_{k=0}^{10s} \mathbf{x}(kT)\mathbf{x}(kT-\tau)^H$$

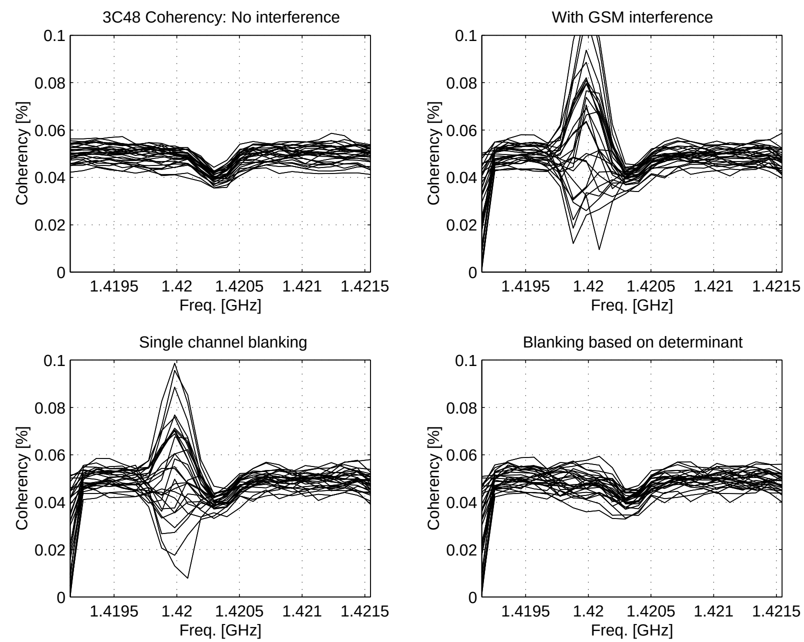
Mitigation techniques

1. Blanking: Simple to implement, good for intermittent signals, single/multichannel, narrowband/wideband, *no substantial effect on image formation*.
2. Spatial filtering: medium complexity, signal independent, multichannel, narrowband signals (wide band signals through sub-band processing).
3. Spatio-temporal processing: High complexity, signals independent, multichannel.
4. Signal exploiting methods: High complexity (different system for each class of signals), single/multichannel, good performance.
5. High order blind methods. High complexity.

Example of balnking results: HI superimposed on GSM signals



Strong GSM signal



Weak GSM signal

Suppose we detect an interferer:

$$\mathbf{R} \simeq \sigma_s^2 \mathbf{a}\mathbf{a}^H + \sigma^2 \mathbf{I} + \mathbf{R}_v$$

and have an estimate $\hat{\mathbf{a}}$

- □ **Temporal filtering:** blanking
- □ **Spatial filtering:**
  - projection $\tilde{\mathbf{R}} = \mathbf{P}_{\hat{\mathbf{a}}}^{\perp} \mathbf{R} \mathbf{P}_{\hat{\mathbf{a}}}^{\perp} = \sigma^2 \mathbf{P}_{\hat{\mathbf{a}}}^{\perp} + \mathbf{P}_{\hat{\mathbf{a}}}^{\perp} \mathbf{R}_v \mathbf{P}_{\hat{\mathbf{a}}}^{\perp}$
  - subtraction of a reference source $\tilde{\mathbf{R}} = \mathbf{R} - \hat{\sigma}_s^2 \hat{\mathbf{a}}\hat{\mathbf{a}}^{\mathrm{H}}$

$\mathbf{R}_v$ can be affected: the 'dirty beam' becomes space-varying

$\Rightarrow$ need to store the effective spatial filter on the 10s data.

Coordinate system

For a small field of view: $\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$, $\qquad \mathbf{s}_0 \perp \boldsymbol{\sigma}$

$$\begin{aligned} \mathbf{s}_0 &= [0,\, 0,\, 1] \\ \boldsymbol{\sigma} &= [\ell,\, m,\, 0] \\ \text{planar array:} \qquad \mathbf{r}_1 - \mathbf{r}_2 &= \lambda[u,\, v,\, 0] \qquad \text{where} \quad \lambda = \frac{c}{f} \end{aligned}$$

Simplified imaging equation

$$V_f(u,v) \;=\; \iint I_f(\ell,m)\, e^{-j2\pi(u\ell+vm)}\, d\ell dm$$

Given V_f, we can compute the brightness image (‘map’) I_f via an inverse Fourier transform.

Assuming that the astronomical skies are a collection of d point sources (maybe unresolved) we obtain:

$$V_f(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1}^{d} e^{-j2\pi f\, \mathbf{s}_i^T(\mathbf{r}_1 - \mathbf{r}_0)/c} I_f(\mathbf{s}_i) e^{j2\pi f\, \mathbf{s}_i^T(\mathbf{r}_2 - \mathbf{r}_0)/c}$$

- $\mathbf{s}_i$ is the coordinate of the i'th source.
- We choose an arbitrary reference point typically the central element of the array, and measure the phase difference with respect to that point.

