Can relativistic corrections to SZ effect alleviate the Planck tension on σ_8?

Mathieu Remazeilles

MANCHESTER 1824

The University of Manchester

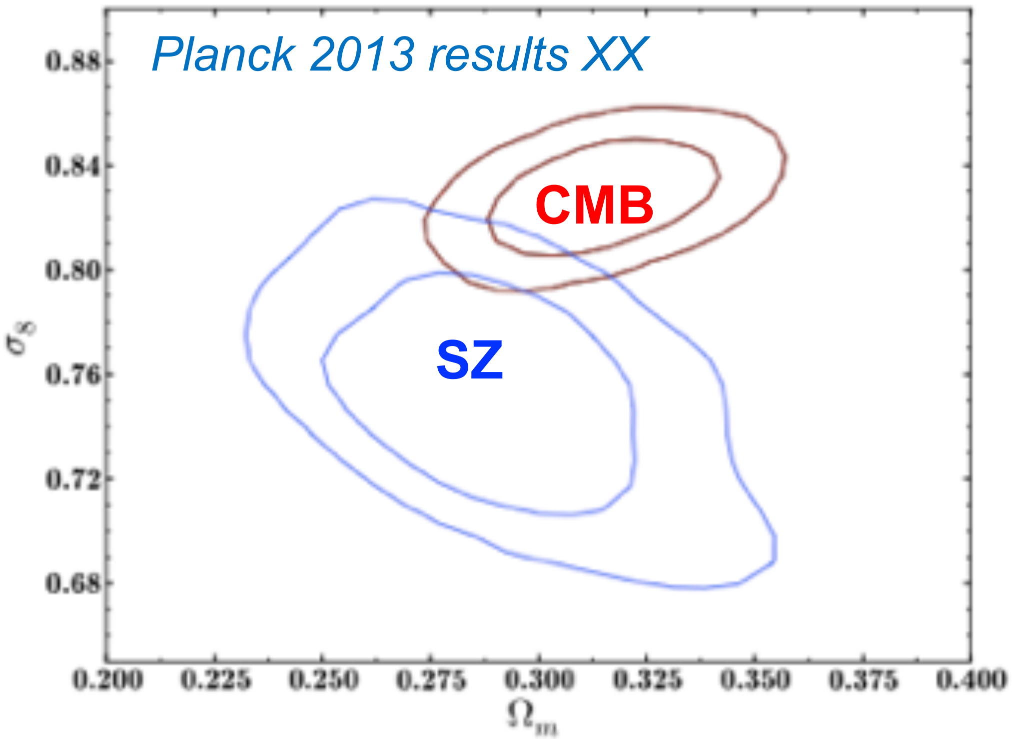
Remazeilles, Bolliet, Rotti, Chluba

arXiv:1809.09666

"Future work with Planck data"

ESAC Madrid, 3-4 Dec 2018

Planck tension on σ_8 between CMB and SZ



$\sigma_8 \simeq 0.82 \pm 0.02$

Planck 2013 results XVI

$\sigma_8 \simeq 0.77 \pm 0.02$

Planck 2013 results XX
Planck 2015 results XXII

- Incompleteness of ΛCDM model?
  Evidence for massive neutrinos?

- Incorrect "mass-bias" between SZ gas and dark matter?
  Hydrodynamical simulations predict: $M_{\mathrm{gas}}\ /\ M_{\mathrm{dark\ matter}} = (1-b) = 0.8$

- Miscalibrated SZ analysis by neglecting relativistic corrections?
  Remazeilles, Bolliet, Rotti, Chluba 2018

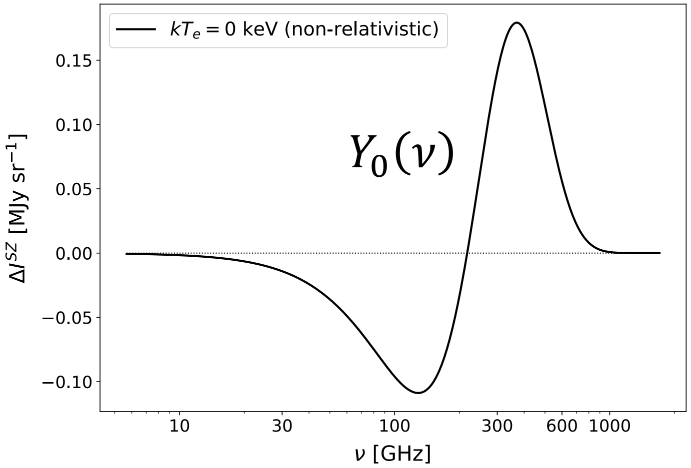
Some facts (1/2)

- So far, relativistic corrections to thermal SZ effect have always been neglected in any cosmological data analysis (e.g. *Planck*)

- The SZ spectral signature adopted for the extraction of Compton-y parameter has always been the non-relativistic limit:

$$\boldsymbol{Y_0(\nu) = x \coth\frac{x}{2} - 4} \quad \text{where} \quad \boldsymbol{x \equiv \frac{h\nu}{kT_{CMB}}}$$

Sunyaev & Zeldovich (1972)



- This is equivalent to saying that the electron gas temperature of all galaxy clusters is $\boldsymbol{T_e \simeq 0}$ **keV**!

Some facts (2/2)

- But galaxy clusters are massive, therefore have hot temperatures:

$$kT_e \simeq 5\,\mathrm{keV}\left[\sqrt{\Omega_{\mathrm{m}}(1+z)^3+\Omega_\Lambda}\,\frac{M_{500}}{3\times 10^{14}\,h^{-1}M_\odot}\right]^{2/3}$$

Arnaud et al 2005
Reichert et al 2011
Erler et al 2018

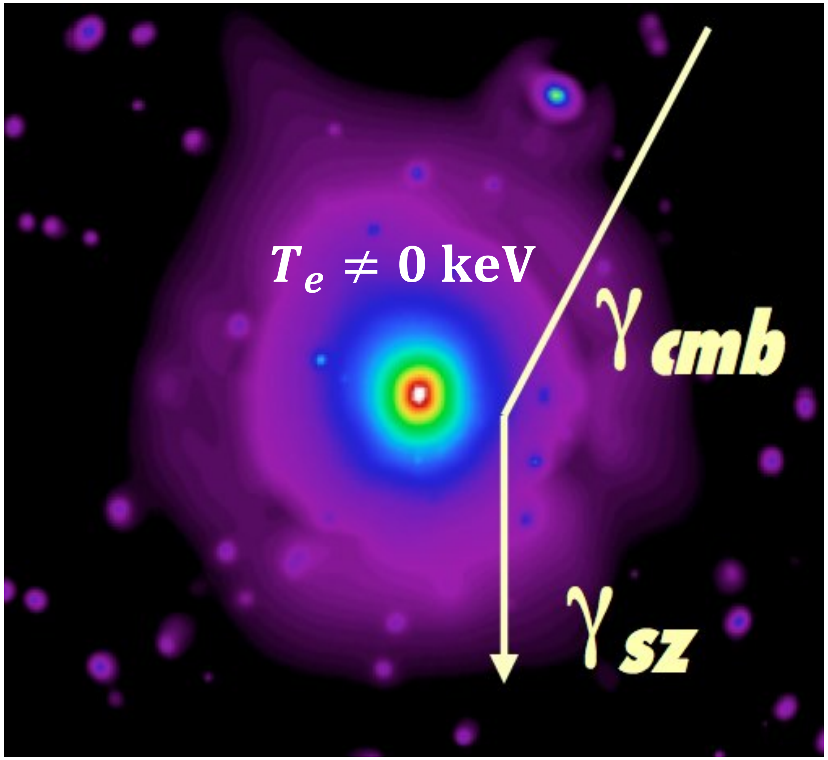
"Temperature-Mass relation"

- So the true SZ spectrum must be different from the non-relativistic limit assumed in *Planck* SZ analysis

→ Miscalibration of Compton-y parameter signal

- While relativistic corrections might be negligible on individual clusters at *Planck* sensitivity, they become in fact relevant on an ensemble of clusters: power spectrum, cluster number counts

Relativistic temperature corrections to the thermal SZ effect



In hot galaxy clusters, relativistic corrections to the thermal SZ effect should be accounted for:

$$\boxed{\begin{aligned}\Delta I^{SZ}(\nu,\vec{\theta}) &= Y(\nu, T_e)\, y(\vec{\theta}; T_e) \\ &= \left(Y_0(\nu) + \delta(\nu, T_e)\right) y(\vec{\theta}; T_e)\end{aligned}}$$

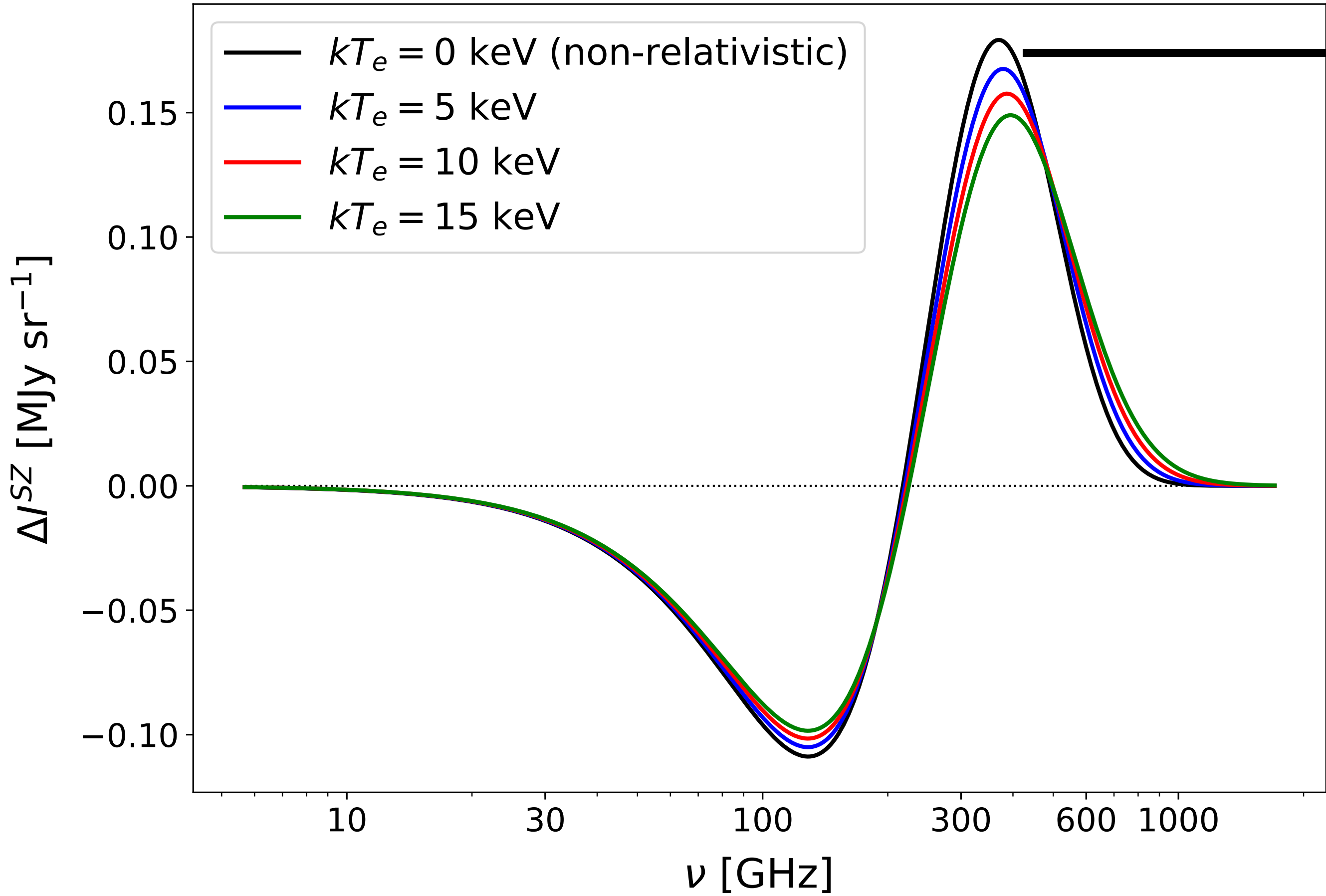
- $Y_0(\nu)$*: non-relativistic spectrum at* $T_e \simeq 0$ *keV:*
- *signal of interest: Compton-*y *parameter*

$$y(\vec{\theta}; T_e) \simeq \sigma_T \int n_e \frac{kT_e(l(\vec{\theta}))}{mc^2}\, dl(\vec{\theta})$$

The spectral signature of SZ emission from galaxy clusters changes with the electron gas temperature

Relativistic corrections to the SZ spectrum

$$\Delta I^{SZ}(\nu, \vec{\theta}) = Y(\nu, T_e)\, y(\vec{\theta})$$

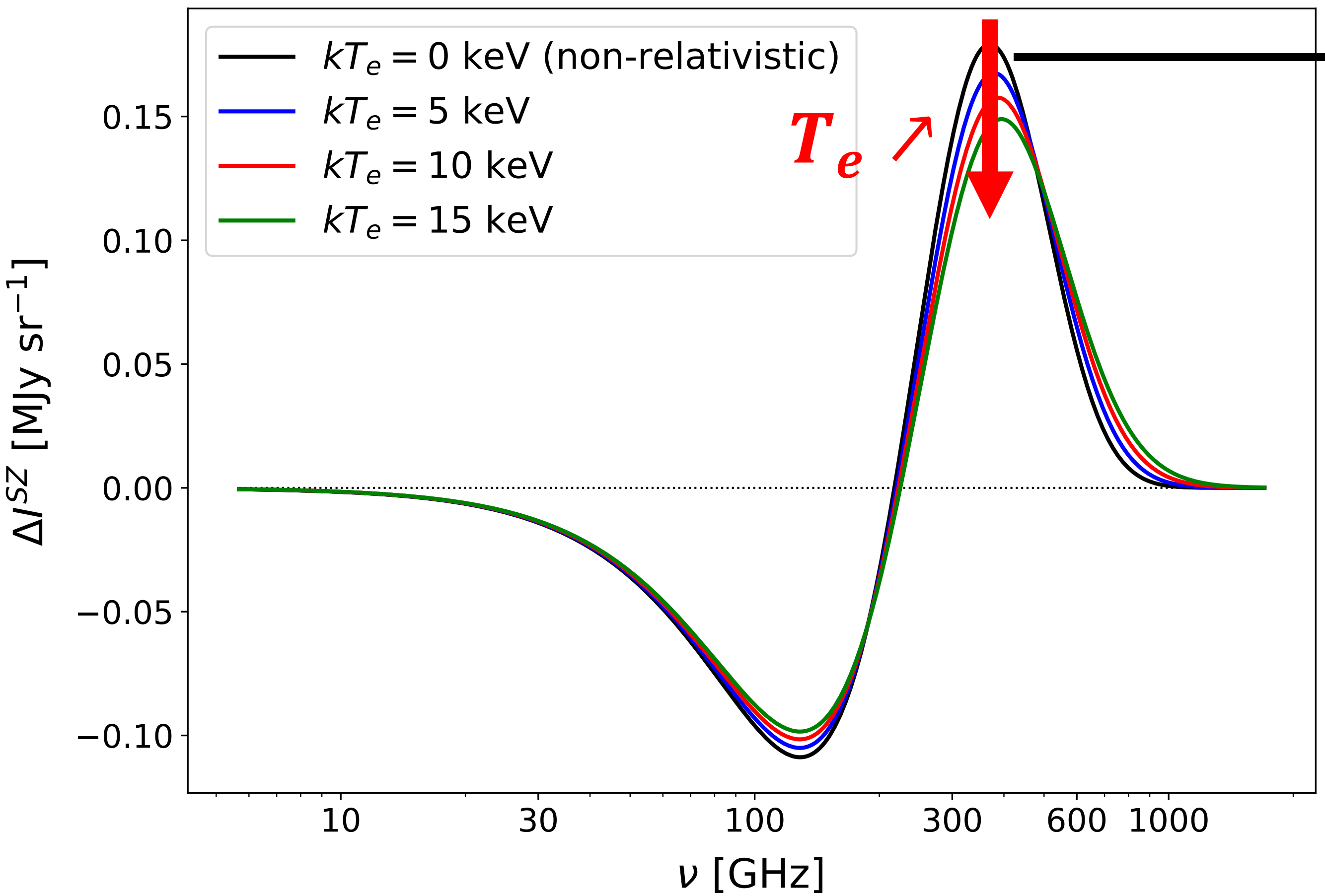


Planck's assumption: $T_e \simeq 0$ keV

The spectral signature of SZ emission from galaxy clusters changes with the electron gas temperature

Relativistic corrections to the SZ spectrum

$$\Delta I^{SZ}(\nu,\vec{\theta}) = Y(\nu, T_e)\, y(\vec{\theta})$$

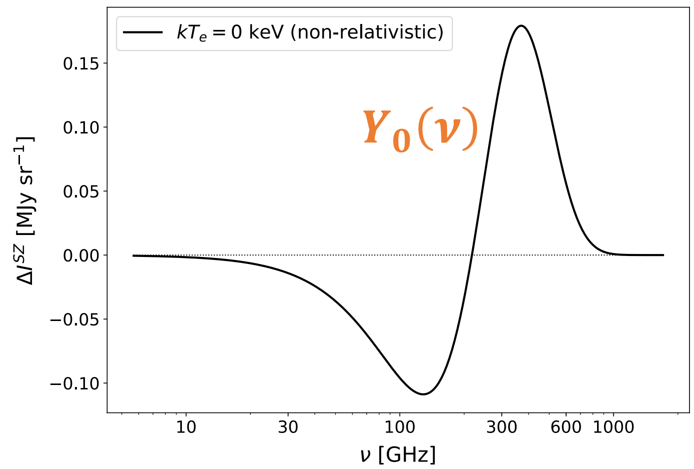


Planck's assumption: $T_e \simeq 0$ keV

- ✓ Relativistic temperature corrections reduce the overall intensity at fixed Compton-y parameter
- ✓ Assuming the non-relativistic spectrum $Y_0(\nu)$ for cosmological SZ analysis leads to an underestimation of the Compton-y parameter

SZ Compton-y signal reconstruction: ILC

$$\underbrace{d(\nu,\vec{\theta})}_{\text{Planck frequency data}} = Y_0(\nu)\ \underbrace{y(\vec{\theta})}_{\text{signal of interest}} + \underbrace{N(\nu,\vec{\theta})}_{\text{foregrounds + noise}}$$



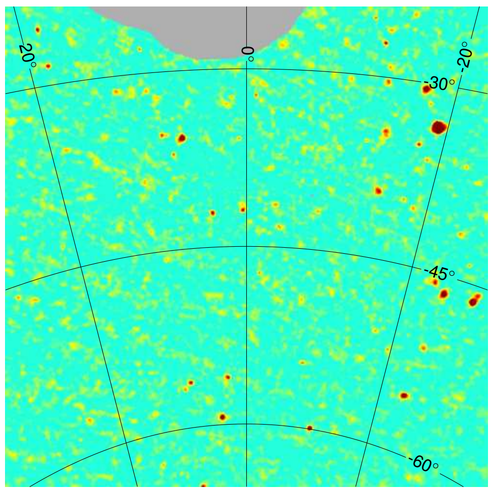
- ILC = weighted linear combination of frequency maps:

$$\hat{y}(\vec{\theta}) = \sum_\nu w(\nu)\, d(\nu,\vec{\theta})$$

$$\text{such that } \begin{cases} \langle \hat{y}^2 \rangle = w^t \langle d d^t \rangle\, w \text{ minimum} & (1) \\ \sum_\nu w(\nu)\, Y_0(\nu) = 1 & (2) \end{cases}$$

- ILC weights : $w^t = \dfrac{Y_0^t C^{-1}}{Y_0^t C^{-1} Y_0} \quad (C \equiv \langle d d^t \rangle)$

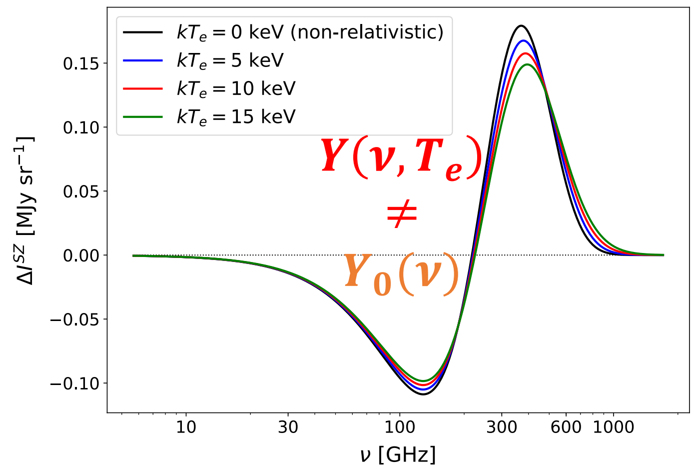
$$\Longrightarrow \hat{y}(\vec{\theta}) = \underbrace{y(\vec{\theta})}_{\substack{\text{recovered signal} \\ (2)}} + \underbrace{w^t N}_{\substack{\text{minimized residuals} \\ \text{by } (1)}}$$