If we look at all baselines measured simultanuously at time t_k as a matrix we obtain

$$\mathbf{R}_{f,k} = \mathbf{A}_{f,k}\mathbf{B}_f\mathbf{A}_{f,k}^H \tag{1}$$

where

$$\mathbf{a}(\mathbf{s}_i, t_k, f) = \begin{bmatrix} e^{-j2\pi f\,\mathbf{s}_i^T(\mathbf{r}_1(t_k)-\mathbf{r}_0(t_k))/c} \\ \vdots \\ e^{-j2\pi f\,\mathbf{s}_i(t_k)^T(\mathbf{r}_p(t_k)-\mathbf{r}_0(t_k))/c} \end{bmatrix}$$

$$\mathbf{A}_{f,k} = [\mathbf{a}(\mathbf{s}_1, t_k, f), \ldots, \mathbf{a}(\mathbf{s}_d, t_k, f)]$$

and

$$\mathbf{B}_f = \begin{bmatrix} I_f(\mathbf{s}_1) & & \\ & \ddots & \\ & & I_f(\mathbf{s}_d) \end{bmatrix}$$

When we have measurement noise which is $\sigma^2\mathbf{I}$ and an unknown complex gain for each antenna element we obtain that asymptotically (from now on we fix f)

$$\mathbf{R}_k = \mathbf{\Gamma}_k \mathbf{A}_k \mathbf{B} \mathbf{A}_k^H \mathbf{\Gamma}_k^H + \sigma^2 \mathbf{I}$$

where

$$\mathbf{\Gamma}_k = \begin{bmatrix} g_{1,k} & & \\ & \ddots & \\ & & g_{p,k} \end{bmatrix}$$

are the calibration coefficients.

$$I_D(\ell,m) \;=\; \sum_l I_l\; B(\ell,m,\ell_l,m_l) \;=\; \sum_l I_l\; B_l(\ell,m)$$

where

$$B_l(\ell,m) \;:=\; \sum_i \sum_j c_{ij}\; e^{-2\pi \jmath (u_j \ell_l + v_j m_l)}\; e^{2\pi \jmath (u_i \ell + v_i m)}\,.$$

Every point source excites a beam centered at its location (ℓ_l, m_l)

Beamforming interpretation

$$I'_D(\mathbf{s}) \;:=\; \sum_k \mathbf{w}_k^{\mathrm{H}}(\mathbf{s})\mathbf{R}_k\mathbf{w}_k(\mathbf{s}) \tag{2}$$

Here, $\mathbf{w}_k(\mathbf{s})$ is the beamformer pointing towards direction $\mathbf{s}$, in classical imaging $\mathbf{w}_k = \mathbf{a}_k(\mathbf{s})$.

1. Find the brightest point in the dirty image.
2. Subtract from the dirty image the dirty beam centered at the location of the peak and multiplied by γ
3. If the residual image is not noiselike goto step (1)
4. Convolve the point sources obtained at steps (1-3) with an ideal synthetic beam.
5. Add the residual image to the generated image.

Finding the brightest point $\mathbf{s}_0$ in the image is equivalent to trying to find a point source using classical Fourier beamforming, i.e., ,

$$\hat{\mathbf{s}}_0 = \arg\max_{\mathbf{s}} \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \left(\mathbf{R}_k - \sigma^2 \mathbf{I}\right) \mathbf{a}_k(\mathbf{s}) \, . \tag{3}$$

Thus, the CLEAN algorithm can be regarded as a generalized classical sequential beamformer, where the brightest points are found one by one, and subsequently removed from $\mathbf{R}_k$ until the LS cost function is minimized.

Removing the estimated source can be performed by subtracting its contribution to the visibility covariance matrices.

LS in the visibility domain

If we forget for a moment the computational issue we can present the LS cost function using direct Fourier transform as the following problem:

$$[\{\hat{\mathbf{s}}_l\}_{l=1}^d, \hat{\mathbf{B}}] = \min_{\{\mathbf{s}_l\}_{l=1}^d, \hat{\mathbf{B}}} \sum_{i=k}^{K} \|\mathbf{R}_K - \mathbf{A}_k \mathbf{B} \mathbf{A}_k^H - \sigma^2 \mathbf{I}\|_F^2 \tag{4}$$

This is just self-cal initialized by $\mathbf{\Gamma} = \mathbf{I}$, where self-cal is:

$$\left[\{\hat{\mathbf{s}}_l\}_{l=1}^d, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}_k\right] = \min_{\{\mathbf{s}_l\}_{l=1}^d, \mathbf{B}, \mathbf{\Gamma}} \sum_{k=1}^{K} \|\mathbf{R}_k - \mathbf{\Gamma}_k \mathbf{A}_k \mathbf{B} \mathbf{A}_k^H \mathbf{\Gamma}_k^H - \sigma^2 \mathbf{I}\|_F^2 \tag{5}$$

We would also like to constrain the matrix $\hat{\mathbf{B}}$ to have positive diagonal, and zeros elsewhere(for non-coherent sources.