Planck 2015 results XXII

Miscalibrated Compton-y signal reconstruction

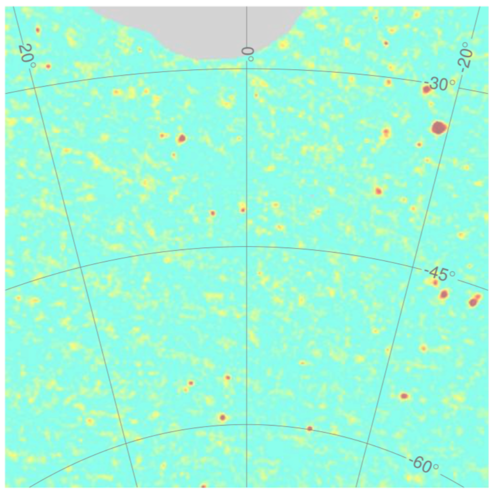
$$\underbrace{\boldsymbol{d}(\boldsymbol{\nu},\vec{\boldsymbol{\theta}})}_{\textit{Planck frequency data}} = {\color{red}\boldsymbol{Y}(\boldsymbol{\nu},\boldsymbol{T_e})}\ \underbrace{\boldsymbol{y}(\vec{\boldsymbol{\theta}})}_{\textit{signal of interest}} + \underbrace{\boldsymbol{N}(\boldsymbol{\nu},\vec{\boldsymbol{\theta}})}_{\textit{foregrounds + noise}}$$



- ILC = weighted linear combination of frequency maps:

$$\hat{\boldsymbol{y}}(\vec{\boldsymbol{\theta}}) = \sum_{\nu} \boldsymbol{w}(\boldsymbol{\nu})\, \boldsymbol{d}(\boldsymbol{\nu},\vec{\boldsymbol{\theta}})$$

$$\text{such that}\quad \begin{cases} \langle \hat{\boldsymbol{y}}^2 \rangle = \boldsymbol{w}^t \langle \boldsymbol{d}\boldsymbol{d}^t \rangle\, \boldsymbol{w} \text{ minimum} & (1) \\ \sum_{\nu} \boldsymbol{w}(\boldsymbol{\nu})\, {\color{orange}\boldsymbol{Y}_0(\boldsymbol{\nu})} = \mathbf{1} & (2) \end{cases}$$



- ILC weights : $\boldsymbol{w}^t = \dfrac{{\color{orange}\boldsymbol{Y}_0^t}\, \boldsymbol{C}^{-1}}{{\color{orange}\boldsymbol{Y}_0^t}\, \boldsymbol{C}^{-1}\, {\color{orange}\boldsymbol{Y}_0}} \qquad (\boldsymbol{C} \equiv \langle \boldsymbol{d}\boldsymbol{d}^t \rangle)$

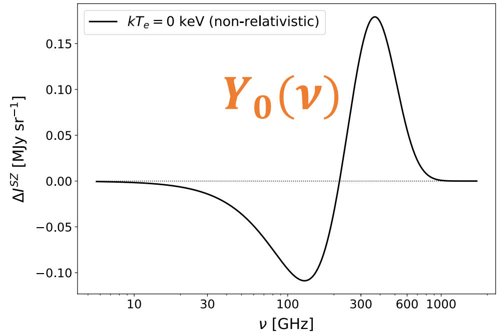
Planck 2015 results XXII

$$\Longrightarrow\ \hat{\boldsymbol{y}}(\vec{\boldsymbol{\theta}}) = \underbrace{\frac{{\color{orange}\boldsymbol{Y}_0^t}\, \boldsymbol{C}^{-1}\, {\color{red}\boldsymbol{Y}(\boldsymbol{T_e})}}{{\color{orange}\boldsymbol{Y}_0^t}\, \boldsymbol{C}^{-1}\, {\color{orange}\boldsymbol{Y}_0}}}_{\textbf{Bias} < 1} \boldsymbol{y}(\vec{\boldsymbol{\theta}}) + \boldsymbol{w}^t \boldsymbol{N}$$

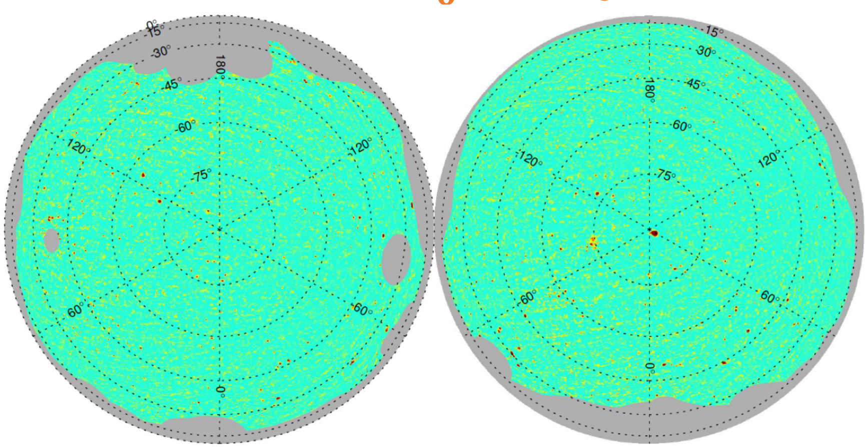
Underestimation of Compton-y signal !

Planck SZ power spectrum is miscalibrated

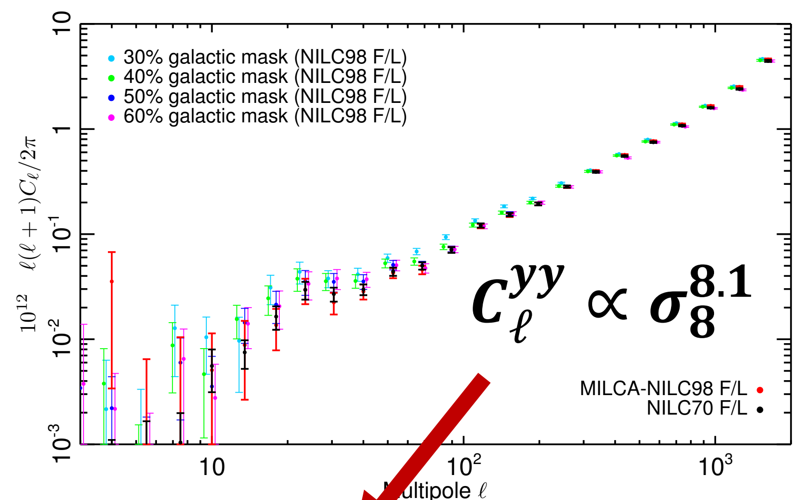
Planck analysis assumes $T_e \simeq 0$ keV for all galaxy clusters by using the **non-relativistic** SZ energy spectrum: $Y_0(\nu)$



NILC: $\dfrac{Y_0^t\, C^{-1}}{Y_0^t\, C^{-1}\, Y_0}$



Planck SZ y-map



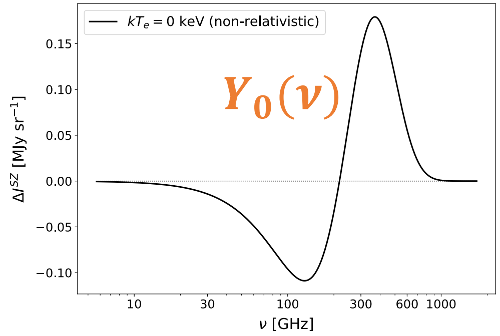
$$C_\ell^{yy} \propto \sigma_8^{8.1}$$

$$\sigma_8 \simeq 0.77 \pm 0.02$$

Planck 2015 results. XXII. A&A 2016

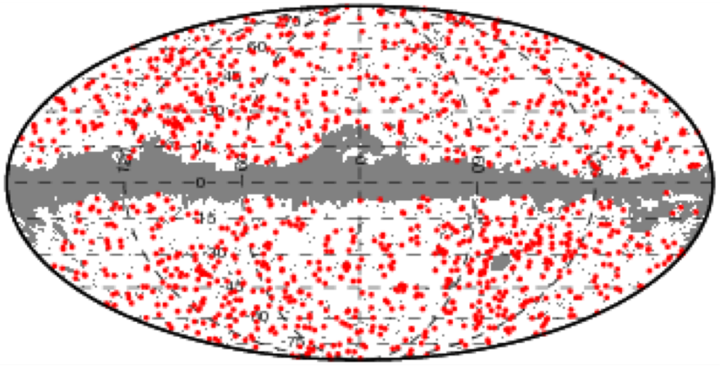
Planck SZ cluster count is miscalibrated

Planck analysis assumes $T_e \simeq 0$ keV for all galaxy clusters by using the **non-relativistic** SZ energy spectrum: $Y_0(\nu)$

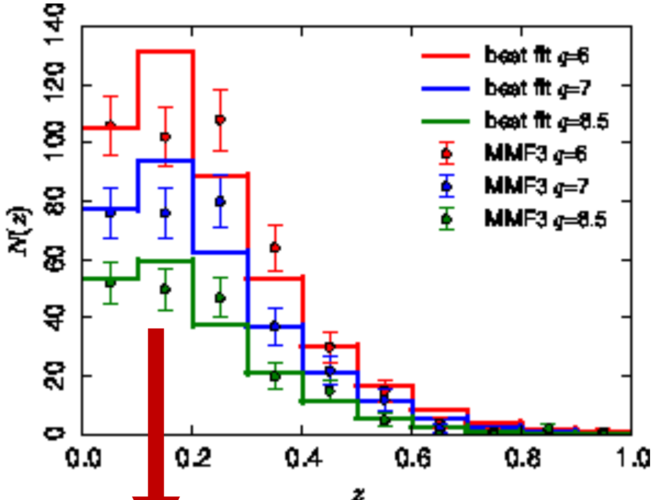


MMF:

$$[(Y_0\bar{y})^t\, N^{-1}\, (Y_0\bar{y})]^{-1}\, (Y_0\bar{y})^t\, N^{-1}$$



Planck SZ catalogue
(clusters detected at
SNR $q > 6$)



$\sigma_8 \simeq 0.77 \pm 0.02$

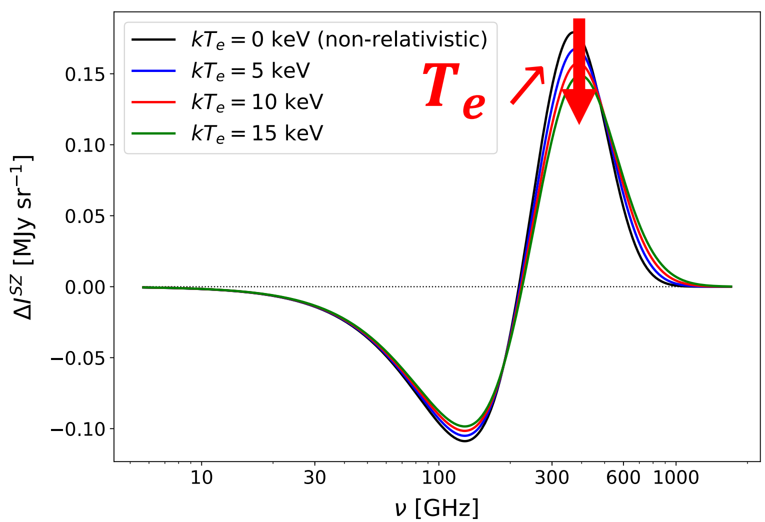
Planck 2015 results. XXIV. A&A 2016

Accounting for relativistic SZ effects will increase the inferred value of σ_8

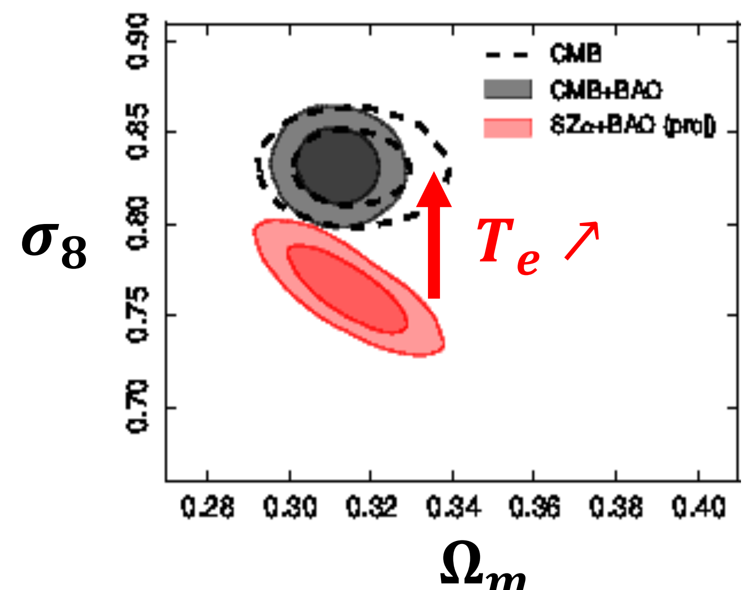
Given the observed intensity:

$$\Delta I^{SZ}(\nu, \vec{\theta}) = Y(\nu, T_e)\, y(\vec{\theta})$$

$$T_e \nearrow \;\Longrightarrow\; Y(\nu, T_e) \searrow \;\Longrightarrow\; y \nearrow \;\Longrightarrow\; \begin{cases} C_\ell^{yy} \nearrow \\ N(z) \nearrow \end{cases} \Longrightarrow\; \sigma_8 \nearrow$$



$\Longrightarrow$



Remazeilles, Bolliet, Rotti, Chluba (2018)

Average temperature of galaxy clusters?

- ✓ It is not $k\bar{T}_e \simeq 0$ keV for sure!

- ✓ $k\bar{T}_e \simeq 6^{+3.8}_{-2.9}$ keV by stacking *Planck* clusters detected at high significance
  Erler et al (2018)

- ✓ $k\bar{T}_e \gtrsim 5$ keV from cluster mass-dependence of thermal SZ power spectrum
  Remazeilles, Bolliet, Rotti, Chluba (2018)

  See hereafter…

Compton y-parameter power spectrum

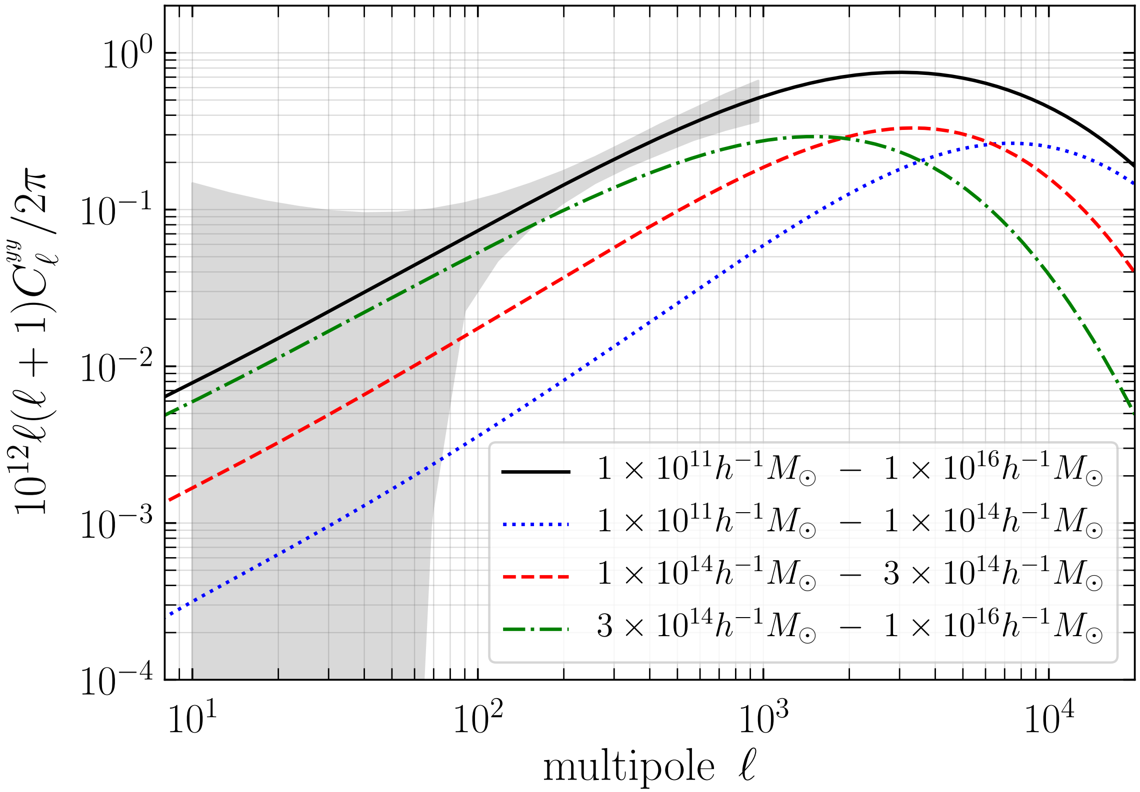
Komatsu & Seljak (2002)

$$C_\ell^{yy} = \int_{z_{min}}^{z_{max}} dz \frac{dV}{dz d\Omega} \int_{\boldsymbol{M_{min}}}^{\boldsymbol{M_{max}}} dM \underbrace{\frac{dn(M,z)}{dM}}_{\text{halo mass function}} \underbrace{|y_\ell(M,z)|^2}_{\text{cluster pressure profile}}$$

Planck 2015 results XXII

$$C_\ell^{yy} \propto \sigma_8^{8.1}$$

Which cluster masses and temperatures are probed by *Planck* ?



Remazeilles, Bolliet, Rotti, Chluba (2018)

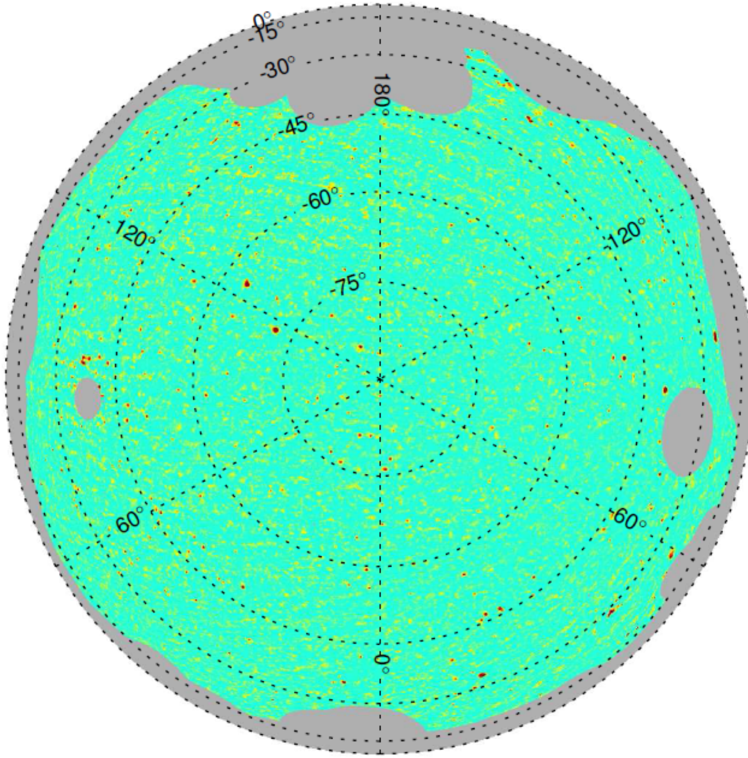
Planck SZ power at $\ell \simeq 10^2$-10^3 is mostly sensitive to massive clusters with:

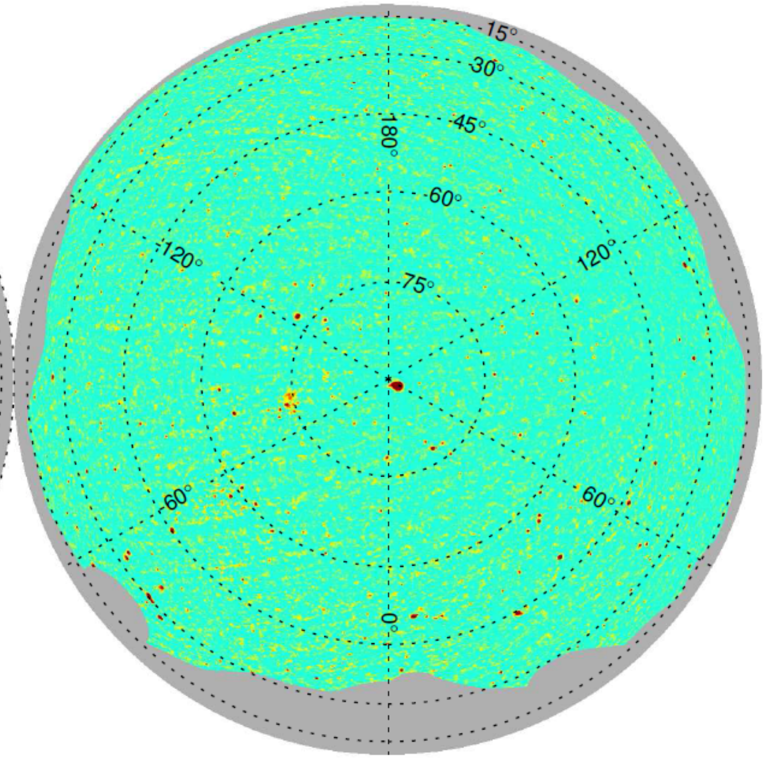
$$\boxed{M \gtrsim 3\times10^{14}\ h^{-1}M_\odot}$$

$\Longrightarrow$ *Planck* mostly sensitive to cluster temperatures: $\boxed{T_e \gtrsim 5\ \mathrm{keV}}$

(temperature-mass relation: $kT_e \simeq 5\ \mathrm{keV}\left[\sqrt{\Omega_\mathrm{m}(1+z)^3+\Omega_\Lambda}\,\frac{M_{500}}{3\times10^{14}\ h^{-1}M_\odot}\right]^{2/3}$*)*

Revisiting the *Planck* NILC y-map





$$\frac{Y_0^t C^{-1}}{Y_0^t C^{-1} Y_0} \longrightarrow \frac{Y(\bar{T}_e)^t C^{-1}}{Y(\bar{T}_e)^t C^{-1} Y(\bar{T}_e)}$$