After Projecting out the interference we are left with modified covariance matrix

$$\tilde{\mathbf{R}}_k := \mathbf{L}_k \mathbf{R}_k \mathbf{L}_k^{\mathrm{H}} = \mathbf{L}_k \left[\mathbf{A}_k(\{\mathbf{s}_l\}) \mathbf{B} \mathbf{A}_k^{\mathrm{H}}(\{\mathbf{s}_l\}) + \sigma^2 \mathbf{I} \right] \mathbf{L}_k^{\mathrm{H}} .$$

This modifies the least squares optimization problem to

$$\left[\{\hat{\mathbf{s}}_l\}, \hat{\mathbf{B}} \right] = \underset{\{\mathbf{s}_l\}, \mathbf{B}, \{\boldsymbol{\Gamma}_k\}}{\arg\min} \sum_{k=1}^{K} \| \mathbf{L}_k \left(\mathbf{R}_k - \mathbf{A}_k(\{\mathbf{s}_l\}) \mathbf{B} \mathbf{A}_k^{\mathrm{H}}(\{\mathbf{s}_l\}) - \sigma^2 \mathbf{I} \right) \mathbf{L}_k^{\mathrm{H}} \|_F .$$

The dirty image can now be redefined as

$$I'_D(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \underbrace{\mathbf{L}_k^{\mathrm{H}} \left(\mathbf{L}_k \mathbf{R}_k \mathbf{L}_k - \sigma^2 \mathbf{L}_k \mathbf{L}_k^{\mathrm{H}} \right) \mathbf{L}_k}_{\mathbf{R}'_k} \mathbf{a}_k(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}'_k \mathbf{a}_k(\mathbf{s}) ,$$

And the modified CLEAN can be done by sequential beamforming and subtraction in the visibility domain.

To reduce the sidelobe response we would like to solve

$$\hat{\mathbf{w}}_k(\mathbf{s}) \;=\; \arg\min_{\mathbf{w}_k} \; \mathbf{w}_k^{\mathrm{H}} \mathbf{R}_k \mathbf{w}_k \qquad \text{such that} \qquad \mathbf{w}_k^{\mathrm{H}} \mathbf{a}_k(\mathbf{s}) = 1\,.$$

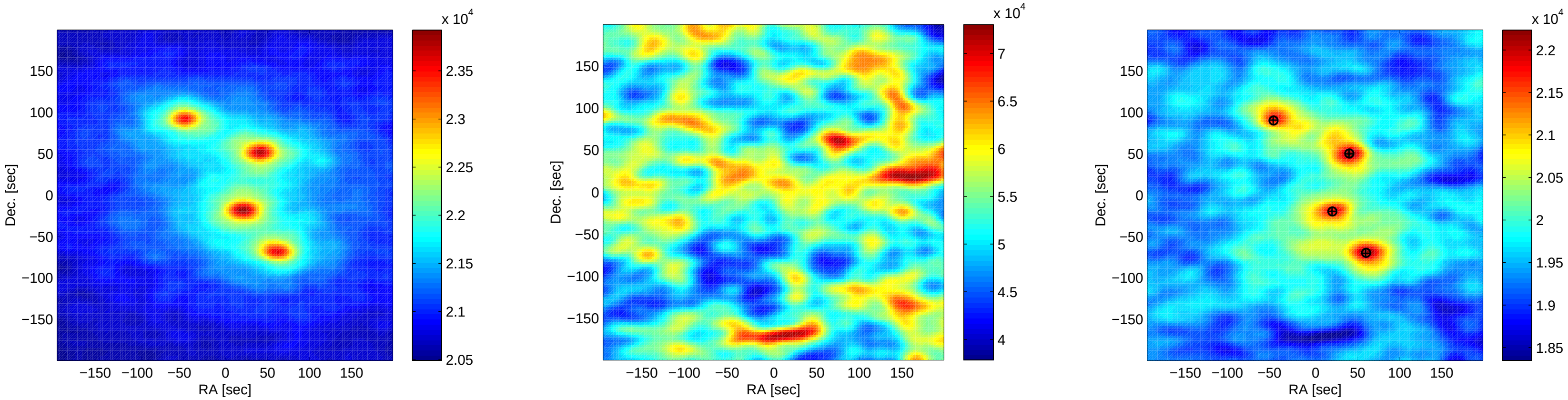
The solution to this problem is

$$\hat{\mathbf{w}}_k = \beta_k \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{s})\,, \qquad \text{where} \qquad \beta_k = \frac{1}{\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{s})}\,.$$

$$I'_D(\mathbf{s}) \;=\; \sum_{k=1}^{K} \frac{1}{\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{s})}\,.$$