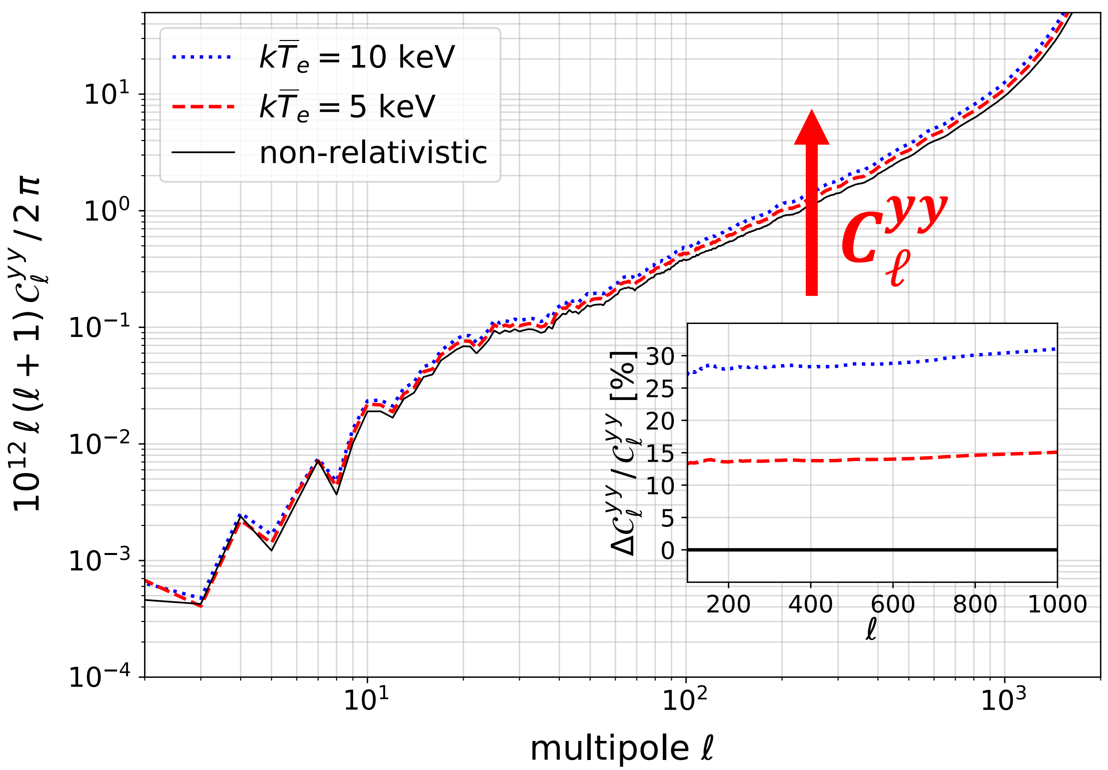
$\bar{T}_e \simeq 0$ keV

$\bar{T}_e = 5$ keV

Planck 2015 results XXII (2016)

Remazeilles, Bolliet, Rotti, Chluba (2018)

Updating the *Planck* y-map power spectrum



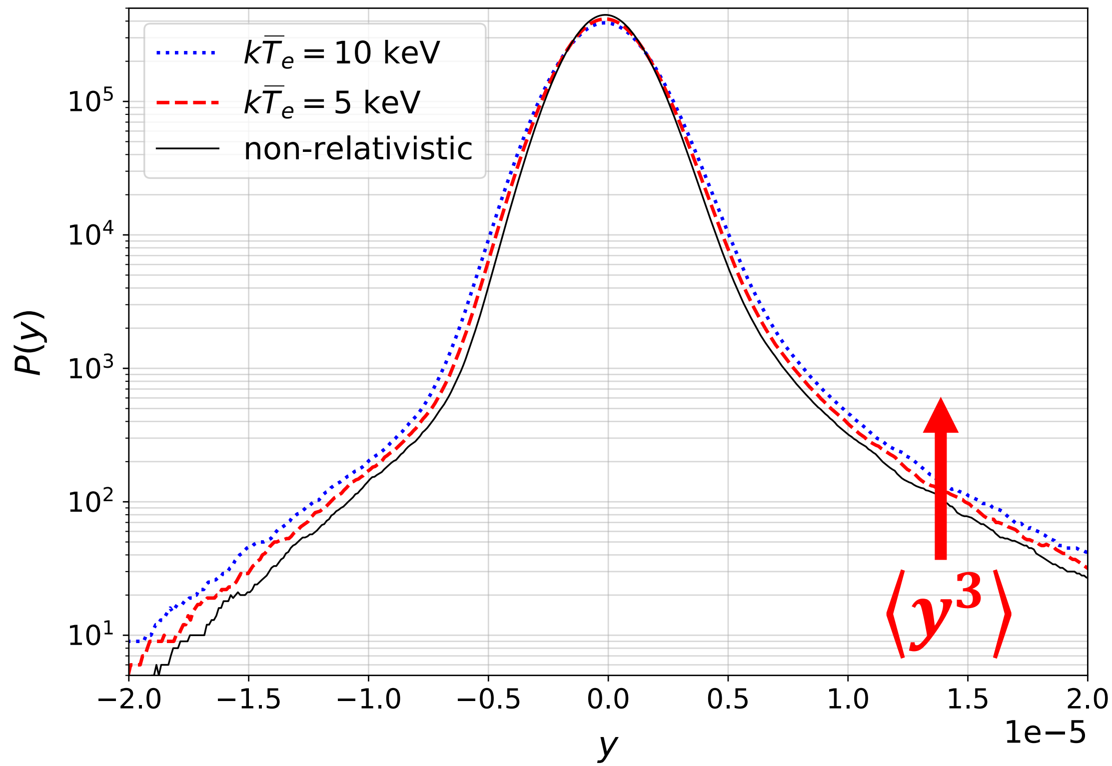
Planck C_ℓ^{yy} increases with average cluster temperature $\overline{T}_e$

$$C_\ell^{yy} \propto \sigma_8^{8.1} \implies \boxed{\frac{\Delta\sigma_8}{\sigma_8} \simeq 0.019\left(\frac{k\overline{T}_e}{5\ \mathrm{keV}}\right)}$$

$\simeq 1\sigma$ increase for $\overline{T}_e \simeq 5$ keV !

Remazeilles, Bolliet, Rotti, Chluba (2018)

Updating the *Planck* y-map PDF



Planck y-map skewness increases with average cluster temperature $\bar{T}_e$

$$\langle y^3 \rangle \propto \sigma_8^{12} \implies \boxed{\frac{\Delta\sigma_8}{\sigma_8} \simeq 0.025\left(\frac{k\bar{T}_e}{5\ \mathrm{keV}}\right)}$$

$\simeq 1\sigma$ *increase*
for $\bar{T}_e \simeq 5$ *keV !*

Remazeilles, Bolliet, Rotti, Chluba (2018)

What is the relevant average temperature of galaxy clusters ?

Moment expansion of relativistic SZ around some pivot temperature $\bar{T}_e$

$$\Delta I^{SZ}(\nu,\vec{\theta}) = Y(\nu,\bar{T}_e)\,y(\vec{\theta}) + \frac{\partial Y(\nu,\bar{T}_e)}{\partial T_e}(T_e-\bar{T}_e)\,y(\vec{\theta}) + \mathcal{O}(T_e^2)$$

$$C_\ell^{SZ} = Y(\nu,\bar{T}_e)^2 \underbrace{\langle\, |y_{\ell m}|^2 \,\rangle}_{C_\ell^{yy}} + \underbrace{2\,Y(\nu,\bar{T}_e)\frac{\partial Y(\nu,\bar{T}_e)}{\partial T_e}\langle\, [(T_e-\bar{T}_e)y]_{\ell m}\, y^*_{\ell m}\,\rangle}_{\text{linear bias in } T_e} + \mathcal{O}(T_e^2)$$

- ✓ Planck's assumption $\bar{T}_e = 0$ is inappropriate: linear bias on C_ℓ^{SZ}
- ✓ The optimal pivot temperature $\bar{T}_e$ is in fact the one that cancels out the linear bias:

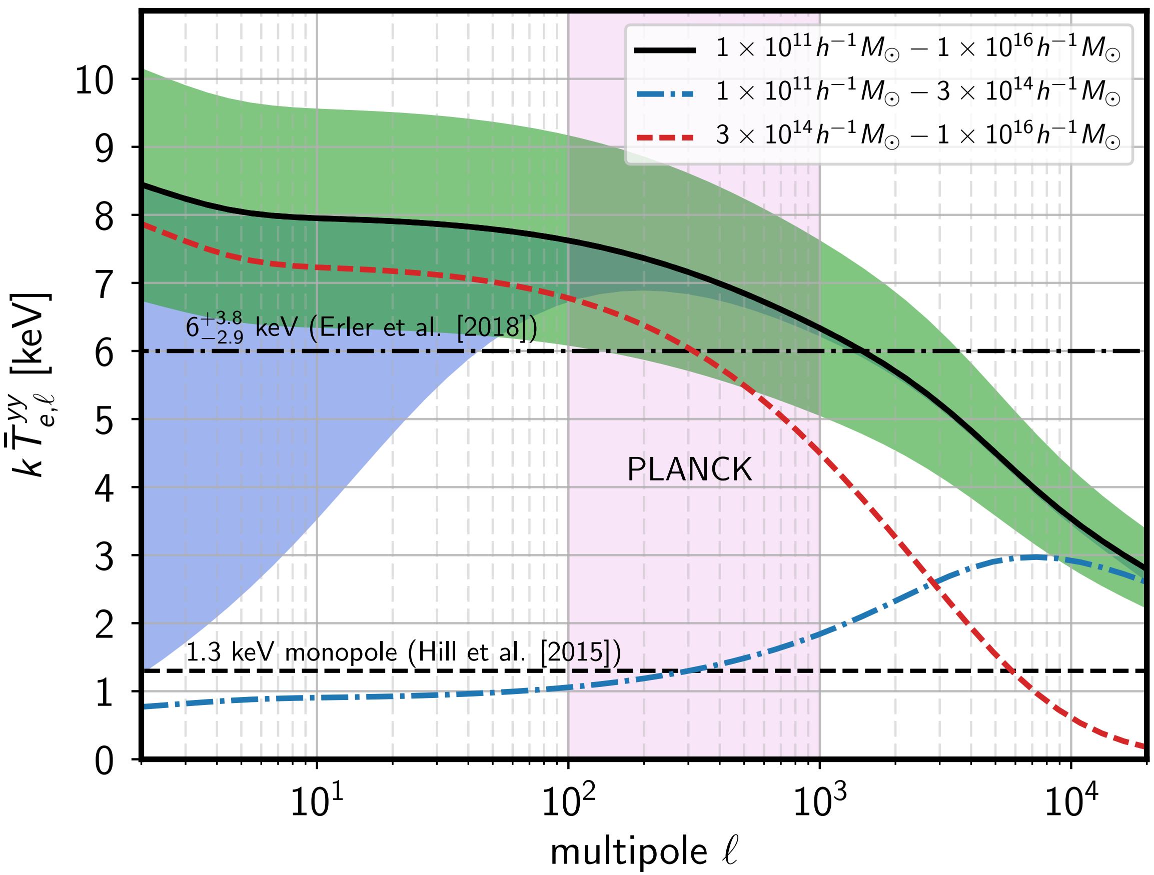
$$\langle\, [(T_e-\bar{T}_e)y]_{\ell m}\, y^*_{\ell m}\,\rangle = 0 \implies \boxed{\bar{T}_e = \frac{\langle T_e y^2\rangle}{\langle y^2\rangle} = \frac{C_\ell^{T_e y, y}}{C_\ell^{yy}}}$$

y^2-weighted average temperature (scale-dependent!)

Remazeilles, Bolliet, Rotti, Chluba (2018)

Relevant average cluster temperature for SZ power spectrum analysis

(theory)



$$\bar{T}_e^{yy}(\ell) = \frac{\langle T_e y^2 \rangle}{\langle y^2 \rangle} = \frac{C_\ell^{T_e y, y}}{C_\ell^{yy}}$$

Remazeilles, Bolliet, Rotti, Chluba (2018)

Mapping out cluster temperatures from data

$$d(\nu,\vec{\theta}) = \underbrace{Y(\nu, T_e = \overline{T}_e)}_{\text{energy spectrum of first component}} \; \underbrace{y(\vec{\theta}; T_e)}_{\text{fluctuations of first component}} + \underbrace{\frac{\partial Y(\nu, T_e = \overline{T}_e)}{\partial T_e}}_{\text{energy spectrum of second component}} \; \underbrace{(T_e(\vec{\theta}) - \overline{T}_e) y(\vec{\theta}; T_e)}_{z(\vec{\theta}; T_e) \text{ fluctuations of second component}} + \underbrace{N(\nu,\vec{\theta})}_{\text{foregrounds + noise}}$$

- Weighted linear combination of frequency data ("Constrained-ILC": *Remazeilles et al 2011*):

$$\hat{z}\,(\vec{\theta}; T_e) = \sum_{\nu} w(\nu)\, d(\nu,\vec{\theta}) \quad \text{such that} \quad \begin{cases} \langle \hat{z}^2 \rangle = w^t \,\langle d d^t \rangle\, w \ \ \text{minimum} \\ \sum_{\nu} w(\nu)\, \partial Y(\nu, \overline{T}_e)/\partial T_e = 1 \\ \sum_{\nu} w(\nu)\, Y(\nu, \overline{T}_e) = 0 \end{cases}$$

- Analytic solution: $w^t = \dfrac{(Y^t C^{-1} Y)\partial_{T_e} Y^t C^{-1} - (\partial_{T_e} Y^t C^{-1} Y) Y^t C^{-1}}{(\partial_{T_e} Y^t C^{-1} \partial_{T_e} Y)(Y^t C^{-1} Y) - (\partial_{T_e} Y^t C^{-1} Y)^2}$ $\quad (C \equiv \langle d d^t \rangle)$ yields to:

$$\hat{z}(\vec{\theta}; T_e) = \left(T_e(\vec{\theta}) - \overline{T}_e\right) y(\vec{\theta}; T_e) + w^t\, N$$

$$\boxed{\textit{Map of cluster temperatures over the sky: } (T_e(\vec{\theta}) - \overline{T}_e) y(\vec{\theta})}$$

Remazeilles et al, in prep.

Mapping out cluster temperatures from data

$$d(\nu,\vec{\theta}) = \underbrace{Y(\nu, T_e = \bar{T}_e)}_{\text{energy spectrum of first component}} \; \underbrace{y(\vec{\theta}; T_e)}_{\text{fluctuations of first component}} + \underbrace{\frac{\partial Y(\nu, T_e = \bar{T}_e)}{\partial T_e}}_{\text{energy spectrum of second component}} \; \underbrace{(T_e(\vec{\theta}) - \bar{T}_e) y(\vec{\theta}; T_e)}_{z(\vec{\theta}; T_e)\ \text{fluctuations of second component}} + \underbrace{N(\nu,\vec{\theta})}_{\text{foregrounds + noise}}$$

- Weighted linear combination of frequency data ("Constrained-ILC": *Remazeilles et al 2011*):

$$\hat{y}(\vec{\theta}; T_e) = \sum_\nu w(\nu)\, d(\nu, \underline{\theta}) \quad \text{such that} \quad \begin{cases} \langle \hat{z}^2 \rangle = w^t \langle d d^t \rangle w \ \ \text{minimum} \\ \sum_\nu w(\nu)\, \partial Y(\nu, \bar{T}_e)/\partial T_e = 0 \\ \sum_\nu w(\nu)\, Y(\nu, \bar{T}_e) = 1 \end{cases}$$

- Analytic solution: $w^t = \dfrac{(\partial_{T_e} Y^t C^{-1} \partial_{T_e} Y)\, Y^t C^{-1} - (Y^t C^{-1} Y) \partial_{T_e} Y^t C^{-1}}{(\partial_{T_e} Y^t C^{-1} \partial_{T_e} Y)(Y^t C^{-1} Y) - (\partial_{T_e} Y^t C^{-1} Y)^2}$ $\quad (\, C \equiv \langle d d^t \rangle \,)$ yields to:

$$\hat{y}(\vec{\theta}; T_e) = y(\vec{\theta}; T_e) + w^t N$$

$$\boxed{\textit{Map of Compton-}y\textit{ parameter over the sky: } y(\vec{\theta})}$$

Remazeilles et al, in prep.

Mapping out cluster temperatures from data

1. **First approach**:

- Cross-power spectrum between the $\boldsymbol{y}$-map and the $\boldsymbol{yT_e}$-map:
$$C_\ell^{yT_e,y} = \langle yT_e, y\rangle$$
- Auto-power spectrum of the $\boldsymbol{y}$-map:
$$C_\ell^{yy} = \langle |y|^2 \rangle$$
- Ratio: $\overline{T}_e^{yy}(\ell) = \dfrac{\langle T_e y^2\rangle}{\langle y^2\rangle} = \dfrac{C_\ell^{T_e y,y}}{C_\ell^{yy}}$

2. **Second approach**: **Analogy with lensing**

- CMB lensing:
$$T^{obs} \simeq T^{true} + \nabla\Phi\, \nabla T^{true}$$
Lensing field (quadratic estimator):
$$\widehat{\Phi}(L) = \int d^2\ell\, K(L,\ell) T^{obs}(\ell)\, T^{obs}(L-\ell)$$
- Suppose we have reconstructed the sum of the $\boldsymbol{y}$- and $\boldsymbol{yT_e}$-maps:
$$y^{obs} \simeq y^{true} + T_e\, y^{true}$$
Temperature field (quadratic estimator):
$$\widehat{T}_e(L) = \int d^2\ell\, W(L,\ell) y^{obs}(\ell)\, y^{obs}(L-\ell)$$

Remazeilles et al, in prep.

Conclusions

- *Planck*'s Compton-y signal (power spectrum and cluster counts), hence σ_8, might have been underestimated by neglecting relativistic SZ effects

- Accounting for relativistic corrections to thermal SZ effect with $kT_e \simeq 5\ \mathrm{keV}$ could alleviate the *Planck* tension on σ_8 by about one-sigma

- The relevant average temperature of clusters for SZ power spectrum analysis is y^2-weighted and scale-dependent with $kT_e \simeq 5\text{-}9\ \mathrm{keV}$ in the range of multipoles $\ell \simeq 10^2\text{-}10^3$ relevant to *Planck*

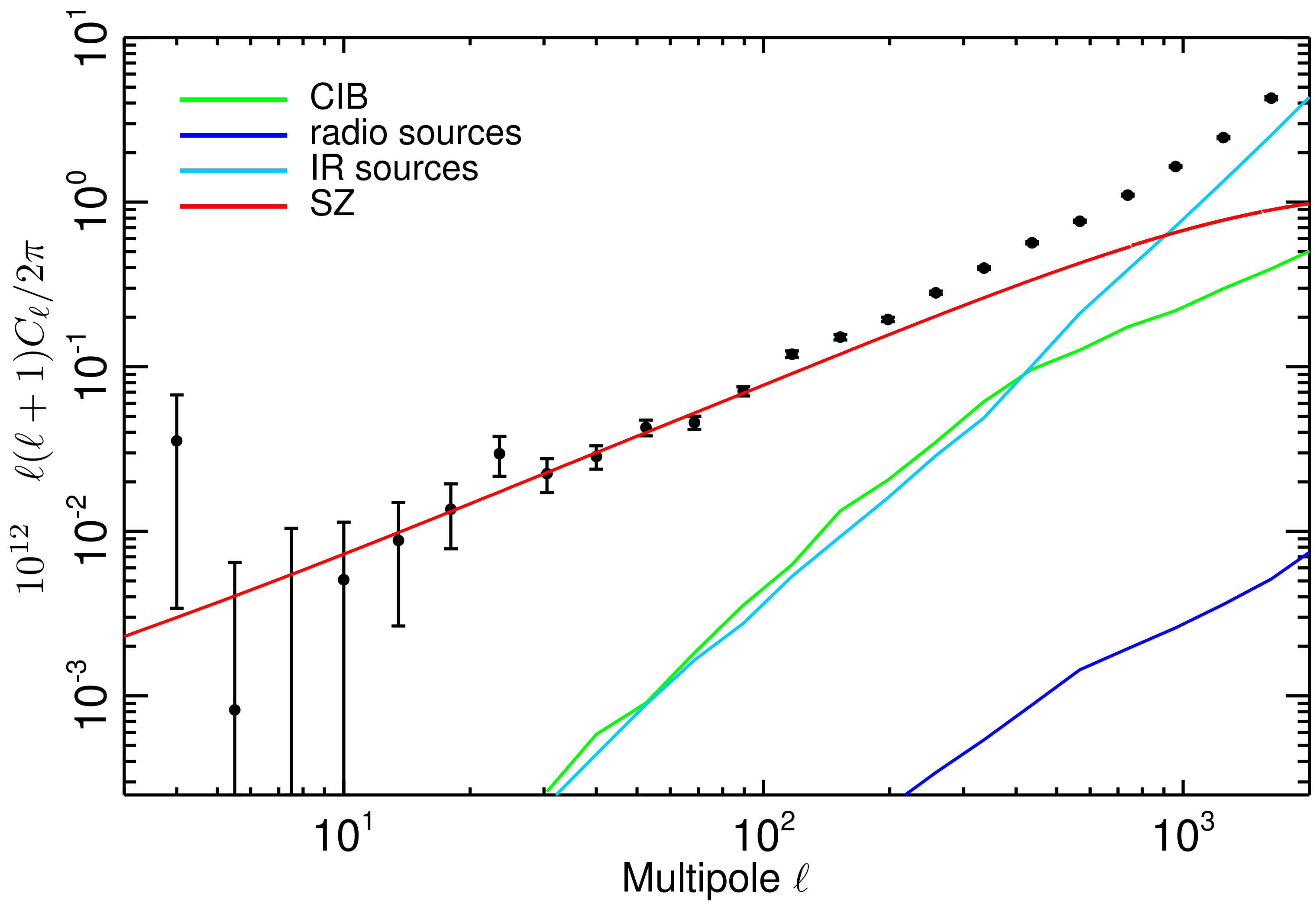
- We expect similar corrections from SZ cluster number count analysis
  Rotti et al, in prep.