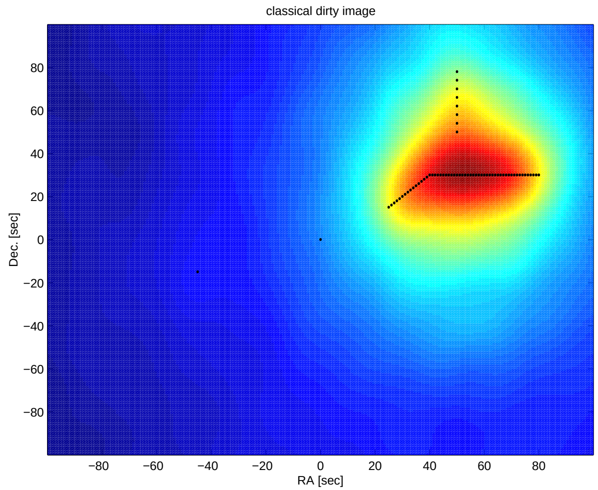
and the locations of the strongest sources are given by the maxima of $I'_D(\mathbf{s})$.

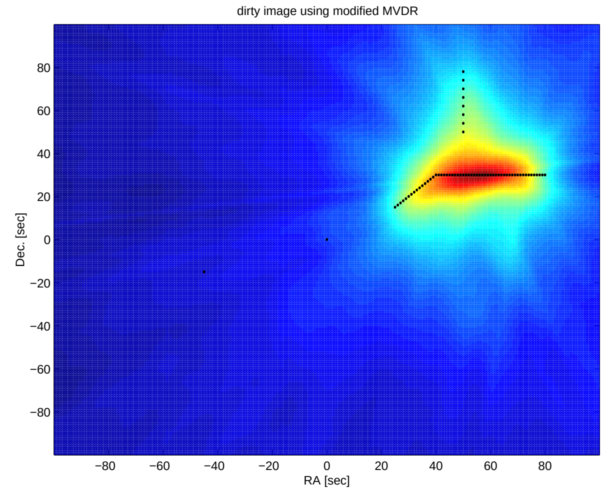
Simulation: The effect of spatial filtering on the image



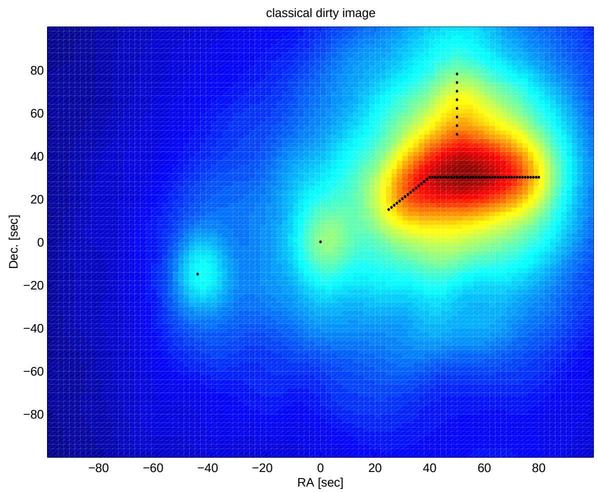
no interference; unsuppressed interference; after spatial filtering.

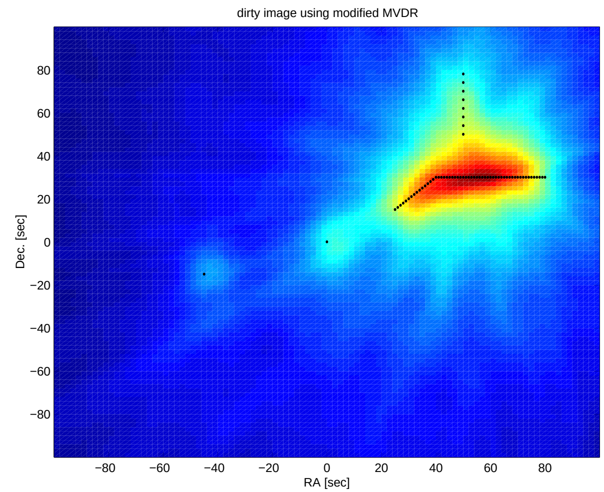
Simulation: Comparing images





Case I





Case II

Classical dirty image

Data driven image