- It is time to include relativistic temperature corrections in the processing of current and future sensitive SZ data.

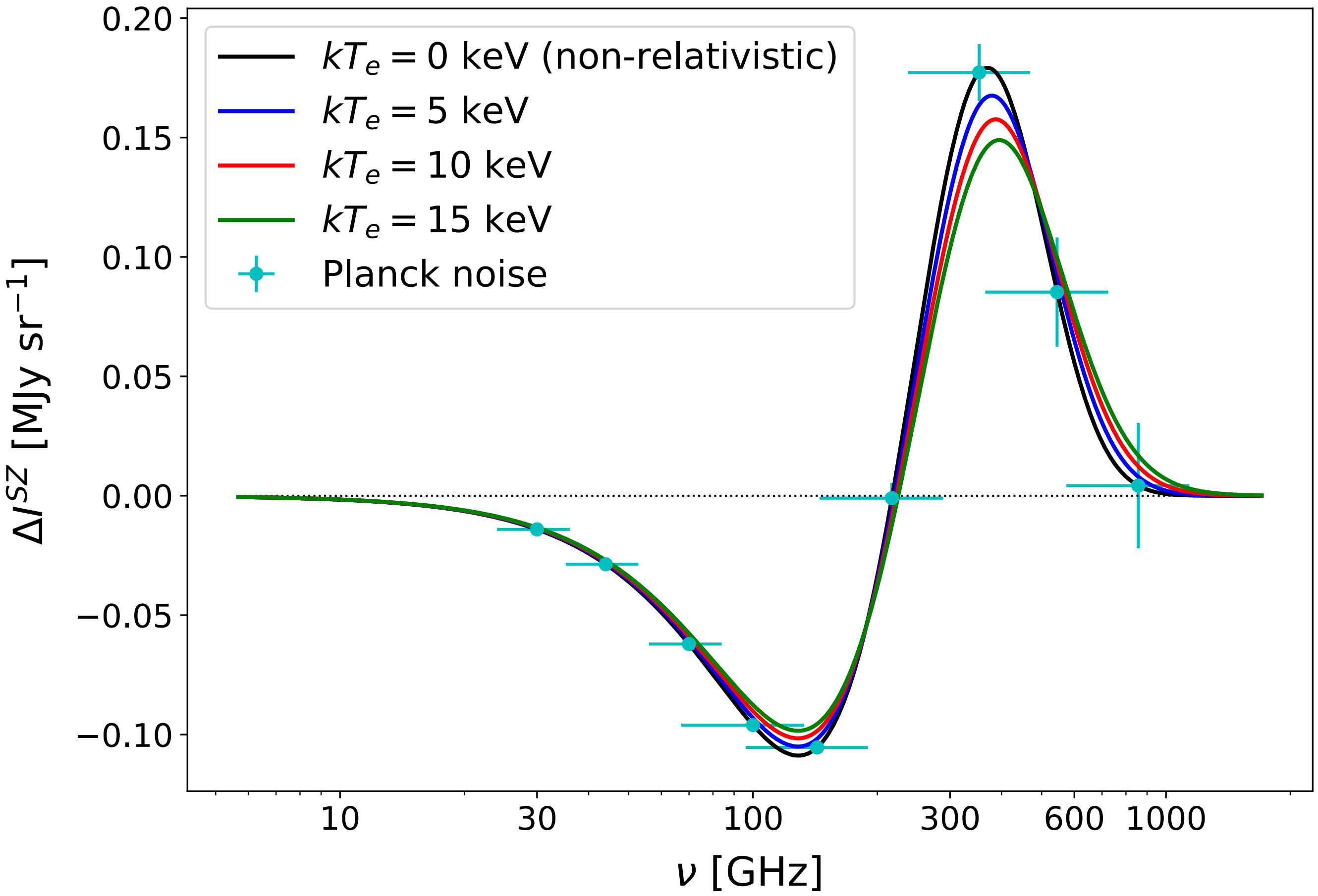
Thank you!

Backup slides

Planck 2015 results XXII



Relativistic SZ vs *Planck* sensitivities



Relativistic SZ formulas

$$\boldsymbol{\Delta I^{SZ}(\nu,\vec{\theta}) = Y(\nu, T_e)\, y(\vec{\theta}; T_e)}$$

- Taylor expansion around $kT_e = 0\ \mathrm{keV}$ – *Itoh et al 1998* ; *Challinor & Lasenby 1998*:

$$\boldsymbol{\Delta I^{SZ}(\nu,\vec{\theta}) \approx \left(Y_0(\nu) + Y_1(\nu)\frac{kT_e}{mc^2} + Y_2(\nu)\left(\frac{kT_e}{mc^2}\right)^2 + \ \ldots \right) y(\vec{\theta}; T_e)}$$

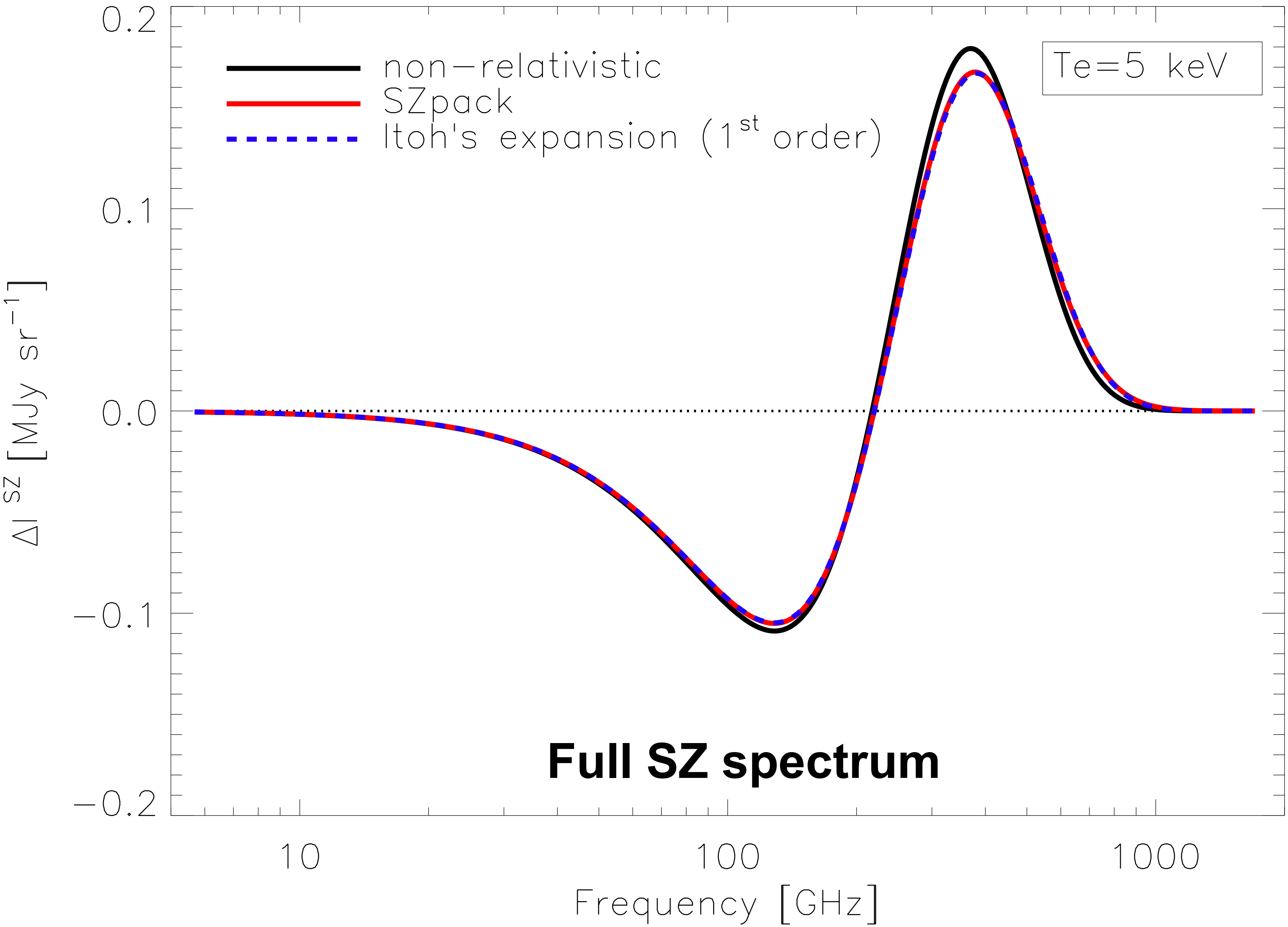
But this expansion does not converge properly for hot clusters ($kT_e > 5\ \mathrm{keV}$)
and it is known that galaxy clusters have $k\bar{T}_e \equiv \langle kT_e \rangle \simeq 5\ \mathrm{keV}$

- Moment expansion around $kT_e \neq 0\ \mathrm{keV}$ from `SZpack` – *Chluba et al 2012*

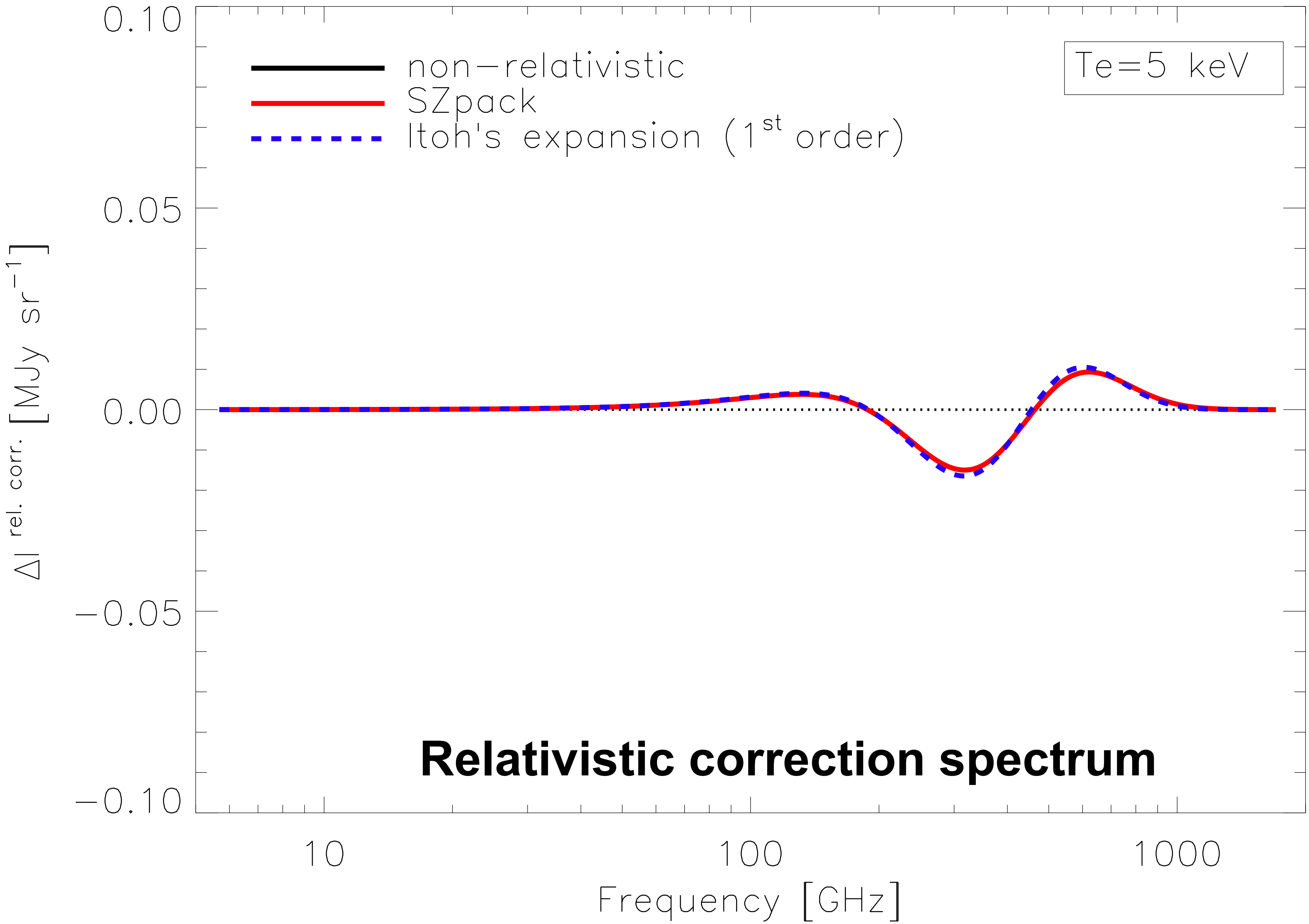
Allows expansion of $\boldsymbol{Y(\nu, T_e)}$ around e.g. $k\bar{T}_e = 5\ \mathrm{keV}$ (or any $k\bar{T}_e$ value):

$$\boldsymbol{\Delta I^{SZ}(\nu,\vec{\theta}) \approx \left(Y(\nu, T_e = \bar{T}_e) + (T_e - \bar{T}_e)\frac{\partial Y(\nu, T_e = \bar{T}_e)}{\partial T_e} + \ \ldots \right) y(\vec{\theta}; T_e)}$$

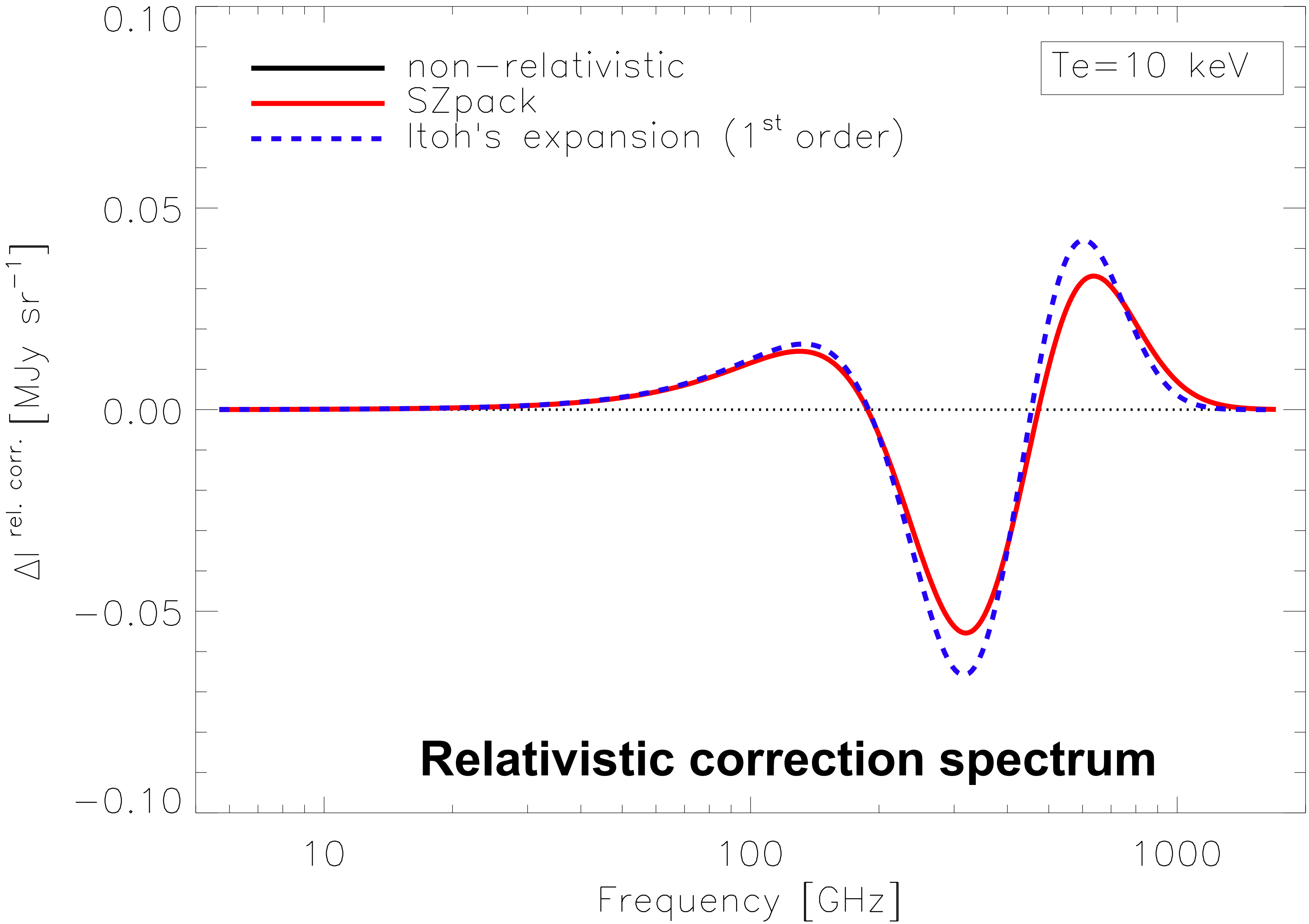
Moment vs Itoh's expansion: $T_e = 5$ keV (1^{st} order)



Moment vs Itoh's expansion: $T_e = 5$ keV (1^{st} order)

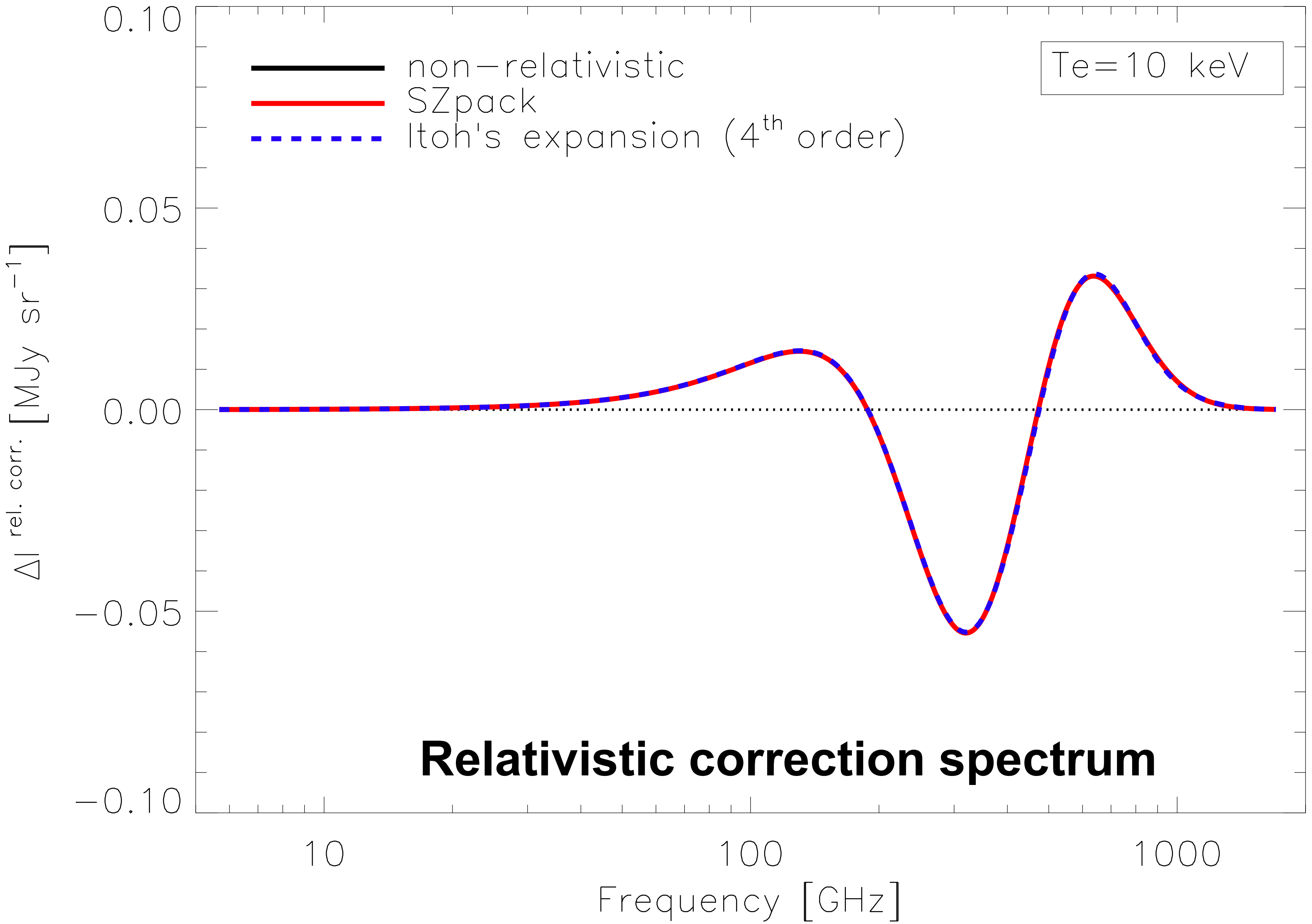


Moment vs Itoh's expansion: $T_e = 10$ keV (1^{st} order)



Relativistic correction spectrum

Moment vs Itoh's expansion: $T_e = 10$ keV (4^{th} order)



Moment vs Itoh's expansion: $T_e = 20$ keV (4^{th} order)